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## Fundamental Real Estate Prices: An Empirical Estimation with International Data

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# Fundamental Real Estate Prices: An Empirical Estimation with International Data\*

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## Abstract

We propose two alternative models to estimate fundamental prices on real estate markets. Both models state that the fundamental price is the sum of the discounted future period costs that arise from owning a house. The first model is based on a no-arbitrage condition between renting and buying a house. It states that the period costs are equal to the rents. The second model interprets the period costs as the result of a market equilibrium between housing demand and supply. We estimate both models for the USA, the UK, Japan, Switzerland, and the Netherlands. We find that observed prices deviate substantially and for long periods from their estimated fundamental values. However, by studying the dynamics of the gaps, we find some evidence that in the long-run actual prices tend to return progressively to their fundamental values. This fact is supported by a forecast analysis. We find that models including our fundamental prices significantly outperform long-term forecasts made by models based on the observed price dynamic only.

*Keywords:* Real Estate Market, Fundamental Price, Long-Term Forecast.

*JEL-Classifications:* R31, C51, D58, C53.

# 1 Introduction

The increase in house prices since the end of the 1990s around the globe has made the housing market a prominent economic topic. The fear is that rising prices have led to a bubble which could burst anytime. The Economist (June, 18th 2005, p. 62) even speaks about "... the biggest bubble in history." There are a number of examples for house-price bubbles which have burst in the past. The latest wave of housing-market crises was around 1990. At that time, real-house prices dropped by more than 25% in countries such as Japan, the United Kingdom, and Switzerland.

Economically, the development of house prices is very relevant. Houses are often the biggest household asset. Therefore, the development of house prices has a non negligible influence on household wealth and consumption and, hence, on GDP. Beyond this, the banking sector is usually highly exposed to the housing market. Banks own property, they lend money to customers that buy properties, and they engage in secondary mortgage markets.<sup>1</sup> Therefore, the burst of a house-price bubble often leads to a banking crisis.<sup>2</sup> This makes it very important to detect price bubbles and to forecast the future development of house prices.

When exploring the question whether houses are overvalued, the simple observation of the development of prices could be misleading. On the one hand, a strong increase in prices could be fundamentally justified. On the other hand, constant prices could lead to an overvaluation if the underlying fundamentals would demand decreasing prices. Hence, we can only decide if there is a bubble or not, once we know the deviation of house prices from their fundamental value.<sup>3</sup> Very often studies of house prices consider fundamentals by using indicators like *price-to-rent* or *price-to-income*.<sup>4</sup> The first one indicates the return on an investment in a house and the second one indicates the affordability of a house. The problem with these indicators is that they consider only

one fundamental factor at a time (here: rent or income). But even if several indicators are considered, it is unclear how to aggregate the signals that they give.

Some studies use more sophisticated indicators to evaluate house prices.<sup>5</sup> For example, instead of observing the price-to-rent ratio, the so called *imputed rent* of a house is compared to the actual rent. The imputed rent reflects the costs that arise from owning a house for one period. Among other things, these costs are influenced by mortgage rates, physical depreciation, and expected capital gains. Usually, expected capital gains are assumed to be equal to their average or it is assumed that there are only nominal capital gains corresponding to the inflation rate.<sup>6</sup> Instead of just looking at the price-to-income ratio, another group of more sophisticated indicators considers income, house prices, and interest rates to estimate the affordability. A disadvantage of those improved indicators is still that most of them only consider the current fundamentals and do not regard their future development.

In this paper we use a two-step approach to calculate our fundamental house prices: First, we define the fundamental house price as the sum of the discounted future imputed rents. This step is based on a standard (e.g. Poterba, 1992, or Himmelberg et al., 2005) definition for the imputed rent. Though, in contrast to other studies, we derive the expected capital gains from the expected future fundamentals. In a second step we use two different interpretations for the imputed rents: The first one is that they are equal to the actual rents and the second one is that they are the result of a market equilibrium between housing demand and supply. The first interpretation is very similar to the "imputed rent-to-rent" approach. In our second interpretation we calculate a fundamental value of (imputed) rents, using income as a measure for the demand for housing. Therefore, we combine the two basic elements of house price assessment: the imputed rent and the influence of income on prices. This is one of the

innovations of the model. Another one is the inclusion of future fundamentals and the calculation of fundamental a value of houses.<sup>7</sup>

On the technical level, the estimation of the fundamental price is based on a methodology developed by Campbell and Shiller (1988). This method assumes that agents are rational and that they form their expectations by using the observed linear dynamic of the fundamental factors. We then compare our estimated fundamental price to the observed one and check if there is any empirical link between them. For that, we first test for the presence of a long-term relation between the fundamental and the observed price with cointegration analysis. Secondly, we study the dynamics of the gap between them with impulse-response functions. Finally, we investigate the out-of-sample forecast performance of models incorporating our fundamental price.

In the next section, we present two models of fundamental house prices. Section 3 presents the data and, in section 4, we estimate both models and compare the fundamental values with the actual prices. Section 5 analyses the link between actual and fundamental house prices. As we will see, in the short run there is no clear-cut relationship between them. However, in the long run actual house prices seem to go back to their fundamental values. Corresponding to this finding, we show in section 6 that the inclusion of fundamental house prices into forecasting models improves their accuracy for long-horizon forecasts. Section 7 offers concluding remarks.

## **2 A Theory of Fundamental House Prices**

### **2.1 The Model**

Our calculation of the fundamental property price ( $P_t$ ) starts with the costs that arise from owning a house for one period. These costs are also known as the *imputed*

*rent* or (in terms of a fraction of the house price) the *user costs* and are subject to various factors. For example Poterba (1984 and 1992), McCarthy and Peach (2004), and Himmelberg et al. (2005) use seven different factors: First, the owner of a house has to pay mortgage or at least loses the interest rate of an alternative investment. Second, the house is subject to a depreciation and third, the owner has to pay for maintenance and repairs. The fourth factor is the property tax she has to pay and the fifth factor is that she bears a risk which has to be compensated by a risk premium. On the other hand the owner of a house can profit from potential capital gains (factor six) and in some countries from a tax deductibility of the mortgage interest payments (factor seven).

In this paper we consider the following factors: The first factor is the mortgage  $m_t P_t$  the owner of a house has to pay in period  $t$ , where  $m_t$  is the mortgage rate and  $P_t$  the price of the house in period  $t$ . House purchases are usually highly leveraged. For the leveraged part of the financing the mortgage rate is the relevant discount rate. Since for most of the time the loan-to-value is well below one, we also have to consider an interest rate of an alternative investment. But corresponding (long-term) interest rates are highly correlated with mortgage rates. Therefore, and for simplicity, we consider the mortgage rate to be relevant for the total price of the house.

The second factor is that in each period the owner of a house has to pay a fixed fraction  $\phi$  of the value of the house. This factor is assumed to be the sum of maintenance costs (as a constant fraction of the house price), constant property taxes, and a constant risk premium.

The third factor is the expected capital gain. In contrast to other papers we will calculate the expected capital gain via the expected house price in the next period ( $E_t(P_{t+1})$ ). We also consider a constant physical depreciation  $(1 - \delta)$  of the house.

Hence, the expected capital gain is  $\delta E_t(P_{t+1}) - P_t$ .

We will not consider a possible tax deductibility of mortgage payments. From a theoretical point of view the effect of the tax deductibility is rather small compared to real world price movements.<sup>8</sup> Furthermore, the relevant marginal income tax rates and the legal requirements vary between countries and over time. This makes it very difficult and time-consuming to employ adequate parameter values in our model estimation.<sup>9</sup>

Altogether, we assume the following imputed rent  $H_t$ :<sup>10</sup>

$$H_t = m_t P_t + \phi P_t - (\delta E_t(P_{t+1}) - P_t). \quad (1)$$

By rearranging equation (1) we get an equation for the price of a house in period  $t$ :

$$P_t = \frac{H_t + \delta E_t(P_{t+1})}{R_t}, \quad (2)$$

where  $R_t = 1 + m_t + \phi$ . As we can see, the price of a house is driven by the imputed rent, the mortgage rate, and the expectation about the future price. Since we want to calculate the fundamental value of a house, we assume that expectations are rational. Therefore, by forward iteration we can rewrite equation (2) to:<sup>11</sup>

$$\begin{aligned} P_t &= \frac{H_t}{R_t} + \frac{\delta E_t(H_{t+1})}{R_t E_t(R_{t+1})} + \frac{\delta^2 E_t(H_{t+2})}{R_t E_t(R_{t+1} R_{t+2})} + \dots \\ &= E_t \left[ \sum_{i=0}^{\infty} \frac{\delta^i H_{t+i}}{\prod_{j=0}^i R_{t+j}} \right]. \end{aligned} \quad (3)$$

Our fundamental house price is driven by the present and expected future imputed rents and interest rates. Thus, before we can calculate it, we have to define how the



imputed rent evolves.

## 2.2 Interpretations of the Imputed Rents

There are two ways to interpret the evolution of the imputed rents: by a no-arbitrage condition and by a market equilibrium. In the following we use both interpretations within our basic model framework and calculate the corresponding fundamental house prices. We use a "•" to indicate the no-arbitrage (or rent) model and a "\*" to indicate the market equilibrium (or supply/demand) model.

### 2.2.1 Rent Model

Following the no-arbitrage view, agents have to choose whether to buy or to rent a house. In equilibrium agents are indifferent between these two options and the imputed rents are equal to the actual rents ( $Q_t$ ). Hence, following our "rent model", the fundamental house price ( $P_t^\bullet$ ) is given by:

$$P_t^\bullet = E_t \left[ \sum_{i=0}^{\infty} \frac{\delta^i Q_{t+i}}{\prod_{j=0}^i R_{t+j}} \right]. \quad (4)$$

The problem with rents as a determining factor for the fundamental house prices is that rents do not have to be fundamental themselves. Furthermore, due to institutional circumstances, (in the short term) the renting market might be disconnected from the buying market. Therefore, we will present an alternative approach to interpret the imputed rents.

### 2.2.2 S/D Model

Another approach is to see imputed rents as the outcome of a market. Following this market view, imputed rents are determined by the supply of and the demand for housing ("S/D model"). The demand is influenced by the utility an individual derives from occupying a house (hence, her utility function) and her budget restriction (hence, the individual income and the costs for housing). In our model we assume that in period  $t$  there are  $N_t$  identical agents. They derive their utility from consumption  $\hat{C}_t$  and housing units  $\hat{D}_t$ .<sup>12</sup> The corresponding utility function is assumed to be:

$$U_t(\hat{D}_t, \hat{C}_t) = \hat{D}_t^\alpha \hat{C}_t^{1-\alpha}, \quad (5)$$

where  $\alpha$  reflects the strength of the preferences for housing compared to the preferences for consumption.

We further assume that each agent has the same income ( $\hat{Y}_t$ ) to spend in period  $t$ . For simplification, we disregard the possibility to save money and to transfer utility into the future. We only consider real values and the price of the consumption good is normalized to one.<sup>13</sup> Hence, the budget restriction in period  $t$  is:

$$\hat{Y}_t \geq H_t \hat{D}_t + \hat{C}_t, \quad (6)$$

since  $H_t$  is the one period price for one housing unit. The utility maximizing amount of housing units per capita is:

$$\hat{D}_t = \alpha \frac{\hat{Y}_t}{H_t}. \quad (7)$$

From this we can calculate the aggregated demand for housing ( $D_t$ ):

$$D_t = N_t \hat{D}_t = \alpha \frac{N_t \hat{Y}_t}{H_t} = \alpha \frac{Y_t}{H_t}. \quad (8)$$

Since each agent spends the fraction  $\alpha$  of his income on housing, the corresponding applies for the whole country: The fraction  $\alpha$  of the aggregated income ( $Y_t$ ) is spent on housing. Hence, the aggregated demand for housing depends on aggregated income and the imputed rents.<sup>14</sup>

It is often assumed that the supply of housing units is fixed or at least very inelastic. Though, for example McCarthy and Peach (2004) explicitly consider that house prices are also influenced by the supply side. They assume that the supply is driven by the construction of new housing units on the one hand and the depreciation of the existing units on the other hand. As already mentioned above, we assume that the depreciation rate per period is  $(1 - \delta)$ . Construction takes one period and leads to  $B_t$  new housing units in period  $t + 1$ .<sup>15</sup> Hence, the supply of housing ( $S_t$ ) develops as follows:

$$S_t = \delta S_{t-1} + B_{t-1} = \delta^t S_0 + \sum_{i=1}^t \delta^{i-1} B_{t-i}. \quad (9)$$

As we can see, today housing supply depends on the depreciation rate, an initial housing stock  $S_0$ , and all previous construction activities.

In equilibrium the demand for housing units from equation (8) must be equal to the supply from equation (9). Hence, we get the following condition for an equilibrium on the housing market:

$$S_t = \alpha \frac{Y_t}{H_t}. \quad (10)$$

From equation (10) we can calculate the equilibrium imputed rents. Therefore, follow-

ing the "S/D model", the fundamental house price ( $P_t^*$ ) is given by:

$$P_t^* = \alpha E_t \left[ \sum_{i=0}^{\infty} \frac{\delta^i Y_{t+i}}{S_{t+i} \prod_{j=0}^i R_{t+j}} \right]. \quad (11)$$

With (4) and (11) we have two equations to calculate the fundamental price of a house. Before we can estimate actual values, we have to transform the equations into a linear form and set up the corresponding estimation equations.

### 2.3 Linearization of the Price Equations

From an empirical point of view, the basic model in equation (3), and therefore the rent and the S/D model that are derived from it, has two major drawbacks: firstly, it is non linear and, secondly, it includes the imputed rent  $H_t$ , which is a non-stationary variable.<sup>16</sup> These two features make the rent and the S/D models tricky to estimate. Therefore, before turning to the empirical study, it is useful to modify the fundamental equations (4) and (11) in order to transform them into linear functions of stationary variables.

We start by defining a new variable  $X_t = P_t/H_t$  (price-to-imputed-rent ratio) and rewrite the original equation (2) as:

$$X_t = \frac{\delta X_{t+1} (H_{t+1}/H_t) + 1}{R_t}. \quad (12)$$

This equation can be linearized by taking a first-order Taylor approximation of its logarithm, which gives:

$$x_t \approx \kappa_1 + \rho (\ln \delta + x_{t+1} + \Delta h_{t+1}) - m_t - \phi, \quad (13)$$

where  $x_t = \ln X_t$ ,  $\Delta h_t = \ln(H_t/H_{t-1})$  and  $\kappa_1$  and  $\rho$  are linearization parameters.<sup>17,18</sup> By forward iteration of the log price-to-imputed-rent ratio  $x_t$  in equation (13), we get:

$$x_t = \sum_{i=0}^{\infty} \rho^i (\rho \Delta h_{t+i+1} - m_{t+i}) + c, \quad (14)$$

where  $c = (\kappa_1 + \rho \ln \delta - \phi) / (1 - \rho)$ . Taking conditional expectation yields:

$$x_t = \sum_{i=1}^{\infty} \rho^i E_t \left( \Delta h_{t+i} - \frac{1}{\rho} m_{t+i} \right) - m_t + c, \quad (15)$$

which corresponds to the linear version of the fundamental model in equation (3). Note that the log price-to-imputed-rent ratio is now a *linear* function of *stationary* variables (i.e. the mortgage rate  $m_t$  and the imputed rent growth rate  $\Delta h_t$ ). From this equation, we can derive the linear version of the rent and the S/D model as explained below.

### 2.3.1 Rent Model

In the rent model we replace the imputed rent  $H_t$  by the actual rent  $Q_t$ . Using the same definition, we get  $\Delta h_t = \Delta q_t$ . Equation (15) thus becomes:

$$x_t^\bullet = \sum_{i=1}^{\infty} \rho^i E_t \left( \Delta q_{t+i} - \frac{1}{\rho} m_{t+i} \right) - m_t + c, \quad (16)$$

where  $x_t^\bullet = p_t^\bullet - q_t$ .

### 2.3.2 S/D Model

In the S/D model, the imputed rent is defined in terms of aggregated income and supply of housing. Taking the log of equation (10) and its first difference yields:

$$\Delta h_t = \Delta y_t - \Delta s_t. \quad (17)$$

Plugging equation (17) in equation (15) gives:

$$x'_t = \sum_{i=1}^{\infty} \rho^i E_t \left( \Delta y_{t+i} - \Delta s_{t+i} - \frac{1}{\rho} m_{t+i} \right) - m_t + c, \quad (18)$$

where  $x'_t = p_t^* - y_t + s_t - \ln \alpha$ . The only unknown variable remaining is the housing supply  $s_t$ , which is defined in equation (9). Taking the log of this equation and its first-order Taylor approximation yields to:

$$s_{t+1} \approx \kappa_2 + \theta s_t + (1 - \theta) b_t, \quad (19)$$

where  $\theta$  and  $\kappa_2$  are linearization parameters.<sup>19</sup> Subtracting  $s_t$  from both sides gives:

$$\Delta s_{t+1} \approx \kappa_2 + (1 - \theta) (b_t - s_t). \quad (20)$$

By forward iteration of equation (19), we get:

$$s_{t+i} = \theta^i s_t + \sum_{j=0}^{i-1} \theta^{i-j-1} (\kappa_2 + (1 - \theta) b_{t+j}). \quad (21)$$

Plugging equation (20) and (21) into equation (18) yields to:

$$x_t^* = \sum_{i=1}^{\infty} \rho^i E_t \left( \Delta y_{t+i} - \lambda b_{t+i} - \frac{1}{\rho} m_{t+i} \right) - \lambda b_t - m_t + c^*, \quad (22)$$

where  $x_t^* = p_t^* - y_t + \varphi s_t$  and  $\varphi$ ,  $\lambda$  and  $c^*$  are functions of the linearization parameters.<sup>20</sup> The log price-to-imputed-rent ration is now a linear function of expected future aggregated income growth rates, future construction activities and future interest rates, which are all stationary variables. The next section explains how the rent and the S/D model are estimated in their linear form.

## 2.4 The Fundamental Price Equations

Both the rent and the S/D model, in equation (16) and (22) respectively, state that the fundamental price is a function of the *expected* fundamentals. To compute them, we must therefore estimate agents' expectations about future fundamentals. We assume that the agents make their forecasts by using a VAR model, which regroups all the observable variables that are relevant to predict the future value of fundamentals.

Concretely, we collect all present and past values (until  $k$  lags) of the observable variables in the vector  $z_t$ . We then estimate the following VAR:

$$z_t = Az_{t-1} + u_t, \quad (23)$$

where  $A$  is the matrix containing the VAR coefficients<sup>21</sup> and  $E_{t-1}(u_t) = 0$ . We can then use the estimated VAR to forecast the future values of  $z_t$  with:

$$E_t(z_{t+i}) = A^i z_t. \quad (24)$$

As the next sections explain, the forecasted values are used to infer the fundamental price.

### 2.4.1 Rent Model Fundamental Price

For the rent model, we use the vector  $z_t^\bullet$ , which regroups all present and past values (until lag  $k$ ) of the observable fundamentals and of the prices:

$$z_t^\bullet = \left[ \hat{x}_t \quad \Delta q_t \quad m_t \quad \dots \quad \hat{x}_{t-k} \quad \Delta q_{t-k} \quad m_{t-k} \right]',$$

where  $\hat{x}_t = p_t - q_t$  is the observable price-to-rent ratio. With this vector, equation (16) can be rewritten as:

$$x_t^\bullet = \sum_{i=1}^{\infty} \rho^i g_1 E_t(z_{t+i}^\bullet) + g_2 z_t^\bullet + c, \quad (25)$$

where  $g_1 = \left[ 0 \quad 1 \quad -1/\rho \quad 0 \quad \dots \quad 0 \right]'$  and  $g_2 = \left[ 0 \quad 0 \quad -1 \quad 0 \quad \dots \quad 0 \right]'$  regroup the coefficients given by equation (16).

Using equation (24), we can rewrite equation (25) to get the final expression for fundamental price according to the rent model:

$$x_t^\bullet = (\rho g_1 A (I - \rho A)^{-1} + g_2) z_t^\bullet + c. \quad (26)$$

Since all the variables of the vector  $z_t^\bullet$  are observable and all coefficients are known (cf. Appendix C), we can compute the fundamental price-to-imputed-rent ratio  $x_t^\bullet$ , and by adding the log rent  $q_t$  to it, we can infer the fundamental price  $p_t^\bullet$ .



### 2.4.2 S/D Model Fundamental Price

Similarly, we can form the vector  $z_t^*$  which regroups all the present and past values of the variables needed in the S/D model, namely the observable log price-to-imputed-rent ratio  $\tilde{x}_t = p_t^* - y_t + \varphi s_t$ , the aggregated income growth rate  $\Delta y_t$ , the log construction  $b_t$  and the mortgage rate  $m_t$ . With this vector, the fundamental equation (22) becomes:

$$x_t^* = (\rho g_1^* A (I - \rho A)^{-1} + g_2^*) z_t^* + c^*, \quad (27)$$

where, similarly to the vector  $g_1$  and  $g_2$  in the rent model, the vectors  $g_1^*$  and  $g_2^*$  regroup the coefficients given by equation (22).

The only problem remaining is that  $s_t$  is not directly observable and therefore  $\tilde{x}_t$ , which is part of the vector  $z_t^*$  is not observable. However, we can get  $s_t$  as explained in Appendix C and then infer the fundamental house prices from  $x_t^*$ . The next sections presents our empirical results.

## 3 Data

We estimate our two fundamental models for various countries. To judge our model we choose countries with different developments of the housing market: the United States (USA), the United Kingdom (UK), Japan (JAP), Switzerland (CH) and the Netherlands (NL). Between 1997 and 2005 real house prices grew very strongly in the UK (+104%), the NL (+74%) and the USA (+54%). On the other hand, real house prices remained almost constant in CH (+4%) and declined in JAP (-28%).

According to equation (4) and (11) we need the following data to compute our two different fundamental house prices: aggregated income ( $Y_t$ ), construction ( $B_t$ ), the mortgage rate ( $m_t$ ), and rents ( $Q_t$ ). Since we want to compare our fundamental

prices with actual house prices, we also need data on the actual development of the house prices ( $P_t$ ). To transform the nominal series into real ones, we need data on the development of the CPI ( $CPI_t$ ).

The construction of house price indices is affected by many problems. One problem is that houses are very heterogenous goods. Thereby, not only the size and the quality of a house matters, but also and most important the location. Therefore, it is very difficult to compare the price of two houses or to assess the market price of a house if it is not on the market.<sup>22</sup> Another problem is the lack of a central trading place. Therefore, it is hard to say if we really can observe a market price of a house even if it has been sold recently. We have to keep these problems in mind when judging the development of the house price indices.

The frequency of most of the data series is quarterly. All the other series we have transformed into quarterly data. The time horizon of the different series varies. Though, they all capture at least one property-price cycle. The main sources are the BIS, the IMF, and the OECD. For more details on the data see Appendix A

## 4 The Fundamental House Prices

To see how high the fundamental house prices in the USA, the UK, Japan, Switzerland and the Netherlands are, we estimate our rent and S/D model. The results of both estimations are presented in Figure 1. As we can see, the development of both series is very similar.<sup>23</sup>

[Insert Figure 1 about here]

For comparison, the actual house prices are displayed in Figure 1 as well. As we can see, actual house prices are much more volatile than their estimated fundamental

values.<sup>24</sup> They often deviate substantially from the corresponding fundamental prices for a long period. This is the case for the fundamental prices according to the rent model as well as for the fundamental prices according to the S/D model. Though, the Rent model has a slightly better fit (in terms of square deviations) to the actual prices.

When we look at the evolution of the actual prices, there are episodes of overvaluation and episodes of undervaluation in each of the countries under consideration. We can easily detect the price bubbles around 1990 in JAP, the UK and CH. After the burst of the bubbles, prices in each of the countries undershoot their fundamental prices and houses became undervalued. At the end of our sample in 2005/06 house prices in the USA, the UK and the NL are overvalued and prices in JAP and CH undervalued. The degree of the over- and undervaluation according to the rent and to the S/D model is shown in Table 1.

[Insert Table 1 about here]

There are various possible reasons for the difference between actual and fundamental house prices. One is that we have assumed that people are rational and forecast the fundamentals up to infinity. These assumptions were made to calculate how high house prices should be. Though, they are rather unrealistic. If people's time horizon is not infinite, they would react much more to the current development of the fundamentals. As a result, prices would be much more volatile than our fundamental house prices. Furthermore, it is doubtful that people base their forecasts solely on rational reasoning. For example, if they look instead at the momentum of the prices, fluctuations of the prices would be much more pronounced and persistent than in our model.

Another possible explanation for the difference between actual and fundamental house prices is that prices are influenced by factors that are not considered in our two

models. For example, we did not consider the possibility that people's risk aversion changes over time or that the governments could influence house prices for example by introducing new tax advantages. Furthermore, our models do not consider the banking sector. The banking sector can play a crucial role for the development of house prices. It is often argued that moral hazard behavior of the Japanese banks was one of the reasons for the development of the housing bubble in the late 1980s in Japan.<sup>25</sup>

## 5 Do Prices go Back to Their Fundamental Values?

As shown in the previous section, the observed market prices can deviate substantially and for long periods from their fundamental values. Given the extent and persistence of these deviations, it is natural to ask if the fundamental and the actual prices are really linked or if they evolve independently? To answer this question, we firstly test if a long-run cointegration relationship exists between these two variables and, secondly, we study the dynamics of the gap between the observed and the fundamental prices by estimating its impulse-response function.

### 5.1 Cointegration Tests

One way to verify whether two economic variables are linked through a long-run equilibrium is to check if they are cointegrated (Engle and Granger, 1987). In our case, we are interested in checking if in the long-run the observed price  $p_t$  is equal to the fundamental value  $p_t^\bullet$  (rent model) or  $p_t^*$  (S/D model). Thus, in our particular case, the hypothetical long-run equilibrium given by the theory is  $p_t = p_t^\bullet$  (or  $p_t = p_t^*$ ). We have to check if this long-run equilibrium is supported by the data - i.e. if the actual and the fundamental price are cointegrated.

When the long-run equilibrium is known from the theory, Hamilton (1994) suggests the following procedure to test for cointegration:

1. Test if both  $p_t^\bullet$  (or  $p_t^*$ ) and  $p_t$  are I(1) (integrated of order one or non-stationary),
2. test if their linear combination  $\Psi_t^\bullet = p_t - p_t^\bullet$  (or  $\Psi_t^* = p_t - p_t^*$ ) is I(0) (integrated of order zero or stationary).<sup>26</sup>

If these two conditions are met, we conclude that the observed price and the fundamental price are linked by a long term equilibrium.

The stationarity of the different series are tested with the ADF test (Dickey and Fuller, 1979), the PP test (Phillips and Perron, 1988) and the KPSS test (Kwiatkowski, Phillips, Schmidt and Shin, 1992). The null hypothesis of the ADF and the PP test is that the tested variable is non-stationary. The null hypothesis for the KPSS test is that the variable is stationary. The results of the tests are presented in Table 2.

[Insert Table 2 about here]

For the observed and the fundamental prices, the results of the ADF and the PP test show that in each country the null hypothesis of a unit root cannot be rejected (at a 1% level for the U.S.A and at a 5% level for the other countries). The KPSS tests show that the null hypothesis of stationarity can be rejected at a 1% level for all prices in Japan and Switzerland, for  $p_t$  and  $p_t^*$  in the USA, and for  $p_t$  in the UK. Overall, for each price at least two out of three tests indicate that the series is I(1). In two thirds of the cases, all three tests conclude that the series is I(1). Given these results, we consider that prices to be I(1) and the first condition to be fulfilled.

For the second condition we have to check whether the gaps are stationary. The two last columns in Table 2 display the results for the different tests. The ADF and

the PP tests do not reject the hypothesis that the gaps are I(1). Thus, according to these tests, the gaps are non-stationary and no long-run equilibrium links the observed and the fundamental prices. However, at a 1% confidence level all KPSS tests do not reject the null hypothesis that gaps are stationary and thus support the existence of a long-run equilibrium. These results are contradictory and, at a 1% confidence interval level, it seems that both I(1) and I(0) processes exist that are able to describe the observed gaps adequately. These results are the same for both the rent and the S/D model. According to these tests it is difficult, from a statistical point of view, to give a definitive answer to the question of whether or not long-run relation between the observed prices and the fundamental values exists.

## 5.2 Impulse-response Functions

As suggested by Hamilton (1994, p. 586), another way of studying the long-run relation between the observed and the fundamental prices is to calculate if a deviation from the fundamental value is likely to persist and, if not, how long it will take to disappear. This can be done by estimating the dynamic of the gaps and checking if this dynamic leads the price back to the fundamental value after a shock. Finally, by computing the impulse-response functions we can get an idea of how long it takes the price to return to the fundamental value.

The dynamic of the gap is estimated by:

$$\Psi_t = \mu + \beta_1 \Psi_{t-1} + \dots + \beta_l \Psi_{t-l} + \varepsilon_t, \quad (28)$$

where the number of lags  $l$  is chosen with the Akaike information criterion. This process is stable if its unit roots are outside the unit circle. In this case, the gap will tend to disappear after a shock, otherwise the gap will remain or increase. Figure 2 displays

the *inverse* unit roots for the different countries. If the inverse roots are inside the unit circle, the estimated process is stable. With the exception of the rent model for the USA, all the inverse roots are inside the unit circle. This means that the gaps will tend to go back to zero after a shock. Note that for each gap, some roots are very close to the boundary of the circle. This can explain why the test presented in the previous section cannot statistically distinguish them from a unit root process.

[Insert Figure 2 about here]

We can now estimate how fast a gap will disappear after a shock by computing the impulse-response functions. These functions are presented in Figure 3. We see that the gaps eventually disappear but they decrease very slowly and can persist for years after the initial shock. This illustrates clearly that, if there is a link between the observed price and the fundamental values, this link only shows up in the very long run and that the short run dynamic of the price can appear to be disconnected from the fundamental price.

[Insert Figure 3 about here]

## **6 Do Fundamental Prices Help to Forecast Future Prices?**

We now study how useful the fundamental price is to forecast future actual prices. Apart from its obvious practical applications, the forecast ability of the fundamental price is also another way to study the link between the price and its fundamental

value. The previous section shows that the one-period price dynamic is very close to be independent from the fundamental price. So close that statistical tests do not unilaterally conclude on the presence (or the absence) of a link between the price and its fundamental value. The forecast ability of the fundamental price shed another light on this link. It has the advantage to study the long run dynamic of the price instead of concentrating on the one-period dynamic only. If the fundamental price is able to give good out-of-sample forecasts for long horizons, we can interpret this as a significant sign in favour of the presence of a link between the price and its fundamental value.

We use our fundamental prices in different ways to forecast the future price and we check which one is the most accurate. The different forecasting models are presented in the next section. For all models, we perform out-of-sample forecasts, which means that we only use the information available at time  $t$  to make a forecast for  $t + h$ . In doing so, we put ourselves in the same position as an agent would be at time  $t$  to make his forecasts. We also vary the forecast horizon to see how the precision of the different models evolve.

## 6.1 Forecasting Models

We compare five different models:

1. **Benchmark model:** We forecast the price by using the observed price dynamic only. For that, we first estimate the equation:

$$\Delta p_t = \beta_0 + \beta_1 \Delta p_{t-1} + \dots + \beta_k \Delta p_{t-k} + \varepsilon_t,$$

on the sample available at time  $t$ . The number of lags  $k$  is determined by the Akaike information criterion. The estimated dynamic is then used to forecast the



future price for different horizons.

2. **Fundamental price model:** We make the assumption that the future price will be equal to the future fundamental price ( $E_t(p_{t+h}) = E_t(p_{t+h}^\bullet)$  for the rent model and  $E_t(p_{t+h}) = E_t(p_{t+h}^*)$  for the S/D model). Then we then forecast the fundamental price by using its observed dynamic in the same way as for the price forecast in the benchmark model.
3. **Fundamental dynamic model:** We assume that the future price will be equal to the future fundamental price plus the future gap ( $E_t(p_{t+h}) = E_t(p_{t+h}^\bullet + \Psi_{t+h}^\bullet)$  or  $E_t(p_{t+h}) = E_t(p_{t+h}^* + \Psi_{t+h}^*)$ ). For that, we forecast the fundamental price as in the fundamental price model. In addition, we forecast the future gap using its estimated one-period dynamic.
4. **Gap model:** We study whether the gap observed at time  $t$  contains some information about the price at time  $t + h$ . For this, we estimate the equation:

$$p_t - p_{t-h} = \beta_0 + \beta_1 \Psi_{t-h},$$

with the sample available at time  $t$  and then use the estimated regression to forecast the price at time  $t + h$ .

5. **Gap and price dynamic model:** We study if the gap observed at time  $t$  can help to explain the forecast error made by the benchmark model. For that, we estimate:

$$e_t = \beta_0^* + \beta_1^* \Psi_{t-h},$$

where  $e_t$  is the error made at time  $t - h$  by the benchmark model for a forecast at the horizon  $h$ . The forecast with this model is equal to benchmark forecast

corrected by the prediction of this equation.

## 6.2 Results

Figures 4 and 5 present the results of the different models for the rent and the S/D models respectively. For each model and for each horizon, we have computed the out-of-sample mean absolute error (MAE). The MAE of each model is then divided by the MAE of the benchmark to compare their performance. If the line for a model is under (over) one, then the model makes, on average, a smaller (bigger) forecast error than the benchmark.

[Insert Figure 4 about here]

[Insert Figure 5 about here]

In the short-run, we see that the benchmark, which is based on the one period price dynamic, performs relatively well. It is the best in almost each country for forecasts up to one year. The information contained in the fundamental price starts to be relevant for forecasts of 2 to 3 years. Starting from this horizon and up to an horizon of 3 to 4 years, the best models are those which combines which combines the information contained in the actual price with the one contained in the gap. For horizons longer than that, the fundamental model forecasts are better than the other options. It seems that, for long horizons, the information contained in the gap do not compensate for the noisier forecasts made with the price dynamic. Thus, for an horizon greater than 3 to 4 years, it seems better to rely on models only using the fundamental price. The only major exception to this conclusion is the Netherlands for which no model can

beat the benchmark. For this country, our fundamental price seems to be completely disconnected from the observed price.

Note that the "fundamental dynamic" model, which uses the one-period dynamic of the gap, is almost always outperformed by models using the value of the gap at time  $t$  ("gap" and "gap and price dynamic" models). It seems that the one-period dynamic of the gap is difficult to predict or too persistent (as suggested by impulse-response function) and that it just adds noise to the benchmark model. In contrast, the level of the gap seems to be a good predictor for the long-run price dynamic. Even a model as simple as the "gap" model outperforms the benchmark in every country for long horizons. Long-term forecasts should therefore take this information into account.<sup>27</sup>

## 7 Conclusion

We have developed two models which enable us to calculate the fundamental value of houses: the rent model and the S/D model. Both estimated models provide very similar results. The SD model seems to have a slightly better fit to the actual prices. As we have shown, there are often substantial and long deviations of the actual house prices from their fundamental values. According to both models, house prices are currently overvalued in the United States, the United Kingdom, and the Netherlands.

We find that the estimated one-period dynamic of the gap between the fundamental and the actual prices is stable but very close to a unit root process. A first consequence of this feature is that unit root tests cannot statistically identify or reject the existence of a long-run link between both prices. A second consequence is that, as the impulse-response functions suggest, the gap decreases very slowly and thus the possible link between the actual and the fundamental price appears only in the long run. Indeed, according to the impulse-response functions, it can take years for the price to go back

to its fundamental value after a shock.

The fundamental prices developed in this paper are also useful forecasting tools. We show, that for horizons longer than 3 or 4 years, forecasting models based on fundamental prices systematically outperform those based on the price dynamic. For horizons between 2 years and 4 years, models that combine the information given by the fundamental price with the actual price are the most accurate. For shorter horizons, forecasting models based on the price dynamic give better results. The fact the models based on the fundamental price give better forecasts for longer horizons is also an evidence of the long-run link between fundamental and actual prices.

There are several explanations for the different development of the actual and the fundamental house prices. One is that there are important fundamental factors that we have not considered. Though, in our view it would be more fruitful to explain the gap by looking at factors others than fundamentals, for example the expectations of the house buyers.

## A Data

In the following we present the data that is used for the estimation of the fundamental house price in the five different countries.

**House Price ( $P_t$ ):** The BIS provides data on residential property prices for all five countries. Thereby, the original sources of the data are country specific. The USA data, for example, is the house price index of OFHEO and for CH the index is provided by Wüest und Partner. In CH it is the price per house (all one family houses), in NL the price per dwelling (existing dwellings), in the UK the price per dwelling (all dwellings), in USA the price per dwelling (existing single family houses) and in JAP the residential land price. All series are transformed into real terms by using the corresponding CPIs. The time horizon of the different series varies between the countries: They start between 1957 Q1 and 1976 Q1 and end between 2005 Q1 and 2006 Q1.

**Rent ( $Q_t$ ):** The Main Economic Indicators of the OECD provide CPI data on rents for all five countries under consideration. To transform the series into real terms, we use the overall CPI (see  $CPI_t$ ). To adjust them seasonally, we use the annual growth rates. The data series start between 1960 Q1 and 1983 Q1 and all end in 2005 Q4.

**Mortgage Rate ( $m_t$ ):** Banks offer a variety of different mortgage products where the main difference lies in the maturity. The popularity of different mortgage products differs between countries. For example in the UK most of the mortgages have a variable mortgage rate. On the other hand, in the USA, most mortgages are long term fixed. (See Miles, 2005, provides an overview.) The BIS is providing relevant mortgage rates for CH, the NL, the UK and USA. For example for CH it is the average variable rate and for USA it is the average rate conventional mortgages. For JAP we use the

lending rates from the International Financial Statistics of the IMF. The data series start between 1957 Q1 and 1984 Q1 and end in 2005 Q3 or Q4.

**Aggregated Income ( $Y_t$ ):** As an indicator for the development of the aggregated income of a country, we use GDP. An alternative variable for income would be the personal disposable income. Since the correlation between GDP and disposable income is very high (in the USA well above 0.99) we decided to use the easy accessible variable GDP. For the NL, USA and the UK we use data from the Main Economic Indicators of the OECD. For CH we extend the OECD data by using a GDP series provided by the Swiss National Bank. For JAP we use GDP Volume index from the International Financial Statistics of the IMF. The data series start between 1948 Q1 and 1977 Q1 and end in 2005 Q4.

**Construction ( $B_t$ ):** The OECD provides construction data for all five countries. For the NL the OECD provides nominal data on construction permits. To convert them into real terms we use BIS data on construction costs. The CH series is extended by a residential construction series from the BIS. We adjust the series for seasonal effects via their annual growth rates.

**CPI ( $CPI_t$ ):** Data on CPI is available in the International Financial Statistics of the IMF from 1957 Q1 until 2006 Q1 on a quarterly basis. For the UK we use CPI: all items, for the USA the CPI: all items city average, for CH the CPI: all country, for JAP the CPI: all Japan-485 items and for the NL the CPI: Wage Earners. We adjust them seasonally by using the annual growth rates.

## B First-order Taylor Approximation

Campbell, Lo and McKinlay (1997) show that it is possible to approximate the logarithm of a sum by a sum of logarithms. First consider:

$$\ln(A + B) = \ln A \left( 1 + \frac{B}{A} \right) = a + \ln(1 + e^{b-a}). \quad (29)$$

The second part of this equation can be approximated by a standard Taylor approximation around its mean. Define  $x = b - a$  and  $f(x) = \ln(1 + e^x)$ . The Taylor approximation yields  $f(x) \approx f(\bar{x}) + f'(\bar{x})(x - \bar{x})$  with  $f'(\bar{x}) = e^{\bar{x}} / (1 + e^{\bar{x}})$ . Plugging this into equation (29) gives the final result:

$$\ln(A + B) \simeq \kappa + (1 - \rho)b + \rho a, \quad (30)$$

with  $\rho = 1 / (1 + e^{\bar{x}})$  and  $\kappa = \ln \rho - (1 - \rho) \ln(1/\rho - 1)$ .

Using this Taylor approximation, we can linearize the price-to-imputed-rent ratio. Let us define  $A = \delta X_{t+1} (H_{t+1}/H_t)$  and  $B = 1$ , the price-to-imputed-rent ratio

$$X_t = \frac{\delta X_{t+1} (H_{t+1}/H_t) + 1}{R_t} \quad (31)$$

becomes

$$x_t \approx \kappa_1 + \rho (\ln \delta + x_{t+1} + \Delta h_{t+1}) - \ln R_t, \quad (32)$$

with  $\rho = 1 / (1 + \exp(-\ln \delta - \bar{x} - \Delta \bar{h}))$  and  $\kappa_1 = -\ln \rho - (1 - \rho) \ln(1/\rho - 1)$ .<sup>28</sup> In addition, when  $m_t$  and  $\phi$  are small, we can approximate  $\ln R_t = \ln(1 + \phi + m_t)$  by

$\phi + m_t$ . This leads to:

$$x_t \approx \kappa_1 + \rho (\ln \delta + x_{t+1} + \Delta h_{t+1}) - m_t - \phi. \quad (33)$$

Similarly, by defining  $A = S_{t-1}$  and  $B = B_{t-1}$ , the supply

$$S_t = \delta S_{t-1} + B_{t-1} \quad (34)$$

becomes

$$s_t \approx \kappa_2 + \theta s_{t-1} + (1 - \theta) b_{t-1}, \quad (35)$$

where  $\theta = 1 / (1 + \exp(\bar{b} - \ln \delta - \bar{s}))$  and  $\kappa_2 = -\ln \theta - (1 - \theta) \ln(1/\theta - 1)$ .

## C Econometric Methodology

To recapitulate the results of section 2.4, the computation of the fundamental prices given by the rent model in equation (26) requires: 1) the linearization parameter  $\rho$ , 2) the VAR coefficients in  $A$  and 3) the constant  $c$ . In addition to this, the S/D model in equation (27) needs 4) the supply series  $s_t$ , 5) the linearization parameter  $\theta$  and 6) and the constant  $c^*$ . The VAR coefficients can be easily estimated with traditional methods. The next sections describe how to get the other elements.

### C.1 Linearization Parameter $\rho$

Recall from Appendix B that the parameter  $\rho$  is defined as

$$\rho = \frac{1}{1 + \exp(-\ln \delta - \bar{x} - \Delta \bar{h})}.$$



where  $\bar{x} = \bar{p} - \bar{h}$ . If both the price and the imputed rent are observable, then it is straightforward to compute  $\rho$ . Unfortunately, in our case, the imputed rents and the prices are not expressed in the same units. Thus, we have that  $\bar{x} = \bar{p} - \bar{h} - \ln \omega$ , where  $\omega$  is the conversion factor of the imputed rent units in terms of price units. This conversion factor is unobservable. To cope with this problem, we estimate  $\rho$  with an OLS regression of equation (33). For example, with the Rent model, we estimate the equation:

$$\hat{x}_t = a + b(\hat{x}_{t+1} + \Delta h_{t+1}) - m_t + \varepsilon_t.$$

The parameter  $a$  gives us an estimation of  $\kappa_1 + \ln \delta - \ln \omega - \phi$  and the parameter  $b$  is our estimation of  $\rho$ .

## C.2 Constants $c$ and $c^*$

To estimate  $c$  and  $c^*$ , we assume that, on average *over time*, the observed log price-to-imputed-rent ratio  $\hat{x}_t$  is equal to the fundamental price  $x_t^\bullet$  for the rent model ( $E(\hat{x}_t) = E(x_t^\bullet)$ ) and that  $\tilde{x}_t$  is equal to  $x_t^*$  for the S/D model ( $E(\tilde{x}_t) = E(x_t^*)$ ).

Taking the unconditional expectation of equations (26) and (27) implies that:

$$c = E(\hat{x}_t) - (\rho g_1 \Phi (I - \rho \Phi)^{-1} + g_2) E(z_t^\bullet) \quad (36)$$

and

$$c^* = E(\tilde{x}_t) - (\rho g_1^* \Phi (I - \rho \Phi)^{-1} + g_2^*) E(z_t^*), \quad (37)$$

respectively.

### C.3 Supply and Linearization Parameter $\theta$

As mentioned in section 2.4.1, the computation of the supply-demand model requires to first estimate the housing supply  $s_t$ , which is not directly observable. Note that the entire series of  $s_t$  can be recovered if the initial  $s_0$  is known. To see how, first note that equation (21) states that  $s_t$  is a function<sup>29</sup>  $f_1$  of  $s_0$  and  $\theta$

$$s_t = \theta^t s_0 + \sum_{j=0}^{t-1} \theta^{t-j-1} (\kappa_2(\theta) + (1-\theta) b_{t+j}). \quad (38)$$

Furthermore, we know<sup>30</sup> that  $\theta$  is a function of the average  $\bar{b} - \bar{s}$

$$\theta = 1 / \left( 1 + \exp \left( \bar{b} - \frac{1}{T} \sum_{t=1}^T s_t - \ln \delta \right) \right). \quad (39)$$

By substitution of  $s_t$ , we have that

$$\theta = \left( 1 + \frac{1}{\delta} \exp \left( \bar{b} - \frac{1}{T} s_0 \sum_{t=1}^T \theta^t - \frac{1}{T} \sum_{t=1}^T \sum_{j=0}^{t-1} \theta^{t-j-1} (\kappa_2(\theta) + (1-\theta) b_{t+j}) \right) \right)^{-1} \quad (40)$$

This equation can be numerically solved for  $\theta$  once the initial supply is known. Thus, the only unknown in this problem is the initial housing supply. Once  $s_0$  is known,  $\theta$  and the entire series of  $s_t$  can be recovered with equation (19).

We can determine  $s_0$  by imposing one additional condition: we impose that the average one-period imputed rent growth rate must be equal to the average renting price growth rate. This is equivalent to saying that, in the long term, the cost of living in a house must grow at the same rate as the cost of living in a rented flat. If this condition is violated one of these two alternatives will be infinitely more costly than

the other. This condition, together with equation (17) yields  $\overline{\Delta s_t} = \overline{\Delta y_t} - \overline{\Delta q_t}$  where  $\overline{\Delta q_t}$  is the average renting price growth rate. As  $\overline{\Delta s_t}$  is also a function of  $s_0$ , it is also possible to compute the  $s_0$  which gives the adequate initial condition. Then, starting from  $s_0$ , the entire series housing supplies can be recovered and used to compute  $x_t^*$ .

## Notes

<sup>1</sup>Beside these exposures Hilbers et al. (2001, p. 6) point out that banks are "financing [...] real estate developers and construction companies, lending to nonbank intermediaries, such as finance companies, that engage in real estate lending [and] relying on real estate to collateralize other kinds of lending."

<sup>2</sup>This relationship is also emphasized by Herring and Wachter (1999).

<sup>3</sup>This is emphasized by Case and Shiller (2003) and Flood and Hodrick (1990).

<sup>4</sup>See also Himmelberg et al. (2005). Case and Shiller (2003) use these indicators in their work on bubbles in the housing market.

<sup>5</sup>For example Himmelberg et al. (2005), OECD (2005) or Weeken (2004).

<sup>6</sup>See e.g. Himmelberger et al. (2005), OECD (2005) or Dougherty and van Order (1982).

<sup>7</sup>Ayuso and Restoy (2006) construct an intertemporal asset pricing model for the price of houses. Though, they do not explicitly model the user costs of housing and do not calculate a fundamental value of rents.

<sup>8</sup>Following Poterba (1984) or Flavin and Yamashita (2002) the tax deductibility of mortgage payments has two effects on the user costs: it reduces the relevant mortgage rate and (because nominal mortgage payments are deductible) inflation rates become relevant. Though, altogether the introduction of the tax deductibility has mainly a level effect on fundamental house prices. If we assume that marginal tax rates are constant over time, the level effect would be completely offset by a change of the parameters of our estimation. But there can be a bigger effect on the development of the house price if there is a change in the fiscal regime. In the Netherlands, for example, the tax deductibility has been limited in recent years to reduce house-price inflation. When we look at the development of the house prices in the Netherlands this attempt was successful. But from a theoretical point of view a limitation of the tax deductibility would have the same effect as a sudden increase of mortgage rate on a new level: House prices should fall. Therefore, the consideration of the tax deductibility of mortgage payments do not help to explain the current level of house prices in the Netherlands.

<sup>9</sup>Poterba (1984, p. 738) also mentions these difficulties.

<sup>10</sup>This formula basically reflects the returns on holding residential real estate in Cho (1996).

<sup>11</sup>Here, we assume that  $\lim_{i \rightarrow \infty} [\delta^i H_{t+i}] / \left[ \prod_{j=0}^i R_{t+j} \right] = 0$ . Therefore, we rule out the possibility of a rational bubble in the housing market.

<sup>12</sup>In Pain and Westaway (1997) and Schwab (1982) utility depends on the same variables.

<sup>13</sup>See also section 3.

<sup>14</sup>Literature offers a range of fundamentals to determine the demand for housing: Case and Shiller (2003), for example consider personal income per capita, population, employment and the unemployment rate. Among other factors Holly and Jones (1997) also consider real income and a demographic factor. Both papers conclude that income is the most important factor. Sendhadji (2002) point out real GDP as an important fundamental because it is a measure for the aggregated level of income per capita and population. Therefore, our model is in line with this literature.

<sup>15</sup>Generally, one could also consider a positive relationship between construction and property prices. In that case, construction would develop endogenous. Since data on construction is available directly, we will treat construction as an exogenous variable.

<sup>16</sup>The imputed rent is not stationary because in the rent and the S/D model, it is defined as a linear function of non-stationary variables, i.e the rents or the GDP, respectively.

<sup>17</sup>Cf. Appendix B for the details on the first order Taylor approximation and the value of the linearisation parameters.

<sup>18</sup>In the rest of this paper, we use small letters to indicate the logarithm of a variable.

<sup>19</sup>See Appendix B for the details on the first order Taylor approximation and the value of the linearisation parameters.

<sup>20</sup>We have that  $\varphi = \frac{1-\rho+\rho\theta-\rho\theta^2}{1-\rho\theta}$ ,  $\lambda = \frac{(1-\theta)(1-\rho)}{1-\rho\theta}$ , and  $c^* = c - \frac{\theta(1-\rho)\rho\kappa_2}{(1-\rho\theta)(1-\rho)} + \ln \alpha$ . The term  $\ln \alpha$  in the constant comes from the definition of  $x'_t$ .

<sup>21</sup>Equation (23) is the companion form of a VAR with  $k$  lags (see Hamilton, 1994, p. 7 for more details).

<sup>22</sup>See Hilbers et al. (2001).

<sup>23</sup>The only exception is for the USA, where the S/D and the Rent model deviates substantially from each other.

<sup>24</sup>With the exception of the S/D model for the USA, which is more volatile than the actual price.

<sup>25</sup>For example Krugman (1998).

<sup>26</sup> $\Psi_t$  corresponds to the gap between the price and its fundamental value.

<sup>27</sup>There are two exceptions to this conclusion: the rent models for Switzerland and for Japan. In these two cases the fundamental dynamic model beat the benchmark and the other models for long forecast horizons.

<sup>28</sup> Note that if  $P_t$  and  $H_t$  are not expressed in the same units (e.g. when they are both indexes), the parameter  $\rho$  is equal to  $\rho = 1 / (1 + \exp(\ln \delta - \bar{x} - \ln \omega_1 - \Delta \bar{h}))$  where  $\omega_1$  is the conversion factor between  $P_t$  and  $H_t$ .

<sup>29</sup>The series of  $b_t$  is observable and  $\kappa_2$  is known if  $\theta$  and  $\delta$  are known. We assume that  $\delta$  is equal to 1% per year, which corresponds to the value usually used by banks.

<sup>30</sup>See Appendix B.

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TABLE 1: OVERVALUATION OF THE DIFFERENT HOUSING MARKETS

	Rent model		S/D model	
	Date	Overvaluation	Date	Overvaluation
United States	2006 Q2	44.1%	2005 Q1	6.4%
United Kingdom	2006 Q2	28.1%	2004 Q4	52.8%
Japan	2005 Q3	-28.2%	2005 Q1	-36.6%
Switzerland	2006 Q2	-21.7%	2004 Q4	-28.0%
Netherlands	2006 Q2	64.6%	2005 Q1	62.8%

TABLE 2: UNIT ROOT TESTS

Country	Test	$p_t$	$p_t^\bullet$	$p_t^*$	$\Psi_t^\bullet$	$\Psi_t^*$
United States	ADF	1.5071	-3.0779*	-2.2342	0.1526	-0.8395
	PP	2.4778	-2.8894	-2.2719	2.7132	-1.0821
	KPSS	0.2122*	0.1432	0.2076*	0.4548	0.7291*
United Kingdom	ADF	-1.0649	-0.7475	-0.7864	-2.1679*	-1.0409
	PP	0.1130	-0.6355	-0.8071	-0.8170	-0.6134
	KPSS	0.2178**	0.1153	0.1379	0.2414	0.2102
Japan	ADF	-2.4787	-1.4116	-2.1710	-0.8654	-1.6411
	PP	-3.0120	-1.6797	-1.8318	-0.5462	-1.4461
	KPSS	0.3266**	0.1843*	0.1797*	0.3915*	0.6960*
Switzerland	ADF	-2.3891	-0.1174	0.6572	-1.3462	-1.1881
	PP	-1.8498	-0.1174	0.0970	-1.0728	-0.8104
	KPSS	0.2117*	0.3027**	0.2576**	0.5131*	0.6322*
Netherlands	ADF	-0.6137	-0.3198	-1.2311	-1.6667	-1.5447
	PP	0.2760	0.0384	-0.5953	-0.3862	-0.9968
	KPSS	0.2960**	0.1186	0.1359	0.4687*	0.3897

\* (\*\*) denotes a rejection of the null hypothesis at 5% (1%) level. Trend included for test on prices, but not for gaps.

FIGURE 1: OBSERVED PRICE VS. FUNDAMENTAL PRICE

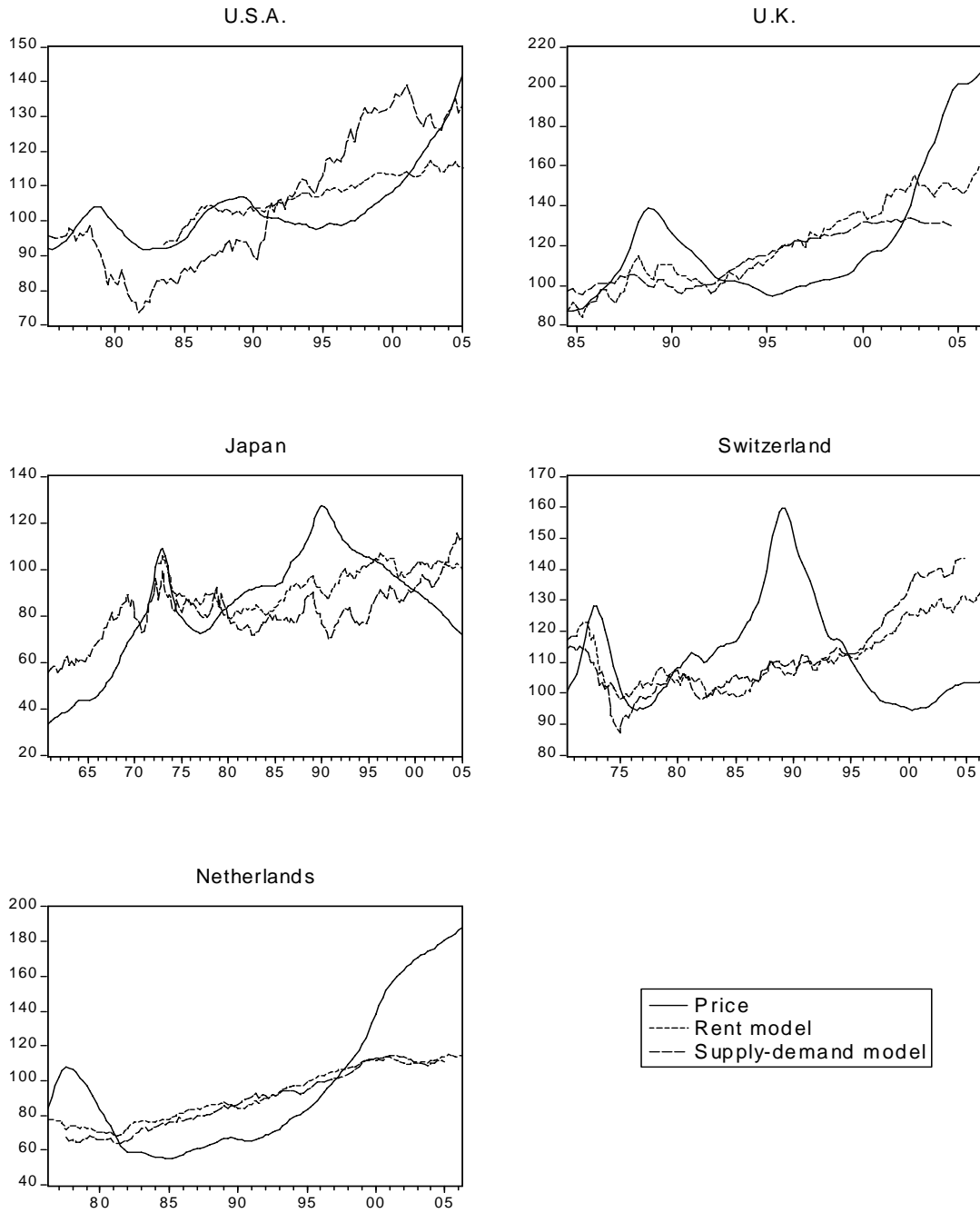


FIGURE 2: INVERSE UNIT ROOTS OF THE GAP DYNAMICS

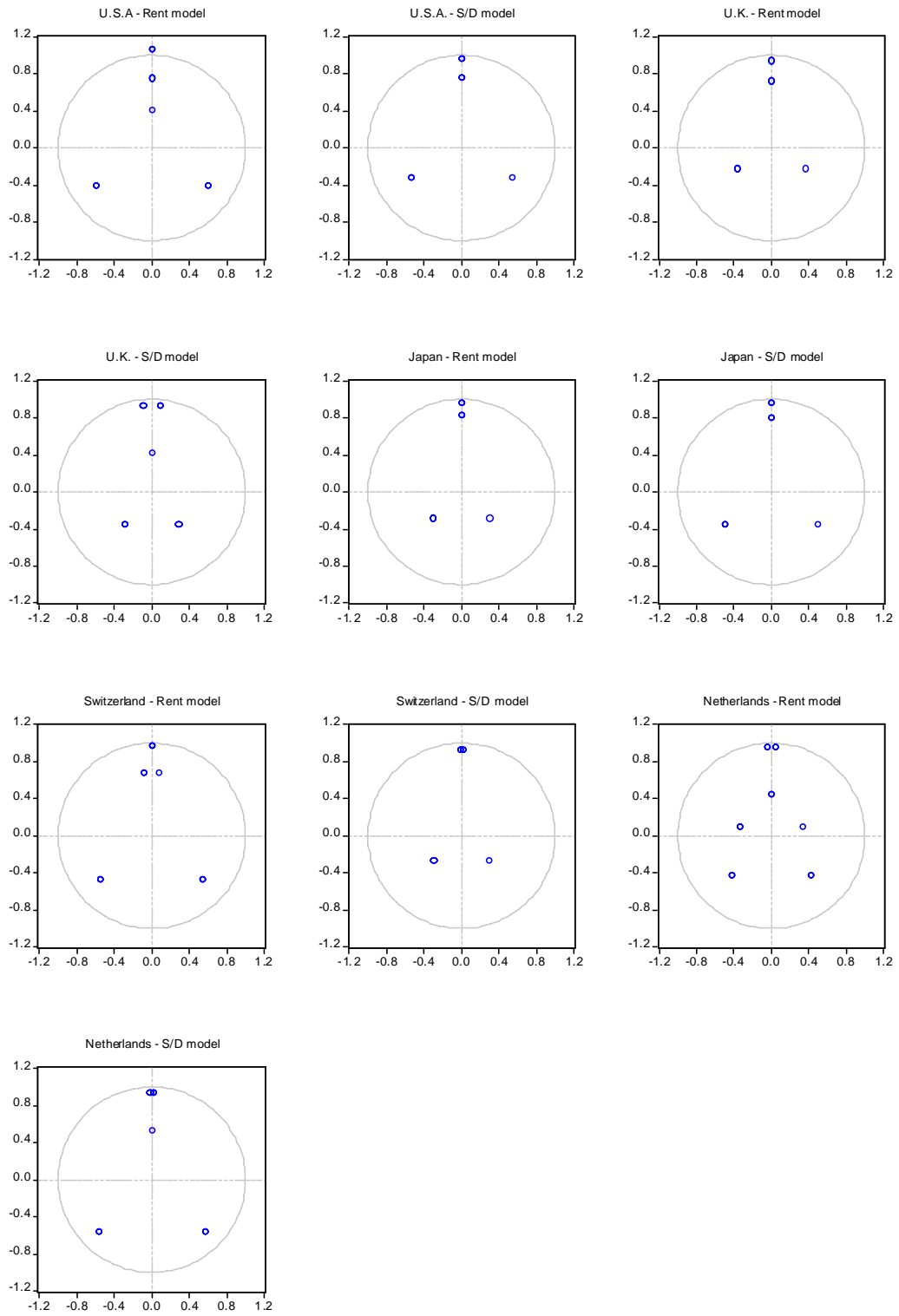


FIGURE 3: IMPULSE-RESPONSE FUNCTIONS OF THE GAPS

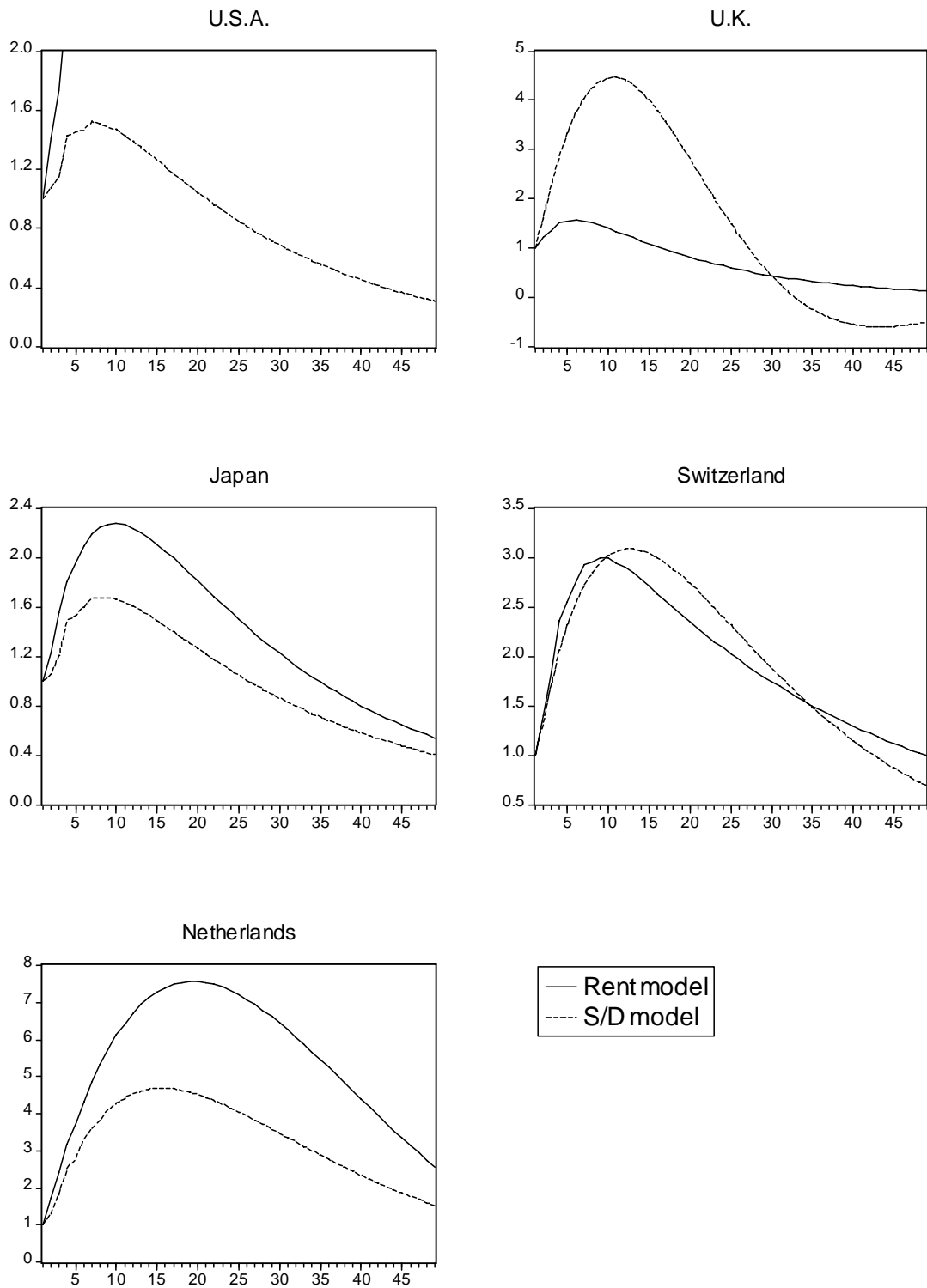


FIGURE 4: COMPARISON OF THE FORECAST PRECISIONS (RENT MODEL)

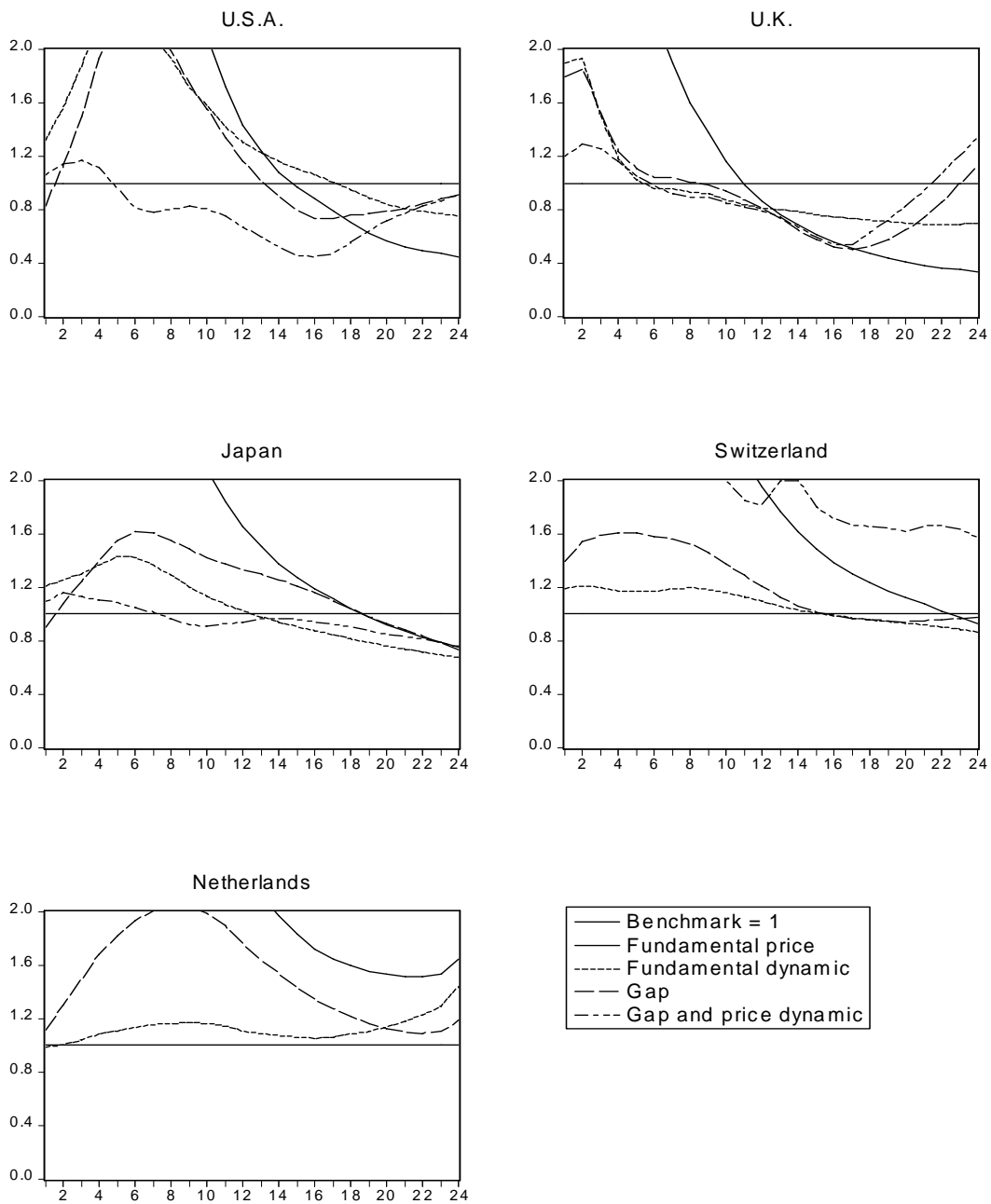


FIGURE 5: COMPARISON OF FORECAST PRECISION (S/D MODEL)

