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Functional Fractional Calculus for System Identification and Controls

With 68 Figures and 11 Tables

 Springer

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