Filomat 29:8 (2015), 1893–1900
DOI 10.2298/FIL1508893A

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Further Discussion on Modified Multivalued α_*-ψ-contractive Type Mapping

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Abstract. In this paper, we investigate the existence of a fixed point for modified multivalued α_*-$\psi-$contractive type mapping in the context of complete metric space. We also construct some examples to illustrate the main result. Our results extend, improve and generalize the results on the topic in the literature.

1. Introduction

Recently, Samet *et al.* [18] introduced the (single-valued) α-ψ-contractive mappings via α-admissible self mappings. In this interesting paper [18], the authors examined the existence and uniqueness of a fixed point of such mappings in the frame of complete metric space. This is one of the significant reports in the recent decade, since the announced results of the paper [18] concluded several existing fixed point results, including well-known Banach contraction mapping principle, as corollaries. Following this initial paper, a number of publications appeared on this subject, see e.g. [1–17]. Among all, we mention the result of Salimi *et al.* [16] in which the authors introduced the notion of modified α-ψ-contractive mappings by the help of another auxiliary function η. As it is expected, the authors [16] established some fixed point theorems for such (single-valued) mappings in the setting of complete metric spaces. Later, Mohammadi and Rezapour [15] and independently, Berzig and Karapınar [9], noticed that modified (single-valued) α-ψ-contractive type mappings can be considered as a particular case of α-ψ-contractive type mappings. After this observation, it is quite natural to ask that whether analog of the results of Mohammadi and Rezapour [15], Berzig and Karapınar [9] in the case of multivalued α_*-$\psi-$contractive type mapping can be obtained.

In this paper, we show that the notion of modified multivalued α_*-$\psi-$contractive type mapping (also called as, multivalued α_*-η-ψ-contractive type mapping) can not be reduced into multivalued α_*-$\psi-$contractive type mapping. In other words, the notion of multivalued α_*-η-ψ-contractive type mappings is a proper generalization of the concept of multivalued α_*-$\psi-$contractive type mappings. In addition, we investigate the existence of a common fixed point theorem for a sequence of multivalued α_*-η-ψ-contractive type mappings.

2010 *Mathematics Subject Classification*. Primary 47H10, Secondary 54H25
Keywords. α_*-admissible maps, α_*-admissible maps with respect to η
Received: 06 November 2013; Accepted: 12 April 2014
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2. Preliminaries

Throughout the paper, the symbols $\mathbb{R}$ and $\mathbb{N}$ denote the real numbers and the natural numbers, respectively. Furthermore, we set $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Let Ψ denote the class of nondecreasing functions, $\psi : [0,\infty) \to [0,\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$, where ψ^n is the n^{th} iterate of the function ψ. It is very well known that for each $\psi \in \Psi$, we have $\psi(t) < t$ for all $t > 0$ and $\psi(0) = 0$. Let (X, d) be a metric space. We denote by $CB(X)$ the class of all nonempty closed and bounded subsets of X and by $CL(X)$ the class of all nonempty closed subsets of X. For every $A, B \in CL(X)$, let

$$H(A,B) = \begin{cases} \max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\}, & \text{if the maximum exists;} \\ \infty, & \text{otherwise.} \end{cases}$$

Such a map H is called generalized Hausdorff metric induced by the metric d.

Samet *et al.* [18] introduced the notions of α-admissible as follows.

Definition 2.1. *[18] Let $\alpha : X \times X \to [0,\infty)$ be a function. A mapping $G : X \to X$ is called α-admissible if*

$$x, y \in X,\ \alpha(x,y) \geq 1 \Rightarrow \alpha(Gx, Gy) \geq 1. \tag{1}$$

In what follows, we state the notion of (single-valued) α-ψ-contractive mappings.

Definition 2.2. *[18] Let (X, d) be a metric space. A mapping $G : X \to X$ is called α-ψ-contractive type mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that*

$$\alpha(x,y)d(Gx,Gy) \leq \psi(d(x,y)) \text{ for each } x, y \in X. \tag{2}$$

The simplicity and applicability of these new notions attracted the attention of many researchers working in this area and many interesting fixed point theorems appeared in the literature, see for example [2–5, 7, 8, 10, 12–14, 16]. One of the earlier generalizations on α-ψ contractive mappings was given by Karapınar *et al.* [12].

Theorem 2.3. *[12] Let (X, d) be a metric space, $\alpha : X \times X \to [0,\infty)$ be mapping and G be an α-admissible selfmap on X such that*

$$\alpha(x,y)d(Gx,Gy) \leq \psi(M(x,y)), \tag{3}$$

for all $x, y \in X$, where $\psi \in \Psi$ and

$$M(x,y) = \left\{d(x,y), \frac{d(x,Gx) + d(y,Gy)}{2}, \frac{d(x,Gy) + d(y,Gx)}{2}\right\}.$$

Suppose that there exists $x_0 \in X$ such that $\alpha(x_0, Gx_0) \geq 1$. If G is continuous or for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}_0$ and $x_n \to x$ we have $\alpha(x_n, x) \geq 1$ for all $n \in \mathbb{N}_0$. Then G has a fixed point.

Now, we recall the notions of α-admissible mapping with respect to η and modified α-ψ-contractive type mapping that was introduced by Salimi *et al*. [16].

Definition 2.4. *[16] Let $\alpha, \eta : X \times X \to [0,\infty)$ be functions. A mapping $G : X \to X$ is called α-admissible with respect to η, if*

$$x, y \in X,\ \alpha(x,y) \geq \eta(x,y) \Rightarrow \alpha(Gx,Gy) \geq \eta(Gx,Gy).$$

Definition 2.5. *[16] Let (X, d) be a metric space. A mapping $G : X \to X$ is called modified α-ψ-contractive type mapping if there exist three functions $\alpha, \eta : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that*

$$\text{for } x, y \in X \text{ with } \alpha(x,y) \geq \eta(x,y) \Rightarrow d(Gx,Gy) \leq \psi(d(x,y)). \tag{4}$$

As it is mentioned above, in [9, 15], the authors point out the fact that the notion of modified (single-valued) α-ψ-contractive type mappings can be considered as a particular case of the concept of (single-valued) α-ψ-contractive type mappings. Hence, the announced results in [16] coincide with the related fixed points results of Samet *et al.* [18], and Karapınar *et al.* [12]. More precisely, in [9, 15] the authors showed that if we define

$$\beta(x,y) = \begin{cases} 1, & \text{if } \alpha(x,y) \geq \eta(x,y); \\ 0, & \text{otherwise}, \end{cases}$$

then (4) becomes

$$\text{for } x, y \in X \ \beta(x,y)d(Gx,Gy) \leq \psi(d(x,y)). \tag{5}$$

Further, G is β-admissible. If we look at (5), we see that indeed we have two cases.

(i) when $\beta = 1$ we have

$$d(Gx,Gy) \leq \psi(d(x,y)). \tag{6}$$

(ii) when $\beta = 0$ we have

$$0 \cdot d(Gx,Gy) \leq \psi(d(x,y)), \ i.e. \ 0 \leq \psi(d(x,y)). \tag{7}$$

Here note that $d(x,y) < \infty$ for all $x, y \in X$. Therefore, from

$$0 \cdot d(Gx,Gy) \leq \psi(d(x,y))$$

we get

$$0 \leq \psi(d(x,y)).$$

Asl *et al.* [8] extended these notions of α-admissible and α-ψ-contractive mappings to multivalued α_*-ψ-contractive mappings as follows.

Definition 2.6. *[8] Let $\alpha : X \times X \to [0,\infty)$ be a function. A mapping $G : X \to CL(X)$ is α_*-admissible if $\alpha(x,y) \geq 1 \Rightarrow \alpha_*(Gx,Gy) \geq 1$, where*

$$\alpha_*(Gx,Gy) = \inf\{\alpha(a,b) : a \in Gx, b \in Gy\}.$$

Definition 2.7. *[8] Let (X,d) be a metric space. A mapping $G : X \to CL(X)$ is called multivalued α_*-ψ−contractive type mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that*

$$\alpha_*(Gx,Gy)H(Gx,Gy) \leq \psi(d(x,y)) \tag{8}$$

for all $x, y \in X$.

Hussain *et al*. [10] extended the notions of α-admissible with respect to η and modified α-ψ-contractive type mappings to multivalued mappings in following way.

Definition 2.8. *[10] Let $\alpha, \eta : X \times X \to [0,\infty)$ be functions, where η is bounded. We say that $G : X \to CL(X)$ is an α_*-admissible mapping with respect to η if we have*

$$x, y \in X, \ \ \alpha(x,y) \geq \eta(x,y) \Rightarrow \alpha_*(Gx,Gy) \geq \eta_*(Gx,Gy), \tag{9}$$

where $\alpha_*(Gx,Gy) = \inf\{\alpha(a,b) : a \in Gx, b \in Gy\}$ and
$\eta_*(Gx,Gy) = \sup\{\eta(a,b) : a \in Gx, b \in Gy\}$.

Definition 2.9. *[10] Let (X, d) be a metric space. A mapping $G : X \to CL(X)$ is called modified multivalued α_*-ψ−contractive type mapping (or, multivalued α_*-η-ψ-contractive type mapping) if there exist three functions $\alpha, \eta : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that*

$$\text{for } x, y \in X \text{ with } \alpha_*(Gx, Gy) \geq \eta_*(Gx, Gy) \Rightarrow H(Gx, Gy) \leq \psi(d(x, y)). \tag{10}$$

By considering the remarks in [9, 15], at the first glance, one would expect that the notion of modified multivalued α_*-η-ψ-contractive type mapping should be particular case of the concept of multivalued α_*-ψ-contractive type mapping. On the other hand, if we look carefully at the contractive conditions (8) and (10), we see that it depends upon the metric H. We observe that if we consider a map $G : X \to CB(X)$ then $H(Tx, Ty) < \infty$ for all $x, y \in X$ and, in this case the fixed point theorems for multivalued α_*-η-ψ-contractive type mappings may follow from the corresponding theorems for multivalued α_*-ψ-contractive type mappings. Note that if T is a single valued map, then $H(Tx, Ty) = d(x, y) < \infty$, for all $x, y \in X$. This is inconsistent with the observations in [9, 15]. For the case $G : X \to CL(X)$, the value of $H(Tx, Ty)$ may be infinite for some choice of $x, y \in X$. Consequently, a multivalued α_*-η-ψ-contractive type mapping may not imply a multivalued α_*-ψ-contractive type mapping, in general. Indeed, if one would define

$$\beta(x, y) = \begin{cases} 1, & \text{if } \alpha(x, y) \geq \eta(x, y); \\ 0, & \text{otherwise,} \end{cases}$$

then (10) appears to reduce into

$$\text{for } x, y \in X \; \beta(x, y)H(Gx, Gy) \leq \psi(d(x, y)), \tag{11}$$

as in [9, 15], we conclude again that G is α_*-admissible mapping with respect to η. Now if we look at (11) we again have two cases:

(i) when $\beta(x, y) = 1$ we have

$$H(Gx, Gy) \leq \psi(d(x, y)),$$

(ii) when $\beta(x, y) = 0$ we have

$$0 \cdot H(Gx, Gy) \leq \psi(d(x, y)). \tag{12}$$

Now here is the point; when G is bounded then

$$0 \cdot H(Gx, Gy) \leq \psi(d(x, y))$$

implies that $0 \leq \psi(d(x, y))$. Otherwise, it is not true. In other words, when G is not bounded it is not possible to define β in above manner.

Following example substantiate our claim.

Example 2.10. *Let $X = \mathbb{R}$ be endowed with the usual metric d. Define $G : X \to CL(X)$ by*

$$Gx = \begin{cases} (-\infty, x] & \text{if } x < 0 \\ [\frac{x}{2}, \infty) & \text{if } x \geq 0 \end{cases}$$

and $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 & \text{if } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and $\eta : X \times X \to [0, \infty)$ by $\eta(x, y) = \frac{1}{2}$ for each $x, y \in X$. Take $\psi(t) = \frac{t}{2}$ for each $t \geq 0$. If $x, y \geq 0$, then $\alpha_*(Gx, Gy) = 1 > \eta_*(Gx, Gy) = \frac{1}{2}$ which implies

$$H(Gx, Gy) = \frac{1}{2}|x - y| = \psi(d(x, y))$$

for otherwise, we have $\alpha_*(Gx, Gy) = 0 < \eta_*(Gx, Gy) = \frac{1}{2}$. *Thus G is modified* α_*-η-ψ-*contractive type mapping. Observe that* $\alpha_*(G(-1), G(1)) = 0$ *and* $H(G(-1), G(1)) = \infty$. *Thus (8) doesn't holds when* $x = -1, y = 1$ *and consequently G is not a multivalued* α_*-ψ-*contractive type mapping.*

Therefore, it is worthwhile to consider fixed point theorems for multivalued α_*-η-ψ-contractive type mappings. Ali *et al*. [5] generalized the Definition 2.8 in the following way.

Definition 2.11. *[5] Let* $G : X \to CL(X)$ *be a multivalued mapping on a metric space* (X, d). *Let* $\alpha, \eta : X\times X \to [0, \infty)$ *be two functions. We say that G is generalized* α_*-*admissible mapping with respect to* η, *if we have*

$$x, y \in X,\ \ \alpha(x, y) \geq \eta(x, y) \Rightarrow \alpha(u, v) \geq \eta(u, v)\ \forall\ u \in Gx \text{ and } v \in Gy. \tag{13}$$

Lemma 2.12. *[2] Let* (X, d) *be a metric space and* $B \in CL(X)$. *Then for each* $x \in X$ *with* $d(x, B) > 0$ *and* $q > 1$, *there exists an element* $b \in B$ *such that*

$$d(x, b) < qd(x, B). \tag{14}$$

3. Main Results

We begin this section with following definition.

Definition 3.1. *Let* $\{G_i : X \to CL(X)\}_{i=1}^{\infty}$ *be a sequence of multivalued mappings on a metric space* (X, d). *Let* $\alpha, \eta : X \times X \to [0, \infty)$ *be two functions. We say that the sequence* $\{G_i\}$ *is* α_*-*admissible with respect to* η, *if we have*

$$x, y \in X,\ \ \alpha(x, y) \geq \eta(x, y) \Rightarrow \alpha(u, v) \geq \eta(u, v)\ \forall\ u \in G_i x \text{ and } v \in G_j y, \tag{15}$$

for each $i, j \in \mathbb{N}$. *In case when* $\alpha(x, y) = 1$ *for all* $x, y \in X$, *the sequence* $\{G_i\}$ *is a* η_*-*subadmissible. In case when* $\eta(x, y) = 1$ *for all* $x, y \in X$, *the sequence* $\{G_i\}$ *is* α_*-*admissible.*

Theorem 3.2. *Let* (X, d) *be a complete metric space and let the sequence* $\{G_i : X \to CL(X)\}_{i=1}^{\infty}$ *be* α_*-*admissible with respect to* η *such that*

$$x, y \in X,\ \alpha(x, y) \geq \eta(x, y) \Rightarrow H(G_i x, G_j y) \leq \psi(d(x, y)), \tag{16}$$

for each $i, j \in \mathbb{N}$ *and* ψ *be strictly increasing function in* Ψ. *Assume that following conditions hold:*

(i) there exist $x_0 \in X$ *and* $y_i \in G_i x_0$ *for each* $i \in \mathbb{N}$ *such that* $\alpha(x_0, y_i) \geq \eta(x_0, y_i)$;

(ii) if $\{x_i\}$ *is a sequence in X with* $x_i \to x$ *as* $i \to \infty$ *and* $\alpha(x_{i-1}, x_i) \geq \eta(x_{i-1}, x_i)$ *for each* $i \in \mathbb{N}$, *then we have* $\alpha(x_{i-1}, x) \geq \eta(x_{i-1}, x)$ *for each* $i \in \mathbb{N}$.

Then, the mappings G_i *for* $i \in \mathbb{N}$, *have a common fixed point.*

Proof. By hypothesis, there exist $x_0 \in X$ and $x_1 \in G_1 x_0$ such that $\alpha(x_0, x_1) \geq \eta(x_0, x_1)$. If $x_1 \in G_i x_1$ for each $i \in \mathbb{N}$, then x_1 is a common fixed point of G_i. Let $x_1 \notin G_2 x_1$. Then from (16), we have

$$0 < d(x_1, G_2 x_1) \leq H(G_1 x_0, G_2 x_1) \leq \psi(d(x_0, x_1)). \tag{17}$$

For $q > 1$ by Lemma 2.12, there exists $x_2 \in G_2 x_1$ such that

$$0 < d(x_1, x_2) < qd(x_1, G_2 x_1) \leq qH(G_1 x_0, G_2 x_1) \leq q\psi(d(x_0, x_1)). \tag{18}$$

Since, ψ is strictly increasing, from (18), we have

$$\psi(d(x_1, x_2)) < \psi(q\psi(d(x_0, x_1))). \tag{19}$$

Put $q_1 = \frac{\psi(q\psi(d(x_0,x_1)))}{\psi(d(x_1,x_2))}$. Then $q_1 > 1$. Since the sequence $\{G_i\}_{i=1}^{\infty}$ is α_*-admissible with respect to η, then $\alpha(x_1, x_2) \geq \eta(x_1, x_2)$. If $x_2 \in G_i x_2$ for each $i \in \mathbb{N}$, then x_2 is a common fixed point of G_i. Let $x_2 \notin G_3 x_2$. Then from (16), we have

$$0 < d(x_2, G_3x_2) \leq H(G_2x_1, G_3x_2) \leq \psi(d(x_1, x_2)). \tag{20}$$

For $q_1 > 1$ by Lemma 2.12, there exists $x_3 \in G_3x_2$ such that

$$\begin{aligned} 0 < d(x_2, x_3) &< q_1 d(x_2, G_3x_2) \\ &\leq q_1 H(G_2x_1, G_3x_2) \\ &\leq q_1\psi(d(x_1, x_2)) = \psi(q\psi(d(x_0, x_1))). \end{aligned} \tag{21}$$

Since, ψ is strictly increasing, from (21), we have

$$\psi(d(x_2, x_3)) < \psi^2(q\psi(d(x_0, x_1))). \tag{22}$$

Put $q_2 = \frac{\psi^2(q\psi(d(x_0,x_1)))}{\psi(d(x_2,x_3))}$. Then $q_2 > 1$. Continuing in the same way, we get a sequence $\{x_i\}$ in X such that $x_i \in G_i x_{i-1}$, $x_i \neq x_{i-1}$, $\alpha(x_{i-1}, x_i) \geq \eta(x_{i-1}, x_i)$ and

$$d(x_i, x_{i+1}) < \psi^{i-1}(q\psi(d(x_0, x_1))) \text{ for each } i \in \mathbb{N}. \tag{23}$$

Let $j > i$, we have

$$d(x_i, x_j) \leq \sum_{n=i}^{n=j-1} d(x_n, x_{n+1}) < \sum_{n=i}^{n=j-1} \psi^{n-1}(q\psi(d(x_0, x_1))).$$

Since $\psi \in \Psi$, then we have

$$\lim_{i,j\to\infty} d(x_i, x_j) = 0. \tag{24}$$

Hence $\{x_{i-1}\}$ is a Cauchy sequence in (X, d). By completeness of (X, d), there exists $x^* \in X$ such that $x_{i-1} \to x^*$ as $i \to \infty$. By hypothesis (ii), we have $\alpha(x_{i-1}, x^*) \geq \eta(x_{i-1}, x^*)$ for each $i \in \mathbb{N}$. From (16), for each $n = 1, 2, \cdots$, we have

$$d(x_i, G_n x^*) \leq H(G_i x_{i-1}, G_n x^*) \leq \psi(d(x_{i-1}, x^*)).$$

Letting $i \to \infty$ in above inequality, we have $d(x^*, G_n x^*) = 0$ for each $n \in \mathbb{N}$. Thus, x^* is a common fixed point of $\{G_i\}$. □

Let us take $G_i = G$ for each $i \in \mathbb{N}$, then Theorem 3.2 reduces to following result:

Theorem 3.3. *Let (X, d) be a complete metric space and let $G : X \to CL(X)$ be a generalized α_*-admissible mapping with respect to η such that*

$$x, y \in X,\ \alpha(x, y) \geq \eta(x, y) \Rightarrow H(Gx, Gy) \leq \psi(d(x, y)), \tag{25}$$

where ψ is strictly increasing function in Ψ. Assume that following conditions hold:

(i) there exist $x_0 \in X$ and $x_1 \in Gx_0$ such that $\alpha(x_0, x_1) \geq \eta(x_0, x_1)$;

(ii) if $\{x_i\}$ is a sequence in X with $x_i \to x$ as $i \to \infty$ and $\alpha(x_{i-1}, x_i) \geq \eta(x_{i-1}, x_i)$ for each $i \in \mathbb{N}$, then we have $\alpha(x_{i-1}, x) \geq \eta(x_{i-1}, x)$ for each $i \in \mathbb{N}$.

Then, G has a fixed point.

Example 3.4. *Let $X = \mathbb{R}$ be endowed with the usual metric d. Define $G : X \to CL(X)$ by*

$$Gx = \begin{cases} (-\infty, 0] & \text{if } x < 0 \\ \{0, \frac{x}{4}\} & \text{if } 0 \leq x \leq 2 \\ [x^2, \infty) & \text{if } x > 2, \end{cases}$$

and $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} \frac{4}{5} & \text{if } x, y \in [0, 2] \\ \frac{1}{2} & \text{otherwise,} \end{cases}$$

and $\eta : X \times X \to [0, \infty)$ by $\eta(x, y) = \frac{3}{4}$ for each $x, y \in X$. Take $\psi(t) = \frac{t}{2}$ for each $t \geq 0$. Then, for each $x, y \in X$ with $\alpha(x, y) \geq \eta(x, y)$, we have

$$H(Gx, Gy) = \frac{1}{4}|x - y| \leq \psi(d(x, y)).$$

Also, G is generalized α_*-admissible mapping with respect to η. For $x_0 = 1$ and $0 \in Gx_0$ we have $\alpha(1, 0) > \eta(1, 0)$. Moreover, for any sequence $\{x_i\}$ in X with $x_i \to x$ as $i \to \infty$ and $\alpha(x_{i-1}, x_i) \geq \eta(x_{i-1}, x_i)$ for each $i \in \mathbb{N}$, we have $\alpha(x_{i-1}, x) \geq \eta(x_{i-1}, x)$ for each $i \in \mathbb{N}$. Therefore, all conditions of Theorem 3.3 are satisfied and G has infinitely many fixed points.

Remark 3.5. *For $G : X \to CL(X)$, contraction condition given in (25) mapping is more general than contraction conditions of following form:*

$$\alpha(x, y)H(Gx, Gy) \leq \psi(d(x, y)), \tag{26}$$

for each $x, y \in X$, where ψ is strictly increasing function in Ψ.

For example, consider G, α, η as defined in Example 3.4. Define $\beta : X \times X \to [0, \infty)$ by

$$\beta(x, y) = \begin{cases} 1 & \text{if } \alpha(x, y) \geq \eta(x, y) \\ 0 & \text{otherwise.} \end{cases}$$

Let $\psi(t) = \frac{t}{2}$. For $x = 2$ and $y = 2.1$, from (26), we have

$$\beta(x, y)H(Gx, Gy) = 0.\infty.$$

As $0.\infty$ is indeterminant form, there is no guaranty, that (26) holds for each $x, y \in X$.

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