Fundamentals of Condensation Heat Transfer: Laminar Film Condensation*

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The paper reviews basic theoretical studies of laminar film condensation since Nusselt (1916). Particular attention is given to those investigations in which the relative importance of the additional complicating factors is revealed and where approximate heat-transfer formulae are provided. For natural convection condensation on vertical plates and horizontal tubes, detailed solutions show that the simple Nusselt results (Eqs. (16) and) are surprisingly accurate over a wide range of conditions. Effects of inertia and convection in the condensate film and vapour shear stress at the condensate surface are generally of minor importance. Relatively simple, accurate, equations, which take account of these factors, are, however, available (Eqs. (30) and (32)). For forced convection condensation, detailed solutions for horizontal plates indicate that inertia and convection are generally unimportant for the normal practical ranges of the variables. A relatively simple result (Eq. (63)), equivalent to the Nusselt equation for natural convection, gives the heat transfer for this case with good accuracy. An interpolation formula (Eqs. (64) and (67)), which satisfies equations (16) and (63) for the natural and forced convection dominated extremes, gives accurate results for condensation in downflow over a vertical plate. For forced convection condensation on a horizontal tube, vapour boundary layer separation and condensate film instability resulting from opposing effects of vapour shear stress and pressure gradient over the downstream half of the tube, present significant difficulties. Approximate equations which either neglect these effects or make conservative allowances are available.

Nomenclature

c_P : isobaric specific heat capacity of condensate
d : tube diameter
F_d : defined in equation (57)
F_x : defined in equation (50)
F_L : defined in equation (51)
G : defined in equation (37)
g : local gravitational acceleration
$H = c_P \Delta T / h_{fg}$
h_{fg} : specific enthalpy of evaporation
h_{fg}^* : defined in equation (20)
$J = k \Delta T / \mu h_{fg}$
K : coefficient in equation (64)
K_1 : coefficient in equation (71)
K_2 : coefficient in equation (73)
k : thermal conductivity of condensate
L : length of plate

m : local condensation mass flux
Nu : mean Nusselt number for flat plate, uniform ΔT
Nu_d : mean Nusselt number for horizontal tube, uniform ΔT
Nu_{Nu} : mean Nusselt number for flat plate from Nusselt theory
$Nu_{Nu,d}$: mean Nusselt number for horizontal tube from Nusselt theory
$Nu_{Nu,x}$: local Nusselt number for flat plate from Nusselt theory
Nu_q : mean Nusselt number for flat plate, uniform q
$Nu_{q,d}$: mean Nusselt number for horizontal tube, uniform q
Nu_x : local Nusselt number for flat plate
P : pressure
P^* : defined in equation (79)
Pr : Prandtl number of condensate
q : heat flux
$\bar{q}$: mean heat flux for flat plate
$\bar{q}_d$: mean heat flux for horizontal tube
Re_{vx} : vapour Reynolds number, $U_\infty \rho_v x / \mu_v$

$\tilde{Re}_d$: "two-phase Reynolds number," $U_\infty \rho d/\mu$

$\tilde{Re}_x$: "two-phase Reynolds number," $U_\infty \rho x/\mu$

$\tilde{Re}_L$: "two phase Reynolds number," $U_\infty \rho L/\mu$

T : temperature

T_r : reference temperature

T_v : vapour temperature

T_w : wall temperature

T_δ : condensate surface temperature

U : x-wise vapour velocity

U' : x-wise vapour velocity at edge of vapour boundary layer

U_∞ : free-stream vapour velocity

u : x-wise condensate velocity

u_δ : x-wise velocity at condensate surface

V : y-wise vapour velocity

v : y-wise condensate velocity

X : defined in equation (70)

x : co-ordinate in streamwise direction along surface

y : co-ordinate normal to surface

z : distance vertically downward

β : coefficient in equation (84)

ΔT : temperature difference across condensate film

$\overline{\Delta T}$: mean temperature difference across condensate film, flat plate

$\overline{\Delta T_d}$: mean temperature difference across condensate film, horizontal tube

δ : condensate film thickness

θ : defined in equation (80)

μ : viscosity of condensate

μ_v : viscosity of vapour

ξ : defined in equation (76)

ρ : density of condensate

ρ_r : reference density of condensate

ρ_v : density of vapour

τ_δ : shear stress at condensate surface

$\varPhi$: angle from top of tube, (see Fig. 1)

$\varPhi_c$: see equation (78)

1. Introduction

Two-phase three-dimensional flows which occur in industrial condensers are extremely complex and involve effects of gravity, interaction between gas and liquid phases and inundation, i. e. condensate from higher or upstream surfaces impinging on lower or downstream surfaces. For profiled surfaces (e.g. finned tubes) surface tension effects are important. Condensate films may be either laminar or turbulent. The presence of non-condensing gases or more than one condensing constituent gives rise to additional complications. In order to develop sound design methods of general applicability, detailed understanding of the processes involved is required. This necessi-

tates prior consideration of simpler cases.

During condensation, temperature differences occur in general in both gas and liquid phases, as well as at the interface between the phases. Following Nusselt (1916), many investigations, aimed at determining the temperature drop across condensate films on wetted surfaces (film condensation) have been carried out. For the case where the condensate flow is laminar, and for both vertical plates and horizontal tubes, the thermal resistance associated with the condensate film can now be predicted very accurately for natural convection and reasonably well for forced convection conditions.

In many cases the temperature drop at the vapour-liquid interface (interfacial resistance) is negligible and equilibrium conditions can be assumed both for pure vapours and mixtures. A notable exception is the case of a liquid metal, particularly at low pressure, where the interface resistance may be many times greater than that of the condensate.

When the condensate does not wet the surface the phenomenon of dropwise condensation may occur. Attention was first drawn to this second ideal mode of condensation, and the associated high heat-transfer coefficients, by Schmidt et al. (1930). Although dropwise condensation is only observed with a few high-surface tension fluids (including water), a great deal of research effort has been put into this area over a period of almost 50 years. The theory of dropwise condensation has been developed to the extent that heat-transfer coefficients can be predicted with good accuracy. Unfortunately the dropwise mode of condensation cannot yet be reliably maintained for long periods under industrial conditions.

Considerable progress has been made towards the theoretical development of multi-constituent condensation and condensation in the presence of a noncondensing gas. Significant results have also been obtained in recent years for cases where surface tension effects are important, in particular, for condensation on low-finned tubes. Less well understood are turbulent film condensation, forced convection condensation inside tubes and condensation on tube banks.

The present paper deals only with laminar film condensation of pure vapours. Many studies of varying degrees of complexity, of the processes occurring in the condensate film, have been undertaken since the pioneering investigations of Nusselt (1916). It is convenient to review these in the light of the most comprehensive approaches, namely those of Koh et al. (1961, 62) and Gaddis (1979). These works cover the cases of natural and forced convection for both flat plate and horizontal tube. Forced convection, in this context, indicates that the vapour velocity is

sufficiently high that the effect of the associated shear stress on the motion of the condensate is comparable with, or greater than, that of gravity; effects of gravity are, however, in general also included.
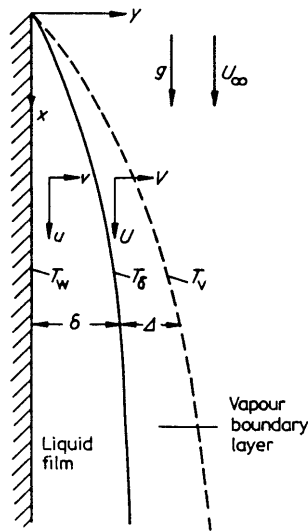
Referring to Fig. 1, for two-dimensional, steady, laminar, boundary-layer flows for the condensate film and saturated vapour, with negligible dissipation and uniform properties, the equations expressing the conservation of mass, momentum and energy are : -
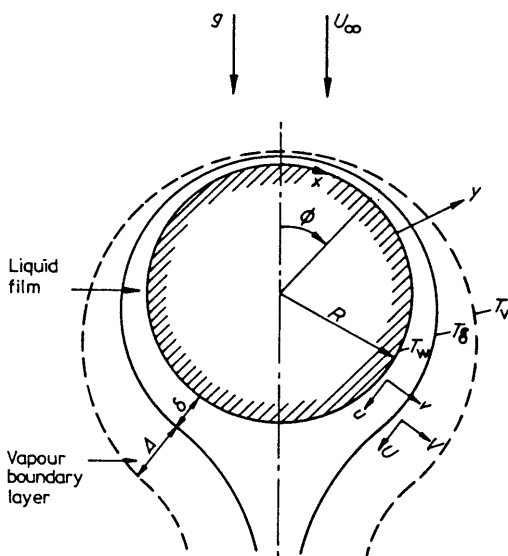For the condensate film :

continuity $\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0$ (1)

momentum $\rho\left[u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}\right]=\mu\dfrac{\partial^2 u}{\partial y^2}-\dfrac{dP}{dx}+\rho g\sin\varPhi$ (2)

energy $\rho c_P\left[u\dfrac{\partial T}{\partial x}+v\dfrac{\partial T}{\partial y}\right]=k\dfrac{\partial^2 T}{\partial y^2}$ (3)

(a) Vertical plate

(b) Horizontal tube

Fig. 1 Co-ordinate systems

For the vapour boundary layer :

continuity $\dfrac{\partial U}{\partial x}+\dfrac{\partial U}{\partial y}=0$ (4)

momentum $\rho_V\left[U\dfrac{\partial U}{\partial x}+V\dfrac{\partial V}{\partial y}\right]=\mu_V\dfrac{\partial^2 U}{\partial y^2}-\dfrac{dP}{dx}$ (5)

For a pure, saturated vapour, the temperature is uniform and the energy equation is not relevant.
The above are subject to boundary conditions : -
At the condensing surface $y=0$:
  zero velocities
  $u=v=0$ (6)
  for uniform wall temperature
  $T=T_W=$ constant (7·a)
  or, for uniform wall heat flux
  $\partial T/\partial y=$ constant (7·b)
At the condensate-vapour interface $y=\delta$:
  conservation of mass[*1]
  $\rho\left[v-u\dfrac{d\delta}{dx}\right]=\rho_V\left[V-U\dfrac{d\delta}{dx}\right]$ (8)
  streamwise velocity continuity
  $u=U$ (9)
  continuity of shear stress
  $\mu\dfrac{\partial u}{\partial y}=\mu_V\dfrac{\partial U}{\partial y}$ (10)

uniform condensate surface temperature (negligible interface resistance)[*2]

  $T=T_\delta=T_V=$ constant (11)

An inconsistency may be noted in that the effect of thickening of the condensate film in the streamwise direction is included in equation (8) but not in equations (9) and (10), where the velocity and shear stress are strictly continuous in the direction tangential to the condensate surface. Since condensate films are in general very thin this leads to negligible error.
In the remote vapour $y\rightarrow\infty$:
  for natural convection $U\rightarrow 0$ (12)
  for forced convection
    flat plate - zero pressure gradient
      $U\rightarrow U_\infty$ (13·a)
    tube - potential flow outside vapour boundary layer
      $U\rightarrow 2U_\infty\sin\varPhi$ (13·b)

Other streamwise velocity distributions, based on pressure measurements, have been used for the tube problem. In all cases the distributions are valid only up to the point of vapour boundary layer separation.
Since δ is not a known quantity the problem is not

[*1]Gaddis (1979) is the slightly less accurate, but more consistent, in using the approximate mass continuity condition : $\rho_V=\rho_V V$.

[*2]For forced convection condensation on a tube this implies neglect of the effect of pressure variation around the tube

closed. The necessary additional condition is provided by the relation between condensation rate and heat transfer. For mass and energy conservation in the film, this gives : -

$$k\left[\frac{\partial T}{\partial y}\right]_\delta = \rho h_{fg}\frac{d}{dx}\left[\int_0^\delta u dy\right] \qquad (14)$$

$$k\left[\frac{\partial T}{\partial y}\right]_0 = \rho\frac{d}{dx}\left[\int_0^\delta\{h_{fg}+c_P(T_\delta-T)\}u dy\right] \qquad (15)$$

Natural Convection

Nusselt's (1916) pioneering approach to this problem was effectively to neglect all but the viscous and gravity terms in equation (2) and all but the conduction term in equation (3). The shear stress at the condensate surface was set to zero so that the conservation equations for the vapour were not required. This gives parabolic and linear velocity and temperature distributions respectively, across the condensate film and, for uniform temperature difference and with the pressure gradient taken as $dP/dz = \rho_v g$, leads to the well-known results for mean Nusselt number : -
For the vertical plate :

$$Nu = \bar{q}L/k\Delta T = 0.943\left[\frac{\rho(\rho-\rho_V)gh_{fg}L^3}{\mu k\Delta T}\right]^{1/4} \qquad (16)$$

where $\quad \bar{q} = \frac{k\Delta T}{L}\int_0^L \delta^{-1}dx \qquad (17)$

For the horizontal tube :[3]

$$Nu_d = \bar{q}_d d/k\Delta T = 0.728\left[\frac{\rho(\rho-\rho_V)gh_{fg}d^3}{\mu k\Delta T}\right]^{1/4} \qquad (18)$$

where $\quad \bar{q}_d = \frac{k\Delta T}{\pi}\int_0^\pi \delta^{-1}d\Phi \qquad (19)$

Since inertia and vapour shear stress act to retard the condensate film, neglect of these effects causes the Nusselt theory to overestimate the heat transfer. Convection/subcooling[4] enhances the heat transfer so that neglect of the convection terms leads to an underestimate of the heat transfer. Errors in the Nusselt theory are therefore to some degree self-compensating.

Bromley (1952) and Rohsenow (1956) have amended Nusselt's result to account approximately for condensate subcooling, using mass-energy balances for an element of the condensate film. Rohsenow's result modifies Nusselt's expression for the Nusselt number by replacing h_{fg} by

$$h_{fg}^* = h_{fg}(1+0.68c_P\Delta T/h_{fg}) \qquad (20)$$

It is not evident, however, that this (generally small) correction, in isolation, gives more accurate results

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[3]The coefficient in equation (18) is sometimes given as 0.725. This stems from a small numerical inaccuracy in (Nusselt, 1916)

[4]It is seen from equation (3) that when the convection terms are dropped, c_P does not enter the problem.

than the unamended theory in all cases.

To obtain equation (18) for the tube, the additional assumption $\delta \ll R$ is required. Since the theory predicts that $\delta \to \infty$ as $\Phi \to \pi$, the result is evidently in error for the lower part of the tube. However, because the predicted and actual local heat fluxes are both small where δ is large, the error in mean heat-transfer rate or Nusselt number is not large.

The same approach was used by Fujii et al. (1972b) for the case of uniform surface heat flux and leads to similar results :
For the vertical plate :

$$Nu_q = qL/\overline{\Delta T}k = 0.943\left[\frac{\rho(\rho-\rho_V)gh_{fg}L^3}{\mu k\overline{\Delta T}}\right]^{1/4} \qquad (21)$$

where $\overline{\Delta T} = \frac{q}{kL}\int_0^L \delta dx \qquad (22)$

For the horizontal tube :

$$Nu_{q,d} = qd/\overline{\Delta T_d}k = 0.695\left[\frac{\rho(\rho-\rho_V)gh_{fg}d^3}{\mu k\overline{\Delta T_d}}\right]^{1/4} \qquad (23)$$

where $\overline{\Delta T_d} = \frac{q}{\pi k}\int_0^\pi \delta d\Phi \qquad (24)$

It is interesting to note that equation (21), for the vertical plate, is identical in form to equation (16), except that the uniform heat flux q in equation (21) replaces the mean heat flux $\bar{q}$ in equation (16) and the mean temperature difference $\overline{\Delta T}$ in equation (21) replaces the uniform ΔT in equation (16).

For the case of the tube, the uniform heat flux theory gives a lower Nusselt number than the uniform temperature difference theory. This is in contrast to single phase flow where uniform heat flux gives a higher Nusselt number. It is to be noted that, in the uniform temperature difference case, the integrand in equation (19) is the reciprocal of δ, whereas equation (24) has δ itself. Thus $\overline{\Delta T_d}$ is significantly influenced by the large values of δ as $\Phi \to \pi$ near the bottom of the tube where the theory becomes inaccurate. This casts doubt on the validity of equation (23). In the investigations discussed below, uniform temperature difference has been used in all cases.

Sparrow and Gregg (1959a, b) were the first to treat the condensate film, for both vertical plate and horizontal tube, on the basis of equations (1-3). The vapour shear stress at the condensate surface was neglected so that equations (4, 5, 8-10) were not required. Using similarity transformations[5,6], it was shown that the result for surface heat transfer could be represented in the form :

$$\frac{Nu}{Nu_{Nu}} = \Phi\left[\frac{c_P\Delta T}{h_{fg}}, Pr\right] \qquad (25)$$

Numerical results were obtained for a range of $c_P\Delta T/h_{fg}$ and various values of Pr. At low values of $c_P\Delta T/h_{fg}$, the results were very close to the simple

Nusselt theory. At high values of $c_p \Delta T/h_{fg}$, somewhat higher values of Nusselt number (in comparison with Nusselt theory predictions) were found for higher values of Pr, indicating the relative importance of convection. At low Pr (liquid metals), lower Nusselt numbers were found indicating that inertia effects outweigh those of convection. Results of Sparrow and Gregg (1959b) are reproduced in Fig. 2. The fact that, for the horizontal tube, the limiting result for $c_p \Delta T/h_{fg} \to 0$ differed slightly from the Nusselt result (Sparrow and Gregg (1959b) give a value of 0.733 rather than 0.728 for the coefficient in equation (18)) is due to a minor numerical inaccuracy in the paper of Herman (1954) used by Sparrow and Gregg (1959b) as discussed by Maekawa and Rose (1988). At the higher values of $c_p \Delta T/h_{fg}$, the increase in Nu_d/N_{Nu_d} for high Pr, and the decrease in Nu_d/Nu_{Nu_d} for low Pr, are due to convection and inertia effects respectively. However, in practice the value of $c_p \Delta T/h_{fg}$ is rarely sufficiently large to make these effects important.
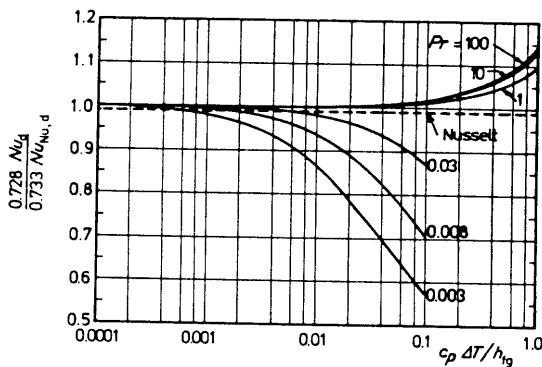
Chen (1961a, b) treated the same problems as

Fig. 2 Natural Convection Condensation
Effect of including inertia and convection terms (after Sparrow and Gregg (1959b))

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*5The similarity variable used in (Sparrow and Gregg 1959a) included c_P which caused Pr to appear in the transformed *momentum equation*. This might be thought inappropriate since c_P, and hence Pr, only enter the problem when the convection terms in the *energy equation* are retained. A slightly modified similarity variable was suggested by Seban in his discussion of (Sparrow and Gregg 1959a). This led to the appearance of Pr in the energy equation from which it is seen that the temperature variation across the condensate film is linear for small Pr, i.e. the convection terms do not contribute in this limiting case.

*6For the case of the cylinder there is, strictly, no similarity solution. The approximation used by Sparrow and Gregg (1959b) is considered to have negligible effect on the total heat transfer.

Sparrow and Gregg and also included the retarding effect of vapour shear stress on the condensate film. The interfacial shear stress was estimated by:

$$\tau_s = -m u_s \tag{26}$$

As discussed later in connection with forced convection condensation, this is the asymptotic value when $m \to \infty$. Use of this approximation avoids the necessity to consider the vapour boundary layer, and thus to bring properties of the vapour into the problem. The mean Nusselt number is again expressible as a function of two condensate film parameters. Chen's results were presented on the basis of equation (25) and also, equivalently, on the basis of equation (27) below

$$\frac{Nu}{Nu_{Nu}} = \Phi\left[\frac{k\Delta T}{\mu h_{fg}}, Pr\right] \tag{27}$$

Chen's results, which extend well beyond the practical range of $c_p \Delta T/h_{fg}$ and $k\Delta T/\mu h_{fg}$, are reproduced in Fig. 3 and apply equally to local and mean values and to both vertical plates and horizontal tubes. It may be noted that

$$\frac{Nu}{Nu_{Nu}} = \Phi\left[\frac{c_p \Delta T}{h_{fg}}\right] \quad \text{for } Pr \to \infty \tag{28}$$

$$\frac{Nu}{Nu_{Nu}} = \Phi\left[\frac{k\Delta T}{\mu h_{fg}}\right] \quad \text{for } Pr \to 0 \tag{29}$$

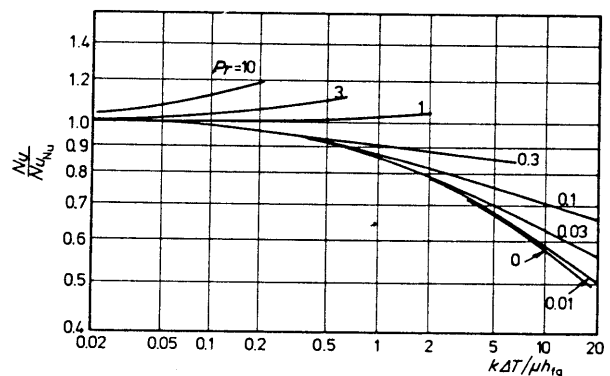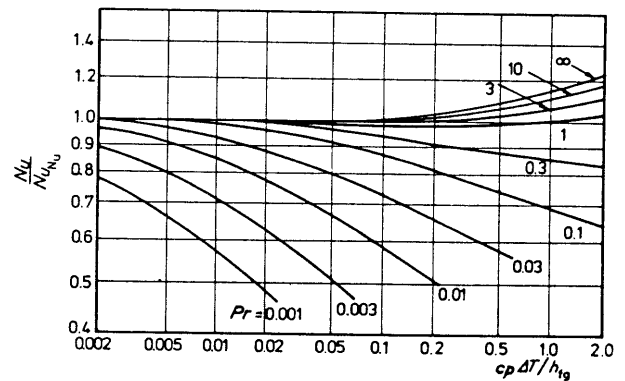so that the parameters of equations (25) and (27) are

Fig. 3 Natural Convection Condensation
Effect of including inertia and convection terms and interface shear stress (after Chen (1961b))

more convenient for high and low Pr respectively. Chen also gave an approximate equation representing his numerical results which may be expressed as

$$\frac{Nu_x}{Nu_{Nu,x}} = \frac{Nu}{Nu_{Nu}} = \frac{Nu_d}{Nu_{Nu,d}}$$

$$= \left[\frac{1 + 0.68H + \dfrac{0.02}{Pr}H^2}{1 + \dfrac{0.85}{Pr}H - \dfrac{0.15}{Pr}H^2} \right]^{1/4}$$

$$= \left[\frac{1 + 0.68PrJ + 0.02PrJ^2}{1 + 0.85J - 0.15PrJ^2} \right]^{1/4} \qquad (30)$$

where $H = c_P \Delta T / h_{fg}$

and $J = k\Delta T / \mu h_{fg}$

Comparison of the results of Sparrow and Gregg with those of Chen shows that the influence of surface shear stress is negligible at higher Prandtl numbers. At low Prandtl numbers, the effect is more significant but quite small except at the highest values of $k\Delta T/\mu h_{fg}$ encountered in practice. Moreover, the highest reported values of $k\Delta T/\mu h_{fg}$ for liquid metals are most probably erroneous, as noted by Denny and Mills (1969a), and caused by gas-side temperature drops, due to non-condensing gas or interphase mass transfer, being included in ΔT.

For the vertical plate, Koh (1961) and Koh et al. (1961) incorporated the interfacial shear stress more accurately using equation (10). This required simultaneous solution of the vapour and condensate equations (1-5) using the interface conditions, equations (8-11). It was shown that the Nusselt number is then expressible as a function of three parameters :

$$\frac{Nu}{Nu_{Nu}} = \varnothing \left[\frac{c_P \Delta T}{h_{fg}}, Pr, \left[\frac{\rho\mu}{\rho_v\mu_v} \right]^{1/2} \right] \qquad (31)$$

The dependence on the third parameter $(\rho\mu/\rho_v\mu_v)^{1/2}$ was, however, found to be insignificant. Results of Koh et al. (1961) are given in Fig. 4 and, since they apparently coincide with those of Chen (1961a), it may be presumed that the effect of surface shear stress is only significant when the condensation rate is sufficiently high to ensure the validity of equation
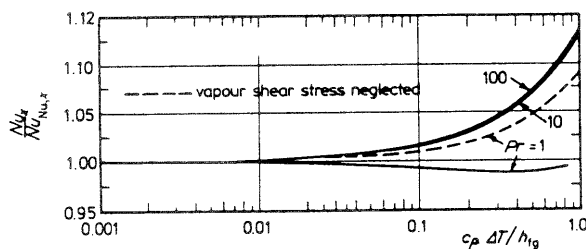
(26).

Maekawa and Rose (1988) have reconsidered the problem solved by Chen (1961a, b) using a similarity approach. They confirmed that Chen's result (equation (30)) was correct to within 1% and gave even more accurate expressions for the Nusselt number. Equation (32) below is valid for all Pr provided H or J is less than about 1. The accuracy is generally within 0.01% with a maximum error of 0.1% near $Pr = 1$ and at H or J near unity.

$$\frac{Nu_x}{Nu_{Nu,x}} = \frac{Nu}{Nu_{Nu}} = \frac{Nu}{Nu_{Nu,d}}$$

$$= 1 + \left[\alpha_1 + \frac{\beta_1}{Pr} \right]H + \left[\alpha_2 + \frac{\gamma_1}{Pr} + \frac{\beta_2}{Pr^2} \right]H^2$$

$$+ \left[\alpha_3 + \frac{\gamma_2}{Pr} + \frac{\gamma_3}{Pr^2} + \frac{\beta_3}{Pr^3} \right]H^3$$

$$= 1 + (\alpha_1 Pr + \beta_1)J + (\alpha_2 Pr^2 + \gamma_1 Pr + \beta_2)J^2$$

$$+ (\alpha_3 Pr^3 + \gamma_2 Pr^2 + \gamma_3 Pr + \beta_3)J^3 \qquad (32)$$

where : $\alpha_1 = 0.168\ 0$

$\alpha_2 = -0.032\ 50$

$\alpha_3 = 0.005\ 716$

$\beta_1 = -0.241\ 8$

$\beta_2 = 0.147\ 6$

$\beta_3 = -0.060\ 92$

$\gamma_1 = 0.035\ 77$

$\gamma_2 = -0.006\ 218$

$\gamma_3 = -0.019\ 80$

The α's and β's are coefficients in the expressions for the limiting cases where $Pr \rightarrow \infty$ and $Pr = 0$ respectively.

Gaddis (1979) has adopted a most comprehensive approach and provided results for the horizontal tube on the basis of equations (1-13). Numerical solutions were obtained for representative cases. These are in general accord with the results of Chen (1961b) and Maekawa and Rose (1988).
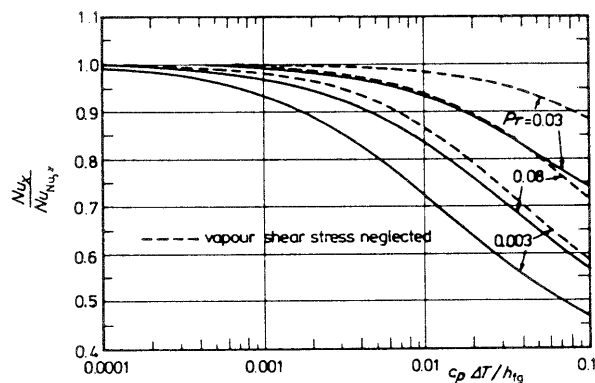
Fig. 4 Natural Convection Condensation 'Exact' solutions including inertia and convection terms and with matched shear stress at vapour-condensate interface (after Koh et al. (1961))

In summary, it may be said that research over the past 30 years has established, for most practical purposes, the validity of the simple Nusselt results *for the condensate film*, for laminar natural convection condensation on a vertical plate and horizontal tube. More accurate equations are now available. However, in most cases and when considering more complex geometries (e.g. finned tubes and tube banks), the simplifying assumptions of the Nusselt (1916) theory should be adequate when treating the condensate film.

Forced Convection

In this case the motion of the condensate film is, by definition, significantly affected by the shear stress from the vapour. Some investigators have sought to obtain expressions for the interface shear stress on the basis of single phase flow (e.g. Nusselt (1916), Sugawara et al. (1956), Le Fevre (1960)). This approach neglects the effect of mass transfer across the interface as well as the streamwise motion of the interface. The former gives results which are valid only in the low condensation rate limit, while the latter generally gives relatively small error. Others have conceived the surface shear stress as a combination of two factors which have been referred to as 'frictional' and 'momentum' shear stresses, τ_f and τ_m (Mayhew et al. (1966, 1973), Nobbs and Mayhew (1976), Nicol and Wallace (1974), Asano et al. (1979)). As noted by South and Denny (1972), τ_f and τ_m are, in fact, the limiting values of the interface shear stress for low and high condensation rates respectively. Various interpolation formulae suggested by the above authors do not approach τ_m for high condensation rate and therefore cannot be strictly accurate.

The correct approach, as for natural convection condensation, requires simultaneous consideration of momentum balances for vapour and condensate with continuity of surface shear stress as an interface matching condition (see equation (10)). This approach was used by Cess (1960) in his treatment of forced convection condensation on a horizontal plate. Cess considered the problem on the basis of the boundary-layer equations but subsequently used approximations with effects equivalent to having neglected the inertia and convection terms in equations (2) and (3). He also implicitly neglected the streamwise velocity of the condensate surface in calculating the shear stress and thus effectively solved the 'Nusselt problem' without gravity but with the surface shear stress condition (equation (10)) and boundary condition equation (13 a). This led to the results : -
for the zero condensation rate limit :

$$\frac{\tau_\delta}{\rho_v U_\infty^2} \cdot Re_{vx}^{1/2} = 0.332 \tag{33}$$

where Re_{vx} is the vapour Reynolds number $U_\infty \rho_v x / \mu_v$, and,
for the infinite condensation rate limit :

$$\frac{\tau_\delta}{\rho U_\infty^2} \cdot \tilde{Re}_x^{1/2} = \frac{1}{2} \left[\frac{k \Delta T}{\mu h_{fg}} \right] \tag{34}$$

where $\tilde{Re}_x$ is the "two-phase Reynolds number" $U_\infty \rho x / \mu$. It may be noted that equation (33) gives the shear stress for boundary-layer flow on a flat plate (without transpiration) and does not involve condensate properties. Equation (34), on the other hand, arises from the asymptotic result

$$\tau_\delta \to m (U_\infty - u_\delta) \tag{35}$$

for $m \to \infty$, together with the assumption $U_\infty \gg u_\delta$, and does not involve vapour properties. Note that equation (34) can be expressed in the form :

$$\frac{\tau_\delta}{\rho_v U_\infty^2} \cdot Re_{vx}^{1/2} = \frac{G}{2} \tag{36}$$

where

$$G = \left[\frac{k \Delta T}{\mu h_{fg}} \right] \cdot \left[\frac{\rho \mu}{\rho_v \mu_v} \right]^{1/2} \tag{37}$$

It is seen from Fig. 5 that the actual surface shear stress is larger than either of the limiting results given by equations (33) and (34) (or (36)). For $G > 0.664$ equation (34) is closer to the true shear stress, while for $G < 0.664$, equation (33) is more accurate.

The results for heat transfer obtained by Cess (1960) using equations (33) and (34) may be written, for the zero condensation rate limit

$$Nu_x \tilde{Re}_x^{-1/2} = 0.436 G^{-1/3} \tag{38}$$

and for infinite condensation rate

$$Nu_x \tilde{Re}_x^{-1/2} = 0.5 \tag{39}$$

These asymptotes, together with numerical solutions of Cess (1960) for the general case, are given in Fig. 6. The fact that the Nusselt number decreases with increasing G, i.e. with increasing condensation rate, while the shear stress increases (see Fig. 5) indicates that the increase in film thickness due to condensation exceeds the decrease associated with increase in shear stress. Sparrow et al. (1967) include a tabulation of
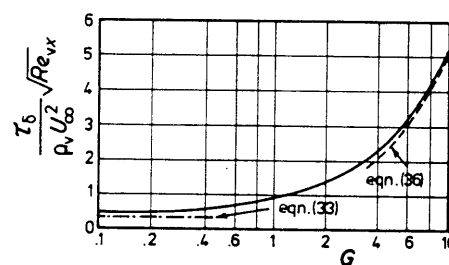
Fig. 5 Forced Convection Condensation on a Horizontal Plate-Condensate surface shear stress (after Cess (1960))

the solution in their table 1 where

$$\frac{Rc_P(T_i - T_w)}{h_{fg}Pr} = G \text{ and } 1/\eta_s = Nu_x \tilde{Re}_x^{-1/2}$$

Koh (1962) has given a very comprehensive treatment of the horizontal plate problem. The inertia and convection terms for the condensate film were retained and the velocity of the condensate surface was not neglected. This led to a relationship for the heat transfer involving two additional dimensionless parameters : -

$$Nu_x \tilde{Re}_x^{-1/2} = \psi\left[\frac{k\Delta T}{\mu h_{fg}}, Pr, \left[\frac{\rho\mu}{\rho_v\mu_v}\right]^{1/2}\right] \quad (40)$$

or, equivalently

$$Nu_x \tilde{Re}_x^{-1/2} = \Phi(G, Pr, (\rho\mu/\rho_v\mu_v)^{1/2}) \quad (41)$$

Extensive numerical solutions, many outside normal practical ranges of the variables, were obtained.

For low Prandtl numbers, the Prandtl number dependence, due to convection, was negligible. The results for Prandtl numbers of 0.03 and 0.003 shown in Fig. 7 are indistinguishable. It is seen from Fig. 7a that for higher values of $k\Delta T/\mu h_{fg}$, where the asymptotic shear stress approximation (equation (35)) is valid, the dependence of $Nu_x\tilde{Re}_x^{-1/2}$ on $(\rho\mu/\rho_v\mu_v)^{1/2}$ is negligible, i.e.
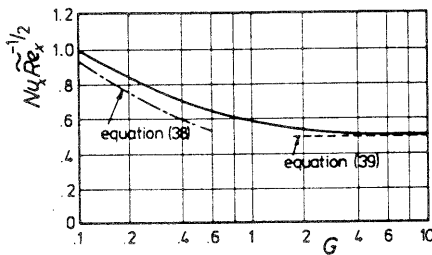

Fig. 6 Forced Convection Condensation on a Horizontal Plate-Heat Transfer (after Cess (1960))

$$Nu_x \tilde{Re}_x^{-1/2} = \psi(k\Delta T/\mu h_{fg}) \quad (42)$$

is valid for low Pr and when $k\Delta T/\mu h_{fg} >$ about 1. It may be noted however that this result differs from equation (39) as found by Cess (1960). As discussed later, this is due to the neglect of the condensate surface velocity by Cess. The inertia terms, included by Koh (1962), are apparently unimportant in forced convection condensation. At low values of $k\Delta T/\mu h_{fg}$, where the surface shear stress approaches the zero condensation rate limit (equation (33)), it is seen from Fig. 7b that $Nu_x\tilde{Re}_x^{1/2}$ depends only on G, the product of $k\Delta T/\mu h_{fg}$ and $(\rho\mu/\rho_v\mu_v)^{1/2}$ i.e.

$$Nu_x \tilde{Re}_x^{-1/2} = \Phi(G) \quad (43)$$

and the results of Koh (1962) are in good agreement with those of Cess (1960).

At higher Prandtl numbers (see Fig. 8), a marked dependence of $Nu_x\tilde{Re}_x^{-1/2}$ on Pr is seen. At lower values of $k\Delta T/\mu h_{fg}$ or G, $Nu_x\tilde{Re}_x^{-1/2}$ falls with increase in either of these parameters as found by Cess (1960). At higher values, $Nu_x\tilde{Re}_x^{-1/2}$ passes through a minimum before rising as convection becomes increasingly important. This only occurs, however, for values of $k\Delta T/\mu h_{fg}$ and G beyond the normal practical ranges. It is also seen in Fig. 8a that for high values of $k\Delta T/\mu h_{fg}$ the dependence on $(\rho\mu/\rho_v\mu_v)^{1/2}$ disappears as the surface shear stress approaches its asymptotic value, i.e.
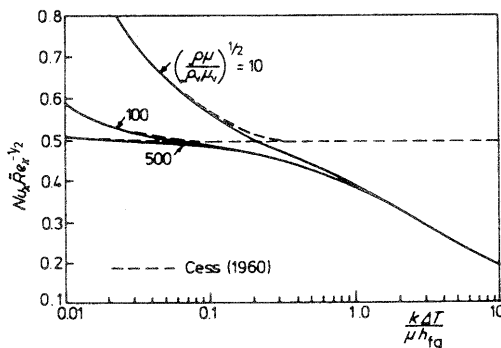
$$Nu_x \tilde{Re}_x^{-1/2} = \psi(k\Delta T/\mu h_{fg}, Pr) \quad (44)$$

At low values of $k\Delta T/\mu h_{fg}$, when the shear stress approaches its zero condensation limit,
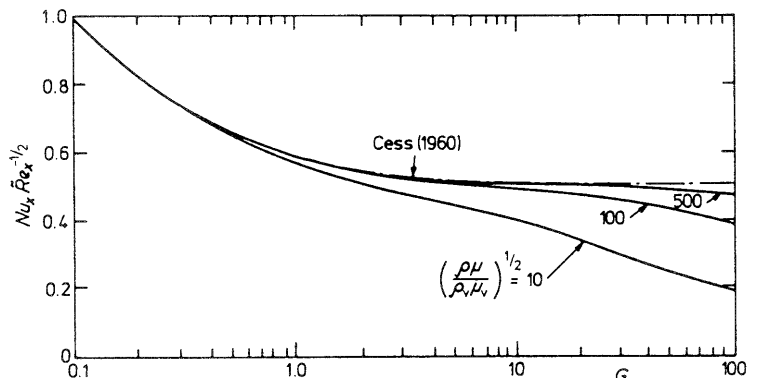
$$Nu_x \tilde{Re}_x^{-1/2} = \Phi(G) \quad (45)$$

as found by Cess (1960) and indicated in Figs. 8b and 8c.

The good agreement between the results of Cess (1960) and Koh (1962) in the normal ranges of the dimensionless variables suggests that, for practical

(a) $Nu_x\tilde{Re}_x^{-1/2}$ vs $k\Delta T/\mu h_{fg}$

(b) $Nu_x\tilde{Re}_x^{-1/2}$ vs G

Fig. 7 Forced Convection Condensation on a Horizontal Plate Dependence of $Nu_x\tilde{Re}_x^{-1/2}$ on $k\Delta T/\mu h_{fg}$, G, $(\rho\mu/\rho_v\mu_v)^{1/2}$ at low Pr. Results for $Pr = 0.03$ and 0.003 indistinguishable (after Koh (1962))

purposes, effects of inertia and convection can be neglected in forced convection condensation. The effect of condensate surface valocity is only significant for low Prandtl numbers and extreme values of $k\Delta T/\mu h_{fg}$ and G.

Shekriladze and Gomelauri (1966) neglected inertia and convection effects in the condensate film when considering condensing vapour flow along horizontal and vertical flat plates and for vapour downflow over a horizontal tube. In addition, they greatly simplified the problem further by adopting the infinite condensation rate asymptotic expression (equation (35)) for the condensate surface shear stress. As may be judged from the results of Cess (1960), this would be valid for $G >$ about 5 but should give quite good accuracy for $G >$ about 2 (see Fig. 6) and would be conservative (i.e. would underestimate the heat transfer) for smaller values of G.

For the horizontal flat plate case, Shekriladze and Gomelauri did not adopt the approximation $U_{\infty} \gg u_s$ in the expression for the surface shear stress and obtained a result which may be expressed:

$$Nu_x \tilde{R}e_x^{-1/2} = \frac{1}{2}\left[1+\frac{k\Delta T}{\mu h_{fg}}\right]^{-1/2} \qquad (46)$$

It is readily seen from equation (46) that the mean heat-transfer coefficient or Nusselt number for a plate of length L is twice the local value at L.

Comparison of equation (46) with equation (39), obtained by Cess (1960) for the same conditions but also assuming $U_{\infty} \gg u_s$, shows that it is permissible to neglect the condensate surface velocity when $k\Delta T/\mu h_{fg} \ll 1$, which is generally true for non-metals. For a value of $k\Delta T/\mu h_{fg}$ of 0.1 equation (39) gives an error in $Nu_x \tilde{R}e_x^{-1/2}$ of about 5%. It may be noted, that for metals, values of $k\Delta T/\mu h_{fg}$ up to around 2 have been reported but, as indicated earlier, erroneous values of ΔT may have been used.

When the uniform heat flux problem was treated in the same way (but in this case taking $U_{\infty} \gg u_s$), Shekriladze and Gomelauri (1966) obtained
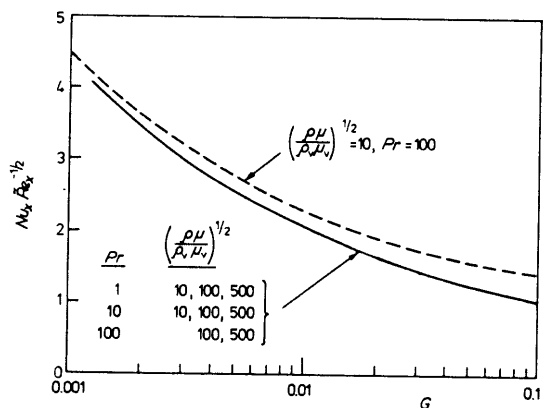
$$Nu_x \tilde{R}e_x^{-1/2} = 1 \qquad (47)$$

i.e. the local heat-transfer coefficient is twice that for the uniform wall temperature case. For the uniform heat flux case, equation (47) shows that the mean heat-transfer coefficient or Nusselt number for a plate of length L is 1.5 times the local value at L.
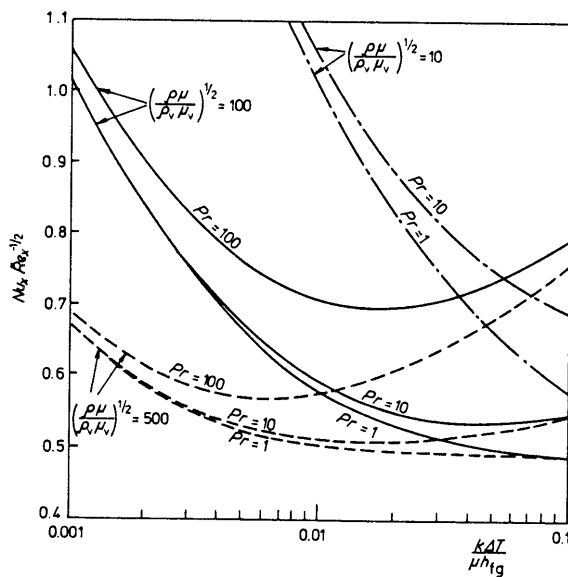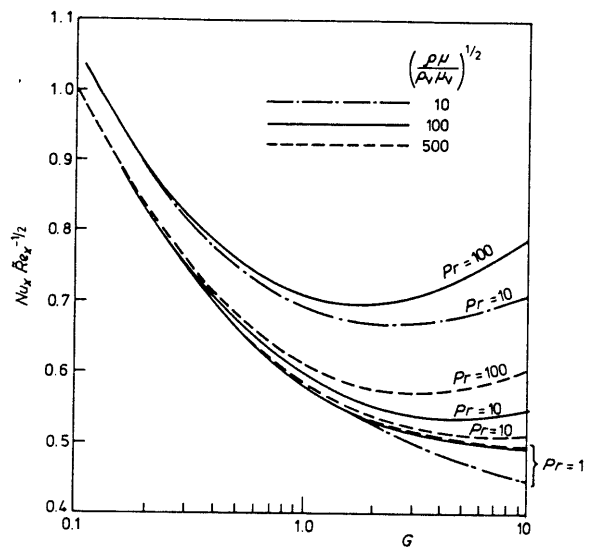
(b)　$Nu_x \tilde{R}e_x^{-1/2}$ vs G　$(G > 0.1)$

(a)　$Nu_x \tilde{R}e_x^{-1/2}$ vs $k\Delta T/\mu h_{fg}$

(c)　$Nu_x \tilde{R}e_x^{-1/2}$ vs G　$(G < 0.1)$

Fig. 8　Forced Convection Condensation on a Horizontal Plate Dependence of $Nu_x \tilde{R}u_x^{-1/2}$ on $k\Delta T/\mu h_{fg}$, G, $(\rho\mu/\rho_v\mu_v)^{1/2}$ and Pr at higher Pr. (after Koh (1962))

For vapour downflow along a vertical flat plate, and with the same assumptions (and taking $U_\infty \gg u_\delta$) Shekriladze and Gomelauri obtained results which may be expressed

$$Nu_x \tilde{Re}_x^{-1/2} = \frac{1}{2} \left[\frac{1+(1+16F_x)^{1/2}}{2} \right]^{1/2} \qquad (48)$$

and, for the mean Nusselt number for a plate of height L

$$Nu \tilde{Re}_L^{-1/2} = \frac{\sqrt{2}}{3} \frac{\{2+(1+16F_L)^{1/2}\}}{\{1+(1+16F_L)^{1/2}\}^{1/2}} \qquad (49)$$

where $F_x = \mu h_{fg} gx/k\Delta T U_\infty^2$ (50)

and $F_L = \mu h_{fg} gL/k\Delta T U_\infty^2$ (51)

As noted above, the fact that u_δ has been neglected implies that equations (48) and (49) are valid only for small $k\Delta T/\mu h_{fg}$.

Shekriladze and Gomelauri (1966) used the same approach to treat the problem of condensation of a vapour flowing normal to a horizontal tube and omitted the pressure gradient in the momentum balance equation for the condensate film (equation (2)). Again they greatly simplified the problem by using the asymptotic expression for the vapour shear stress at the condensate surface

$$\tau_\delta = m(U' - u_\delta) \qquad (52)$$

where U' is the tangential velocity at the outer 'edge' of the vapour boundary layer. Shekriladze and Gomelauri δ neglected u_δ in equation (52) and took, on the basis of potential flow outside the vapour boundary layer,

$$U' = 2U_\infty \sin \Phi \qquad (53)$$

so that $\tau_\delta = 2U_\infty m \sin \Phi$ (54)

Equation (54) indicates that τ_δ is positive for all Φ so that the vapour boundary layer does not separate. This is due to the fact that equation (52) is strictly only valid for infinite condensation rate (i.e. infinite *suction* at the condensate surface). Use of equation (54) for τ_δ permits a solution (determination of the condensate film thickness) for the whole surface of the tube. However, since in general, vapour boundary layer separation will occur at some position over the downstream side of the tube, results obtained will only be valid up to the separation point. At high condensation rates this can, however, theoretically include most or all of the tube surface.

When gravity is omitted, this gives

$$Nu_d \tilde{Re}_d^{-1/2} = 0.9 \qquad (55)$$

To include the effect of gravity, Shekriladze and Gomelauri proposed a simple interpolation formula which approximately satisfies equation (55) at high vapour velocity and equation (18) at low vapour velocity. This may be expressed

$$Nu_d \tilde{Re}_d^{-1/2} = 0.644\{1+(1+1.69F_d)^{1/2}\}^{1/2} \qquad (56)$$

where $F_d = \mu h_{fg} gd/k\Delta T U_\infty^2$ (57)

Again it is noted that since u_δ was assumed small in

comparison with U_∞, the validity of equations (55) and (56) requires that $k\Delta T/\mu h_{fg}$ be small.

Denny and Mills (1969b) treated the horizontal cylinder problem on the same basis as Shekriladze and Gomelauri (1966), but incorporated effects of gravity and vapour shear stress simultaneously. An approximate equation for the total heat-transfer coefficient, which includes tabulated functions of angle, was obtained.

Jacobs (1966) and later Fujii and Uehara (1972) attempted to solve the vertical flat plate problem more accurately by considering also the vapour boundary layer and matching the shear stress at the vapour-condensate interface rather than adopting an approximate expression for τ_δ. An integral treatment, using a quadratic velocity profile, was used for the vapour boundary layer and the condensate and vapour streamwise velocities were also matched at the interface. Fujii and Uehara (1972), while acknowledging that their method followed that of Jacobs (1966), pointed out that an incorrect boundary condition ($V = 0$ at the edge of the vapour boundary layer) had been used in the earlier work. Their results were represented approximately by

$$Nu \tilde{Re}_L^{-1/2} = \{0.656(1.2+G^{-1})+0.79F_L\}^{1/4} \qquad (58)$$

Comparison of equations (58) and (49) indicates that calculation of τ_δ by matching shear stress at the vapour-condensate interface leads to the appearance of the additional parameter G in equation (58).

For forced convection only, Fujii and Uehara (1972) obtained

$$Nu_x \tilde{Re}_x^{-1/2} = 0.45G^{-1/3} \qquad (59)$$

for $G \ll 1$, which is in quite good agreement with the 'exact' result obtained by Cess (1960) (see equation (38)). Fujiii and Uehara (1972) also gave

$$Nu_x \tilde{Re}_x^{-1/2} \to 0.5 \qquad (60)$$

for $G \to 10$. It may be noted that Cess (1960), who unlike Fujii and Uehara, implicitly used $u_\delta \ll U_\infty$, also obtained equation (60) for *infinite* G (see equation (39)).

Careful scrutiny of Fujii and Uehara's results suggests that equation (60) is only obtained if $u_\delta/U_\infty \ll 1$. When this restriction is lifted, one finds

$$Nu_x \tilde{Re}_x^{-1/2} \to 0.5 \left[1 + \frac{k\Delta T}{\mu h_{fg}} \right] \qquad (61)$$

for $G \to \infty$, i.e. as obtained by Shekriladze and Gomelauri (1966), see equation (46).

On the basis of equations (59) and (60), Fujii and Uehara (1972) proposed the interpolation formula

$$Nu_x \tilde{Re}_x^{-1/2} = 0.45(1.2+G^{-1})^{1/3} \qquad (62)$$

for $G < 10$.

A better formula would appear to be

$$Nu_x \tilde{R}e_x^{-1/2} = 0.436 \left[\frac{1.508}{\left[1 + \frac{k\Delta T}{\mu h_{fg}}\right]^{3/2}} + \frac{1}{G} \right]^{1/3} \quad (63)$$
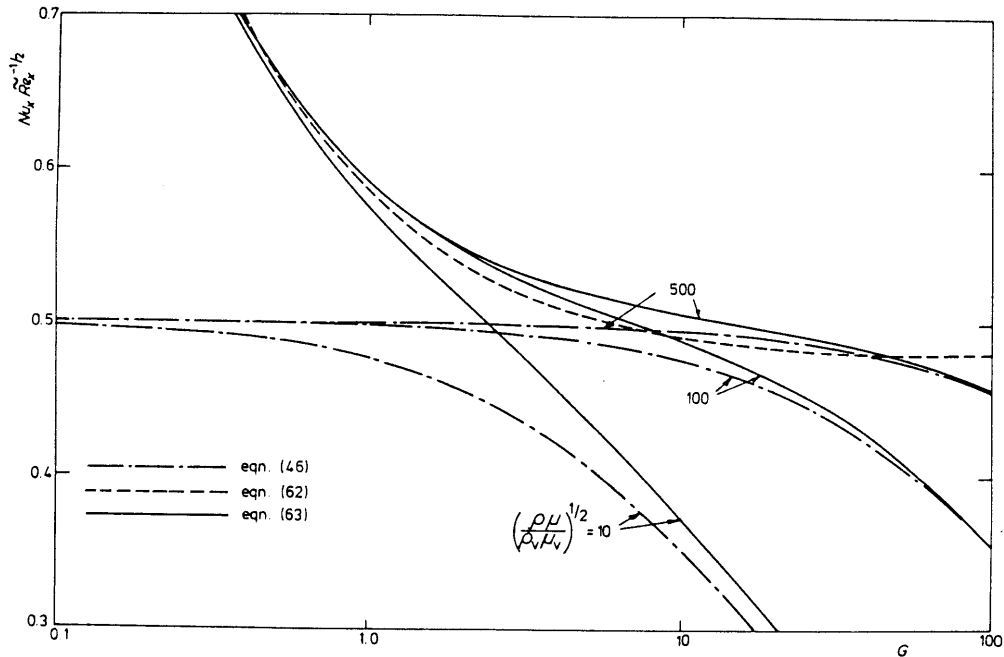
for all values of G. The coefficient outside the bracket in Eq. (63) has been chosen to agree

with equation (38) of Cess (1960) when $G \rightarrow 0$. The inside coefficient has been chosen so that equation (63) agrees with equation (46) of Shekriladze and Gomelauri (1966) when $G \rightarrow \infty$.

Equations (46), (62) and (63) are compared in
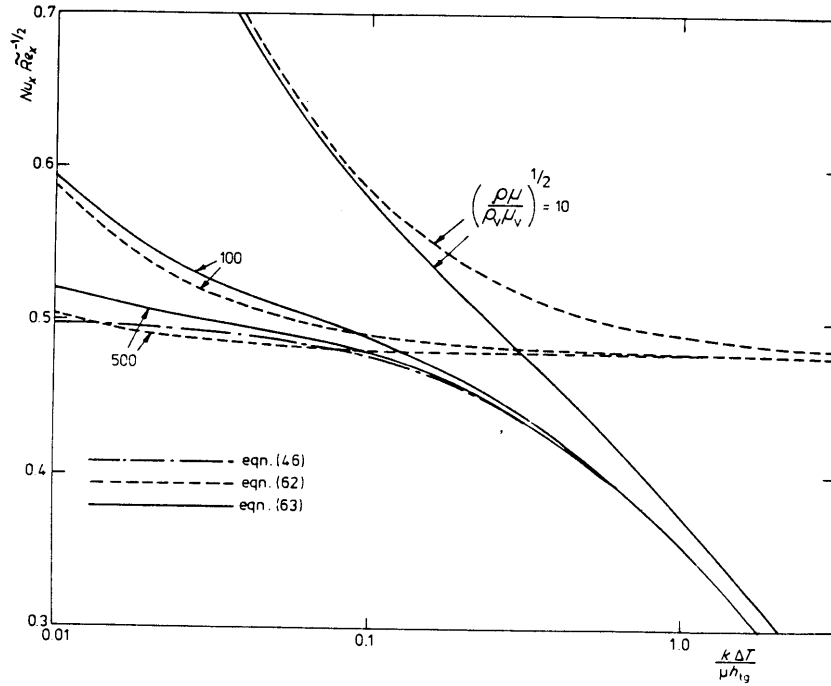
(a) $Nu_x \tilde{R}e_x^{-1/2}$ vs $k\Delta T/\mu h_{fg}$

(b) $Nu_x \tilde{R}e_x^{-1/2}$ vs G

Fig. 9 Forced Convection Condensation on a Horizontal Plate
Comparison of equations (46), (62) and (63)

Fig. 9. Equation (63) closely follows the exact solutions of Koh (1962) for low Pr (see Fig. 7), where effects of convection (considered neither by Cess (1960) nor Shekriladze and Gomelauri (1966) nor Fujii and Uehara (1972)) are unimportant. It may be seen that at low values of $k\Delta T/\mu h_{fg}$ or G, equation (62) of Fujii and Uehara (1972), which accounts more accurately for the interface shear stress, has the correct behaviour, whereas equation (46) of Shekriladze and Gomelauri (1966) approaches a constant value of $Nu_x\tilde{R}e_x^{-1/2}$ of 0.5. At high values of $k\Delta T/\mu h_{fg}$ or G, where the asymptotic shear stress approximation of Shekriladze and Gomelauri becomes more accurate, equation (46) behaves correctly, while equation (62) indicates $Nu_x\tilde{R}e_x^{-1/2}$ approaches a constant value owing to neglect of the condensate surface velocity in calculating the surface shear stress.

For the combined forced and free convection (vertical plate) case, Fujii and Uehara (1972) obtained the interpolation formulae

$$Nu_x\tilde{R}e_x^{-1/2}=K(1+0.25K^{-4}F_x)^{1/4} \qquad (64)$$

where $K=0.45(1.2+G^{-1})^{1/3}$ \qquad (65)

and, for the average Nusselt number for a plate of height L

$$Nu_L\tilde{R}e_L^{-1/2}=2K(1+(\sqrt{2}/3K)^4F_L)^{1/4} \qquad (66)$$

For reasons indicated above, equations (64) and (66) would be significantly in error for $k\Delta T/\mu h_{fg}$ greater than about 0.1. A better result, valid for larger values of $k\Delta T/\mu h_{fg}$, would appear to be obtained by taking

$$K=0.436\left[\frac{1.508}{\left[1+\dfrac{k\Delta T}{\mu h_{fg}}\right]^{3/2}}+\frac{1}{G}\right]^{1/3} \qquad (67)$$

Fujii et al. (1972a) treated the problem of the horizontal tube with vertical vapour downflow, using the approach of Jacobs (1966), in the same way as for the vertical plate case, i.e. inertia, convection and pressure gradient terms were omitted in the momentum equation for the condensate. The integral form of momentum equation for the vapour boundary layer was used with a quadratic velocity profile. Following Shekriladze and Gomelauri (1966), equation (53) (potential flow) was used for the streamwise velocity at the 'edge' of the vapour boundary layer. Use of a quadratic vapour velocity profile dictates that the velocity gradient in the vapour boundary layer is always positive, so that, as in the solution of Shekriladze and Gomelauri (1966), vapour boundary layer separation does not occur.

Figure 10 shows an example of the variation of heat-transfer coefficient with angle obtained by Fujii et al. (1972a). The fact that the local coefficient is generally higher than corresponding values obtained using the approach of Sugawara et al. (1956) and Shekriladze and Gomelauri (1966), who used, respec-

tively, the zero and infinite condensation rate limiting values of the shear stress at the condensate surface, reflects the fact that the actual surface shear stress is larger than both of the asymptotic values. The slope discontinuity in the case of the Sugawara et al. (1956) solution occurs at the point of separation of the vapour boundary layer, beyond which the surface shear stress was set to zero. Comparison with the Nusselt solution shows that, at high vapour velocity, the heat transfer is weighted more heavily towards the upper, upstream part of the tube. Inaccuracies arising from error for the lower part of the tube (relatively thick condensate film and vapour boundary layer separation) therefore have a weaker influence on the mean heat-transfer coefficient for the whole tube in the forced convection case.[*]

For forced convection, the results of Fujii et al. (1972a) were summarized by

$$Nu_d\tilde{R}e_d^{-1/2}=0.90(1+G^{-1})^{1/3} \qquad (68)$$

for $G<10$.

This suggests, as in the case of the flat plate (Fujii and Uehara (1972)), that equation (68) is valid provided $u_s\ll U_\infty$ which requires $k\Delta T/\mu h_{fg}$ to be small, i.e. less than about 0.1.
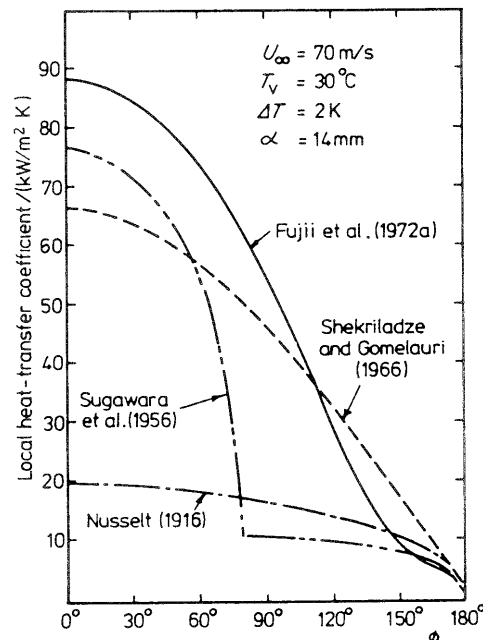
Fig. 10 Local Heat-Transfer Coefficients for Forced Convection Condensation of Steam on a Horizontal Tube (after Fujii et al. (1972a))

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[*]Note that the reverse would be true if uniform surface heat flux were used rather than uniform surface temperature.
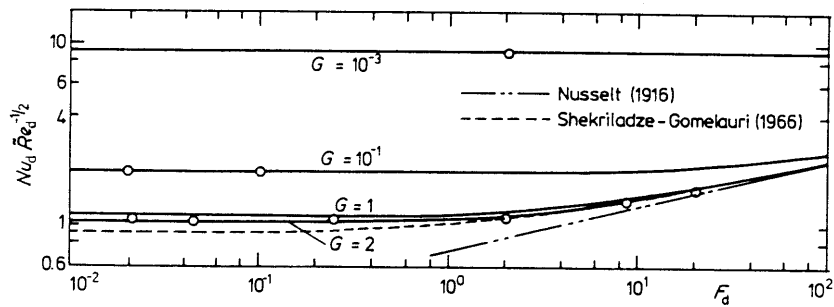
Fig. 11 Condensation Heat Transfer in Downflow over a Horizontal Tube
Comparison of equation (69) with numerical solutions (after Fujii et al. (1972a))

For the combined forced and natural convection case, an interpolation formula:

$$Nu_d \tilde{R}e_d^{-1/2} = X \left[1 + \frac{0.276 F_d}{X^4} \right]^{1/4} \qquad (69)$$

where $X = 0.90\{1 + G^{-1}\}^{1/3}$ (70)

was proposed. Equation (69) satisfies equations (18) and (68) for the low and high vapour velocity extremes and agrees with the numerical solutions of Fujii et al. (1972a) to within about 5% as illustrated in Fig. 11.

Shekriladze (1977) indicated that the results of Shekriladze and Gomelauri (1966), obtained when using the asymptotic (high condensation rate) shear stress, could be modified so as to be valid also at low condensation rates, and gave equations for the mean Nusselt number which may be expressed* : -

(a)　for vapour downflow over a vertical plate:

$$Nu_L \tilde{R}e_L^{-1/2} = \frac{\sqrt{2}}{3} K_1 \frac{(2 + \sqrt{1 + 16 F_L K^{-4}})}{(1 + \sqrt{1 + 16 F_L K_1^{-4}})^{1/2}} \qquad (71)$$

where $K_1 = (1 + 0.66 G^{-1})^{1/3}$ (72)

(b)　for vertical vapour downflow (without separation) over a horizontal tube

$$Nu_d \tilde{R}e_d^{-1/2} = 0.64 K_2 (1 + \sqrt{1 + 1.69 F_d K_2^{-4}}) \qquad (73)$$

where $K_2 = (1 + G^{-1})^{1/3}$ (74)

It is not clear precisely how equations (71) and (73) were obtained (a key reference appears to have been omitted) but comparison shows that they agree, generally to within a few percent, with the corresponding equations of Fujii and Uehara (1972) and Fujii et al. (1972a), (equations (66) and (69) respectively).

Gaddis (1979) treated the problem of vertical downflow over a horizontal tube in a very comprehensive manner, including the inertia, convection and pressure gradient terms in the momentum equation

for the condensate film and did not assume $u_s \ll U_\infty$. Potential flow was assumed outside the vapour boundary layer. Nusselt numbers at the top stagnation point were obtained numerically for a range of conditions corresponding to steam, liquid metals and viscous fluids.

Mean Nusselt numbers for the whole tube were also calculated where not precluded by boundary layer separation. Under certain conditions, the condensate film thickness increased rapidly with angle on the lower part of the tube which also limited the range of the solution. Ferreira (1973) had earlier considered this case on essentially the same basis as Gaddis. Ferriera, however, was primarily concerned with non-condensing gas effects. His calculations for steam and steam-air mixtures were carried out only at one Reynolds number and no reference to the infinite film thickness phenomenon noted by Gaddis was made.

Fujii et al. (1979) and Fujii (1981) treated the horizontal tube problem more generally by considering vapour approach velocities other than vertically downwards. Inertia, convection and pressure gradient terms were, however, neglected. The surface shear stress was obtained on the basis of an integral theory of Truckenbrodt (1956) which also enabled determination of the angle at which vapour boundary layer separation occurred. Various flow regimes outside the vapour boundary layer were considered, the limiting cases being regarded as potential flow and 'Roshko' flow. The latter is a streamwise velocity distribution based on pressure measurements of Roshko (1954) for flow over a porous tube with suction. Local and mean Nusselt numbers were obtained numerically for uniform surface heat flux as well as for uniform wall temperature. As pointed out earlier in relation to natural convection condensation, and discussed by Lee and Rose (1982), the mean Nusselt number for the case of uniform heat flux is less reliable since the

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*Shekriladze's (1977) equation would, in fact, give K_1 in the denominator of equation (71) to the power one. This is thought to be a printing error.
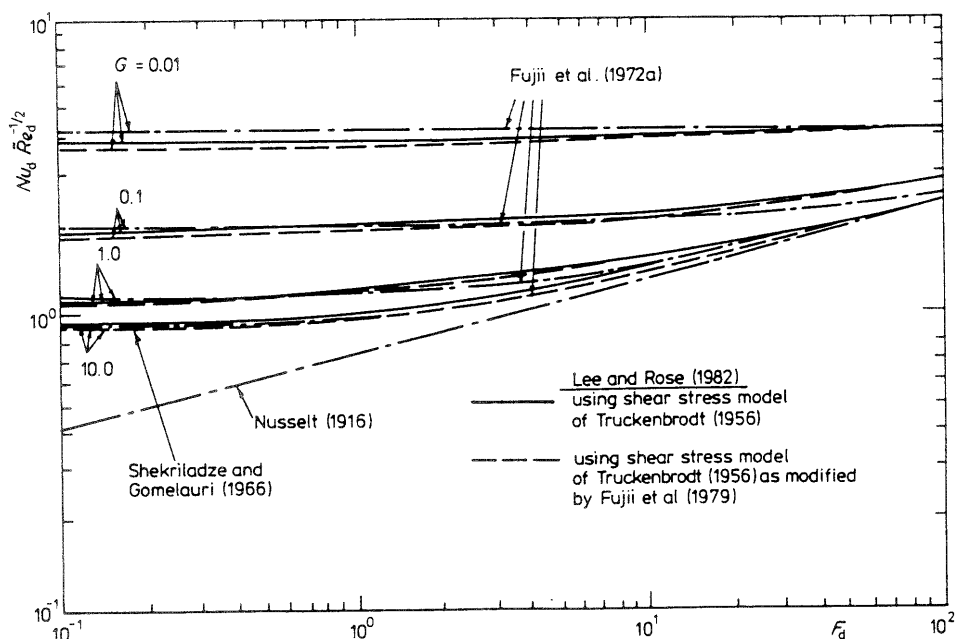
Fig. 12 Condensation Heat-Transfer in Downflow over a Horizontal Tube
Results re-calculated by Lee and Rose (1982) using the method of Fujii et al. (1979) (after Lee and Rose (1982))

average value is more strongly affected by the rearmost part of the tube where the theory is least accurate. For uniform wall temperature and vertical vapour downflow, the results of Fujii et al. (1979) appeared to differ substantially from the earlier integral theory of Fujii et al. (1972) where vapour boundary layer separation was not provided for.

Lee and Rose (1982) noted an error and repeated the calculations of Fujii et al. (1979) for the case of the vertical vapour downflow. Their results showed that the more elaborate approach gave Nusselt numbers in quite close agreement with those of the earlier theory (see Fig. 12). On the basis of their solutions, and neglecting heat-transfer beyond the separation point, Lee and Rose proposed a conservative equation for the mean Nusselt number :

$$Nu_d \overline{Re}_d^{-1/2} = \xi(1 + 0.281 F_d/\xi^4)^{1/4} \qquad (75)$$

where $\xi = 0.88(1 + 0.74 G^{-1})^{1/3}$ $\qquad$ (76)

Equation (75) has the correct behaviour for the limiting cases of low velocity $(F_d \to \infty)$ and high velocity, high condensation rate $(F_d \to 0, G \to \infty)$, and agrees well with the numerical solutions for intermediate values.

For the case of vertical vapour downflow, Rose (1984) examined in some detail the effect of including the pressure gradient in the momentum equation for the condensate film. In other respects the problem was simplified by ignoring inertia and convection

terms, by using the potential flow velocity distribution outside the vapour boundary layer, by taking the asymptotic (infinite condensation rate) value for the condensate surface shear stress and by taking $U' > u_\delta$. It was shown that when

$$\rho_v U_\infty^2 / \rho g d > 1/8 \qquad (77)$$

the rate of increase of film thickness with angle became infinite at some position on the lower half of the tube (where the pressure gradient opposes the shear stress and gravity) so that solutions could not be obtained for the whole tube. Calculated film thickness profiles are given in Fig. 13. (It is interesting to note that similar results had been obtained in some cases by Gaddis (1979) as noted above.) The angle, Φ_c, at which the condition occurred was given, to within 0.6%, by

$$\Phi_c = \arccos\left\{\frac{-(1 + 21.5\,\theta P^{*1.05})}{(1 + 21.5 P^{*1.05})}\right\} \qquad (78)$$

where $P^* = \rho_v h_{fg} \mu / \rho k \Delta T$ $\qquad$ (79)

$\qquad \theta = F_d/8P^* = \rho g d/8\rho_v U_\infty^2 \leqslant 1$ $\qquad$ (80)

For $\theta > 1$ $(\rho_v U_\infty^2 / \rho g d < 1/8)$, where solutions could be obtained for the whole surface, the mean Nusselt number for the upper half of the tube was increased and that for the lower half decreased by including the pressure gradient term. The mean coefficient for the whole tube was, however, generally within 1% of that found when ignoring the pressure term. For the general case Rose (1984) gave :

$$Nu_d\tilde{R}e_d^{-1/2}$$

$$= \frac{0.64(1+1.81P^*)^{0.209}(1+G^{-1})^{1/3}+0.728F_d^{1/2}}{(1+3.51F_d^{0.53}+F_d)^{1/4}}$$

$$(81)$$

Equation (81) includes effects of pressure gradient, shear stress and gravity and includes a correction for the surface shear stress approximation. It approaches the Nusselt result at low vapour velocity and for high vapour velocity takes account only of the heat transfer for the upstream half of the tube and should therefore be conservative.

For the case considered earlier by Shekriladze and Gomelauri (1966) (asymptotic shear stress and neglect of pressure gradient), Rose (1984) obtained

$$Nu_d\tilde{R}e_d^{-1/2} = \frac{0.9+0.728F_d^{1/2}}{(1+3.44F_d^{1/2}+F_d)^{1/4}} \qquad (82)$$

which agreed with the numerical solutions for all F_d to within 0.4%.
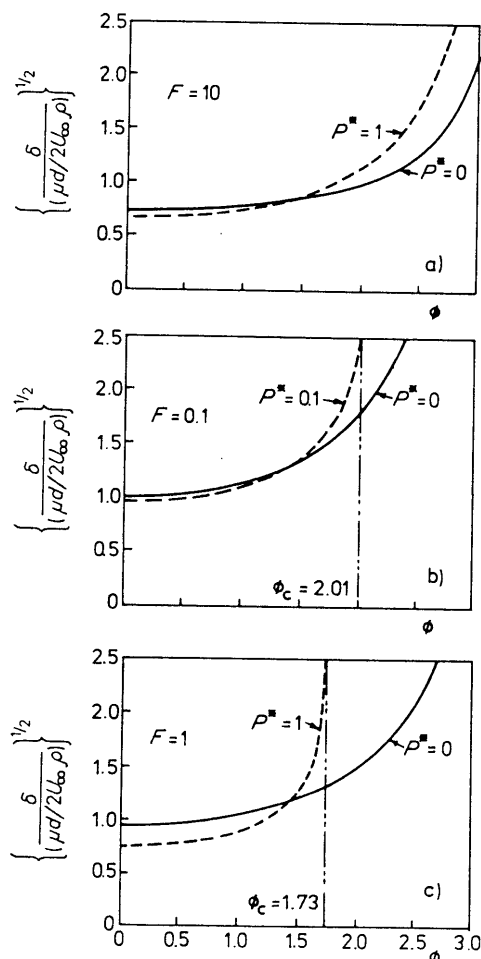
Honda and Fujii (1984) undertook a detailed

Fig. 13   Condensation Heat-Transfer in Downflow over a Horizontal Tube
Profiles of condensate film thickness (after Rose (1984))

"conjugate" solution of the horizontal tube problem in which heat transfer in the condensate film and tube wall were considered simultaneously, with prescribed conditions in the vapour and coolant. The coolant-side heat-transfer coefficient was considered uniform around the tube surface. The approach to the vapour-side problem was much the same as in (Fujii et al. 1979). In view of the fact that both outer tube surface and heat flux varied with angle around the tube, two definitions of average vapour-side heat-transfer coefficient were adopted: (1) the average heat flux divided by the average vapour-side temperature difference, and (2) that obtained by subtracting coolant-side and wall resistances (for uniform radial conduction) from the overall resistance (overall temperature difference divided by mean heat flux).

Honda and Fujii (1984) obtained numerical solutions for steam, R113 and R114. Their main conclusions were that, while differences of local behaviour were apparent, the mean vapour-side coefficient was not strongly dependent on the vapour flow direction and that the two types of mean coefficient did not differ greatly. The effect of variable wall temperature was generally to reduce the mean outside coefficient but, as expected, was small for the case where the condensate resistance dominates (refrigerant) so that the outside wall temperature approaches more closely the uniform (with angle) coolant temperature as found experimentally by Lee et al. (1983).

Summarising, forced convection condensation is appreciably more complicated than the natural convection problem. The main complicating factors are the importance of the vapour shear stress and, for the case of the tube, the fact that solutions cannot be obtained for the whole surface in the presence of vapour boundary layer separation or condensate film instability. However, inertia and convection effects in the condensate film can, for practical purposes, generally be neglected. Also for forced convection, the proportion of the heat transfer carried by the upstream part of the tube where solutions can be obtained, is greater, so that inaccuracies resulting from approximations for the downstream part have smaller effect on the mean coefficient for the whole tube. While some of the more detailed numerical approaches of Gaddis (1979) and Honda and Fujii (1984) may be more complete, equation (81) should provide an adequate value of the vapour-side coefficient for a horizontal tube for most practical purposes. This equation is correct at low vapour velocity and conservative at high vapour velocity where account is only taken of heat transfer on the upper half of the tube.

Effects of superheat and variable properties

Minkowycz and Sparrow (1966), for the case of natural convection condensation on a vertical flat plate, evaluated the effect of vapour superheat by including the energy equation for the vapour along with equations (1-12). Numerical results were obtained for steam which showed that superheats of 56 K and 220 K give heat flux enhancements less than about 1% and 5% respectively. Similar results were reported for local coefficients in forced convection condensation of superheated steam on a horizontal cylinder by Ferreira (1973).

Only a few investigations of the effects of fluid property variations have been made. Minkowycz and Sparrow (1966) solved numerically the Nusselt problem for the vertical flat plate with quiescent. They made calculations for T_V in the range 27°C to 100°C and for values of $(T_V - T_W)$ between 1 K and 44 K and concluded that when the condensate viscosity, density and thermal conductivity in the Nusselt theory were evaluated at a reference temperature

$$T_r = T_W + 0.31(T_V - T_W) \qquad (83)$$

the heat-transfer results virtually coincided with the numerical variable property values.

Poots and Miles (1967) carried out numerical solutions for condensation of steam at atmospheric pressure and for values of $T_V - T_W$ in the range 0 to 100 K. A reference temperature expression was obtained which involved three coefficients. Values of the coefficients were determined for use with equations of Nusselt (1916), Rohsenow (1956) and Chen (1961a) for the flat plate free convection problem.

Denny and Mills (1969a) considered the forced convection, vertical plate, variable property problem and obtained numerical solutions for ten condensing fluids for various vapour temperatures, velocities and values of $T_V - T_W$. Mean values of the constant β in the reference temperature equation

$$T_r = T_W + \beta(T_V - T_W) \qquad (84)$$

for use with an approximate equation based on the Nusselt (1916) and Shekriladze and Gomelauri (1966) assumptions were given. Values of β ranged from 0.07 for carbon tetrachloride to 1.0 for propane with a value of 0.33 for steam. It was suggested that the same reference temperatures might also be adequate for the horizontal tube case.

Conclusion

For natural convection condensation and for both vertical plate and horizontal tube, more detailed analyses have shown that the Nusselt number for the condensate film, normalised with respect to the value given by the simple Nusselt theory, can be expressed

as a function of three parameters : -

(i) $k\Delta T/\mu h_{fg}$, which arises from consideration of inertia effects,

(ii) $c_P\Delta T/h_{fg}$, which is due to convection, (note that the ratio of (ii) to (i) above is the Prandtl number, which is generally used with either one or other of the parameters in presenting results),

(iii) $(\rho\mu/\rho_V\mu_V)^{1/2}$, which results from inclusion of the effect of surface shear stress by matching shear stresses on either side of the vapour-condensate interface. When the shear stress is approximated by the asymptotic (infinite condensation rate) value, this parameter is no longer present. Moreover, it has been found that the effect of interface shear stress is only significant for condensation rates sufficiently high to ensure the validity of the approximation.

In most practical cases, the Nusselt result should be adequate. However, relatively simple, accurate expressions (equations (30) and (32)) are available which cover the more extreme cases where effects of inertia, convection and vapour shear stress are significant.

For forced convection condensation, in most practical circumstances, inertia and convection have small effect on the heat-transfer coefficient for the thinner condensate film. This is fortunate since other complicating factors are present, particularly in the case of the horizontal tube. When inertia and convection effects are omitted it is found, for the horizontal plate (i. e. in the absence of gravity), that

$$Nu_x \tilde{Re}_x^{-1/2} = \Phi\left\{ \frac{k\Delta T}{\mu h_{fg}}\left[\frac{\rho\mu}{\rho_V\mu_V} \right]^{1/2}, \frac{k\Delta T}{\mu h_{fg}} \right\} \qquad (85)$$

The parameter $\dfrac{k\Delta T}{\mu h_{fg}}\left[\dfrac{\rho\mu}{\rho_V\mu_V} \right]^{1/2}$ arises from the condition of continuity of shear stress across the interface and $k\Delta T/\mu h_{fg}$ from velocity continuity. At higher values of $\dfrac{k\Delta T}{\mu h_{fg}}\left[\dfrac{\rho\mu}{\rho_V\mu_V} \right]^{1/2}$, when the surface shear stress can be approximated by the infinite condensation rate asymptote, this parameter is no longer present. Furthermore, at low values of $k\Delta T/\mu h_{fg}$ when the condensate surface velocity can be neglected when considering the vapour boundary layer, this parameter also has negligible effect on the heat transfer. Evidently both conditions can be satisfied at large values of $(\rho\mu/\rho_V\mu_V)^{1/2}$ (i.e. provided the condensing vapour is well below its critical pressure). When neither parameter plays a significant role

$$Nu_x \tilde{Re}_x^{-1/2} = \text{constant} \qquad (86)$$

as given in equation (39), and the heat-transfer coefficient for the condensate is proportional to the square root of the vapour velocity. This approximation holds with good accuracy when $G > 5$ and $k\Delta T/\mu h_{fg} < 0.01$, and is reasonably good when $G > 1$ and $k\Delta T/\mu h_{fg} < 0.1$. The latter conditions are satisfied

in many practical situations.

For the vertical plate, when gravity enters the problem, the additional parameter $F_x(=\mu h_{fg}gx/k\Delta TU_\infty^2)$ arises. Whth the above-mentioned approximations (i. e. those which lead to equation (86) for the horizontal plate), we have, for the vertical plate,

$$Nu_x\tilde{Re}_x^{-1/2}= \Psi(F) \tag{87}$$

as given in equation (48).

For the case of the horizontal tube, additional complications arise. The effect of the pressure gradient enhances heat transfer over the forward part of the tube but, under certain circumstances, leads to an instability problem on the rear part where the pressure gradient opposes the vapour drag force. Also, in general, vapour boundary-layer separation occurs, making the prediction of heat transfer for the rear part of the tube less certain. An approximate result (equation (81)) has been given which takes account of the effects of vapour shear stress, gravity and pressure gradient in the condensate film. This approaches the Nusselt expression for low vapour velocity and neglects heat transfer to the downstream half of the tube at high velocity. The equation should therefore be conservative at high velocity, but not excessively so, since a larger proportion of the heat transfer occurs over the upstream half of the tube under forced convection conditions.

Fig. 14 compares data of 12 investigations using 4 fluids with equation (81). The theoretical lines are for the limiting values of G and P^* of the experiments. It may be noted that the lower bound of the theoretical result overpredicts $Nu_d\tilde{Re}_d^{-1/2}$ for the high velocity

steam data. This is thought to result from the relatively strong variation in wall temperature around the tube which occurs in the case of steam where the thermal resistance on the condensing side is relatively small. It may be noted that the 'conjugate' theory of Honda and Fujii (1984) predicts lower values in these circumstances. In practice however, when the vapour-side resistance is not large, a moderate overestimate of the vapour-side heat-transfer coefficient has relatively small effect on the calculated overall coefficient.

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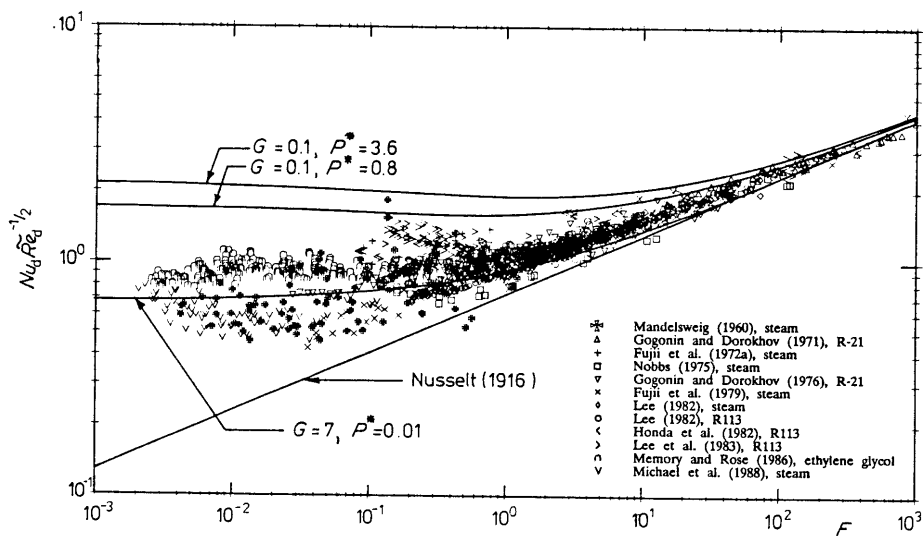
Fig. 14 Condensation in Downflow over Horizontal Tubes Comparison of equation (81) with experimental data for extreme values of G and P^*

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