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Further evidence for a stiff nuclear equation of state from a transverse-momentum analysis of Ar(1800 MeV/nucleon) + KCl

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The novel momentum analysis technique introduced by Danielewicz and Odyniec can be used to detect and exhibit collective flow in the light system Ar(1800 MeV/nucleon) + KCl where the usual kinetic energy flow analysis fails. The microscopic Vlasov-Uehling-Uhlenbeck theory which includes the nuclear mean field, two-body collisions, and Pauli blocking is used to study this phenomenon. The resulting transverse momentum transfers turn out to be quite sensitive to the nuclear equation of state. From a comparison with experimental data, evidence is presented for a rather stiff nuclear equation of state. The cascade model is unable to describe the data.

Early indications for the occurrence of a collective sidewards flow in relativistic nucleus-nucleus collisions have been reported for asymmetric reactions (C + Ag and Ne + U) in particle track and solid state detector experiments,[1] but only recently has this phenomenon been unambiguously observed in the 4π exclusive event by event analysis[2] of near central collisions of heavy nuclei, i.e., Nb(400 MeV/nucleon) + Nb (Ref. 3) and Ar(770 MeV/nucleon) + Pb.[4] Experimental data for systems as heavy as Au + Au and U + U continue to accumulate and support these results.[5] The collective sidewards flow had first been predicted on the basis of nuclear fluid dynamics.[6] In contrast, microscopic intranuclear cascade models, which have been successful in describing inclusive data,[7] only predict flow when unbound nuclei expand due to Fermi motion.[8,9] On the other hand, the microscopic Vlasov-Uehling-Uhlenbeck (VUU) theory used in the present work and the classical equations of motion approach, both of which incorporate a repulsive nuclear equation of state at high densities, have successfully reproduced the sidewards peaking observed experimentally.[9–11] Light systems have not exhibited any signatures of collective sidewards flow when the kinetic energy flow analysis is applied.[12]

In the present work it is shown that transverse flow effects are predicted even in light systems in a microscopic approach based on the VUU theory.[9,11,13,14] This theory enables simultaneously the investigation of the influence of the nuclear equation of state and the Pauli principle directly within the context of a microscopic model. To study the effects of the nuclear equation of state, or rather the nuclear compressional potential energy $E_{cp}(\rho)$, on the reaction dynamics, we use two distinct forms for the density dependent potential field $U(\rho)=\partial(\rho E_{cp})/\partial\rho$ in the VUU theory:

$$\text{stiff,}\quad U(\rho) = -124\rho/\rho_0 + 70.5(\rho/\rho_0)^2\ \text{MeV}\ , \tag{1a}$$

$$\text{medium,}\quad U(\rho) = -356\rho/\rho_0 + 303(\rho/\rho_0)^{7/6}\ \text{MeV}\ . \tag{1b}$$

These are simplified local Skyrme interactions with compressibility coefficients $K = 380$ and 200 MeV, respectively. In Fig. 1 we plot the compressional energy $E_c(\rho)$, which is the sum of E_{cp} plus the degenerate Fermi energy $E_F(\rho)$, for the stiff ($K = 380$ MeV) and medium ($K = 200$ MeV) Skyrme equations of state and compare them with the equation of state extracted recently from pion multiplicity data.[15] Note that the simplified iterative procedure applied in the recent chemical and cascade model analysis of the GSI-LBL Streamer Chamber group,[15] which was used to extract a nuclear matter equation of state from the differences of the calculated pion multiplicities to the observed pion yields, results in an $E_c(\rho)$ which agrees rather closely with our stiff equation of state. In fact, in the present self-consistent VUU theory this stiff equation of state has simultaneously reproduced the observed pion yields as well as the sidewards flow angular distributions in heavier systems.[9] Medium energy collisions have also been successfully studied within this approach.[14] Recall the Vlasov equation with Uehling-Uhlenbeck's collision integral, which respects the

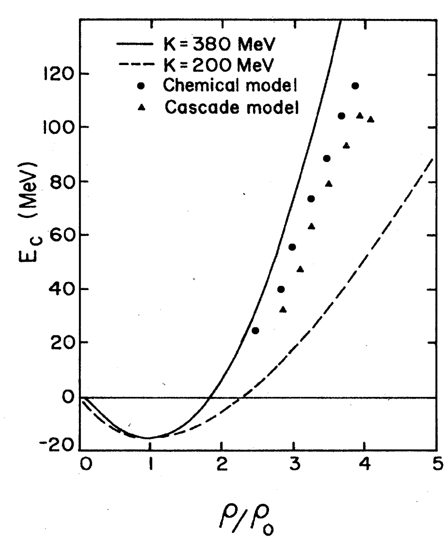

FIG. 1. The nuclear equation of state with $K = 200$ MeV and $K = 380$ MeV as used in the Vlasov-Uehling-Uhlenbeck theory compared with values extracted from pion yields (Ref. 15).

Pauli principle:

$$\frac{\partial}{\partial t}f+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}f-\nabla U\cdot\frac{\partial}{\partial \mathbf{p}}f=-\int\frac{d^3p_2d^3p_1'd^3p_2'}{(2\pi)^6}\sigma v_{12}[ff_2(1-f_1')(1-f_2')-f_1'f_2'(1-f)(1-f_2)]\delta^3(p+p_2-p_1'-p_2') \quad . \quad (2)$$

The classical equations of motion of a large number of marker particles, representing the single particle distribution function $f(r,p,t)$, are integrated numerically to solve the Vlasov equation [the left hand side (LHS) of Eq. (2)] and the collision term is treated in a Monte Carlo framework that is reminiscent of the cascade model.[9,11,13,14] Relativistic kinematics is used throughout the present theory, just as in the cascade model. Protons, neutrons, deltas, and pions of different isospin are included separately with their experimental scattering cross sections.[9]

In the classical equations of motion, and the Vlasov-Uehling-Uhlenbeck theory, the collective flow is caused by a combination of the collisions and the nuclear compressional energy. For central impact parameters in symmetric systems, well defined peaks occur in the flow angular distribution.[2,3,9–11] In asymmetric systems the flow distribution is broad for small impact parameters so that finite flow angles are best observable for intermediate impact parameter collisions.[4,11]

However, for light systems ($A_T\approx A_P\leq 40$) and high energies ($E_{\rm lab}>1$ GeV/N), flow effects are not observed when the standard kinetic energy flow tensor analysis is used:[12]

$$K_{ij}=\Sigma_\nu p_i(\nu)p_j(\nu)/2m(\nu) \quad , \quad (3)$$

where i and j denote the Cartesian components (x,y,z) and ν is a charged particle label. In fact, the experimental flow angular distributions for the reaction Ar (1800 MeV/nucleon, $b<2.4$ fm)+KCl (Refs. 12 and 16) [Fig. 2(a)] are peaked at zero degrees, as the cascade model[7,8] [Fig. 2(b)] predicts. But also the present Vlasov-Uehling-Uhlenbeck approach, which does predict finite flow angles for heavier systems at lower energies,[9] does not yield any observable sidewards maxima in the flow angle distributions [see Figs. 2(c) and 2(d)]; even less so can we distinguish between hard [Fig. 2(c)] and medium [Fig. 2(d)] equations of state when the standard kinetic energy flow tensor analysis is used: All flow angle distributions are peaked at zero degrees. Therefore, one might be tempted to hastily conclude that flow effects do not occur for light systems.

However, Danielewicz and Odyniec[16] have recently proposed a novel transverse momentum analysis technique that provides a much more sensitive test for collective flow. They analyze the transverse momentum spectrum $p_x(y)$ where

$$y=\tfrac{1}{2}\ln(E+p_{\rm par})/(E-p_{\rm par}) \quad (4)$$

is the rapidity, E is the total energy of the fragment, $p_{\rm par}$ is the momentum in the beam (here the $z-$) direction, and p_x is the projection of the transverse momenta into the scattering plane. Danielewicz and Odyniec have been able to determine the scattering plane in the experimental data by controlling the finite multiplicity distortions carefully. They have tested their method by subjecting events generated via the intranuclear cascade model, i.e., events where the actual reaction plane has been given, to their procedure for determining the reaction plane from data and find good agreement. In the following we compare the data in the extracted scattering plane with the theoretical results in the given $(x-z)$ scattering plane.

Danielewicz and Odyniec[16] detected collective flow effects in the streamer chamber data[12] for Ar(1800 MeV/nucleon) +KCl using this technique [see Fig. 3(a)]. There is a transverse momentum accumulation at both the projectile and target rapidities $y=\pm0.86$ in the center of momentum frame. They report[16] that the collective flow effects are weaker than in the hydrodynamic model, but much stronger than in the cascade[7] [see Fig. 3(b)]. It is important to point out that the intranuclear cascade model fails to reproduce the data, even though it appeared to be consistent when the kinetic energy flow analysis had been applied. The highly increased sensitivity of this new technique has more recently been used to predict the presence of collective flow for O (600 MeV/nucleon)+O within the context of the time dependent Dirac equation with relativistic mean field dynamics.[17]

We have applied this novel transverse momentum analysis technique to the Vlasov-Uehling-Uhlenbeck results for the reaction Ar(1800 MeV/nucleon, $b<2.4$ fm)+KCl studied experimentally. We find that the peak in the transverse momentum spectrum $p_x(y)$ depends linearly on the nuclear equation of state: the cascade model predicts $p_x^{\rm max}\approx 25$ MeV/c/nucleon [Fig. 3(b)]; the medium equation of state in the Vlasov-Uehling-Uhlenbeck approach predicts $p_x^{\rm max}\approx 50$ MeV/c/nucleon [Fig. 3(d)]; and the stiff equation of state yields $p_x^{\rm max}\approx 100$ MeV/c/nucleon [Fig. 3(c)]. Only the latter is in agreement with the data. This result is sup-

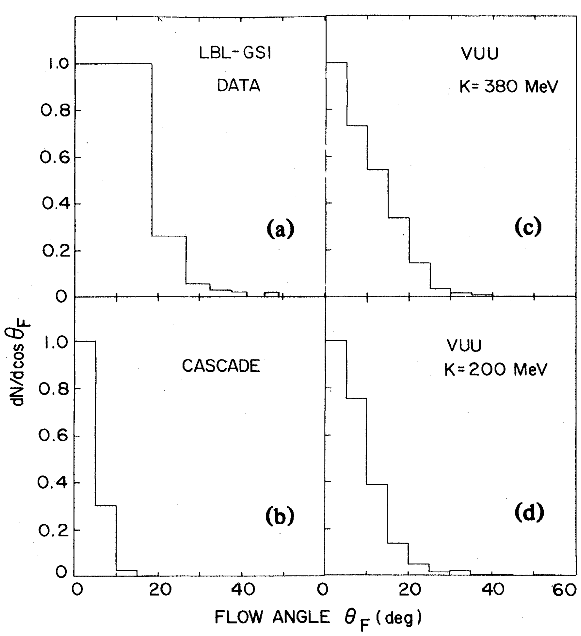

FIG. 2. Kinetic energy flow angular distributions for Ar(1800 MeV/nucleon)+KCl for (a) the experimental data (Ref. 12), (b) the intranuclear cascade model (Ref. 7), (c) the VUU approach with the stiffer equation of state, and (d) the VUU approach with the softer equation of state.

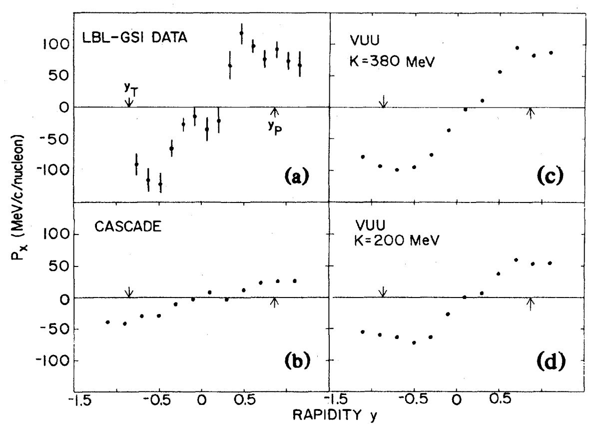

FIG. 3. In plane transverse momentum vs rapidity for Ar(1800 MeV/nucleon)+KCl for (a) the experimental results (Ref. 12) based on the Streamer Chamber data (Ref. 16), (b) the intranuclear cascade model (Ref. 7), (c) the VUU approach with the stiffer equation of state, and (d) the VUU approach with the softer equation of state.

ported by the previous finding[9] that the stiff equation of state reproduces best the pion yields observed in the streamer chamber at this energy (1800 MeV/nucleon) and also at lower energies, down to 360 MeV/nucleon. It is interesting to remark that this equation of state agrees rather well with the one extracted phenomenologically from the pion data.[15]

In summary, a novel transverse momentum analysis has been applied to collisions of Ar(1800 MeV/nucleon)+KCl. The intranuclear cascade model, lacking compressional energy, is unable to produce the transverse momenta of -100 MeV/c/nucleon at the beam and target rapidities; there is only a small effect of the order of 25 MeV/c/nucleon. With the Vlasov-Uehling-Uhlenbeck theory, a soft nuclear equation of state produces about 50 MeV/c/nucleon of transverse momentum at y_P and y_T: This is greater than with the cascade model, but still clearly inconsistent with the data. The theory reproduces the measured transverse momentum spectrum only with the stiffer nuclear equation of state; this is in quantitative agreement with the equation of state derived via the present VUU approach from the pion yields of the GSI-LBL Streamer Chamber group.

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