Further evidence for asymptotic safety of quantum gravity

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Asymptotic safety

- A fully-fledged quantum field theory may exist fundamentally provided the short distance fluctuations of the quantum fields lead to an (interacting) fixed point
- In gravity for the metric field an interacting fixed point is required
- Residual interactions in the UV modify the power counting of interaction terms
- Well-known in asymptotically free theories, otherwise only in exceptional cases
- No natural small expansion parameter and non-perturbative techniques required

Testing asymptotic safety

- Assume that interaction terms with increasing canonical mass dimension remain increasingly irrelevant at an interacting UV fixed point

$$\beta_i = -d_i \lambda_i + \text{quantum correction}$$

- This hypothesis can be falsified and therefore allows for systematic tests of the asymptotic safety conjecture
- Feasible: polynomial $f(R)$-truncations
  - Offers sufficient complexity
  - Interaction terms sorted by canonical mass dimension
  - Similarities to local potential approximation for scalar field theories
  - Of phenomenological relevance for cosmology

RG flow of F(R)-gravity

Flow equation C. Wetterich (1993)

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

Ansatz

$$\Gamma_k = \int d^4x \sqrt{\det g_{\mu\nu}}\, k^4 f(R)/16\pi \; + S_{GF} + S_{GH}$$

M. Reuter (1996); M. Reuter, O. Lauscher (2002); D. Litim (2004);
A. Codello, R. Percacci, C. R. (2007,2008 $\Rightarrow$ same conventions);
P. Machado, F. Saueressig (2007); A. Bonanno, A. Contillo, R. Percacci (2011);
D. Benedetti, F. Caravelli (2012); D. Benedetti (2013); J. Dietz, T. Morris (2013); I. Bridle, J. Dietz, T. Morris (2014)

RG equation with optimised cutoff D. Litim (2004)

$$(\partial_t + 4 - 2R\,\partial_R)\, f = I[f]$$

$$I[f] = I_0[f] + I_1[f] \cdot \partial_t f' + I_2[f] \cdot \partial_t f''$$

Quantum fixed points ($\partial_t f = 0$)

Polynomial expansion around $R = 0$

$$f(R) = \sum_{n=0}^{\infty} \lambda_n R^n$$

with free boundary conditions

$$\lambda_N = 0\ ;\ \lambda_{N+1} = 0$$

- Region where the heat-kernel expansion is most reliable
- β_n depends on couplings up to λ_{n+2}
- $\beta_n = 0$ gives fixed points
- Solving $\beta_n = 0$ provides us with an expression for λ_{n+2}
- Doing that subsequently, we can eliminate all but two couplings (λ_0 and λ_1)

Fixed point conditions

- Two-parameter family of fixed point candidates for $n \geq 2$:

$$\lambda_n = \lambda_n(\lambda_0, \lambda_1) = P_n / Q_n$$

- Recursive relations are extremely involved!
  P_n, Q_n are polynomials with up to around 45000 terms!
- Sets limit on computability, here up to $N = 35$

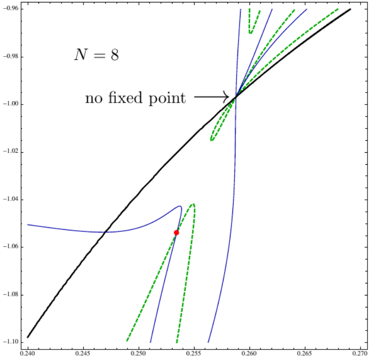
Fixed point conditions:

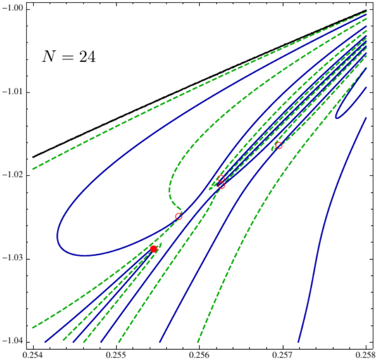
$$\begin{aligned} P_N(\lambda_0, \lambda_1) &= 0\,;\; P_{N+1}(\lambda_0, \lambda_1) = 0 \\ Q_N(\lambda_0, \lambda_1) &\neq 0\,;\; Q_{N+1}(\lambda_0, \lambda_1) \neq 0 \end{aligned}$$

Consistency conditions

- Identify the stable roots for each approximation order
- In principle, there are a large number of potential fixed point candidates in the complex plane.
- In practice, we only find a small number of real solutions at any order, and a unique one which consistently persists from order to order.
- Guiding principle for the identification of a fixed point:
  - Consistency condition I: fixed point coordinates at expansion order N should not differ drastically from those at order $N-1$
  - Consistency condition II: universal eigenvalues at expansion order N should not differ drastically from those at order $N-1$

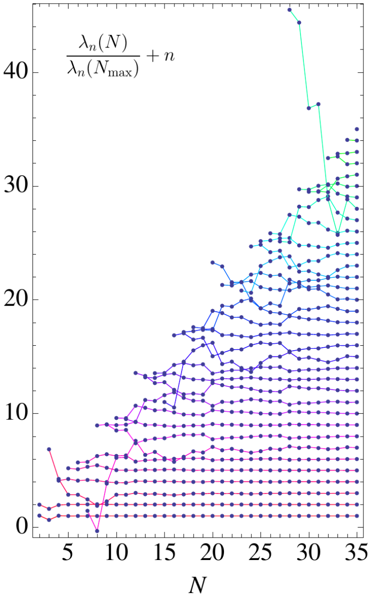
Nullclines for fixed points



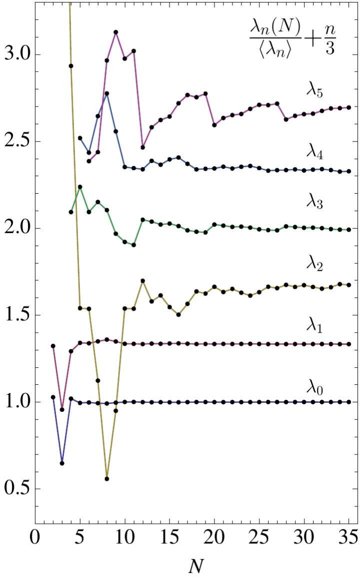


Blue lines: $P_8 = 0$, $P_{24} = 0$
Dashed green lines: $P_9 = 0$, $P_{25} = 0$
Black lines: $Q_8 = 0$, $Q_{24} = 0$; Q_9, Q_{25} out of range
Full red point: fixed point fulfilling consistency condition
Empty red point: fixed point failing consistency condition

Fixed point results

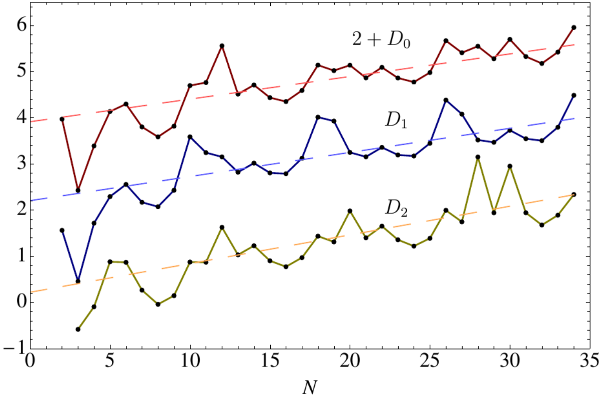


Convergence of the first polynomial couplings



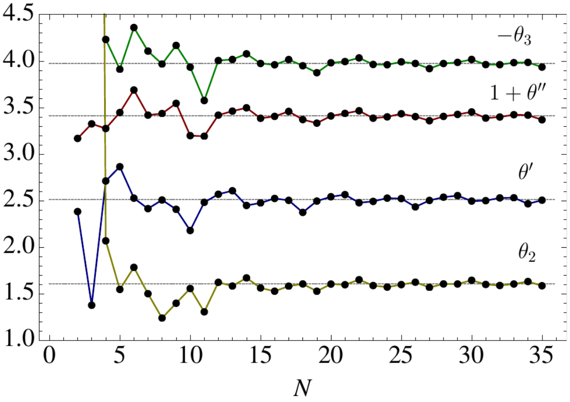
Rate of convergence of the three leading couplings

$$10^{-D_n} \equiv |1 - \lambda_n(N)/\lambda_n(N_{\max})|$$



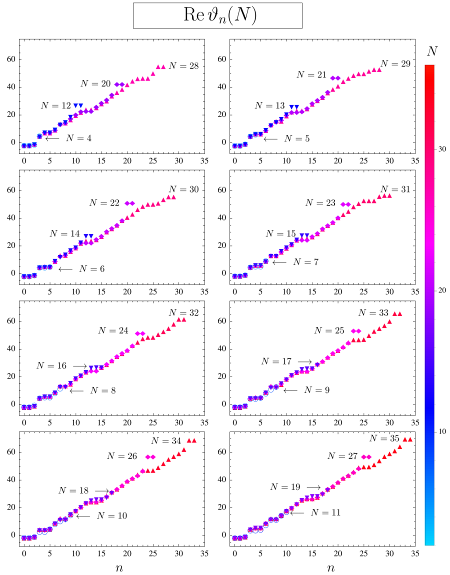
The accuracy in the fixed point couplings increases steadily by roughly one decimal place for $N \to N + 20$.

Convergence of first few exponents



- Fast convergence
- Oscillations: eight-fold periodicity pattern as known from scalar field theory D. Litim, L. Vergara (2003)

Convergence of eight-fold periodicity pattern



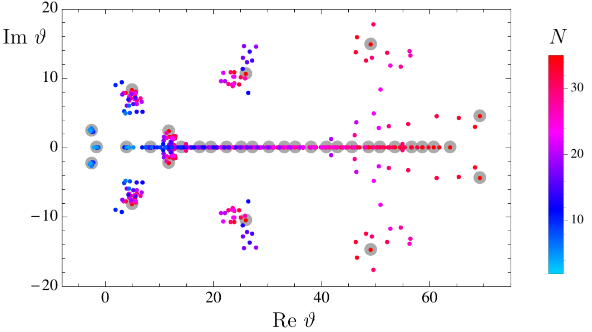
Accuracy reached for the three leading couplings

Periodicity pattern for signs of couplings: $(+ + + + - - - -)$

$$\langle X \rangle = \frac{1}{8} \sum_{N=N_{max}-7}^{N_{max}} X(N)$$

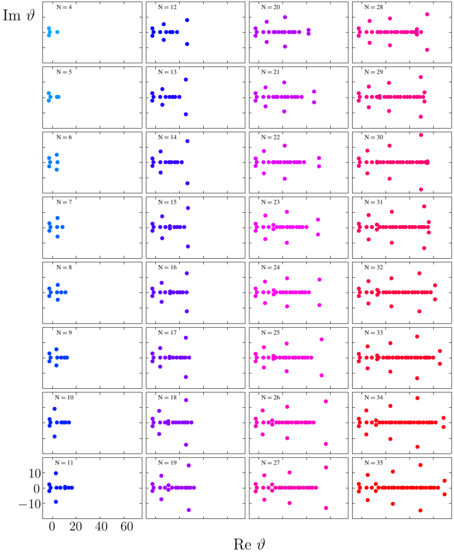
$$\begin{aligned}
\langle \lambda_0 \rangle &= 0.25574 && \pm\, 0.015\% \\
\langle \lambda_1 \rangle &= -1.02747 && \pm\, 0.026\% \\
\langle \lambda_2 \rangle &= 0.01557 && \pm\, 0.9\% \\
\langle \lambda_3 \rangle &= -0.4454 && \pm\, 0.70\% \\
\langle \lambda_4 \rangle &= -0.3668 && \pm\, 0.51\% \\
\langle \lambda_5 \rangle &= -0.2342 && \pm\, 2.5\%
\end{aligned}$$

Eigenvalue distribution in the complex plane



- Gray-filled circles: eigenvalues ϑ_n at order $N = 35$
- Small coloured circles: eigenvalues for $4 \leq N \leq 35$
- Most eigenvalues are real
- The imaginary parts show slower convergence

Order-by-order evolution of eigenvalue spectrum



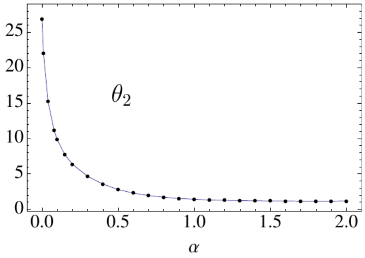
So far

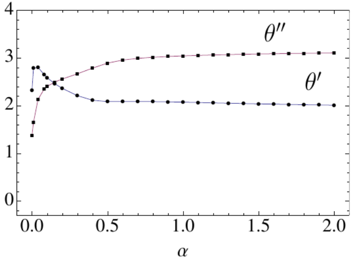
$$f(R) = \sum_{n=0}^{\infty} \lambda_n R^n$$

$$\lambda_N = 0 \; ; \; \lambda_{N+1} = 0$$

- Stable convergent behaviour towards fixed point values
- Characteristic: appearance of complex scaling exponents
- Higher-derivative truncation with Weyl curvature: only real scaling exponents D. Benedetti, P. Machado, F. Saueressig (2009)
- Slow convergence of dimensionless coupling λ_2

R^2-gravity with higher-order information





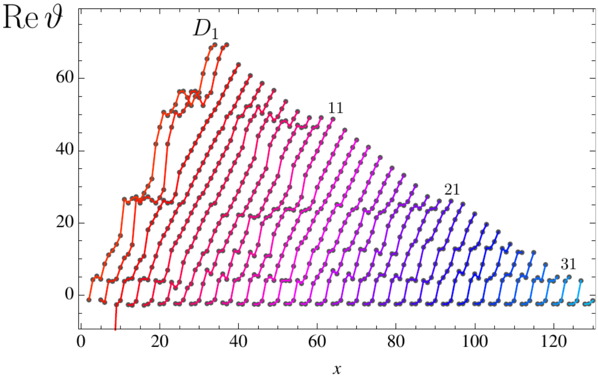
Splice-in information about higher-order couplings

$$
\begin{aligned}
\lambda_N &= \alpha \cdot \lambda_N^{np} \\
\lambda_{N+1} &= \alpha \cdot \lambda_{N+1}^{np}
\end{aligned}
$$

- θ_2 decreases quickly, curves are essentially flat around $\alpha = 1$
- Scaling exponents end up within 15 % of their asymptotic values

Is the mass dimension a good guiding principle?

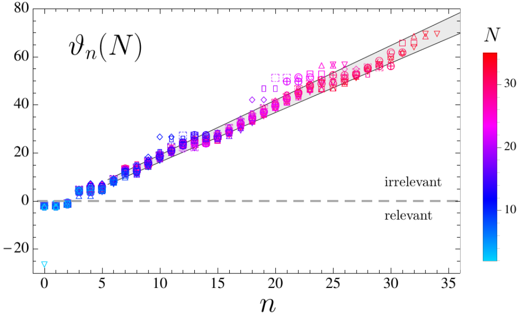
Bootstrap for asymptotic safety



D_1 connects the largest eigenvalue at approximation order $N_{\max}$ with the largest at order $N_{\max} - 1$, and so forth.

The positive slope of all curves D_i indicates that the working hypothesis is satisfied on average, although not for each and every order.

Near-Gaussianity



$$\vartheta_n = a \cdot n - b$$

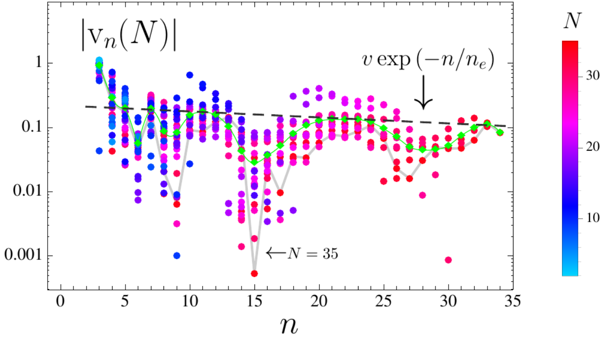
$$a_G = 2\ ;\ \ b_G = 4$$

$$a_{\mathrm{UV}} = 2.17 \pm 5\%\ ;\ \ b_{\mathrm{UV}} = 4.06 \pm 10\%$$

$\Rightarrow$ Can be used to extrapolate to larger N

Relative variation of the non-perturbative eigenvalues

$$\mathrm{v}_n(N) = 1 - \mathrm{Re}\,\vartheta_n(N)/\vartheta_{\mathrm{G},n}$$



Gray line: data at order $N = 35$

Green line: mean val. for each n; $v = 0.220 \pm 0.003$; $n_e = 46.68 \pm 0.92$

Summary

- Stable picture in the polynomial f(R)-approximation
- Slow convergence requires going to very high order
- Near-Gaussianity establishes mass dimension as a good guiding principle
- Agreement with all previous results so far
- Generalise beyond f(R)-approximation in the future