

# FUNDAMENTAL GROUPS OF QUASI GRAPHS OF GROUPS

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Received 26:11:2007 : Accepted 05:12:2009

## Abstract

A graph is called a quasi graph if the possibility of an edge of the graph being equal to its inverse is not excluded. Quasi HNN groups are a new generalizations of HNN groups. In this paper we introduce the concepts of a quasi graph of groups and its fundamental group, and show that the fundamental group of a quasi graph of groups is a quasi HNN group. The embedding theorem for the fundamental group of a quasi graph of groups is formulated and proved. Furthermore, we find the structures of groups induced by the vertices of a quasi graph of groups.

**Keywords:** Quasi graphs, Quasi graphs of groups, Quasi HNN groups, Fundamental groups of quasi graphs of groups.

*2000 AMS Classification:* 20 E 07; 20 E 08; 20 F 65.

## 1. Introduction

In [6, p. 37], Serre and Bass introduced the concepts of graphs of groups and their fundamental groups under the condition that the edge of a graph does not equal its inverse. Then they showed that the fundamental group of a graph of groups is an HNN group of base the tree product of all of the vertex groups (see [2, p. 139]). For further information on fundamental groups of graphs of groups we refer the readers to H. Bass [1, p. 6], G. Baumslag [2, p. 131] or D. E. Cohen [3, p. 198]. In this paper we generalize the above concepts in the case where an edge of the graph can be equal to its inverse. This paper is divided into 5 sections. In section 2, we introduce the concepts of quasi graphs of groups and formulate the fundamental groups of the quasi graph of groups. In section 3, we form and prove the embedding theorem for fundamental groups of quasi graphs of groups. In section 4, we form the structures of the fundamental groups of quasi graphs of groups relative to maximal subtrees of the graphs. In section 5, we find the structures of groups induced by the vertices of quasi graphs of groups.

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