Functional Encryption for Inner Product Predicates from Learning with Errors

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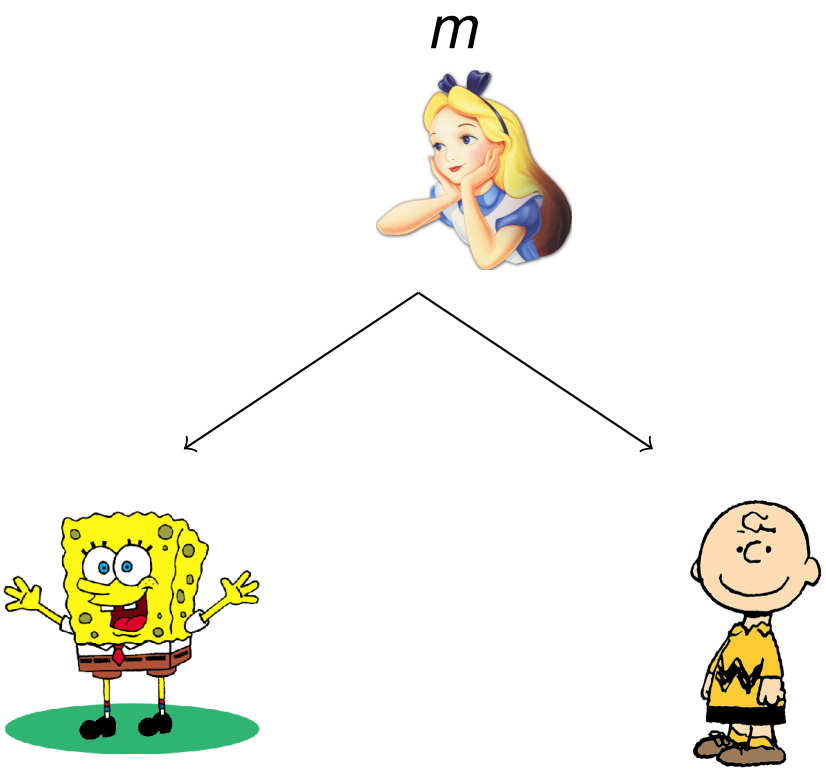
Seoul, Korea
5 December 2011

Traditional Public-Key Encryption [DH76,RSA78,...]



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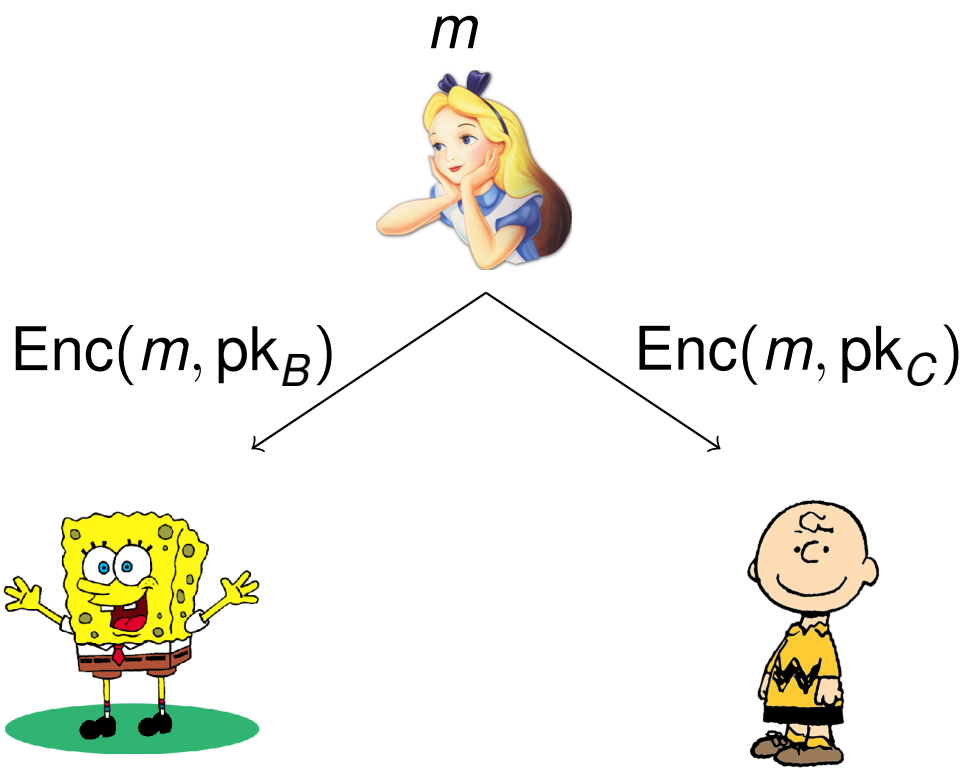

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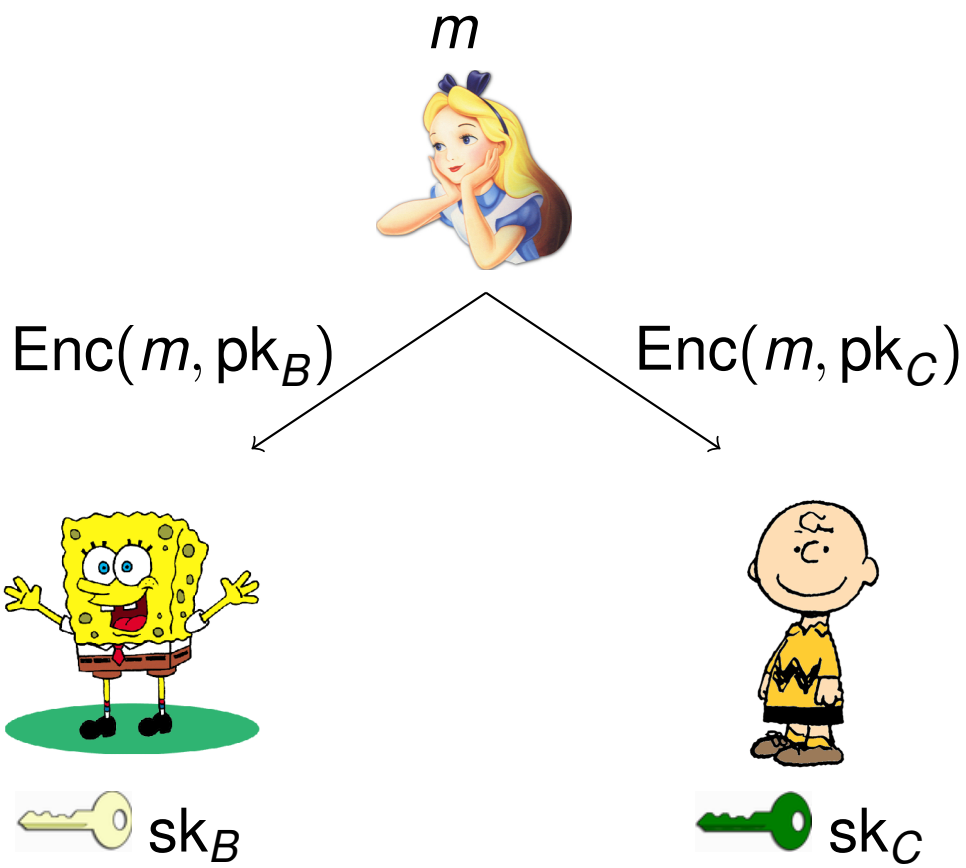

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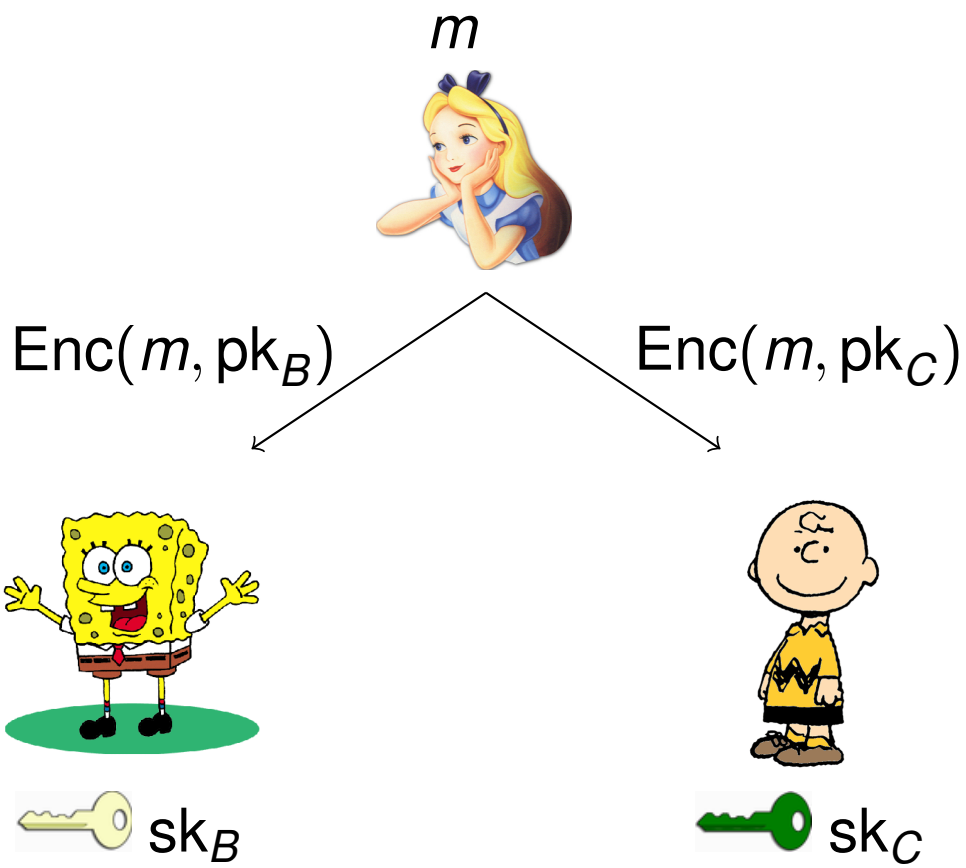
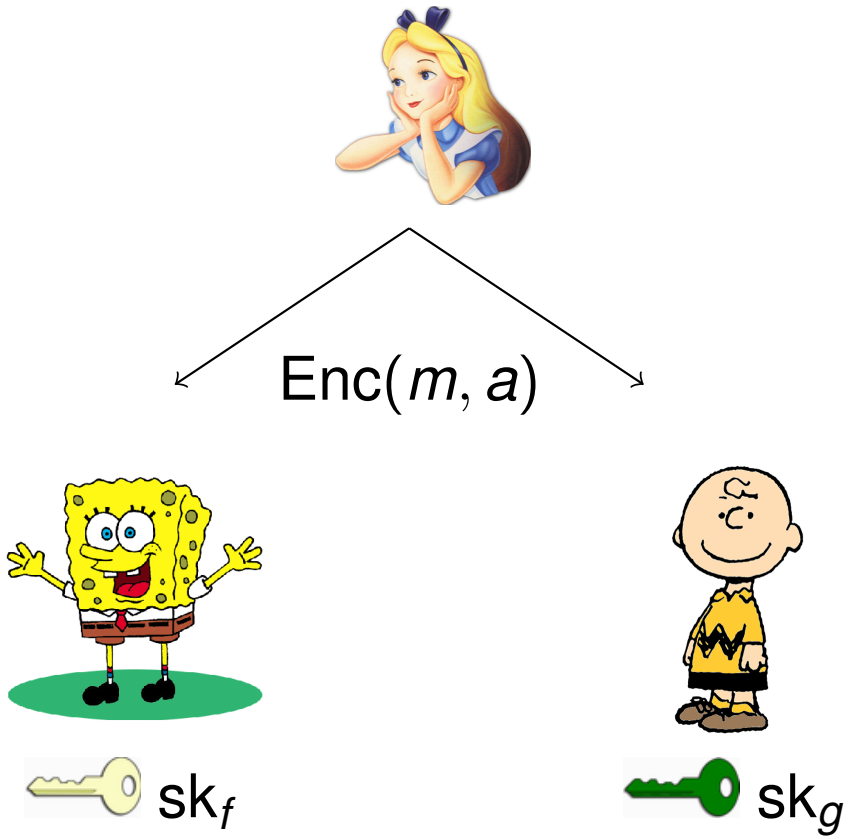


- m must be encrypted separately to each user.
- Recipient set must be decided in advance.

Functional Encryption [SW05,GPSW05,BW07,KSW08,...]



- Ciphertext equipped with attribute a.
- sk equipped with predicate f.
- User with sk_f can decrypt iff $f(a) = 1$.

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Prior Work on Functional Encryption

Identity-based encryption is functional encryption for equality predicates.

- Ciphertexts & keys equipped with identity *id*.
- Decrypt succeeds iff (key *id*) $=$ (CT *id*).
- Achieved using pairings, QR, and lattices.
  [BF01,BB04ab,...], [C01,BGH07], [GPV08,CHKP10,ABB10ab]

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Inner product predicates [KSW08,OT09,LOSTW10,...]:

- CT $\leftrightarrow$ vector $\vec{w}$; key $\leftrightarrow$ vector $\vec{v}$
- Key for $\vec{v}$ can decrypt CT for $\vec{w}$ iff $\langle \vec{v}, \vec{w} \rangle = 0$.
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[KSW08]: Inner product predicates allow us to instantiate range, conjunction, disjunction, and polynomial evaluation predicates.

Our Contribution

Functional encryption for inner product predicates based on the *learning with errors* (LWE) assumption.

- Achieves functionality of [KSW08].
- Worst-case reduction, (conjectured) quantum security.
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Privacy property: CT attribute is hidden from users who cannot decrypt ("*weakly attribute hiding*").

- [KSW08] construction hides attribute from all users.
- Open problem: achieve same privacy property from LWE.

Building Blocks

Lattice-based PKE [GPV08 “dual Regev”]:

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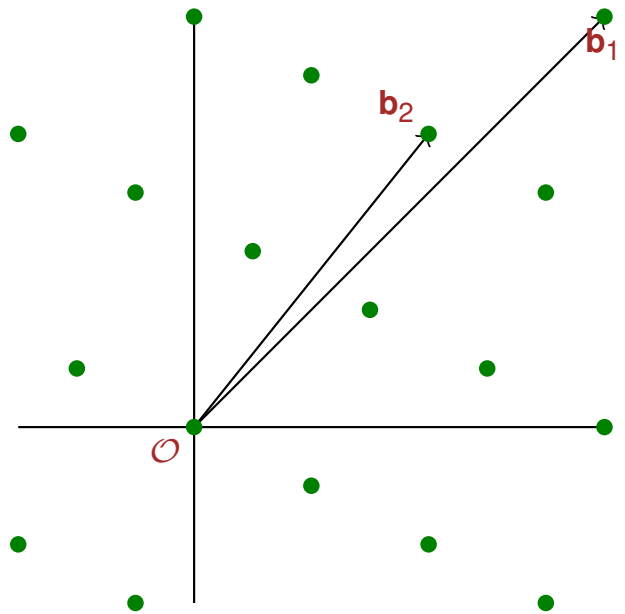

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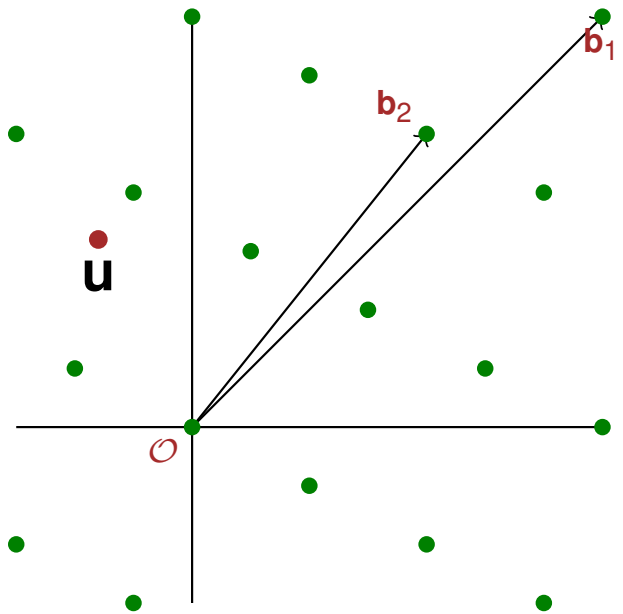

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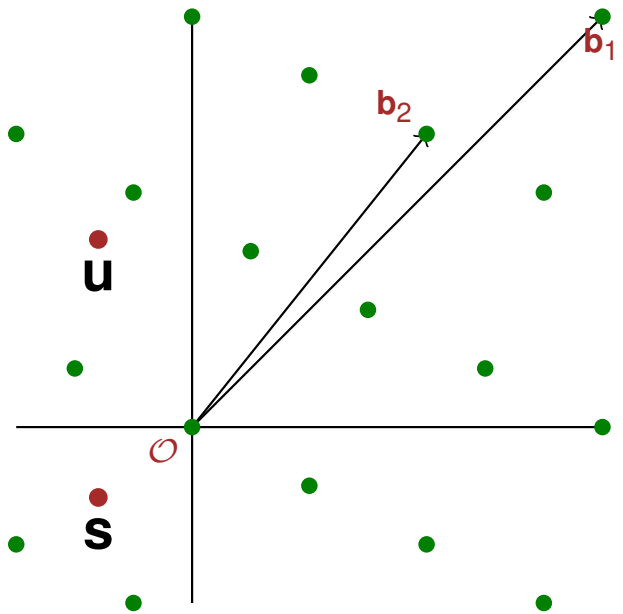

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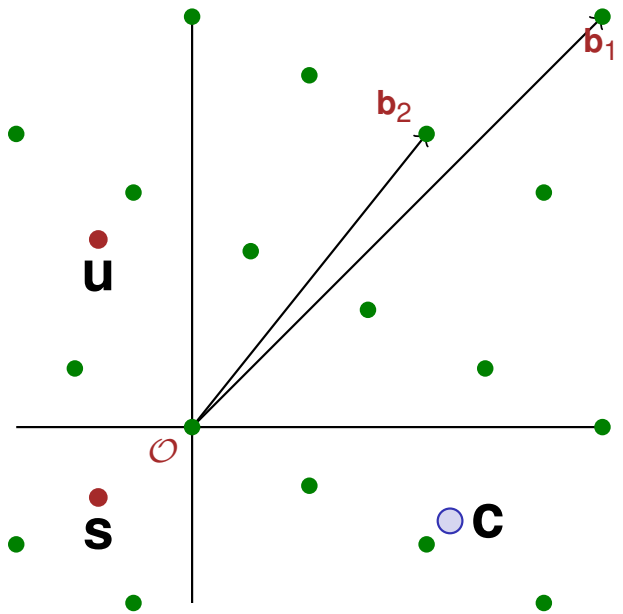

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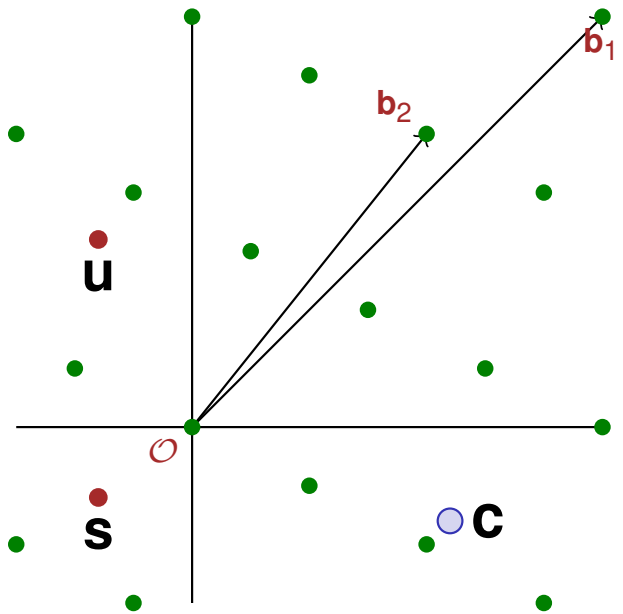

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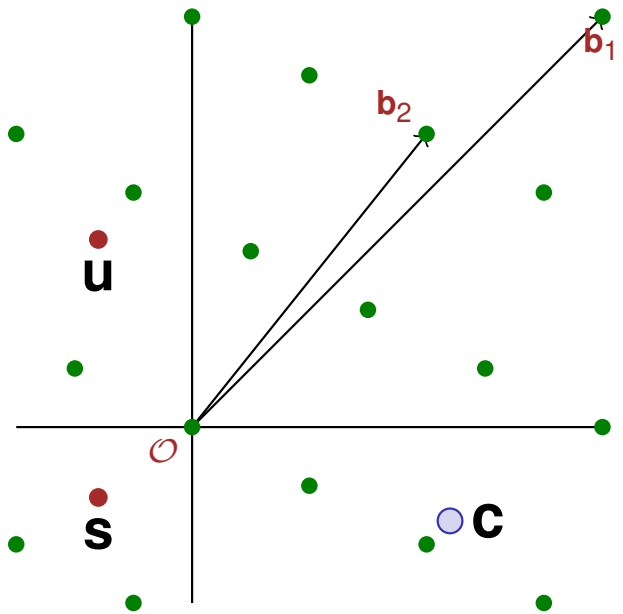

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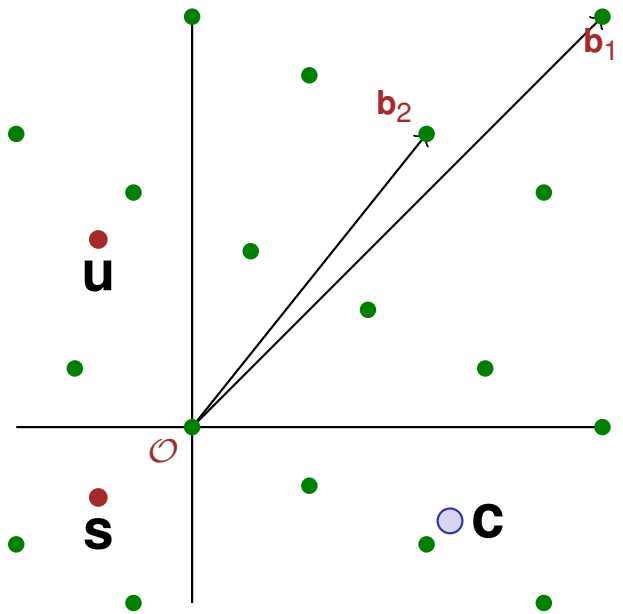

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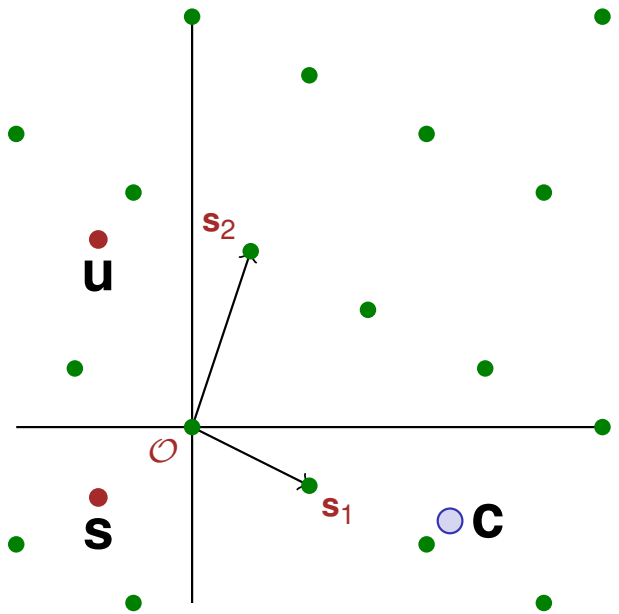

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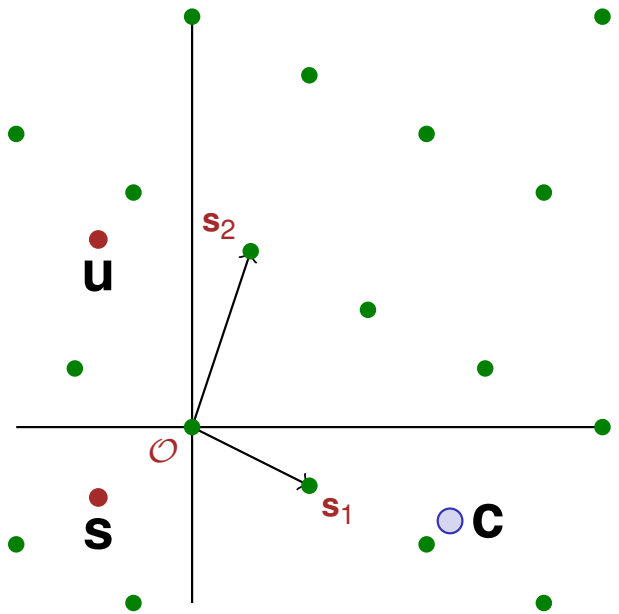

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[A99,AP09]: Can generate a random lattice Λ along with short basis of $\Lambda^\perp$ = trapdoor for Λ.

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Conclude: can't require CT lattice to match sk lattice.

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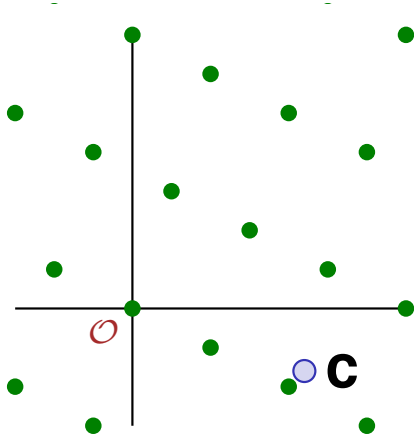
Encrypt relative to
attribute lattice $\Lambda_{\vec{w}} \subset \mathbb{Z}^r$



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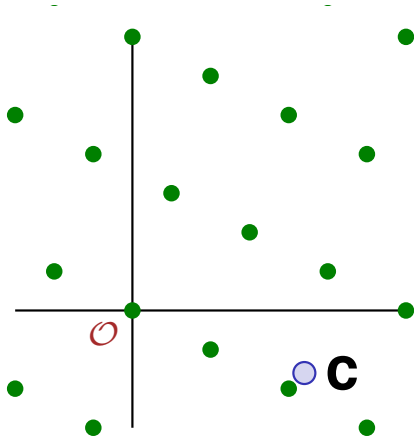

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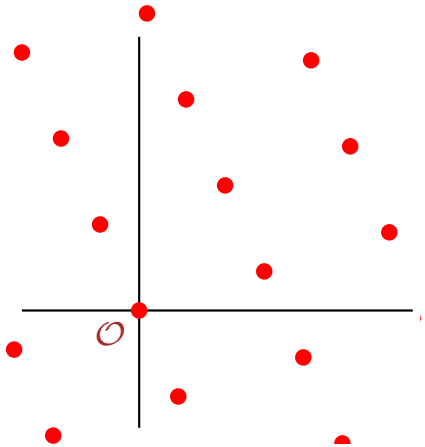

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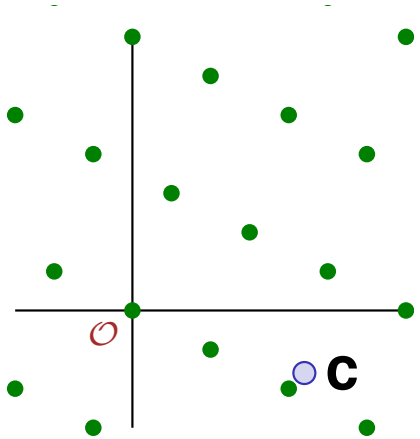

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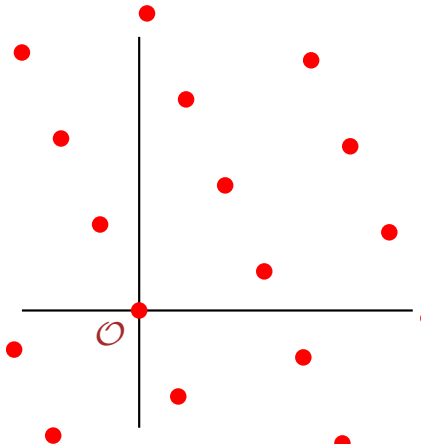

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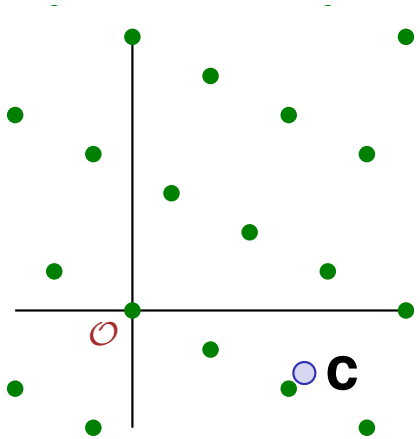

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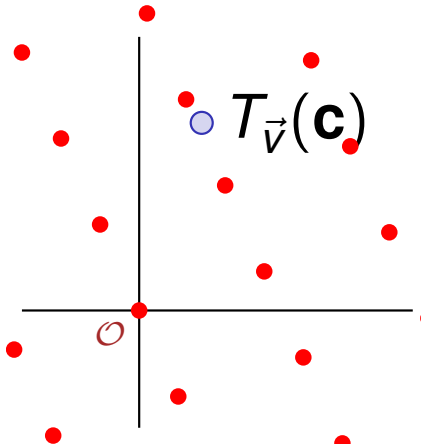

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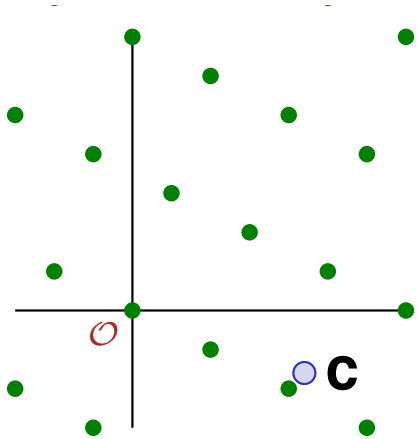

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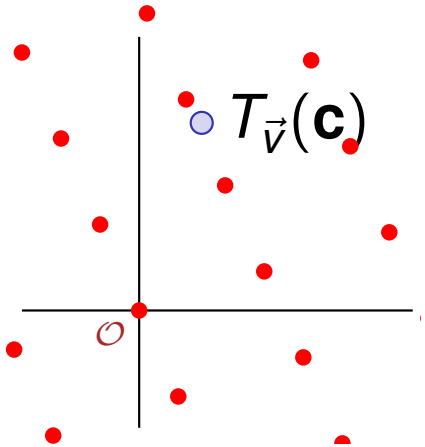

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$\Rightarrow$ key for $\Lambda_{\vec{v}}$ can decrypt $T_{\vec{v}}(\mathbf{c})$.

What Lattices are Used?

Regev/GPV lattice Λ defined by matrix $\mathbf{A}_0 \in \mathbb{Z}_q^{n \times m}$, $n < m$:

$$\Lambda = \Lambda_q(\mathbf{A}_0) = \left\{ \mathbf{v} \in \mathbb{Z}^m : \mathbf{v} \bmod q = \mathbf{r}^t \cdot \mathbf{A}_0 \text{ for some } \mathbf{r} \in \mathbb{Z}_q^n \right\}$$

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[ABB10a] IBE: to encrypt to identity *id*, use lattice

$$\Lambda_{id} = \Lambda_q(\mathbf{A}_0 \parallel \mathbf{A}_1 + H(id)\mathbf{B}) \subset \mathbb{Z}^{2m}.$$

- public $\mathbf{A}_0, \mathbf{A}_1, \mathbf{B} \in \mathbb{Z}_q^{n \times m}$.
- $H\colon \{0,1\}^* \to \mathbb{Z}_q^{n \times n}$ is a hash function.

Secret key for Λ_{id} can be computed using trapdoor for $\mathbf{A}_0$.

A Functional Encryption Scheme

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To compute CT for vector $\vec{w} = (w_1, \ldots, w_\ell)$, use lattice

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Then

$$T_{\vec{v}}(\Lambda_{\vec{w}}) = \Lambda_q(\mathbf{A}_0 \parallel \sum v_i\mathbf{A}_i + \langle\vec{v}, \vec{w}\rangle\mathbf{B})$$

So sk for $\Lambda_{\vec{v}}$ can decrypt $T_{\vec{v}}(CT)$ iff $\langle\vec{v}, \vec{w}\rangle = 0$ (and $\vec{v}$ is short).

(Selective) Security Model

Challenger

Adversary

(Selective) Security Model

attributes $\vec{w}_0, \vec{w}_1$ ⟵

Challenger

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(Selective) Security Model

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attributes $\vec{w}_0, \vec{w}_1$

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Definition

Scheme is *weakly attribute hiding* if $\left| \Pr[b' = b] - \frac{1}{2} \right|$ is negligible for all efficient $\mathcal{A}$.

Security Theorem

Learning With Errors (LWE) assumption [R05]

For fixed $\mathbf{s} \in \mathbb{Z}_q^n$, "noisy inner products" with $\mathbf{s}$ are indistinguishable from random:

$$\left\{\mathbf{a}_i, \langle \mathbf{s}, \mathbf{a}_i \rangle + e_i\right\}_{i=1}^{m} \approx_c \left\{\mathbf{a}_i, r_i\right\}_{i=1}^{m}$$

for random $\mathbf{a}_i \in \mathbb{Z}_q^n$, small $e_i \in \mathbb{Z}$, and random $r_i \in \mathbb{Z}_q$.

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Theorem

If the LWE assumption holds, then our inner product encryption scheme is weakly attribute hiding.

Proof Idea

CT lattice: $\Lambda_{\vec{w}} = \Lambda_q(\mathbf{A}_0 \parallel \mathbf{A}_1 + w_1\mathbf{B} \parallel \cdots \parallel \mathbf{A}_\ell + w_\ell\mathbf{B})$.
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Embed LWE challenge in the matrix $\mathbf{A}_0$.

- If LWE challenge is "noisy inner products" $\langle \mathbf{s}, \mathbf{a}_i \rangle + e_i$, obtain real CT.
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Adversary that breaks system can break LWE assumption.

Open Questions

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Thank you!