

Fundamentals of Astrodynamics and Applications

Third Edition

David A. Vallado

with technical contributions by

Wayne D. McClain



Space Technology Library

Published Jointly by

Microcosm Press

Hawthorne, CA



Springer

New York, NY

Table of Contents

Chapter 1	Equations of Motion	1
1.1	History	1
1.1.1	Ancient Era	1
1.1.2	The Copernican Revolution	7
1.1.3	Kepler's Laws	9
1.1.4	Newton's Laws	10
1.1.5	Other Early Astrodynamical Contributions	12
1.2	Geometry of Conic Sections	12
1.2.1	Basic Parameters	12
1.3	Two-Body Equation	20
1.3.1	Assumptions for the Two-Body Equation	23
1.3.2	Specific Angular Momentum	23
1.3.3	Specific Mechanical Energy	25
1.3.4	Kepler's First Law (Trajectory Equation)	27
1.3.5	Kepler's Second and Third Laws	29
1.3.6	Velocity Formulas	32
1.4	Three-body and n -body Equations	33
1.4.1	Inertial, Relative, and Barycentric Formulas	33
1.4.2	Ten Known Integrals	37
1.4.3	General Three-Body Problem	40
Chapter 2	Kepler's Equation and Kepler's Problem	49
2.1	Historical Background	49
2.2	Kepler's Equation	51
2.2.1	Alternate Formulation for Eccentric Anomaly	57
2.2.2	Formulation for the Parabolic Anomaly	58
2.2.3	Formulation for the Hyperbolic Anomaly	60
2.2.4	Universal Formulation	66
2.2.5	Solutions of Kepler's Equation	71
2.2.6	Summary and Related Formulas	85
2.3	Kepler's Problem	87
2.3.1	Solution Techniques	88
2.4	Satellite State Representations	103
2.4.1	Classical Orbital Elements (Keplerian)	104
2.4.2	Two-line Element Sets	113
2.4.3	Other Element Sets	116
2.4.4	Canonical Elements	118
2.5	Application: Orbital Elements from r and v	119
2.6	Application: r and v from Orbital Elements	124
2.7	Application: Groundtracks	128
2.8	Application: Find Time of Flight (FINDTOF)	131
Chapter 3	Coordinate and Time Systems	137
3.1	Historical Background	137
3.2	The Earth	139
3.2.1	Location Parameters	141
3.2.2	Shape of the Earth	142
3.2.3	Gravitational Model	149

3.3	Coordinate Systems	153
3.3.1	Interplanetary Systems	158
3.3.2	Earth-based Systems	159
3.3.3	Satellite-based Systems	162
3.4	Coordinate Transformations	166
3.4.1	Coordinate Rotation	167
3.4.2	Rotating Transformations	173
3.4.3	Common Transformations	174
3.4.4	Application: Converting IJK (ECEF) to Latitude and Longitude	176
3.5	Time	181
3.5.1	Solar Time and Universal Time	184
3.5.2	Sidereal Time	191
3.5.3	Atomic Time	196
3.5.4	Dynamical Time	197
3.5.5	Coordinate Time	200
3.5.6	Conversions	201
3.6	Time Conversions	203
3.6.1	DMS to Rad / Rad to DMS	203
3.6.2	HMS to Rad / Rad to HMS	204
3.6.3	HMS to Time of Day/ Time of Day to HMS	205
3.6.4	YMD to Day of Year/ Day of Year to YMD	206
3.6.5	YMDHMS to Days / Days to YMDHMS	207
3.6.6	Julian Date to Gregorian Date	208
3.7	Transforming Celestial and Terrestrial Coordinates	209
3.7.1	IAU-2000 Resolutions	217
3.7.2	Velocity Transformations	227
3.7.3	IAU-76 / FK5 Reduction	228
3.7.4	Other Reductions	236
3.7.5	Computational Considerations	240
3.8	Earth Models and Constants	242
3.8.1	Canonical Units	242
Chapter 4	Observations	247
4.1	Introduction	247
4.2	Obtaining Data	248
4.2.1	Quantity of Data	251
4.2.2	Types of Data	252
4.2.3	Example Applications	253
4.3	Introduction to Sensor Systems	254
4.4	Observation Transformations	258
4.4.1	Geocentric Right Ascension and Declination	260
4.4.2	Topocentric Right Ascension and Declination	263
4.4.3	Azimuth-Elevation	265
4.4.4	Practical Az-El Conversions	270
4.4.5	Transformations for Ecliptic Latitude and Longitude	272
4.4.6	Practical Aspects of Surveillance	274
Chapter 5	Celestial Phenomena	279
5.1	Solar Phenomena	279
5.1.1	Application: Sun Position Vector	279
5.1.2	Application: Sunrise, Sunset, and Twilight Times	283

5.2	Lunar Phenomena	287
5.2.1	Application: Moon Position Vector	288
5.2.2	Application: Moon Rise and Set Times	292
5.2.3	Phases of the Moon	296
5.3	Celestial Applications	296
5.3.1	Application: Planetary Ephemerides	297
5.3.2	Eclipses	300
5.3.3	Application: Sight and Light	307
5.3.4	Ground Illumination	311
5.3.5	Miscellaneous Phenomena	313
Chapter 6	Orbital Maneuvering	319
6.1	Historical Background	319
6.2	Introduction	320
6.3	Coplanar Maneuvers	322
6.3.1	Hohmann and Bi-elliptic Transfers	324
6.3.2	Comparing Hohmann and Bi-elliptic Transfers	330
6.3.3	Transfers Using the One-Tangent Burn	333
6.3.4	General Transfers	339
6.4	Noncoplanar Transfers	339
6.4.1	Introduction	339
6.4.2	Inclination-Only Changes	344
6.4.3	Changes in the Right Ascension of the Ascending Node	347
6.4.4	Changes to Inclination and the Ascending Node	350
6.5	Combined Maneuvers	352
6.5.1	Minimum-Inclination Maneuvers	353
6.5.2	Fixed- Δv Maneuvering	356
6.6	Circular Rendezvous	360
6.6.1	Circular Coplanar Phasing	360
6.6.2	Circular Noncoplanar Phasing	367
6.7	Continuous-Thrust Transfers	373
6.7.1	Introduction	374
6.7.2	Orbit Raising	377
6.7.3	Low-Thrust, Noncoplanar Transfers	383
6.8	Relative Motion	389
6.8.1	Position Solutions for Nearly Circular Orbits	394
6.8.2	Trend Analysis	398
6.8.3	Accuracy Analysis	412
Chapter 7	Initial Orbit Determination	419
7.1	Historical Background	419
7.2	Observations of Range, Azimuth, and Elevation	422
7.2.1	Application: SITE-TRACK	422
7.3	Angles-only Observations	429
7.3.1	Laplace's Method	431
7.3.2	Gauss's Technique	435
7.3.3	Double r-iteration	439
7.4	Mixed Observations	445
7.4.1	Range and Range-Rate Processing	446
7.4.2	Range-only Processing	448

7.5	Three Position Vectors and Time	450
7.5.1	Gibbs Method	450
7.5.2	Herrick-Gibbs	457
7.6	Two Position Vectors and Time—Lambert’s Problem	464
7.6.1	Lambert—Minimum Energy	465
7.6.2	Lambert—Gauss’s Solution	472
7.6.3	Lambert’s Problem—Thorne’s Solution	476
7.6.4	Lambert—Universal Variables	485
7.6.5	Lambert Solution—Battin Method	490
7.7	Application: Targeting Problem	495
Chapter 8	Special Perturbation Techniques	515
8.1	Historical Background	515
8.2	Introduction to Perturbations	516
8.3	Encke’s Formulation	521
8.4	Cowell’s Formulation	523
8.5	Numerical Integration Methods	524
8.5.1	Implementing an Integrator and Determining Step Size	533
8.6	Disturbing Forces	536
8.6.1	Gravity Field of a Central Body	536
8.6.2	Atmospheric Drag	549
8.6.3	Third-Body Perturbations	571
8.6.4	Solar-Radiation Pressure	574
8.6.5	Other Perturbations	580
8.7	Forming Numerical Solutions	587
8.7.1	Application: Simplified Acceleration Model	589
8.7.2	Application: Complex Acceleration Model	592
8.8	Practical Considerations	595
8.8.1	Verifying and Validating the Propagator	596
8.8.2	Physical Data and Sources	597
Chapter 9	General Perturbation Techniques	605
9.1	Historical Background	605
9.2	Introduction	609
9.2.1	The Method of Perturbations	613
9.3	Variation of Parameters	615
9.3.1	Lagrangian VOP (Conservative Effects)	617
9.3.2	Gaussian VOP (Nonconservative and Conservative Effects)	624
9.4	Hamilton’s Formulation	633
9.5	Disturbing-Potential Formulations	637
9.5.1	Gravity Potential in Terms of the Satellite’s Orbital Elements	637
9.5.2	Third-Body Potential in Terms of the Satellite’s Orbital Elements	639
9.5.3	Tidal-Motion Potential in terms of the Satellite’s Orbital Elements	641
9.6	Linearized Perturbations and Effects	642
9.6.1	Central-Body Analysis	643
9.6.2	Drag Analysis	667
9.6.3	Third-Body Analysis	675
9.6.4	Solar-Radiation Analysis	681
9.7	Forming Analytical Solutions	685
9.7.1	Application: Perturbed Two-Body Propagation	686