

Functional Renormalization with fermions and tetrads

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Functional Renormalization with fermions and tetrads,
P.D., R. Percacci - arXiv:1209.3649v2

Overview of the talk

- ① Scheme dependence and fermions
 - A puzzling ambiguity and its solution
 - General discussion
- ② Constraints on matter content
 - One loop results
- ③ FRGE and tetrad gravity
 - Type Ia cutoff
 - Type Ib cutoff
 - Type IIa cutoff
 - Type IIb cutoff
- ④ Discussion

A puzzling ambiguity

The modified effective action for a fermion field is:

$$\Gamma = -\text{Tr} \log (\not{D}) = -\frac{1}{2} \text{Tr} \log (\not{D}^2) \longrightarrow \Gamma_k = -\frac{1}{2} \text{Tr} \log \left(-\nabla^2 + \frac{R}{4} + R_k \right)$$

We will consider:

$$\text{Type I: } R_k (-\nabla^2) \quad \text{or} \quad \text{Type II: } R_k (-\nabla^2 + \frac{R}{4})$$

In the literature¹:

$$\begin{aligned} \text{Type I: } \frac{d\Gamma_k}{dt} &= -\frac{1}{8\pi^2} \int d^d x \sqrt{g} \left[Q_2 \left(\frac{\partial_t R_k}{P_k} \right) + \left(\frac{1}{6} Q_1 \left(\frac{\partial_t R_k}{P_k} \right) - \frac{1}{4} Q_2 \left(\frac{\partial_t R_k}{P_k^2} \right) \right) R \right] \\ \text{Type II: } \frac{d\Gamma_k}{dt} &= -\frac{1}{8\pi^2} \int d^d x \sqrt{g} \left[Q_2 \left(\frac{\partial_t R_k}{P_k} \right) - \frac{1}{12} R Q_1 \left(\frac{\partial_t R_k}{P_k} \right) \right] \end{aligned}$$

¹ A. Codello, R. Percacci, C. Rahmede, Ann. Phys.**324** (2009) 44, [arXiv:0805.2909(hep-th)]

Optimized cutoff

Cutoff function shape:

$$R_k(z) = (k^2 - z)\theta(k^2 - z)$$

In the two schemes:

$$\text{Type I : } = \frac{1}{48\pi^2} \int d^4x \sqrt{g} \left(-6k^4 - \frac{1}{2}k^2 R \right)$$

$$\text{Type II : } = \frac{1}{48\pi^2} \int d^4x \sqrt{g} \left(-6k^4 + k^2 R \right)$$

Also with exponential cutoff

Cutoff function shape:

$$R_k(z) = \frac{ze^{-az/k^2}}{1 - e^{-az/k^2}}$$

In the two schemes:

$$\text{Type I : } \frac{n_D}{8\pi^2} \int d^4x \sqrt{g} \left(-\frac{4\zeta(3)}{a^2} k^4 - \frac{\pi^2 - 9}{24a} k^2 R \right)$$

$$\text{Type II : } \frac{n_D}{8\pi^2} \int d^4x \sqrt{g} \left(-\frac{4\zeta(3)}{a^2} k^4 + \frac{\pi^2}{3a} k^2 R \right)$$

Also with Kähler fermions

$$\{\phi, \phi_\mu, \phi_{\mu\nu}, \phi_{\mu\nu\rho}, \phi_{\mu\nu\rho\sigma}\} \quad \Longleftrightarrow \quad 4 \times \psi$$

FRGE:

$$\frac{d\Gamma_k}{dt} = -2 \frac{1}{2} \text{Tr}_{(0)} \left(\frac{\partial_t R_k(\Delta^{(0)})}{P_k(\Delta^{(0)})} \right) - 2 \frac{1}{2} \text{Tr}_{(1)} \left(\frac{\partial_t R_k(\Delta^{(1)})}{P_k(\Delta^{(1)})} \right) - \frac{1}{2} \text{Tr}_{(2)} \left(\frac{\partial_t R_k(\Delta^{(2)})}{P_k(\Delta^{(2)})} \right)$$

with a general cutoff

$$\text{Type I : } = -4 \times \frac{1}{8\pi^2} \int d^d x \sqrt{g} \left[Q_2 \left(\frac{\partial_t R_k}{P_k} \right) + \left(\frac{1}{6} Q_1 \left(\frac{\partial_t R_k}{P_k} \right) - \frac{1}{4} Q_2 \left(\frac{\partial_t R_k}{P_k^2} \right) \right) R \right]$$

$$\text{Type II : } = -4 \times \frac{1}{8\pi^2} \int d^d x \sqrt{g} \left[Q_2 \left(\frac{\partial_t R_k}{P_k} \right) - \frac{1}{12} R Q_1 \left(\frac{\partial_t R_k}{P_k} \right) \right]$$

Spectral Sum

$$\Gamma = -\text{Tr} \log (\not{D}) = -\frac{1}{2} \text{Tr} \log \left(-\nabla^2 + \frac{R}{4} \right)$$

$$\Gamma = -\text{Tr} \log (\not{D}) \longrightarrow \Gamma_k = -\text{Tr} \log \left(|\not{D}| + R_k^D (|\not{D}|) \right)$$

On a spherical background

$$\begin{aligned} \frac{d\Gamma_k}{dt} &= -\text{Tr} \left[\frac{\partial_t R_k^D (|\not{D}|)}{P_k^D (|\not{D}|)} \right] = -\sum_n m_n \frac{\partial_t R_k^D (|\lambda_n|)}{P_k^D (|\lambda_n|)} \\ &= -\sum_{\pm} \sum_n m_n \theta(k - |\lambda_n|) = \frac{V(4)}{48\pi^2} (-6k^4 + k^2 R) \end{aligned}$$

Defining a good cutoff

$$R_k \overset{?}{\longleftrightarrow} R_k^D$$

Formally

$$\Gamma_k = -\frac{1}{2}\text{Tr} \log \left(|\not{D}| + R_k^D (|\not{D}|) \right)^2 = -\frac{1}{2}\text{Tr} \log \left(-\nabla^2 + \frac{R}{4} + \underbrace{2|\not{D}|R_k^D (|\not{D}|) + R_k^D (|\not{D}|)^2}_{=R_k} \right)$$

Inverting

$$R_k^D(z) = -z + \sqrt{z^2 + R_k}$$

- must be continuous and monotonically decreasing in z^2 and k ,
- it must go rapidly to zero for $z^2 > k^2$,
- it must tend to a positive value for $z^2 \rightarrow 0$
- it must tend to zero for $k \rightarrow 0$.

Type I vs Type II

Type II cutoff: $R_k(-\nabla^2 + R/4) = R_k(\not{D}^2) \equiv R_k(z^2)$

$$R_k^D(z) = -z + \sqrt{z^2 + R_k(z^2)} \quad \text{is always good}$$

With R_k optimized
cutoff shape

$$R_k^D(z) = (k - z)\theta(k - z)$$

Type I cutoff $R_k(-\nabla^2) = R_k(\not{D}^2 - R/4) \equiv R_k(z^2 - R/4)$.

With R_k optimized
cutoff shape

$$R_k^D(z) = \left(\sqrt{k^2 + \frac{R}{4}} - z \right) \theta \left(\sqrt{k^2 + \frac{R}{4}} - z \right)$$

With R_k exponential
cutoff shape

$$R_k^D(z) = -z + \sqrt{z^2 + \left(z^2 - \frac{R}{4} \right) \frac{e^{-a(z^2 - \frac{R}{4})/k^2}}{1 - e^{-a(z^2 - \frac{R}{4})/k^2}}}$$

for both cases

$$\lim_{k \rightarrow 0} R_k^D(z) = \left(\sqrt{\frac{R}{4}} - z \right) \theta \left(\sqrt{\frac{R}{4}} - z \right)$$

Constraints on Matter from Asymptotic Safety

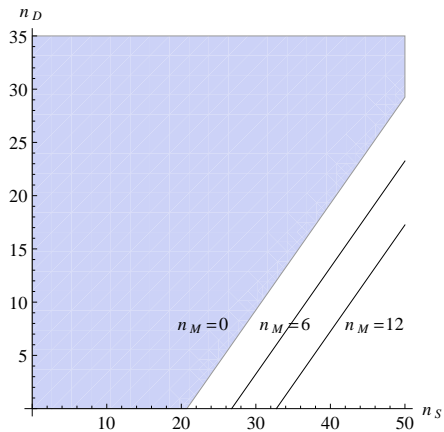
One loop approximation:

$$\begin{aligned}\partial_t \tilde{G} = & 2\tilde{G} - \frac{1}{2\pi} \left(\frac{13}{6} Q_1 \left(\frac{\partial_t R_k}{P} \right) + \frac{57}{6} Q_2 \left(\frac{\partial_t R_k}{P^2} \right) \right) \tilde{G}^2 \\ & + \frac{1}{6\pi} (n_S + 2n_D - 4n_M) Q_1 \left(\frac{\partial_t R_k}{P} \right) \tilde{G}^2\end{aligned}$$

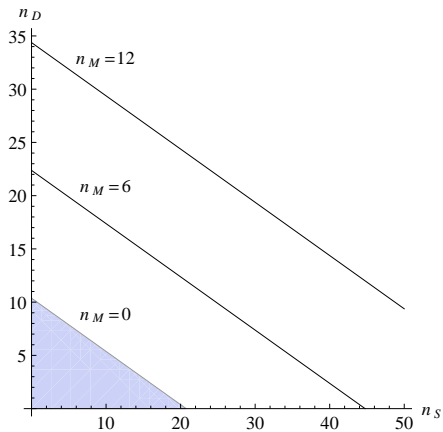
Requiring a positive FP:

$$n_S + 2n_D - 4n_M - \left[\frac{13}{2} + \frac{57}{2} Q_2 \left(\frac{\partial_t R_k}{P_k^2} \right) / Q_1 \left(\frac{\partial_t R_k}{P_k} \right) \right] < 0$$

Type I:



Type II:



Tetrad Gravity

- This has been discussed recently by Harst and Reuter [3] (Type Ia cutoff, $\alpha = 1$)
- Einstein-Hilbert truncation with the usual diffeomorphism ghosts and gauge fixing (and symmetric vielbein)
- $O(d)$ ghost contribution depending on a free ghost mass parameter μ

$$S_{\text{gh}} = -\int dx \bar{e} (\bar{C}^\mu, \bar{\Sigma}'^{\mu\nu}) \begin{pmatrix} \sqrt{2}\xi^{-1}(\delta^\mu_\rho \bar{D}^2 + \bar{R}^\mu_\rho) & 0 \\ 2\xi^{-\frac{1}{2}}\mu \delta^\mu_\rho \bar{D}^\nu & 2\xi^{-\frac{1}{2}}\mu^2 \delta_\rho^{[\mu} \delta_{\sigma]}^{\nu]} \end{pmatrix} \begin{pmatrix} C^\rho \\ \Sigma'^{\rho\sigma} \end{pmatrix}$$

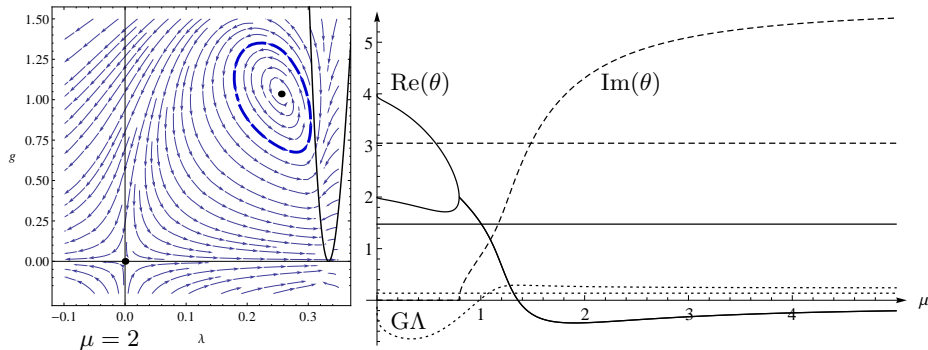
- Substituting the relation $h_{\mu\nu} = 2\varepsilon_{(\mu\nu)} + \varepsilon_{(\mu}{}^\rho \varepsilon_{\nu)\rho}$ gives

$$\Gamma_k^{(2)}[e, \bar{e}] = 2 \frac{\delta\Gamma^{EH}}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \varepsilon_{\mu\nu} \varepsilon_{\rho\sigma} + \zeta \frac{\delta\Gamma^{EH}}{\delta g_{\mu\nu}} \varepsilon_\mu{}^\rho \varepsilon_{\nu\rho}$$

- We used a Type Ib and IIb cutoff

³ U. Harst and M. Reuter, JHEP **1205** (2012) 005 [arXiv:1203.2158 [hep-th]]

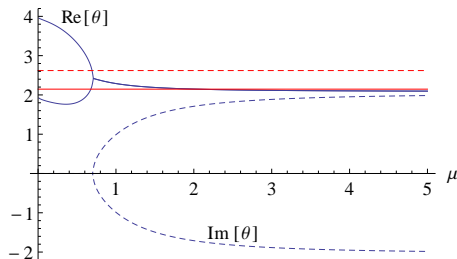
Gauge $\alpha = 1$, dependence on μ :



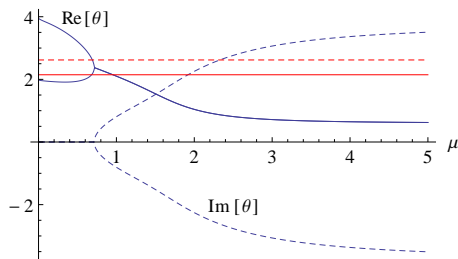
³ U. Harst and M. Reuter, JHEP **1205** (2012) 005 [arXiv:1203.2158 [hep-th]]

Dependence on μ :

$\alpha = 0$



$\alpha = 1$



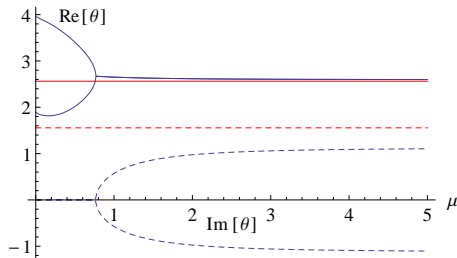
Type IIb cutoff

$$(h_{\mu\nu}^{TT}, \xi, \sigma, h)$$

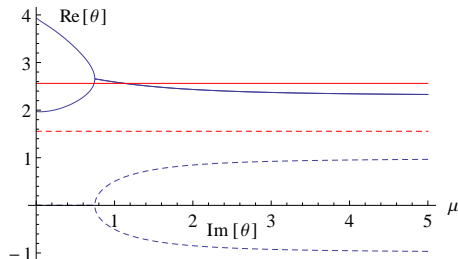
$$R_k(-\nabla^2 + CR)$$

Dependence on μ :

$\alpha = 0$



$\alpha = 1$

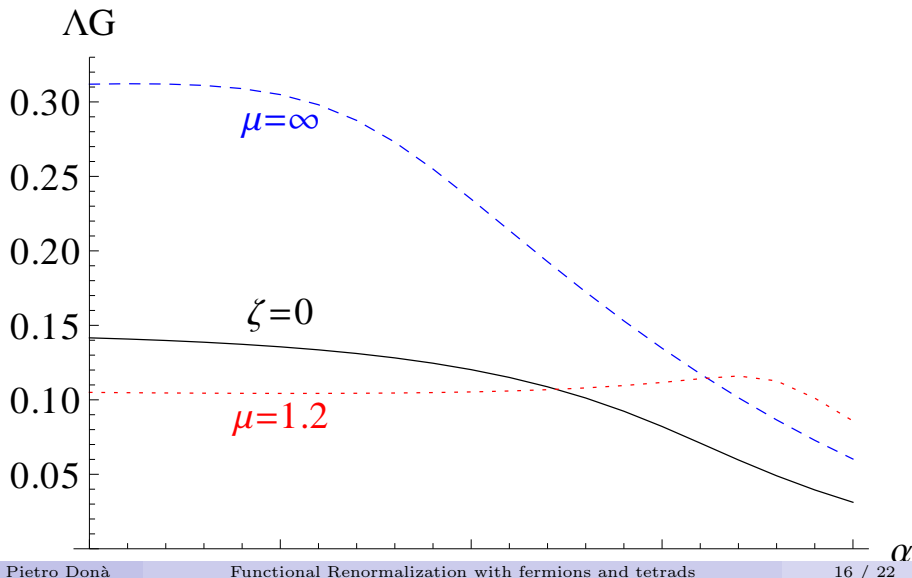


Type Ib cutoff

$$(h_{\mu\nu}^{TT}, \xi, \sigma, h)$$

$$R_k(-\nabla^2)$$

Dependence on α :



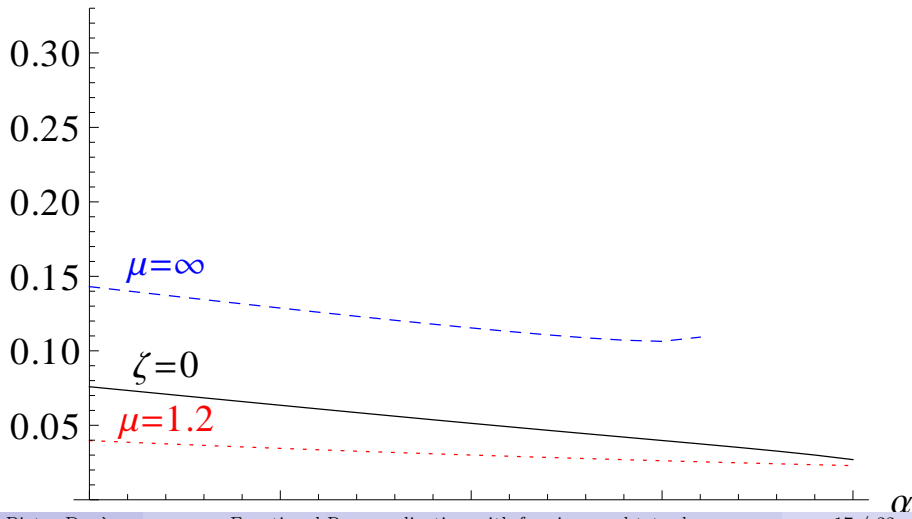
Type IIb cutoff

$$(h_{\mu\nu}^{TT}, \xi, \sigma, h)$$

$$R_k(-\nabla^2 + CR)$$

Dependence on α :

ΛG

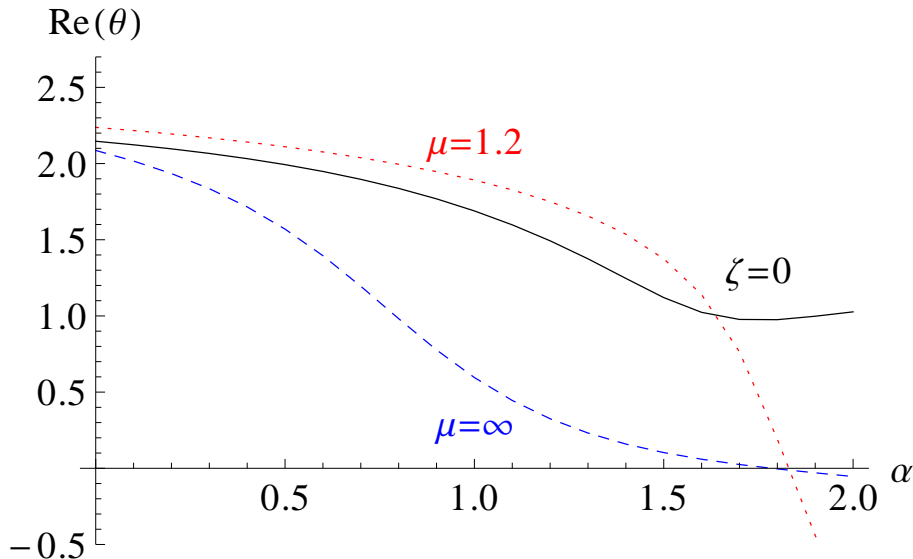


Type Ib cutoff

$$(h_{\mu\nu}^{TT}, \xi, \sigma, h)$$

$$R_k(-\nabla^2)$$

Dependence on α :

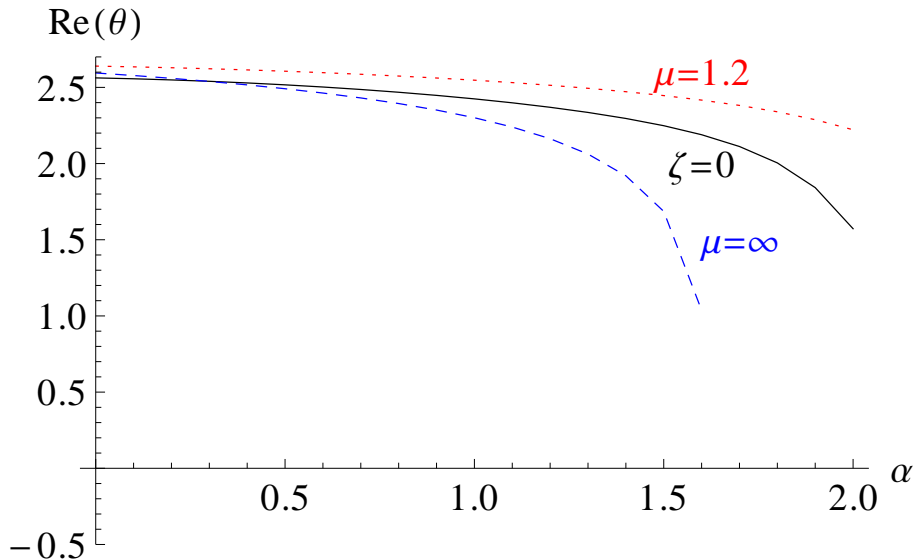


Type IIb cutoff

$$(h_{\mu\nu}^{TT}, \xi, \sigma, h)$$

$$R_k(-\nabla^2 + CR)$$

Dependence on α :



$$\text{Type IIa cutoff} \quad (h_{\mu\nu}^{TT}, h) \quad R_k(-\nabla^2 + CR)$$

This scheme coincides with IIb for $\alpha = 1$ [4].

Type IIa:

$$\frac{d\Gamma_k}{dt} = \sum_{\lambda_n} \frac{\partial_t R_k(\lambda_n) - \eta R_k(\lambda_n)}{P_k(\lambda_n) - 2\Lambda}$$

Type IIb:

$$\begin{aligned} \frac{d\Gamma_k}{dt} = & \sum_n \frac{\partial_t R_k(\lambda_n^{TT}) - \eta R_k(\lambda_n^{TT})}{P_k(\lambda_n^{TT}) - 2\Lambda} + \sum_n \frac{\partial_t R_k(\lambda_n^\xi) - \eta R_k(\lambda_n^\xi)}{P_k(\lambda_n^\xi) - 2\Lambda} \\ & + \sum_n \frac{\partial_t R_k(\lambda_n^\sigma) - \eta R_k(\lambda_n^\sigma)}{P_k(\lambda_n^\sigma) - 2\Lambda} + \sum_n \frac{\partial_t R_k(\lambda_n^h) - \eta R_k(\lambda_n^h)}{P_k(\lambda_n^h) - 2\Lambda} \end{aligned}$$

⁴C. Rahmede, PhD thesis, SISSA (2008).

Discussion

- ① Problems with bad cutoffs
 - “squaring” is OK but must use Type II
- ② Constraints on matter content from asymptotic safety
 - in one loop approximation bounds on n_S, n_F , SM is OK
- ③ Tetrad formalism
 - extended the results of Harst and Reuter to more general cutoff schemes and gauges
 - cutoffs of Type b are less sensitive to μ for $\alpha = 0, 1$
 - pathologies arise again for $\alpha \gtrsim 2$

Thank you