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FURTHER EVIDENCE ON THE GREAT CRASH, THE OIL PRICE SHOCK,
AND THE UNIT ROOT HYPOTHESIS

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ABSTRACT

Recently Perron (1989) has carried out tests of the unit root hypothesis against the alternative hypothesis of trend stationarity with a break in the trend occurring at the Great Crash of 1929 or at the 1973 oil price shock. His analysis covers the Nelson–Plosser macroeconomic data series as well as a post–war quarter real GNP series. His tests reject the unit root null hypothesis for most of the series.

This paper takes issue with the assumption used by Perron that the Great Crash and the oil price shock can be treated as exogenous events. A variation of Perron’s test is considered in which the break point is estimated rather than fixed. We argue this test is more appropriate than Perron’s, since it circumvents the problem of data–mining.

The asymptotic distribution of the "estimated break point" test statistic is determined. The data series considered by Perron are reanalyzed using this test statistic. The empirical results make use of the asymptotics developed for the test statistic as well as extensive finite sample corrections obtained by simulation. The effect on the empirical results of fat–tailed and temporally dependent innovations is investigated. In brief, by treating the break point as endogenous, we find that there is less evidence against the unit root hypothesis than Perron finds for many of the data series, but stronger evidence against it for several of the series, including the Nelson–Plosser industrial production, nominal GNP, and real GNP series.

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1. INTRODUCTION

A major debate concerning the dynamic properties of macroeconomic and financial time series has been going on since Nelson and Plosser (1982) published their stimulating article in the *Journal of Monetary Economics* nearly a decade ago. The primary issue involves the long-run response of a trending data series to a current shock to the series. The traditional view holds that current shocks only have a temporary effect and that the long-run movement in the series is unaltered by such shocks. Nelson and Plosser challenged this view and argued, using statistical techniques developed by Dickey and Fuller (1979, 1981), that current shocks have a permanent effect on the long-run level of most macroeconomic and financial aggregates. Others, including Campbell and Mankiw (1987,1988), Clark (1987), Cochrane (1988), Shapiro and Watson (1988) and Christiano and Eichenbaum (1989), have argued that current shocks are a combination of temporary and permanent shocks and that the long-run response of a series to a current shock depends on the relative importance or "size" of the two types of shocks.

Recent research has cast some doubt on Nelson and Plosser's conclusions. In particular, Perron (1988,1989) argues that if the years of the Great Depression are treated as points of structural change in the economy and the observations corresponding to these years are removed from the noise functions of the Nelson and Plosser data, then a "flexible" trend stationary representation is favored by eleven of the fourteen series. Similarly, Perron shows that if the first oil crisis in 1973 is treated as a point of structural change in the economy then one can reject the unit root hypothesis in favor of a trend stationary hypothesis for postwar quarterly real GNP. These results imply that the only observations (shocks) that have had a permanent effect on the long-run level of most macroeconomic aggregates are those associated with the Great Depression and the first oil price crisis.¹

We enter this debate by taking issue with the unit root testing procedure used by Perron (1989) (hereafter referred to as Perron). In particular, we examine the sensitivity of Perron's results to his exogeneity assumption concerning the Great Depression and the 1973 oil crisis. A skeptic of Perron's approach would argue that Perron's choices of break points are based on prior observation of the data and hence problems associated with "pre-testing" are applicable to his methodology. Simple visual inspection of the Nelson and Plosser data shows that there is an obvious jump down for most of the series occurring in 1929. Due to the sudden change in the data at 1929, Perron chooses to treat the drop in the Nelson and Plosser series as an exogenous event. This jump, however, could be interpreted as a realization from the tail of the distribution of the underlying data-generating process. This interpretation views the Great Depression as a shock or a combination of shocks from the underlying errors.

Similarly, an examination of the postwar quarterly GNP data shows a slowdown in GNP growth after the oil crisis in 1973. Analogous to his treatment of the Nelson and Plosser data, Perron's statistical model handles the slowdown in growth after the 1973 oil crisis as an event external to the domestic economy. While it seems reasonable to regard the formation of OPEC as an exogenous event, there are other big events such as the 1964 tax cut, the Viet Nam War and the financial deregulation in the 1980's that could also be viewed *ex ante* as possible exogenous structural break points. Perron's preference for the 1973 oil price crisis is undoubtedly influenced by his prior examination of the data.

If one takes the view that these events are endogenous then the correct unit root testing procedure would have to account for the fact that the break points in Perron's regressions are data dependent. The null hypothesis of interest in these cases is a unit root process with drift that excludes any structural change. The relevant alternative hypothesis is still a trend stationary process that allows for a one-time break in the trend function. Under the alternative, however, we assume that we do not know exactly when the break point occurs. Instead, a data dependent algorithm is used to proxy Perron's subjective

procedure to determine the break points. Such a procedure transforms Perron's unit root test which is conditional on a known break point into an unconditional unit root test.

We develop a unit root testing procedure that allows for an estimated break in the trend function under the alternative hypothesis. Using our procedure on the data series analyzed by Perron, we find less conclusive evidence against the unit root hypothesis than Perron finds. In particular, using our asymptotic critical values we cannot reject the unit root hypothesis at the 5% level for four of the ten Nelson and Plosser series for which Perron rejects the hypothesis, viz., real per capita GNP, GNP deflator, money stock, and real wages. We still reject the unit root hypothesis, however, for six of the series. Further, contrary to Perron, we cannot reject the unit root null at the 5% or 10% level for the post-war quarterly real GNP series.

We also investigate the accuracy of our asymptotic approximations by computing the exact finite sample distributions of our test statistics for the two data sets by Monte Carlo methods, assuming normal ARMA innovations. Here we find that our asymptotic critical values are more liberal than the finite sample critical values. Using the finite sample critical values, we cannot reject the unit root hypothesis at the 5% level for three more of the series for which Perron rejects, viz., employment, nominal wages and common stock prices (although the latter two are very close to being rejected at the 5% level). We can, however, still reject the unit root null at the 5% level for the real GNP and nominal GNP series and we can reject the unit root null at the 1% level for the industrial production series.

For the series that we reject the unit root null using our finite sample critical values, we investigate the possibility that the distributions of the innovations driving these series have tails thicker than the normal distribution. Our estimates of the kurtosis of these series lead us to believe that Student- t innovations may be more appropriate than normal innovations for some of these series. We recompute the finite sample distributions using Student- t ARMA innovations, with degrees of freedom determined by equating

sample kurtosis values to theoretical kurtosis values. Although the percentage points of the finite sample distributions using the t -innovations are uniformly larger (in absolute value) than the corresponding percentage points assuming normality, our unit root testing conclusions remain the same as in the normal case. Thus, our finite sample results for these series are robust to some relaxations of the normality assumption.

Last, we consider the effects of relaxing the assumption of finite variance by computing the finite sample distributions of our test statistics using stable ARMA innovations. Our conclusion is that it would take only slightly more than infinite variance for us not to reject the unit root hypothesis for all of the series. On the other hand, the estimates of kurtosis do not indicate that the series have infinite variance innovations.

The approach of this paper is similar to that taken by Christiano (1988). Christiano's results, however, are based solely on bootstrap methods. The latter have questionable reliability in regression models with dependent errors and small sample sizes. Christiano also limits his analysis to the postwar quarterly real GNP series.

The asymptotic distribution theory developed here is quite similar to that of Banerjee, Lumsdaine, and Stock (1989), although our empirical applications are substantially different. Our asymptotic theory was developed simultaneously and independently of the theory presented in Banerjee *et al.*

The outline of this paper is as follows. Section 2 reviews Perron's unit root testing methodology and presents our testing strategy. Section 3 contains the requisite asymptotic distribution theory for our unit root test in time series models with estimated structural breaks. We derive the asymptotic distributions for the test statistics, tabulate their critical values, and compare the latter to the critical values used by Perron. In Section 4 we apply our results to the Nelson and Plosser data and the postwar quarterly real GNP data. Section 5 investigates the finite sample distributions of the test statistics by Monte Carlo methods. This section determines the difference in test size between the finite sample distributions and the asymptotic distributions and determines the effect of

fat-tailed innovations on the finite sample distributions of our test statistics. Section 6 contains our concluding remarks.

2. MODELS AND METHODOLOGY

Perron develops a procedure for testing the null hypothesis that a given series $\{y_t\}_1^T$ has a unit root with drift and that an exogenous structural break occurs at time $1 < T_B < T$ versus the alternative hypothesis that the series is stationary about a deterministic time trend with an exogenous change in the trend function at time T_B . He considers three parameterizations of the structural break under the null and the alternative. Following the notation in Perron, the unit root null hypotheses are:

$$\text{Model (A)} \quad y_t = \mu + dD(T_B)_t + y_{t-1} + e_t ,$$

$$\text{Model (B)} \quad y_t = \mu_1 + y_{t-1} + (\mu_2 - \mu_1)DU_t + e_t ,$$

$$\text{Model (C)} \quad y_t = \mu_1 + y_{t-1} + dD(T_B)_t + (\mu_2 - \mu_1)DU_t + e_t ,$$

where $D(T_B)_t = 1$ if $t = T_B + 1$, 0 otherwise; $DU_t = 1$ if $t > T_B$, 0 otherwise; $A(L)e_t = B(L)v_t$, $v_t \equiv \text{iid}(0, \sigma^2)$, with $A(L)$ and $B(L)$ p^{th} and q^{th} order polynomials in the lag operator respectively. Model (A) permits an exogenous change in the level of the series, Model (B) allows an exogenous change in the rate of growth and Model (C) admits both changes.

The trend stationary alternative hypotheses considered are:

$$\text{Model (A)} \quad y_t = \mu_1 + \beta t + (\mu_2 - \mu_1)DU_t + e_t ,$$

$$\text{Model (B)} \quad y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + e_t ,$$

$$\text{Model (C)} \quad y_t = \mu + \beta_1 t + (\mu_2 - \mu_1)DU_t + (\beta_2 - \beta_1)DT_t^* + e_t ,$$

where $DT_t^* = t - T_B$ if $t > T_B$ and 0 otherwise. As with the unit root hypotheses, Model (A) allows for a one time change in the level of the series and, appropriately, Perron

calls this the "crash" model. The difference $\mu_2 - \mu_1$ represents the magnitude of the change in the intercept of the trend function occurring at time T_B . Perron labels Model (B) the "changing growth" model and the difference $\beta_2 - \beta_1$ represents the magnitude of the change in the slope of the trend function occurring at time T_B . Model (C) combines changes in the level and the slope of the trend function of the series.

Perron proposes Model (A) (the "crash" model) for all of the Nelson and Plosser series except the real wage and common stock price series for which he suggests Model (C). He submits Model (B) as the representation for the postwar quarterly real GNP series. His arguments for these representations are based primarily on visual inspection of the data.

Perron employs an adjusted Dickey-Fuller (ADF) type unit root testing strategy (see Dickey and Fuller (1981) and Said and Dickey (1984)). His tests for a unit root in Models (A), (B) and (C) involve the following augmented regression equations:

$$y_t = \hat{\mu}^A + \hat{\theta}^A DU_t + \hat{\beta}^A t + \hat{d}^A D(T_B)_t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^k \hat{c}_j^A \Delta y_{t-j} + \hat{e}_t, \quad (1)$$

$$\tilde{y}_t^B = \hat{\alpha}^B \tilde{y}_{t-1}^B + \sum_{j=1}^k \hat{c}_j^B \Delta \tilde{y}_{t-j}^B + \hat{e}_t, \quad (2)$$

$$y_t = \hat{\mu}^C + \hat{\theta}^C DU_t + \hat{\beta}^C t + \hat{\gamma}^C DT_t^* + \hat{d}^C D(T_B)_t + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^k \hat{c}_j^C \Delta y_{t-j} + \hat{e}_t, \quad (3)$$

where $\{\tilde{y}_t^B\}$ are the residuals from a regression of y_t on a constant, a time trend and DT_t^* . The k extra regressors in the above regressions are added to eliminate possible nuisance parameter dependencies in the limit distributions of the test statistics caused by temporal dependence in the disturbances. The number k of extra regressors is determined by a test of the significance of the estimated coefficients \hat{c}_j^i ($i = A, B, C$) (as described below).

To formally test for the presence of a unit root, Perron considers the following statistics computed from (1)–(3):

$$t_{\hat{\alpha}^i}(\lambda) \quad (i = A, B, C), \quad (4)$$

where (4) represents the standard t -statistic for testing $\alpha^i = 1$. These statistics depend

on the location of the break fraction (or break point), $\lambda = T_B/T$, and we exhibit this dependence explicitly since this notation will be useful for the analysis that follows. Perron's test for a unit root using (4) can be viewed as follows: reject the null hypothesis of a unit root if

$$t_{\hat{\alpha}}(\lambda) < \kappa_{\alpha}(\lambda), \quad (5)$$

where $\kappa_{\alpha}(\lambda)$ denotes the size α critical value from the asymptotic distribution of (4) for a fixed $\lambda = T_B/T$. Perron derives the asymptotic distributions for these statistics under the above null hypotheses and tabulates their critical values for a selected grid of λ values in the unit interval. Based on the critical values for (4), he rejects the unit root hypothesis at the 5% level of significance for all of the Nelson and Plosser data series except consumer prices, velocity and interest rates. He also rejects the unit root hypothesis at the 5% level for the postwar quarterly real GNP series.²

We construe Perron's test statistic (4) in a different manner. Perron's null hypotheses take the break fraction λ to be exogenous. We question this exogeneity assumption and instead treat the structural break as an endogenous occurrence. That is, we do not remove the Great Crash and the 1973 oil price shock from the noise functions of the appropriate series. Our null hypothesis for the three models is

$$y_t = \mu + y_{t-1} + e_t, \quad (6)$$

which is an integrated process with drift.

Since we consider the null that the series $\{y_t\}$ is integrated without an exogenous structural break, we view the selection of the break point, λ , for the dummy variables in Perron's regressions (1)–(3) as the outcome of an estimation procedure designed to fit $\{y_t\}$ to a certain trend stationary representation. That is, we assume that the alternative hypothesis stipulates that $\{y_t\}$ can be represented by a trend stationary process with a one time break in the trend occurring at an unknown point in time.³ The goal is to

estimate the break point that gives the most weight to the trend stationary alternative. Our hope is that an explicit algorithm for selecting the break points for the series will be consistent with Perron's (subjective) selection procedure.

One plausible estimation scheme, consistent with the above view, is to choose the break point that gives the least favorable result for the null hypothesis (6) using the test statistic (4). That is, λ is chosen to minimize the one-sided t -statistic for testing $\alpha^i = 1$, $i = A, B, C$, when small values of the statistic lead to rejection of the null. Let $\hat{\lambda}_{\inf}^i$ denote such a minimizing value for model i . Then, by definition,

$$t_{\hat{\alpha}^i}[\hat{\lambda}_{\inf}^i] = \inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda) \quad (i = A, B, C), \quad (7)$$

where Λ is a specified closed subset of $(0,1)$.

With the null model defined by (6) we no longer need the dummy variable $D(T_B)_t$ in (1) and (3). Therefore, following Perron's ADF testing strategy, the regression equations we use to test for a unit root are:

$$y_t = \hat{\mu}^A + \hat{\theta}^A DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^k \hat{c}_j^A \Delta y_{t-j} + \hat{e}_t, \quad (1')$$

$$\tilde{y}_t^B(\hat{\lambda}) = \hat{\alpha}^B \tilde{y}_{t-1}^B(\hat{\lambda}) + \sum_{j=1}^k \hat{c}_j^B \Delta \tilde{y}_{t-j}^B(\hat{\lambda}) + \hat{e}_t, \quad (2')$$

$$y_t = \hat{\mu}^C + \hat{\theta}^C DU_t(\hat{\lambda}) + \hat{\beta}^C t + \hat{\gamma}^C DT_t^*(\hat{\lambda}) + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^k \hat{c}_j^C \Delta y_{t-j} + \hat{e}_t \quad (3')$$

where $DU_t(\lambda) = 1$ if $t > [T\lambda]$, 0 otherwise; $DT_t^*(\lambda) = t - [T\lambda]$ if $t > [T\lambda]$, 0 otherwise; $\{\tilde{y}_t^B(\hat{\lambda})\}$ are the residuals from a regression of y_t on a constant, a time trend and $DT_t^*(\hat{\lambda})$.⁴ We put "hats" on the λ parameters in (1')–(3') to emphasize that they correspond to estimated values of the break fraction.

Table 1 reports the values of $\hat{T}_B (= [T\hat{\lambda}])$ that correspond to $\hat{\lambda}_{\inf}^i$ and the minimum values of $t_{\hat{\alpha}^i}(\lambda)$ obtained from the procedure defined in (7) for the data series analyzed by Perron. The break points and minimum t -statistics were determined as follows. For each series, (1'), (2') or (3') was estimated by OLS with the break fraction, $\lambda = T_B/T$, ranging from $j = 2/T$ to $j = (T-1)/T$.⁵ For each value of λ , the number

of extra regressors, k , was determined using the same procedure as in Perron and the t -statistic for testing $\alpha^i = 1$ was computed.⁶ The minimum t -statistics reported are the minimums over all $T-2$ regressions and the break years are the years corresponding to the minimum t -statistics.

From Table 1 we see that the break year that minimizes the one-sided t -statistic for testing $\alpha^A = 1$ does, in fact, correspond to the year of the Great Depression, 1929, for the eight series that Perron rejects the unit root hypothesis. The three series with estimated break points not consistent with Perron's choice are consumer prices, velocity and the interest rate. These are also the series for which Perron does not reject the unit root hypothesis. The break years for these series are 1873, 1949 and 1932 respectively. The estimated break date for the velocity series corresponds to the widely noted leveling off of the series in the mid to late 40's.

For the postwar quarterly real GNP series, the minimizing break point occurs in the second quarter of 1973. Perron's choice of 1973:I produces the third smallest t -statistic. The numerical difference between the t -statistics for these two dates, however, is very small. The break years corresponding to the minimum t -statistics for the Model (C) series do not coincide with the year of the Depression. The estimated break year for the common stock price series is 1936 and the break year for the real wage series is 1940. As these results show, our break point algorithm is generally, though not completely, consistent with the subjective selection procedure used by Perron for the Nelson and Plosser series and the postwar quarterly real GNP series.

When we treat the selection of λ as the outcome of an estimation procedure we can no longer use Perron's critical values to test the unit root hypothesis. To see this, consider the minimum t -statistic break point estimation procedure. With this definition of the break fraction, our interpretation of Perron's unit root test becomes: reject the null of a unit root if

$$\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda) < \kappa_{\inf, \alpha}^i \quad (i = A, B, C), \quad (8)$$

where $\kappa_{\inf, \alpha}^i$ denotes the size α left tail critical value from the asymptotic distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda)$. By definition, the left tail critical values in (8) are as least as large in absolute value as those computed for an arbitrary fixed λ . If one takes this unconditional perspective, then Perron's unit root tests are biased towards rejecting the unit root null hypothesis because he uses critical values that are too small (in absolute value). The extent of this size distortion depends on the magnitude of the difference between the critical values defined in (8) and those defined in (5). To determine this difference, the asymptotic distributions of the test statistics $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda)$ ($i = A, B, C$) are required. These distributions are derived in the next section.

3. ASYMPTOTIC DISTRIBUTION THEORY

The asymptotic distributions of the minimum t-statistics may be compactly expressed in terms of standardized Brownian motions. Following Phillips (1988a), Park and Phillips (1988) and Ouliaris, Park and Phillips (1988), define $W^i(\lambda, r)$ to be the stochastic process on $[0, 1]$ that is the projection residual in $L_2[0, 1]$ of a Brownian motion projected onto the subspace generated by the following: (a) $i = A : 1, r, du(\lambda, r)$; (b) $i = B : 1, r, dt^*(\lambda, r)$; (c) $i = C : 1, r, du(\lambda, r), dt^*(\lambda, r)$; where $du(\lambda, r) = 1$ if $r > \lambda$ and 0 otherwise, and $dt^*(\lambda, r) = r - \lambda$ if $r > \lambda$ and 0 otherwise. Here, $L_2[0, 1]$ denotes the Hilbert space of square integrable functions on $[0, 1]$ with inner product $(f, g) = \int_0^1 fg$ for $f, g \in L_2[0, 1]$. For example, in Model (A), $W^A(\lambda, r)$ is the L_2 projection residual from the continuous time regression

$$W(r) = \hat{\alpha}_0 + \hat{\alpha}_1 r + \hat{\alpha}_2 du(\lambda, r) + W^A(\lambda, r). \quad (9)$$

That is, $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$ solve

$$\min_{\alpha_0, \alpha_1, \alpha_2} \int_0^1 |W(r) - \alpha_0 - \alpha_1 r - \alpha_2 \int_0^1 W(r) dr|^2 dr. \quad (10)$$

Notice that if we allow $\lambda = 0$ or 1 , the above minimization problem, and the minimization problems for Models (B) and (C), do not have unique solutions due to the singularity of the matrix defining the normal equations.

The following theorem gives the asymptotic distributions for the minimum t -statistics in terms of $W^i(\lambda, r)$.

THEOREM 1: Let $\{y_t\}$ be generated under the null hypothesis (6) and let the errors $\{e_t\}$ be iid, mean zero, variance σ^2 random variables with $0 < \sigma^2 < \infty$. Let $t_{\hat{\alpha}^i}(\lambda)$ denote the t -statistic for testing $\alpha^i = 1$ computed from either (1'), (2') or (3') with $k = 0$ for Models $i = A, B$ and C respectively. Let Λ be a closed subset of $(0, 1)$. Then,

$$\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda) \Rightarrow \inf_{\lambda \in \Lambda} \left[\int_0^1 W^i(\lambda, r)^2 dr \right]^{-1/2} \left[\int_0^1 W^i(\lambda, r) dW(r) \right] \text{ as } T \rightarrow \infty$$

for $i = A, B$ and C .

The proof is given in Appendix A.

The limiting distributions presented in Theorem 1 are for the case where the disturbances are independent and there are no extra lag terms in the regression equations (1')–(3'). If we allow the disturbances to be correlated and heterogeneously distributed, then the asymptotic distributions in the theorem become nonstandard in that they depend on the nuisance parameters $\sigma^2 = \lim_{T \rightarrow \infty} E T^{-1} (\sum_1^T e_t)^2$ and $\sigma_e^2 = \lim_{T \rightarrow \infty} E T^{-1} \sum_1^T e_t^2$.

Two approaches have been employed in the time series literature to eliminate this nuisance parameter dependency. One approach is due to Phillips (1987). His technique is based on the result that if consistent estimators of σ^2 and σ_e^2 are available then one can derive a nonparametric transformation of the test statistics whose limiting distributions are independent of the population parameters σ^2 and σ_e^2 . The other approach is the ADF approach referred to above. It is based on the addition of extra lags of first differences of

the data as regressors. The number of extra regressors must increase with the sample size at a controlled rate. With the ADF procedure, the errors are restricted to the class of ARMA(p,q) processes. Since we follow Perron and use the ADF approach, we consider the following assumption.

ASSUMPTION 1: (a) $A(L)e_t = B(L)v_t$, $A(L)$ and $B(L)$ are p^{th} and q^{th} order polynomials in the lag operator L and satisfy the standard stationarity and invertibility conditions.

(b) $\{v_t\}$ is a sequence of iid(0, σ^2) random variables with $E|v_t|^{4+\delta} < \infty$ for some $\delta > 0$.

(c) $k \xrightarrow{P} \infty$ and $T^{-1}k^3 \xrightarrow{P} 0$ as $T \rightarrow \infty$.

When the error sequence $\{e_t\}$ satisfies Assumption 1, we conjecture, based on arguments outlined in Said and Dickey (1984), that the limiting distributions of the test statistics computed from the ADF regression equations (1')–(3') are free of nuisance parameter dependencies and have the limiting distributions presented in the theorem. As in Perron, we do not give a proof of the efficacy of the ADF procedure, but we use it in the empirical applications below.

Critical values for the limiting distributions in the theorem are obtained by simulation methods. That is, the integral functions in the theorem are approximated by functions of sums of partial sums of independent normal random variables. The method used is described in Appendix B.

The critical values for the limiting distributions of the minimum t-statistics and for the t-statistics used by Perron are presented in Tables 2A – 4B.⁷ Estimates of their densities are plotted in Figure 1. As expected, for a given size of a left-tailed test, the critical values for $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda)$ are larger in absolute value (more negative) than the critical values obtained by Perron for any fixed value of the break fraction λ . The biggest difference occurs for the Model (A) densities. At the 5% level the critical value for $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^A}(\lambda)$ is

−4.80 and the average value, over λ , of Perron's critical values is −3.74. Thus, at the 5% level, our critical value is roughly 24% larger (in absolute value) than Perron's and at the 1% level our critical value is about 23% larger. For the Model (B) densities, our 5% critical value is −4.42 and Perron's average critical value is −3.84. For the Model (C) densities, our 5% critical value is −5.08 and Perron's average value is −4.07.

We can now address the magnitude of the size distortion of Perron's test statistics incurred by ignoring the pre-test information concerning the location of the trend break. Table 5 gives the actual asymptotic sizes of tests based on the statistic $\inf_{\lambda \in \Lambda} t_i(\lambda)$ that use Perron's 5% critical values. We see that the size distortion is quite dramatic for Models (A) and (C), where the actual sizes of Perron's 5% tests are 55.1% and 34.5% respectively. The size distortion for Model (B) is more moderate with an actual size of 14.2%. The density plots in Figure 1 clearly illustrate this distortion. For all models the asymptotic densities of the minimum t-statistics are shifted to the left of the Perron densities. The densities for the minimum t-statistics also have thinner tails than the Perron densities.

4. EMPIRICAL APPLICATIONS

We now apply the unit root test developed in the previous sections to the data series analyzed by Perron.⁸ We analyze the natural logarithm of all the data except for the interest rate series, which is analyzed in levels form. Tables 6A–C present the estimated regressions for all of the series using the regression equations (1')–(3'). t-statistics are in parentheses. The t-statistic for $\hat{\alpha}^i$ is for testing the hypothesis that $\alpha^i = 1$ ($i = A, B, C$).

These results are somewhat different from the results in Perron's Table 7 for two reasons. First, the break years defining the dummy variables are estimated according to (7) instead of being fixed at 1929 or 1973:I. This has relevance only for the series whose

estimated break years are different from the ones used by Perron. Second, we do not impose a structural break under our null hypothesis, and hence, the variable $D(T_B)_t$ is not included in the regressions. This affects only the Model (A) and Model (C) regressions. For this effect, the most notable change in the regression results is that the estimated t-statistics for testing $\alpha^i = 1$ ($i = A$ and C) increased (in absolute value) for a majority of the series, see footnote 4. Often this increase was substantial. For example, the absolute change in the t-statistic for the real per capita GNP series is 0.52 ($-4.61 - (-4.09)$), which is roughly 13%. For most of the affected series, this change favors the trend stationary alternative.

The results of our unit root tests are also presented graphically in Figure 2, which contains time plots of the natural logarithm of the fourteen data series. Superimposed on the time plot of each series are the estimated t-statistics (in absolute value) for testing $\alpha^i = 1$ for each possible break date $T_B = [T\lambda]$, a line indicating the appropriate asymptotic 5% critical value (in absolute value) for the minimum t-statistic, and a line depicting the appropriate 5% critical value from Perron's asymptotic distributions for a fixed break date. Also superimposed on the time plots is a line labeled "Finite Sample 5% C.V.", which will be explained later.

Consider first the results for the Model (A) series, presented in Table 6A. From Table 1, we know that Perron's break fraction for eight of the eleven series corresponds to the break fraction associated with the minimum t-statistic for testing $\alpha^A = 1$. This can also be seen graphically from Figure 2 where, clearly, the largest t-statistic (in absolute value) for these series occurs at $T_B = 1929$. These eight series are also the ones for which Perron rejects the unit root null hypothesis at a 5% significance level using his critical values for a fixed break point. Now, treating the break fraction as the outcome of the estimation procedure defined by (7) and using the critical values from Table 2A, we can reject the unit root null at the 1% level for the real GNP, nominal GNP and industrial production series. We can reject the unit root null at the 2.5% level for the nominal wage series,

at the 5% level for the employment series, and at the 10% level for the real per capita GNP series. We *cannot* reject the unit root null at the 5% or 10% level, however, for the GNP deflator, consumer prices, money stock, velocity and interest rate series. In fact, the p-values for these series, computed from the asymptotic distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^A}(\lambda)$, are .278, .951, .174, .737 and .999 respectively. Thus, by endogenizing the break point selection procedure, we reverse Perron's test conclusions for the GNP deflator and nominal money stock series, and weaken the evidence against the unit root hypothesis for the remaining series.

Next, consider the results for the Model (B) series presented in Table 6B. The estimated break date for the postwar quarterly real GNP series occurs one quarter after Perron's choice of 1973:I, so it seems reasonable to apply our methodology to this series. Using the critical values from Table 3A, we find, contrary to Perron, that we cannot reject the unit root null at the 5% or 10% level. The asymptotic p-value for the t-statistic is .131.

Finally, the results for the Model (C) series are given in Table 6C. For these series the estimated break years do not coincide with Perron's choices. Nevertheless, using our estimated break points for these series and the critical values from Table 4A, we reject the unit root null for the common stock price series at the 1% level but, contrary to Perron, we cannot reject the unit root null at the 1%, 5% or 10% level for the real wage series. The asymptotic p-value for the t-statistic is .119.

Table 11 compares the p-values computed from Perron's fixed- λ distributions to the p-values computed from our asymptotic distributions, as well as p-values from distributions that will be explained below. The table clearly shows the effects of incorporating the pre-test trend break information on the asymptotic distributions of the unit root tests. In sum, by endogenizing Perron's break point selection procedure, we reverse his conclusions for five of the eleven series for which he rejects the unit root null hypothesis at 5%

and for four of the eleven series for which he rejects at 10%.⁹ On the other hand, even after adjusting for pre-test examination of the data, we reject the unit root null for six series using our 5% asymptotic "estimated break point" critical values and for seven series using 10% critical values.

5. FINITE SAMPLE RESULTS

The sample sizes for the series under consideration range from $T = 62$ to $T = 111$. In addition, there appears to be considerable temporal dependence in the data. In consequence, our asymptotic critical values may differ from the appropriate finite sample critical values. In this section we investigate this possibility by computing the finite sample distributions of our test statistics, under specific distributional assumptions, by Monte Carlo methods.

To compute the finite sample distributions of the minimum t -statistics one has to make specific assumptions concerning the underlying error sequence $\{e_t\}$ for each series. First, we suppose the errors driving the data series are normal ARMA(p, q) processes. In this case, the first differences of the series are normal ARMA(p, q) processes, possibly with nonzero mean, under the null hypothesis. To determine p and q , we fit ARMA(p, q) models to the first differences of each series and we use the model selection criteria of Akaike (1974) and Schwartz (1978) to choose the optimal ARMA(p, q) model with $p, q \leq 5$. The Akaike criterion minimizes $2\ln L + 2(p+q)$, where L denotes the likelihood function. The Schwartz criterion minimizes $2\ln L + (p+q)\ln T$, where T is the sample size. The Schwartz criterion penalizes extra parameters more heavily than does the Akaike criterion. We then treat the optimal estimated ARMA(p, q) models as the true data generating processes for the errors of each of the series.

Tables 7A and 7B present the chosen models for each of the data series. In most cases the Akaike and Schwartz criteria select the same model. ARMA(1,0) models are

selected by both criteria for the real GNP, nominal GNP, real per capita GNP, GNP deflator and money stock series, whereas ARMA(0,1) models are selected by both criteria for the employment, nominal wages, velocity and real wages series. In addition, an ARMA(0,5) model is selected by both criteria for the industrial production series. The Akaike criterion favors an ARMA(5,0) model for stock prices, an ARMA(3,0) model for interest rates, an ARMA(1,1) model for consumer prices and an ARMA(0,3) model for postwar quarterly real GNP; the Schwartz criterion chooses ARMA(0,1), ARMA(2,0), ARMA(0,1) and ARMA(1,0) models for these series respectively. In those cases where the two criteria choose different models, we select the most parsimonious model.

To determine the finite sample distributions of our test statistics under the null hypothesis with the above error distributions, we perform the following Monte Carlo experiment. For each series, we construct a pseudo sample of size equal to the actual size of the series using the optimal ARMA(p,q) models described above with iid $N(0, \sigma^2)$ innovations, where σ^2 is the estimated innovation variance of the optimal ARMA(p,q) model. Then, for each $j = 2, \dots, T-1$, we set $\lambda = j/T$, determine k as in footnote 6, and compute $t_{\hat{\alpha}_i}(\lambda)$ using either (1'), (2') or (3'). Our test statistic is then determined to be the minimum t -statistic over all $T-2$ regressions. We repeat this process 5000 times and the critical values for the finite sample distributions are obtained from the sorted vector of replicated statistics.

Tables 8A–C display the percentage points of the finite sample distributions of the minimum t -statistics for all of the data series under the assumption of normal ARMA(p,q) errors.¹⁰ The salient feature of these critical values is that they are all uniformly larger (in absolute value) than the corresponding asymptotic critical values. At the 5% level, the Model (A) finite sample critical values range from -5.12 to -5.38 , the average of which is 9.2% larger (in absolute value) than the corresponding asymptotic critical value. At the same level, the Model (B) finite sample critical value is -4.86 and the Model (C) finite

sample critical values are -5.63 and -5.68 . These finite sample critical values are 9.0% and 11.8% larger (in absolute value), respectively, than their corresponding asymptotic values. For the Model (A) series, the difference between the finite sample and asymptotic critical values abates for the series with larger sample sizes. For the two Model (C) series, however, the critical values are nearly identical even though the sample size for the real wage series is 71 and the sample size for the common stock price series is 100. Furthermore, for the series with comparable sample sizes, the finite sample distributions generated from different ARMA models are fairly similar. The latter result suggests that the ADF methodology works fairly well in finite samples for our data set.

Using the finite sample distributions of the Model (A) t -statistics, the actual sizes of the asymptotic 5% tests range from 10.7% to 16.0%, producing an average size distortion of 8.2%. The size of the Model (B) asymptotic 5% test is 13.8% and the average size for the Model(C) t -statistics is 16.0%. These size distortions are presented graphically in Figure 1, where we see that the finite sample densities of the minimum t -statistics are shifted to the left of the asymptotic densities of the minimum t -statistics.

Assuming that the fitted ARMA models of the Nelson and Plosser series and the postwar quarterly real GNP series are correct, we can use the Monte Carlo generated finite sample distributions of our test statistics to test these series for a unit root. From the above discussion we know that the asymptotic tests are too liberal, allowing us to reject the unit root null too often. This effect can be seen graphically in Figure 2, which shows the finite sample 5% critical values lying above the corresponding asymptotic critical values. Using the finite sample distributions, we can no longer reject the unit root null at the 5% level for the employment, nominal wage and common stock price series. On the other hand, we can reject the unit root null at the 1% level for the industrial production series, we can reject the null at the 2.5% level for the nominal GNP series and we can reject the null at the 5% level for the real GNP series. The p -values computed from the above finite sample distributions are presented, for comparison with the previous asymptotic results, in

column three of Table 11. Thus, after endogenizing the break point selection procedure and correcting for small sample biases we do not reject the unit root hypothesis for eight of the eleven series for which Perron rejects the hypothesis. In accordance with Perron, however, we do reject the unit root null for the real GNP, nominal GNP and industrial production series.

For the above series for which we do reject the unit root hypothesis, we investigate the effect of relaxing the normality assumption on the finite sample distributions of our test statistics. In particular, the large changes in the series at the estimated break points suggest that the distributions of the innovations underlying the series may have fatter tails than the normal distribution. In repeated samples under a distribution with a higher probability of generating tail events than the normal, our break point selection procedure will tend to produce larger (in absolute value) t -statistics than in the normal case. Therefore, with fat-tailed innovations, we expect the finite sample distributions of our test statistics to shift further to the left.

To assess the normality assumption, Table 9 gives the estimated skewness and kurtosis values for the residuals from the optimal ARMA models for the first differences of the logarithms of the data series. Most of the series exhibit mild negative skewness. The estimated kurtosis values for real GNP, industrial production and employment are only slightly larger than three (the kurtosis for a standard normal random variable), whereas the the values for nominal GNP, nominal wages and common stock prices are considerably larger than three.¹¹ Hence, there is some evidence of leptokurtosis for some of the series.

A plausible family of distributions close to the normal but with thicker tails is the Student- t family with η degrees of freedom. To determine the appropriate degrees of freedom, we use a modified method of moments approach.¹² In particular, for each series under consideration, we compute by Monte Carlo the means of the sample kurtosis statistic using the appropriate ARMA(p,q) model with iid Student- t innovations for various values of η . We then determine the t -distribution for each series by finding the closest match

between the observed sample kurtosis and the finite sample mean kurtosis values. The finite sample mean values of the sample kurtosis and the degrees of freedom of the selected t -distributions are given in the third and fourth columns of Table 9. Four degrees of freedom are chosen for nominal GNP, five are chosen for nominal wages, six for common stock prices, nine for both industrial production and real GNP and ten for employment.

Table 10A gives the percentage points of the finite sample distributions of our test statistics for the above series. Table 11 (column four) reports the p -values computed from these distributions. The percentage points obtained using the Student- t innovations are uniformly larger than the corresponding percentage points determined from normal innovations. Our test conclusions based on the Student- t distributions, however, remain essentially the same as in the normal case. That is, we reject the unit root null at the 1% level for the industrial production series, we reject at the 5% level for the real GNP and nominal GNP series and we reject at the 10% level for the common stock price series. We no longer reject at the 10% level for the nominal wage series, but we are still close to rejecting since the p -value is only 11.7%. Thus, our rejections of the unit root hypothesis for these series are not very sensitive to the relaxation of the normality assumption.

Lastly, we briefly investigate the effect that infinite variance innovations would have on our test results, although the sample kurtosis estimates are not indicative of innovations whose tails are that fat. We compute the finite sample distribution of our test statistic, using the parameters of the nominal GNP series, under the assumption of ARMA errors with innovations that follow a stable distribution with characteristic exponent α .¹³ Table 10B reports the percentage points of this distribution for various values of α . From these results, we see that if the innovations have only slightly less than two moments finite (e.g., $\alpha = 1.9$) one cannot reject the unit root hypothesis at the 5% level for any of the series.

6. CONCLUDING REMARKS

In this paper, we transform Perron's unit root test that is conditional on structural change at a known point in time into an unconditional unit root test. We also take into consideration the effects of fat-tailed innovations on the performance of the tests. Our analysis is motivated by the fact that the break points used by Perron are data dependent and plots of drifting unit root processes often are very similar to plots of processes that are stationary about a broken trend for some break point. The null hypothesis that we believe is of most interest is a unit root process without any exogenous structural breaks and the relevant alternative hypothesis is a trend stationary process with possible structural change occurring at an unknown point in time.

We systematically address the effects of endogenizing the break point selection procedure on the asymptotic distributions and finite sample distributions of Perron's test statistics for a unit root. Using our "estimated break point" asymptotic distributions, we find less conclusive evidence against the unit root hypothesis than Perron finds for many of the data series. We reverse his conclusions for five of the eleven Nelson and Plosser series for which he rejects the unit root hypothesis at the 5% level, and we reverse his unit root rejection for the postwar quarterly real GNP series. When we take into consideration small sample biases and the effects of fat-tailed (but not infinite variance) innovations, we reverse his conclusions for three more of the Nelson and Plosser series.

The reversals of some of Perron's results should not be construed as providing evidence for the unit root null hypothesis, since the power of our test against Perron's trend stationary alternatives is probably low for small to moderate changes in the trend functions. Rather, the reversals should be viewed as establishing that there is less evidence against the unit root hypothesis for many of the series than the results of Perron indicate. On the other hand, for some of the series (industrial production, nominal GNP, and real GNP), we reject the unit root hypothesis even after endogenizing the break point selection

procedure and accounting for moderately fat-tailed errors. For these series, our results provide stronger evidence against the unit root hypothesis than that given by Perron.

APPENDIX A

One way to establish the convergence result in the theorem is to first show that the finite dimensional distributions of $t_{\hat{\alpha}}^i(\lambda)$, indexed by λ , converge; i.e., for any finite number J of λ values one must show that $(t_{\hat{\alpha}}^i(\lambda_1), \dots, t_{\hat{\alpha}}^i(\lambda_J))'$ converges weakly to $(L(\lambda_1), \dots, L(\lambda_J))'$. Next one must show that the sequence of probability measures associated with $t_{\hat{\alpha}}^i(\lambda)$ is tight. If the above two conditions hold then we have the weak convergence result $t_{\hat{\alpha}}^i(\cdot) \Rightarrow L(\cdot)$. Then, provided $\inf_{\lambda \in \Lambda} L(\lambda)$ is a continuous functional of $L(\cdot)$ a.s. $[L(\cdot)]$ we get the desired result: $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}^i(\lambda) \Rightarrow \inf_{\lambda \in \Lambda} L(\lambda)$.

Establishing the finite dimensional convergence of $t_{\hat{\alpha}}^i(\lambda)$ is trivial given Perron's results. Showing tightness, however, is a difficult task. We avoid the problem of establishing tightness by using a different method of proof from the "fidi plus tightness" method. The method we use appeals directly to the continuous mapping theorem (CMT). The idea is to express the test statistic in the theorem as a functional, say $g(\cdot, \cdot, \cdot, \cdot, \cdot)$, of the partial sum process $X_T(\cdot)$, a rescaled version of the deterministic regressors $Z_T(\cdot, \cdot)$, the process $T^{-1/2} \sum_1^T Z_T(\cdot, t/T) e_t$, the average squared innovations σ_T^2 and an estimate $s^2(\cdot)$ of the error variance. If we have joint weak convergence of the process $(X_T(\cdot), Z_T(\cdot, \cdot), T^{-1/2} \sum_1^T Z_T(\cdot, t/T) e_t, \sigma_T^2, s^2(\cdot))'$ to a process $(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))'$ and if g is continuous with respect to $(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))'$ on a set C with $P\{(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))' \in C\} = 1$, then $g(X_T(\cdot), Z_T(\cdot, \cdot), T^{-1/2} \sum_1^T Z_T(\cdot, t/T) e_t, \sigma_T^2, s^2(\cdot)) \Rightarrow g(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))$ by the CMT.

Let $S_t = \sum_1^t e_j$ ($S_0 = 0$) and $X_T(r) = \sigma^{-1} T^{-1/2} S_{[Tr]}$, $(j-1)/T \leq r < j/T$ for $j = 1, \dots, T$, where $\sigma^2 = \lim_{T \rightarrow \infty} T^{-1} E S_T^2$ and $[Tr]$ denotes the integer part of Tr . Here, as in the theorem, we assume that the disturbances are iid so that $\sigma^2 = E e_1^2 = \sigma_e^2 \in (0, \infty)$. Under these assumptions, the disturbances $\{e_t\}$ satisfy an

invariance principle. Specifically, as processes indexed by $r \in [0,1]$, we have

$$X_T(\cdot) \Rightarrow W(\cdot) \text{ as } T \rightarrow \infty,$$

where $W(r)$ denotes a standard Brownian motion or Wiener process on $[0,1]$. In addition, $\sigma_T^2 = T^{-1} \Sigma_1^T e_t^2 \xrightarrow{P} \sigma^2$. As in the theorem, we take Λ to be a closed subset of $(0,1)$.

Throughout what follows " \Rightarrow " denotes weak convergence and " \equiv " denotes equivalence in distribution. For notational convenience we shall often denote $W(r)$ by W . Similarly, we will often write integrals with respect to Lebesgue measure such as $\int_0^1 W(r) dr$ as $\int_0^1 W$.

We consider least squares regressions of the form

$$y_t = \hat{\beta}^i(\lambda)' z_{tT}^i(\lambda) + \hat{\alpha}^i(\lambda) y_{t-1} + \hat{e}_t(\lambda) \quad (t = 1, \dots, T)$$

for Models $i = A, B$ and C . The vector $z_{tT}^i(\lambda)$ encompasses the deterministic components of the regression equation and it depends explicitly on the location of the break fraction and the sample size. For example, in Model (A) we have $z_{tT}^A(\lambda)' = (1, t, DU_t(\lambda))$, where $DU_t(\lambda) = 1$ if $t > [T\lambda]$ and 0 otherwise.

Let $Z_T^i(\lambda, r) = \delta_T^i z_{[Tr]T}^i(\lambda)$ denote a rescaled version of the deterministic regressors, where δ_T^i is a diagonal matrix of weights. For each $i = A, B, C$, there is a nonrandom function $Z^i(\lambda, r)$ such that $Z_T^i(\lambda, r) \rightarrow Z^i(\lambda, r)$ as $T \rightarrow \infty$ uniformly over $(\lambda, r) \in \Lambda \times [0,1]$. For example, in Model A we have

$$\delta_T^A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

and $Z_T^A(\lambda, r) \rightarrow Z^A(\lambda, r) = (1, r, du(\lambda, r))'$, where $du(\lambda, r) = 1$ if $r > \lambda$, 0 otherwise.

The coefficient $\hat{\alpha}^i(\lambda)$ and its t -statistic are invariant with respect to the value of the drift μ in the null model (6). Therefore, without loss of generality, we set $\mu = 0$ in (6).

The normalized bias for testing the null hypothesis $\alpha^i = 1$ is given by

$$T(\hat{\alpha}^i(\lambda) - 1) = \left[T^{-2} \Sigma_1^T y_{t-1}^i(\lambda)^2 \right]^{-1} \left[T^{-1} \Sigma_1^T y_{t-1}^i(\lambda) e_t \right]$$

and the t -statistic for testing $\alpha^i = 1$ is given by

$$\begin{aligned} t_{\hat{\alpha}^i(\lambda)} &= \left[T^{-2} \Sigma_1^T y_{t-1}^i(\lambda)^2 \right]^{1/2} T(\hat{\alpha}^i(\lambda) - 1) / s(\lambda) \\ &= \left[T^{-2} \Sigma_1^T y_{t-1}^i(\lambda)^2 \right]^{-1/2} \left[T^{-1} \Sigma_1^T y_{t-1}^i(\lambda) e_t \right] / s(\lambda), \end{aligned}$$

where $y_t^i(\lambda) = y_t - z_{tT}^i(\lambda)' \left[\Sigma_1^T z_{sT}^i(\lambda) z_{sT}^i(\lambda)' \right]^{-1} \Sigma_1^T z_{sT}^i(\lambda) y_s$ and $s^2(\lambda) = T^{-1} \Sigma_1^T (y_t - \hat{\beta}^i(\lambda)' z_{tT}^i(\lambda) - \hat{\alpha}^i(\lambda) y_{t-1})^2$ for Models $i = A, B$ and C . For brevity we drop the superscript i and only consider Model (A) for which $z_{tT}^A(\lambda)' = (1, t, DU_t(\lambda))$. The proofs for Models (B) and (C) are analogous and are therefore omitted.

The test statistic of interest is

$$\inf_{\lambda \in \Lambda} t_{\hat{\alpha}(\lambda)} = \inf_{\lambda \in \Lambda} \left[T^{-2} \Sigma_1^T y_{t-1}(\lambda)^2 \right]^{-1/2} \left[T^{-1} \Sigma_1^T y_{t-1}(\lambda) e_t \right] / s(\lambda),$$

which we may write as a function of $X_T \left[= X_T(\cdot) \right]$, $Z_T \left[= Z_T(\cdot, \cdot) \right]$, $T^{-1/2} \Sigma_1^T Z_T e_t \left[= T^{-1/2} \Sigma_1^T Z_T(\cdot, t/T) e_t \right]$, σ_T^2 and $s^2 \left[= s^2(\cdot) \right]$ plus an asymptotically negligible term:

$$\inf_{\lambda \in \Lambda} t_{\hat{\alpha}(\lambda)} = g(X_T, Z_T, T^{-1/2} \Sigma_1^T Z_T e_t, \sigma_T^2, s^2) + o_{p\lambda}(1), \quad (A1)$$

where g is defined below. The symbol " $o_{p\lambda}(1)$ " denotes any random variable $\zeta(\lambda)$ such that $\sup_{\lambda \in \Lambda} |\zeta(\lambda)| \xrightarrow{P} 0$.

It will be useful to re-express g as the following composite functional

$$\begin{aligned} g(X_T, Z_T, T^{-1/2} \Sigma_1^T Z_T e_t, \sigma_T^2, s^2) \\ = h^* \left[h \left[H_1[\sigma X_T, Z_T], H_2[\sigma X_T, Z_T, T^{-1/2} \Sigma_1^T Z_T e_t, \sigma_T^2], s^2 \right] \right], \end{aligned} \quad (A2)$$

where H_1 maps a function on $[0,1]$ and a function on $\Lambda \times [0,1]$ into a function on Λ , H_2 maps a function on $[0,1]$, a function on $\Lambda \times [0,1]$, a function on Λ , and a positive real number into a function on Λ , h maps three functions on Λ into a function on Λ , and h^* maps a function on Λ into a real number. Specifically, for any real function $m = m(\cdot)$

on Λ ,

$$h^*(m) = \inf_{\lambda \in \Lambda} m(\lambda), \quad (A3)$$

and for any functions $m_1 = m_1(\cdot)$, $m_2 = m_2(\cdot)$ and $m_3 = m_3(\cdot)$ on Λ ,

$$h[m_1, m_2, m_3](\cdot) = m_1(\cdot)^{-1/2} m_2(\cdot) / m_3(\cdot). \quad (A4)$$

The functionals H_1 and H_2 are the functional analogs of the sample moments $T^{-2} \Sigma_1^T y_{t-1}(\lambda)^2$ and $T^{-1} \Sigma_1^T y_{t-1}(\lambda) e_t$ plus an $o_{p\lambda}(1)$ term. In particular,

$$\begin{aligned} T^{-2} \Sigma_1^T y_{t-1}(\lambda)^2 &= T^{-2} \Sigma_1^T \left\{ y_{t-1} - z_{tT}(\lambda)' \left[\Sigma_1^T z_{sT}(\lambda) z_{sT}(\lambda)' \right]^{-1} \Sigma_1^T z_{sT}(\lambda, s) y_s \right\}^2 \\ &= T^{-1} \Sigma_1^T \left\{ T^{-1/2} S_{t-1} \right. \\ &\quad \left. - z_{tT}(\lambda)' \delta_T \left[T^{-1} \Sigma_1^T \delta_T z_{sT}(\lambda) z_{sT}(\lambda)' \delta_T \right]^{-1} T^{-1} \Sigma_1^T \delta_T z_{sT}(\lambda) T^{-1/2} S_{s-1} \right\}^2 + o_{p\lambda}(1) \quad (A5) \\ &= \int_0^1 \left\{ \sigma X_T(r) - Z_T(\lambda, r)' \left[\int_0^1 Z_T(\lambda, s) Z_T(\lambda, s)' ds \right]^{-1} \int_0^1 Z_T(\lambda, s) \sigma X_T(s) ds \right\}^2 dr + o_{p\lambda}(1) \\ &= H_1[\sigma X_T, Z_T](\lambda) + o_{p\lambda}(1) \end{aligned}$$

and

$$\begin{aligned} T^{-1} \Sigma_1^T y_{t-1}(\lambda) e_t &= T^{-1} \Sigma_1^T \left\{ y_{t-1} - z_{tT}(\lambda)' \left[\Sigma_1^T z_{sT}(\lambda) z_{sT}(\lambda)' \right]^{-1} \Sigma_1^T z_{sT}(\lambda) y_s \right\} e_t \\ &= T^{-1} \Sigma_1^T S_{t-1} e_t \\ &\quad - T^{-1/2} \Sigma_1^T e_t z_{tT}(\lambda)' \delta_T \left[T^{-1} \Sigma_1^T \delta_T z_{sT}(\lambda) z_{sT}(\lambda)' \delta_T \right]^{-1} T^{-1} \Sigma_1^T \delta_T z_{sT}(\lambda) T^{-1/2} S_{s-1} \\ &\quad + o_{p\lambda}(1) \quad (A6) \\ &= (1/2)(\sigma^2 X_T(1) - \sigma_T^2) \\ &\quad - T^{-1/2} \Sigma_1^T e_t Z_T(\lambda, t/T)' \left[\int_0^1 Z_T(\lambda, s) Z_T(\lambda, s)' ds \right]^{-1} \int_0^1 Z_T(\lambda, s) \sigma X_T(s) ds + o_{p\lambda}(1) \\ &= H_2[\sigma X_T, Z_T, T^{-1/2} \Sigma_1^T Z_T e_t, \sigma_T^2](\lambda) + o_{p\lambda}(1). \end{aligned}$$

For the analysis that follows, we require the following lemma.

LEMMA A1: $T^{-1/2} \Sigma_1^T Z_T(\cdot, t/T) e_t \Rightarrow \sigma \int_0^1 Z(\cdot, r) dW(r)$.

PROOF: The individual components of the vector $T^{-1/2} \Sigma_1^T Z_T(\lambda, t/T) e_t$ are $T^{-1/2} \Sigma_1^T e_t$, $T^{-3/2} \Sigma_1^T t e_t$ and $T^{-1/2} \Sigma_{[T\lambda]+1}^T e_t$ respectively. By straightforward manipulations, we may express the above sums as functions of X_T . That is, $T^{-1/2} \Sigma_1^T e_t = \sigma X_T(1)$, $T^{-3/2} \Sigma_1^T t e_t = \sigma [X_T(1) - \int_0^1 X_T(r) dr]$ and $T^{-1/2} \Sigma_{[T\lambda]+1}^T e_t = \sigma X_T(1) - X_T(\lambda)$. By joint convergence and the CMT, we have that $[T^{-1/2} \Sigma_1^T Z_T(\cdot, t/T) e_t] \Rightarrow [\sigma W(1), \sigma [W(1) - \int_0^1 W], \sigma (W(1) - W(\cdot))]' = \sigma \int_0^1 Z(\cdot, r) dW(r)$. \square

Note that the convergence result of Lemma A1 holds jointly with $X_T(\cdot) \Rightarrow W(\cdot)$. Further, using arguments similar to those used below it can be shown that $s^2(\lambda) = \sigma^2 + o_{p\lambda}(1)$. Since $Z_T(\cdot, \cdot)$ has the degenerate limiting distribution $Z(\cdot, \cdot)$, and σ_T^2 and $s^2(\cdot)$ have the degenerate limit distributions σ^2 and $\sigma^2 1(\cdot)$, where $1(\cdot)$ is the constant function equal to 1 $\forall \lambda \in \Lambda$, it follows that $(X_T(\cdot), Z_T(\cdot, \cdot), T^{-1/2} \Sigma_1^T Z_T(\cdot, t/T) e_t, \sigma_T^2, s^2(\cdot))'$ converges weakly to $(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))'$. Hence the desired result follows from the CMT provided (A2) defines a continuous functional with probability one with respect to the limit process $(W(\cdot), Z(\cdot, \cdot), \int_0^1 Z(\cdot, r) dW(r), \sigma^2, \sigma^2 1(\cdot))'$. In what follows, continuity is defined using the uniform metric on the space of functions on Λ and on the space of functions on $[0, 1]$.

We prove the continuity of g in a series of steps. The first step establishes continuity of H_1 at (W, Z) and H_2 at $(W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2)$ a.s. $[W]$. The second step establishes continuity of $h[m_1, m_2, m_3](\cdot)$ at $(m_1, m_2, m_3) = (H_1[\sigma W, Z], H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2], \sigma^2 1)$ a.s. $[W]$. The last step establishes the continuity of $h^*(m)$ at all real functions m on Λ . The continuity of g then follows from the continuity of a composition of continuous functions and the result of the theorem follows from the CMT.

LEMMA A2: *The functions H_1 and H_2 defined in (A5) and (A6) are continuous at (W, Z) and $(W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2)$, respectively, with W -probability one.*

PROOF: From (A5) we see that the functional $H_1[\sigma W, Z](\lambda)$ is simply the sum of products of the functions $\int_0^1 W^2$, $\int_0^1 Z(\lambda, r) W(r) dr$, and $\left[\int_0^1 Z(\lambda, r) Z(\lambda, r)' dr \right]^{-1}$, each of which is being viewed as a map that maps $\sigma W(\cdot)$ on $[0, 1]$ and $Z(\cdot, \cdot)$ on $\Lambda \times [0, 1]$ to a function on Λ . From (A6) we see that $H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2](\lambda)$ is similarly defined with the addition of the terms $W(1)$, $\int_0^1 Z(\lambda, r) dW(r)$, and σ^2 . Continuity of H_1 with respect to (W, Z) and H_2 with respect to $(W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2)$ follows from continuity of each of the above functions with respect to (W, Z) ($(W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2)$ respectively) with W -probability one provided each function is bounded over Λ ; i.e., provided $\sup_{\lambda \in \Lambda} \left| \int_0^1 Z(\lambda, r) W(r) dr \right| < \infty$ and likewise for the other functions. Let $\int_0^1 ZZ' = \int_0^1 Z(\lambda, r) Z(\lambda, r)' dr$. The function $\left[\int_0^1 ZZ' \right]^{-1}$ will be continuous and bounded over $[0, 1]$ provided $\inf_{\lambda \in \Lambda} \det \left[\int_0^1 ZZ' \right] > 0$. Since $\det \left[\int_0^1 ZZ' \right] = (1/3)(1-\lambda) - (1/4)(1-\lambda^2)^2 - (1/4)(1-\lambda)\lambda^2 + (1/4)(1-\lambda)(1-\lambda^2) - (1/3)(1-\lambda)^2$, $\inf_{\lambda \in \Lambda} \det \left[\int_0^1 ZZ' \right] > 0$ if Λ is a closed subset of $(0, 1)$, which we assume. For the integral functions involving W , consider $\int_0^1 W^2$ for example. Let W and \tilde{W} be two Wiener processes on $[0, 1]$ such that for some $\delta > 0$ $\sup_{r \in [0, 1]} |W(r) - \tilde{W}(r)| < \delta$. Then

$$\begin{aligned} \left| \int_0^1 W^2 - \int_0^1 \tilde{W}^2 \right| &\leq \sup_{r \in [0, 1]} |W(r)^2 - \tilde{W}(r)^2| \\ &\leq \sup_{r \in [0, 1]} |W(r) - \tilde{W}(r)| \cdot \sup_{r \in [0, 1]} |W(r) + \tilde{W}(r)| \\ &\leq \delta \cdot \sup_{r \in [0, 1]} |W(r) + \tilde{W}(r)|. \end{aligned}$$

Since W and \tilde{W} are continuous functions with probability one on the compact set $[0, 1]$,

$\sup_{r \in [0, 1]} |W(r) + \tilde{W}(r)| < \infty$ with probability one and continuity follows on a set with

W -probability one. Similar proofs hold for $\int_0^1 Z(\lambda, r) W(r) dr$ and $\int_0^1 Z(\lambda, r) dW(r)$, using

the fact that the latter function can be written as an explicit function of $W(\cdot)$ as in the proof of Lemma A1. \square

REMARK: The functions $H_1[\sigma W, Z](\lambda)$ and $H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2](\lambda)$ may be expressed as $\sigma^2 \int_0^1 W(\lambda, r)^2 dr$ and $\sigma^2 \int_0^1 W(\lambda, r) dW(r)$, respectively, where $W(\lambda, r)$ is the limit expression of the projection residual $y_t(\lambda)$; i.e.,

$$W(\lambda, r) = W(r) - Z(\lambda, r)' \left[\int_0^1 Z(\lambda, s) Z(\lambda, s)' ds \right]^{-1} \int_0^1 Z(\lambda, s) W(s) ds. \quad (A7)$$

LEMMA A3: The function h defined in (A4) is continuous at $(m_1, m_2, m_3) = (H_1[\sigma W, Z], H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2], \sigma^2 1)$ with W -probability one.

PROOF: Since $h[m_1, m_2, m_3](\cdot) = m_1(\cdot)^{-1/2} m_2(\cdot) / m_3(\cdot)$, h is continuous at $(m_1, m_2, m_3) = (H_1[\sigma W, Z], H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2], \sigma^2 1)$ with W -probability one provided $\sigma^2 > 0$ and $\inf_{\lambda \in \Lambda} |H_1[\sigma W, Z](\lambda)| > 0$ with probability one. Suppose $H_1[\sigma W, Z](\lambda) = 0$ with positive W -probability. Then, since $H_1[\sigma W, Z](\lambda)$ is continuous in λ with W -probability one and Λ is compact, there exists a $[0, 1]$ -valued random variable λ_0 such that for those realizations of W for which $\inf_{\lambda \in \Lambda} |H_1[\sigma W, Z](\lambda)| = 0$ we have $H_1[\sigma W, Z](\lambda_0) = 0$ and $\lambda_0 \in \Lambda$, for other realizations of W $\lambda_0 = 0$, and $\lambda_0 > 0$ with positive probability. In consequence, on a set with positive probability $W(\lambda_0, r) = 0 \forall r \in [0, 1]$. From the definition of $W(\lambda_0, r)$ given in (A7) above, this implies that

$$\begin{aligned} W(r) &= Z(\lambda_0, r)' \left[\int_0^1 Z(\lambda_0, s) Z(\lambda_0, s)' ds \right]^{-1} \int_0^1 Z(\lambda_0, s) W(s) ds \\ &= (1, r, du(\lambda_0, r)) \cdot C(W, \lambda_0) \end{aligned} \quad (A8)$$

$\forall r \in [0, 1]$, where $C(W, \lambda_0)$ is a (3×1) vector, independent of r , with elements $C_1(W, \lambda_0)$, $C_2(W, \lambda_0)$ and $C_3(W, \lambda_0)$ respectively. Now consider any $0 \leq r_1 < r_2 < r_3 < \inf\{\lambda : \lambda \in \Lambda\}$. By definition of the Wiener process, the increments $W(r_3) - W(r_2)$ and $W(r_2) - W(r_1)$ are independent. On the other hand, by (A8), $W(r_3) - W(r_2) = C_2(W, \lambda_0)(r_3 - r_2)$ and $W(r_2) - W(r_1) = C_2(W, \lambda_0)(r_2 - r_1)$ on a set

with positive probability. This implies that these increments are not independent, which is a contradiction. Hence we conclude that $\lambda_0 = 0$ with probability one and the desired result follows. \square

LEMMA A4: *The function h^* defined in (A3) is continuous at all functions m on Λ .*

PROOF: Given $\epsilon > 0$, let m and \tilde{m} be two functions on Λ such that $\sup_{\lambda \in \Lambda} |m(\lambda) - \tilde{m}(\lambda)| < \epsilon$. Then the result follows from the inequality

$$|\inf_{\lambda \in \Lambda} m(\lambda) - \inf_{\lambda \in \Lambda} \tilde{m}(\lambda)| \leq \sup_{\lambda \in \Lambda} |m(\lambda) - \tilde{m}(\lambda)| < \epsilon. \square$$

The proof of the theorem follows from Lemmas A1–A4, the continuity of a composition of continuous functions and the CMT. The expression for the limit distribution given in the theorem may be verified by using the integral representations of $H_1[\sigma W, Z](\cdot)$ and $H_2[\sigma W, Z, \int_0^1 Z(\cdot, r) dW(r), \sigma^2](\cdot)$ described in the above remark.

APPENDIX B

This appendix details the approach used to approximate the limiting distributions in Theorem 1. It is instructive to outline the steps of the approximation since our methodology for approximating the limiting distributions differs slightly from the procedure used by Perron. First we generate $N = 1000$ iid $N(0,1)$ random variables, $\{e_t\}$, and form the $(N \times 1)$ vector of partial sums, S . Then for each value of $\lambda = j/N$, where j runs from 2 to 999, we create the data matrix $X^i(\lambda) = (Z^i(\lambda), S_{-1})$, where $Z^i(\lambda)$ contains the deterministic components of the regressions, and construct the projection residual vector $S^i(\lambda) = (I - X^i(\lambda)(X^i(\lambda)'X^i(\lambda))^{-1}X^i(\lambda)')S$ for each model $i = A, B, C$. We then form sample moments that converge as $N \rightarrow \infty$ to the functions of the standardized Wiener processes that are involved in the expressions in the theorem. That is, we form

$$\begin{aligned} N^{-1}\Sigma_1^N S^i(\lambda)_{j-1}e_j & \left[\Rightarrow \int_0^1 W^i(\lambda;r)dW \text{ as } N \rightarrow \infty \right] \text{ and} \\ N^{-2}\Sigma_1^N S^i(\lambda)_{j-1}^2 & \left[\Rightarrow \int_0^1 W^i(\lambda;r)^2 dr \text{ as } N \rightarrow \infty \right]. \end{aligned}$$

Using these values we form the approximate expressions for the limiting distributions of the statistics for a fixed value of λ , e.g.,

$$t_{\alpha j}^i(\lambda) = \left[N^{-2}\Sigma_1^N S^i(\lambda)_{j-1}^2 \right]^{-1/2} \left[N^{-1}\Sigma_1^N S^i(\lambda)_{j-1}e_j \right].$$

We do this for each value of $\lambda = j/N$ for $1 < j < N$ and from these $N-2$ expressions we define λ_{\inf}^i to be the value of λ that minimizes the above expression. The test statistic approximations evaluated at these values of λ give the corresponding approximate limiting distributions of the test statistics. This process gives us one observation from the asymptotic distributions of the test statistics. We repeat this process 5000 times and obtain the critical values for the limiting distributions from the sorted vector of replicated statistics.

FOOTNOTES

¹Phillips (1988b) argues, on the other hand, that the need for structural shifts to eliminate the statistical evidence in favor of the unit root hypothesis actually provides support for this hypothesis because such adjustments attach unit weight, and hence persistence, to certain observations.

²Perron also considers the normalized bias statistic, $T(\hat{\alpha}(\lambda) - 1)$. This statistic cannot be used to test the unit root hypothesis with the ADF methodology, however, since its asymptotic distribution depends on an infinite number of nuisance parameters. On the other hand, it can be used if Phillips type corrections are employed. Since we use the ADF methodology we will not consider this statistic.

³Several recent papers in the econometric literature consider the problem of testing for *structural change* with unknown change point, see Ploberger, Krämer, and Kontrus (1989), Andrews (1989), Chu (1989), and Hansen (1990). The problem considered here differs from that considered in the aforementioned papers. The problem considered here is one of testing for a *unit root* against the alternative of stationarity with structural change at some unknown point.

⁴Except for one series, the effect of excluding $D(T_B)_t$ from (1') and (3') is to increase (in absolute value) the magnitude of the t-statistic for testing $\alpha^i = 1$. The actual changes in the t-statistics for the series with estimated break points equal to Perron's break points are: -.55 (real GNP), -.40 (nominal GNP), -.52 (real per capita GNP), -.48 (industrial production), -.44 (employment), -.08 (GNP deflator), .11 (nominal wages), -.05 (money stock). For the series with estimated break points different from Perron's choices, the changes in the t-statistics are: -1.48 (consumer prices), -1.73 (velocity), -.53 (interest rates), -.01 (quarterly real GNP), -.74 (common stock prices) and -.46 (real wages).

⁵This range corresponds to our choice of $\Lambda = [.001, .999]$. In fact, the results are not sensitive to this particular choice of Λ .

⁶It is important to note that the number of extra regressors, k , required for the ADF regressions was allowed to vary for each tentative choice of λ . We determined k using the same selection procedure as that used by Perron. That is, working backwards from $k = \bar{k}$, we chose the first value of k such that the t-statistic on \hat{c}_k was greater than 1.6 in absolute value and the t-statistic on \hat{c}_ℓ for $\ell > k$ was less than 1.6. For the Nelson and Plosser series we set $\bar{k} = 8$ and for the postwar quarterly real GNP series we set $\bar{k} = 12$. These are the same values of \bar{k} used by Perron (although a typographical error in his paper erroneously indicates that he used $\bar{k} = 12$ for the Nelson and Plosser series).

⁷The critical values for Perron's test statistics presented in our Tables 2B, 3B and 4B were generated from projection residual approximations instead of the approximations used in Perron's Theorem 2 to give more accurate comparisons with the critical values derived in this paper. The two techniques give approximately the same results and any difference can be attributed to simulation error.

⁸The Nelson and Plosser data were generously provided by Charles Nelson. The postwar quarterly real GNP series (GNP82) was extracted from the Citibase databank.

⁹Of course, our inability to reject the unit root null hypothesis for these series should not be interpreted as an acceptance of the unit root hypothesis.

¹⁰The finite sample distributions of our test statistics are sensitive to the procedure used to determine k , the number of lags of first differences of the data used in the regressions (1')-(3'). In particular, when k is fixed at some value, say k^* , for each tentative choice of the break fraction λ instead of being allowed to vary, the fixed- k finite sample distributions of the minimum t -statistics are much closer to the appropriate asymptotic distributions than the random- k finite sample distributions. Furthermore, this result obtains regardless of the value of k^* chosen (for $k^* \leq 8$ for the Nelson and Plosser data and $k^* \leq 12$ for the postwar quarterly real GNP data). For example, the 1%, 2.5%, 5% and 10% points (based on 5000 repetitions) of the fixed- k distributions for the nominal GNP series are: (1) $k^* = 2$: -5.55, -5.21, -4.89, -4.62; (2) $k^* = 4$: -5.52, -5.18, -4.94, -4.63; (3) $k^* = 6$: -5.56, -5.19, -4.91, -4.60; (4) $k^* = 8$: -5.61, -5.17, -4.88, -4.60. These percentage points are, on average over k^* , 10% smaller (in absolute value) than the random- k percentage points reported in Table 8A. The p -values for nominal GNP computed from the above four fixed- k distributions are .005, .003, .005 and .005, whereas the asymptotic p -value is .003 and the p -value computed from the random- k distribution is .017.

¹¹For a sequence of iid normal random variables, the sample kurtosis has standard error equal to $(24/T)^{1/2}$. Using this formula, the estimated kurtosis values for the nominal GNP, nominal wages and common stock prices are 4.3, 2.3 and 5.7 standard deviations larger than the kurtosis values of a normal random variable.

¹²We originally tried the traditional method of moments approach to estimate η by using the fact that the kurtosis of a Student- t random variable with η degrees of freedom is $3 + 6/(\eta-4)$. Using this method, six degrees of freedom were determined for nominal GNP and common stock prices, eight were chosen for nominal wages, twelve for both industrial production and employment and sixteen for real GNP. Our test conclusions based on these t -distributions are the same as in the normal case.

¹³The characteristic function for a symmetric stable random variable Z is of the form $\phi(u) = \exp(-d|u|^\alpha)$, where $\alpha \in (0,2]$ is referred to as the characteristic exponent and $d^{1/\alpha} \in [0,\infty)$ is the scale parameter. If $\alpha = 2$ then $Z \sim N(0,2d)$. If $\alpha = 1$, then Z has the Cauchy distribution with probability density $f(z) = (d/\pi)[d^2 + z^2]^{-1}$. The use of stable ARMA models is discussed by Brockwell and Davis (1987), chapter 12. The authors would like to thank Mico Loretan for generously providing the GAUSS code to simulate stable random variables.

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TABLE 1
Minimum t-Statistics

Series ⁱ (i = A,B,C)	Rank					
	1		2		3	
	t-stat	year	t-stat	year	t-stat	year
Real GNP ^A	-5.58 ^{***}	1929	-4.34	1928	-3.89	1927
Nominal GNP ^A	-5.82 ^{***}	1929	-4.36	1927	-4.23	1928
Real Per Capita GNP ^A	-4.61 ^{***}	1929	-4.29	1928	-4.09	1927
Industrial Production ^A	-5.95 ^{***}	1929	-5.40	1928	-5.09	1927
Employment ^A	-4.95 ^{***}	1929	-4.71	1928	-4.38	1927
GNP Deflator ^A	-4.12 ^{**}	1929	-3.90	1928	-3.82	1930
Consumer Prices ^A	-2.76	1873	-2.69	1872	-2.64	1864
Nominal Wages ^A	-5.30 ^{***}	1929	-5.10	1930	-4.61	1928
Money Stock ^A	-4.34 ^{***}	1929	-4.34	1928	-4.32	1930
Velocity ^A	-3.39	1949	-3.35	1947	-3.21	1946
Interest Rate ^A	-0.98	1932	-0.96	1965	-0.91	1967
Quarterly Real GNP ^B	-3.99 ^{**}	1973:II	-3.99	1974:I	-3.98	1973:I
Common Stock Prices ^C	-5.61 ^{***}	1936	-5.60	1937	-5.52	1939
Real Wages ^C	-4.74 ^{***}	1940	-4.67	1941	-4.59	1931

Notes: The minimum t-statistics were determined as follows. For each series, equation (1'), (2') or (3') was estimated with the break point, T_B , ranging from $t = 2$ to $t = T-1$. For each regression, k was determined as in footnote 6 and the t-statistic for testing $\alpha^i = 1$ was computed. The minimum t-statistic reported is the minimum over all $T-2$ regressions.

The symbols *, ** and *** indicate that the unit root hypothesis is rejected at the 10%, 5% and 1% levels, respectively, using Perron's critical values.

TABLE 2A

Percentage Points of the Asymptotic Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}^A(\lambda)$

1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
-5.34	-5.02	-4.80	-4.58	-3.75	-2.99	-2.77	-2.56	-2.32

TABLE 2B

Percentage Points of the Asymptotic Distribution of $t_{\hat{\alpha}}^A(\lambda)$ for a Fixed λ

λ	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
0.1	-4.30	-3.93	-3.68	-3.40	-2.35	-1.38	-1.09	-0.78	-0.46
0.2	-4.39	-4.08	-3.77	-3.47	-2.45	-1.45	-1.14	-0.90	-0.54
0.3	-4.39	-4.03	-3.76	-3.46	-2.42	-1.43	-1.13	-0.83	-0.51
0.4	-4.34	-4.01	-3.72	-3.44	-2.40	-1.26	-0.88	-0.55	-0.21
0.5	-4.32	-4.01	-3.76	-3.46	-2.37	-1.17	-0.79	-0.49	-0.15
0.6	-4.45	-4.09	-3.76	-3.47	-2.38	-1.28	-0.92	-0.60	-0.26
0.7	-4.42	-4.07	-3.80	-3.51	-2.45	-1.42	-1.10	-0.82	-0.50
0.8	-4.33	-3.99	-3.75	-3.46	-2.43	-1.46	-1.13	-0.89	-0.57
0.9	-4.27	-3.97	-3.69	-3.38	-2.39	-1.37	-1.04	-0.74	-0.47

Notes: λ = time of break relative to total sample size. Percentage points are based on 5000 repetitions.

TABLE 3A

Percentage Points of the Asymptotic Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}^B(\lambda)$

1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
-4.93	-4.67	-4.42	-4.11	-3.23	-2.48	-2.31	-2.17	-1.97

TABLE 3B

Percentage Points of the Asymptotic Distribution of $t_{\hat{\alpha}}^B(\lambda)$ for a Fixed λ

λ	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
0.1	-4.27	-3.94	-3.65	-3.36	-2.34	-1.35	-1.04	-0.78	-0.40
0.2	-4.41	-4.08	-3.80	-3.49	-2.50	-1.48	-1.18	-0.87	-0.52
0.3	-4.51	-4.17	-3.87	-3.58	-2.54	-1.59	-1.27	-0.97	-0.69
0.4	-4.55	-4.20	-3.94	-3.66	-2.61	-1.69	-1.37	-1.11	-0.75
0.5	-4.55	-4.20	-3.96	-3.68	-2.70	-1.74	-1.40	-1.18	-0.82
0.6	-4.57	-4.20	-3.95	-3.66	-2.61	-1.71	-1.36	-1.11	-0.78
0.7	-4.51	-4.13	-3.85	-3.57	-2.55	-1.61	-1.28	-0.97	-0.67
0.8	-4.38	-4.07	-3.82	-3.50	-2.47	-1.49	-1.16	-0.87	-0.54
0.9	-4.26	-3.96	-3.68	-3.35	-2.33	-1.34	-1.04	-0.77	-0.43

Notes: λ = time of break relative to total sample size. Percentage points are based on 5000 repetitions.

TABLE 4A

Percentage Points of the Asymptotic Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}}^C(\lambda)$

1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
-5.57	-5.30	-5.08	-4.82	-3.98	-3.25	-3.06	-2.91	-2.72

TABLE 4B

Percentage Points of the Asymptotic Distribution of $t_{\hat{\alpha}}^C(\lambda)$ for a Fixed λ

λ	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
0.1	-4.38	-4.01	-3.75	-3.45	-2.38	-1.44	-1.11	-0.82	-0.45
0.2	-4.65	-4.32	-3.99	-3.66	-2.67	-1.60	-1.27	-0.98	-0.67
0.3	-4.78	-4.46	-4.17	-3.87	-2.75	-1.78	-1.46	-1.15	-0.81
0.4	-4.81	-4.48	-4.22	-3.95	-2.88	-1.91	-1.62	-1.35	-1.04
0.5	-4.90	-4.53	-4.24	-3.96	-2.91	-1.96	-1.69	-1.43	-1.07
0.6	-4.88	-4.49	-4.24	-3.95	-2.87	-1.93	-1.63	-1.37	-1.08
0.7	-4.75	-4.44	-4.18	-3.86	-2.77	-1.81	-1.47	-1.17	-0.79
0.8	-4.70	-4.31	-4.04	-3.69	-2.67	-1.63	-1.29	-1.04	-0.64
0.9	-4.41	-4.10	-3.80	-3.46	-2.41	-1.44	-1.12	-0.80	-0.50

Notes: λ = time of break relative to total sample size. Percentage points are based on 5000 repetitions.

TABLE 5

Model	Critical Value	Size
A	-3.68	0.551
B	-3.96	0.142
C	-4.24	0.345

TABLE 6A

Tests for a Unit Root: Model (A)

$$\text{Regression: } y_t = \hat{\mu}^A + \hat{\theta}^A \text{DU}(\hat{\lambda})_t + \hat{\beta}^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^k \hat{c}_j^A \Delta y_{t-j} + \hat{e}_t$$

Series	T	\hat{T}_B	k	$\hat{\mu}^A$	$\hat{\theta}^A$	$\hat{\beta}^A$	$\hat{\alpha}^A$	S(\hat{e})
Real GNP	62	1929	8	3.514 (5.62)	-.195 (-4.92)	.027 (5.71)	.267 (-5.58)***	.05
Nominal GNP	62	1929	8	5.040 (5.85)	-.311 (-5.12)	.032 (5.97)	.532 (-5.82)***	.07
Real Per Capita GNP	62	1929	7	3.584 (4.62)	-.117 (-3.41)	.012 (4.69)	.494 (-4.61)*	.056
Industrial Production	111	1929	8	.122 (4.46)	-.317 (-5.12)	.034 (5.91)	.290 (-5.95)***	.088
Employment	81	1929	7	3.564 (4.97)	-.051 (-3.14)	.006 (4.79)	.651 (-4.95)**	.029
GNP Deflator	82	1929	5	.641 (4.17)	-.091 (-3.23)	.007 (4.14)	.786 (-4.12)	.044
Consumer Prices	111	1873	2	.217 (2.79)	-.055 (-2.51)	.001 (3.27)	.941 (-2.76)	.043
Nominal Wages	71	1929	7	2.126 (5.35)	-.161 (-4.16)	.017 (5.32)	.660 (-5.30)**	.054
Money Stock	82	1929	6	.288 (4.76)	-.064 (-2.54)	.011 (4.25)	.823 (-4.34)	.044
Velocity	102	1949	0	.224 (2.99)	.095 (3.09)	-.002 (-2.95)	.840 (-3.39)	.064
Interest Rate	71	1932	2	.065 (.31)	-.444 (-2.55)	.013 (3.09)	.945 (-.98)	.272

TABLE 6B

Tests for a Unit Root: Model (B)

Regression: $y_t = \tilde{\mu}^B + \tilde{\beta}^B t + \tilde{\gamma}^{DT*}(\hat{\lambda})_t + \tilde{y}_t^B(\hat{\lambda})$;

$$\tilde{y}_t^B(\hat{\lambda}) = \hat{\alpha} \tilde{y}_{t-1}^B(\hat{\lambda}) + \sum_{j=1}^k \hat{c}_j^B \Delta \tilde{y}(\hat{\lambda})_{t-j} + \tilde{e}_t$$

Series	T	\hat{T}_B	k	$\tilde{\mu}^B$	$\tilde{\beta}^B$	$\tilde{\gamma}^B$	$\hat{\alpha}^B$	S(\tilde{e})
Quarterly Real GNP	159	73:II	10	6.978 (1150)	.009 (97.3)	-.003 (-11.4)	.857 (-3.99)	.010

TABLE 6C

Tests for a Unit Root: Model (C)

Regression: $y_t = \hat{\mu}^C + \hat{\theta}^C DU(\hat{\lambda})_t + \hat{\beta}^C t + \hat{\gamma}^C DT^*(\hat{\lambda})_t + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^k \hat{c}_j^C \Delta y_{t-j} + \hat{e}_t$

Series	T	\hat{T}_B	k	$\hat{\mu}^C$	$\hat{\theta}^C$	$\hat{\beta}^C$	$\hat{\gamma}^C$	$\hat{\alpha}^C$	S(\hat{e})
Common Stock Prices	100	1936	1	.471 (5.12)	-.226 (-3.25)	.007 (4.83)	.021 (4.80)	.642 (-5.61)***	.139
Real Wages	71	1940	8	2.678 (4.81)	.085 (4.33)	.012 (4.49)	.008 (3.68)	.115 (-4.74)	.030

Notes: t-statistics are in parentheses. The t-statistic for $\hat{\alpha}^i$ is for testing $\alpha^i = 1$. k is determined as in footnote 6.

The symbols *, ** and *** denote significance of the test of $\alpha^i = 1$ at the 10%, 5% and 1% levels, respectively, using the critical values from TABLE 2A, 3A or 4A.

TABLE 7A
Selected ARMA Models

$$\text{Model: } \Delta y_t = \hat{\mu} + \hat{\theta}\Delta y_{t-1} + e_t + \hat{\psi}e_{t-1}$$

Series	Model	$\hat{\theta}$	$\hat{\psi}$	$\hat{\mu}$	$\hat{\sigma}$	AIC	SBIC	Q(x)
Real GNP	(1,0)	.341 (2.78)	— (—)	.029 (2.50)	.061	-165	-161	Q(22) 18.06
Nominal GNP	(1,0)	.440 (3.76)	— (—)	.055 (1.36)	.089	-120	-116	Q(22) 23.57
Real Per Capita GNP	(1,0)	.331 (2.69)	— (—)	.016 (1.36)	.062	-164	-160	Q(22) 17.60
Employment	(0,1)	— (—)	.388 (3.72)	.016 (2.38)	.036	-302	-278	Q(22) 18.53
GNP Deflator	(1,0)	.434 (4.27)	— (—)	.20 (2.19)	.047	-262	-257	Q(22) 21.55
Consumer Prices	(0,1)	— (—)	.655 (9.21)	.012 (1.73)	.046	-365	-360	Q(22) 25.78
Nominal Wages	(0,1)	— (—)	.474 (4.44)	.040 (3.81)	.061	-192	-188	Q(22) 34.90
Money Stock	(1,0)	.622 (7.06)	— (—)	.059 (4.28)	.048	-257	-253	Q(22) 22.19
Velocity	(0,1)	— (—)	.116 (1.16)	-.012 (-1.55)	.068	-254	-248	Q(22) 21.53
Quarterly Real GNP	(1,0)	.368 (4.94)	— (—)	.005 (6.08)	.010	-992	-986	Q(28) 19.80
Stock Prices	(0,1)	— (—)	.313 (3.22)	.029 (1.40)	.156	-84.6	-79.5	Q(22) 21.41
Real Wages	(0,1)	— (—)	.205 (1.72)	.018 (3.50)	.036	-263	-258	Q(22) 10.80

Notes: All models were estimated using PROC ARIMA in SAS. t-statistics are in parentheses. AIC denotes the Akaike information criterion, SBIC denotes the Schwartz criterion and Q(x) denotes the Box-Pierce statistic.

TABLE 7B
Selected ARMA Models

$$\text{Model: } \Delta y_t = \hat{\mu} + \sum_1^p \hat{\theta}_i \Delta y_{t-i} + e_t + \sum_1^q \hat{\psi}_i e_{t-i}$$

Series	Model	$\hat{\theta}_i$	$\hat{\psi}_i$	$\hat{\mu}$	$\hat{\sigma}$	AIC	SBIC	Q(x)
Industrial Production	(0,5)	—	.033	.043	.095	-198	-182	Q(18)
		(—)	(.36)					13.42
		—	-.087					
		(—)	(-.98)					
		—	-.022					
		(—)	(-.24)					
		—	-.199					
		(—)	(-2.27)					
Interest Rates	(2,0)	—	-.402					
		(—)	(-4.46)					
		.177	—	.079	.282	24.5	31.3	Q(21)
		(1.43)	(—)	(1.10)				14.98
		.367	—					
		(2.86)	(—)					

TABLE 8A
Percentage Points of the Finite Sample Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^A}(\lambda)$
Assuming Normal ARMA Innovations

Series/Model	T	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
Asymptotic	∞	-5.34	-5.02	-4.80	-4.58	-3.75	-2.99	-2.77	-2.56	-2.32
Real GNP ARMA(1,0)	62	-6.03	-5.65	-5.35	-4.99	-3.96	-2.90	-2.47	-2.07	-1.51
Nominal GNP ARMA(1,0)	62	-6.12	-5.67	-5.38	-5.05	-4.00	-2.90	-2.53	-2.14	-1.52
Real P.C. GNP ARMA(1,0)	62	-6.03	-5.63	-5.32	-5.01	-3.99	-2.92	-2.52	-2.23	-1.62
Ind. Prod. ARMA(0,5)	111	-5.73	-5.41	-5.14	-4.86	-3.88	-3.01	-2.74	-2.52	-2.15
Employment ARMA(0,1)	81	-5.92	-5.55	-5.26	-4.95	-3.97	-3.00	-2.66	-2.26	-1.71
GNP Deflator	82	-5.85	-5.50	-5.21	-4.87	-3.91	-2.99	-2.62	-2.33	-1.82
CPI ARMA(0,1)	111	-5.76	-5.46	-5.14	-4.85	-3.88	-2.97	-2.68	-2.34	-1.95
Nominal Wages ARMA(0,1)	71	-5.93	-5.69	-5.33	-5.02	-4.01	-2.96	-2.55	-2.16	-1.90
Money Stock ARMA(1,0)	82	-5.91	-5.49	-5.19	-4.90	-3.96	-2.94	-2.60	-2.22	-1.71
Velocity ARMA(0,1)	102	-5.67	-5.37	-5.12	-4.85	-3.88	-3.02	-2.75	-2.47	-2.16
Interet Rate ARMA(3,0)	71	-5.90	-5.64	-5.30	-5.00	-3.99	-2.95	-2.60	-2.31	-1.96

TABLE 8B

Percentage Points of the Finite Sample Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^B}(\lambda)$
Assuming Normal ARMA Innovations

Series/Model	T	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
Asymptotic	∞	-4.93	-4.67	-4.42	-4.11	-3.23	-2.48	-2.31	-2.17	-1.97
Quarterly Real GNP ARMA(1,0)	159	-5.41	-5.16	-4.86	-4.59	-3.54	-2.70	-2.51	-2.34	-2.21

TABLE 8C

Percentage Points of the Finite Sample Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^C}(\lambda)$
Assuming Normal ARMA Innovations

Series/Model	T	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
Asymptotic	∞	-5.57	-5.30	-5.08	-4.82	-3.98	-3.25	-3.06	-2.91	-2.72
Stock Prices ARMA(0,1)	100	-6.30	-5.93	-5.63	-5.31	-4.30	-3.30	-3.09	-2.85	-2.64
Real Wages ARMA(0,1)	71	-6.25	-5.92	-5.68	-5.38	-4.32	-3.36	-3.04	-2.81	-2.57

Note: Percentage points are based on 5000 repetitions.

TABLE 9
Skewness and Kurtosis Values for Residuals from ARMA Model for Δy_t

Series	Sample Skewness	Sample Kurtosis	Finite Sample Kurtosis	df
Real GNP	-.317	3.400	3.426	9
Nominal GNP	-1.146	5.868	4.804	4
Real Per Capita GNP	-.235	3.257		
Industrial Production	-.737	3.722	3.815	9
Employment	-.424	3.469	3.420	10
GNP Deflator	-.907	9.739		
Consumer Prices	1.023	6.852		
Nominal Wages	-.304	4.658	4.283	5
Money Stock	-.270	4.636		
Velocity	-.329	2.927		
Interest Rate	.751	4.413		
Quarterly Real GNP	-.061	3.828		
Common Stock Prices	-.390	4.324	4.351	6
Real Wages	-.040	3.161		

Note: The column labeled "Finite Sample Kurtosis" gives the mean kurtosis value obtained from the finite sample distribution of the sample kurtosis using Student-t ARMA innovation with degrees of freedom given in the adjacent column.

TABLE 10A
Percentage Points of the Finite Sample Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^i}(\lambda)$
Assuming Student-t ARMA Innovations

Series	df	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
Nominal GNP	4	-7.56	-6.40	-5.86	-5.31	-4.05	-3.01	-2.63	-2.20	-1.84
Real GNP	9	-6.16	-5.75	-5.39	-5.04	-3.98	-2.92	-2.57	-2.13	-1.56
Ind. Prod.	9	-5.94	-5.60	-5.29	-4.91	-3.95	-3.03	-2.73	-2.48	-1.93
Employment	10	-5.98	-5.67	-5.27	-5.01	-3.98	-3.06	-2.66	-2.32	-1.90
Nominal Wages	5	-7.26	-6.31	-5.81	-5.39	-4.11	-3.12	-2.82	-2.40	-1.86
Stock Prices	6	-6.66	-6.09	-5.84	-5.46	-4.29	-3.37	-3.12	-2.91	-2.71

Notes: The column labeled "df" gives the degrees of freedom of a Student-t random variable which gives the closest match between the observed sample kurtosis and the finite sample mean kurtosis value. Percentage points are based on 1000 repetitions.

TABLE 10 B
Percentage Points of the Finite Sample Distribution of $\inf_{\lambda \in \Lambda} t_{\hat{\alpha}^A}(\lambda)$
Assuming Stable ARMA Innovations

α	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
2.0	-6.12	-5.65	-5.37	-5.05	-4.00	-2.90	-2.53	-2.15	-1.51
1.9	-9.99	-6.69	-6.00	-5.33	-4.08	-2.92	-2.43	-1.95	-1.52
1.8	-12.0	-9.14	-7.05	-5.87	-4.11	-2.94	-2.51	-1.96	-1.54
1.5	-49.3	-18.2	-12.2	-9.32	-4.57	-3.00	-2.57	-1.70	-1.50
1.0	-221	-93.0	-42.9	-20.8	-5.72	-3.02	-2.45	-1.52	-1.01
0.5	-15622	-4518	-893	-205	-7.18	-3.42	-2.32	-1.41	-0.73

Notes: α is the characteristic exponent of a standard stable random variable. $\alpha = 2$ corresponds to a normal variate and $\alpha = 1$ corresponds to a Cauchy variate. The ARMA innovations use the nominal GNP parameters. Percentage points are based on 5000 repetitions.

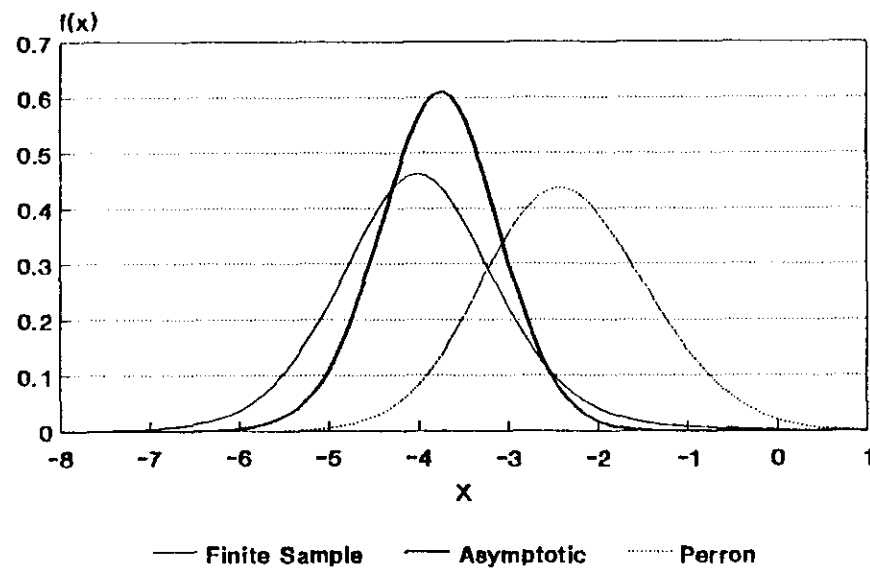
TABLE 11
One Sided P-Values for the Minimum t-Statistics

Series	t-stat	Perron's P-Value	Asymptotic P-Value	F.S.N. P-Value	F.S.T. Value
Real GNP	-5.58	.000***	.003***	.029**	.035** (9)
Nominal GNP	-5.82	.000***	.001***	.017**	.050* (4)
Real Per Capita GNP	-4.61	.003***	.091*	.216	
Industrial Production	-5.95	.000***	.000***	.005***	.009*** (9)
Employment	-4.95	.001***	.031**	.101	.124 (10)
GNP Deflator	-4.12	.017**	.278	.392	
Consumer Prices	-2.76	.340	.951	.939	
Nominal Wages	-5.30	.000***	.012**	.053*	.117 (5)
Money Stock	-4.34	.008***	.174	.293	
Velocity	-3.39	.104	.737	.774	
Interest Rate	-.98	.939	.999	.999	
Quarterly Real GNP	-3.99	.033**	.131	.286	
Common Stock Prices	-5.61	.000***	.009***	.055*	.075* (6)
Real Wages	-4.74	.005***	.119	.298	

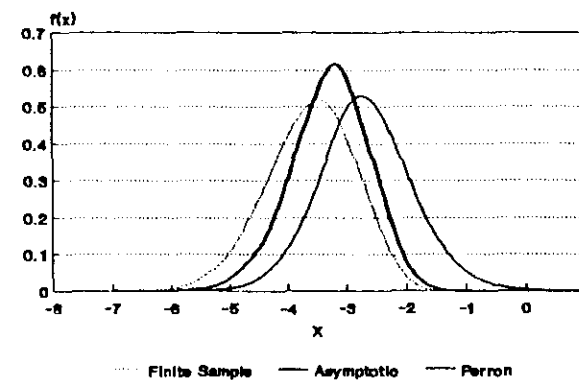
Notes: The symbols *, **, and *** denote rejection at the 10%, 5% and 1% levels respectively. The column labeled "Perron's P-Value" gives the p-values computed from Perron's fixed λ distributions for the appropriate λ value, the column labeled "F.S.N" gives the p-values computed from the finite sample distributions using normal innovations and the column labeled "F.S.T" gives the p-values computed from the finite sample distributions using Student-t innovations. The degrees of freedom for the t-distribution p-values are in parentheses.

Figure 1
Density Plots

Model (A) Densities



Model (B) Densities



Model (C) Densities

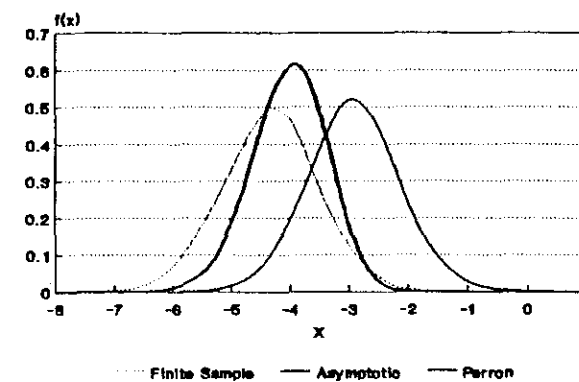
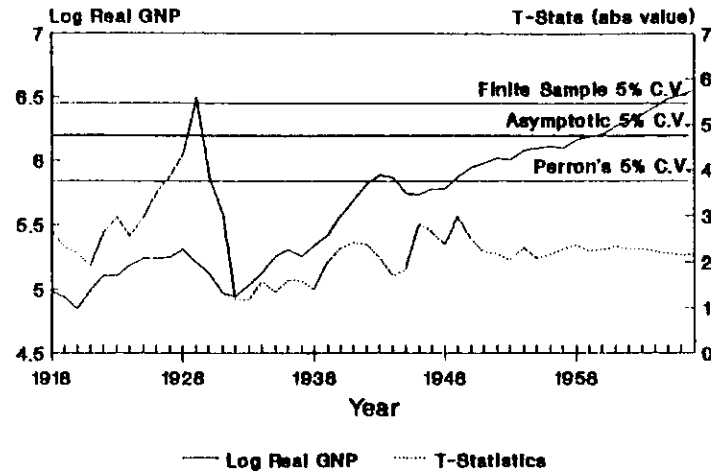


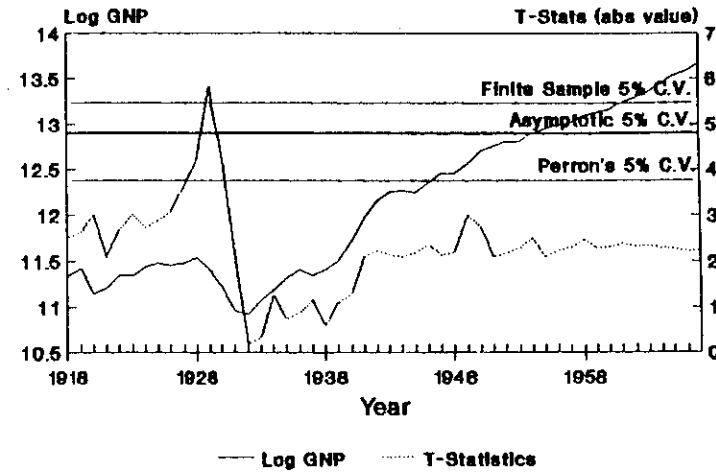
Figure 2

Time Plots of T-Statistics

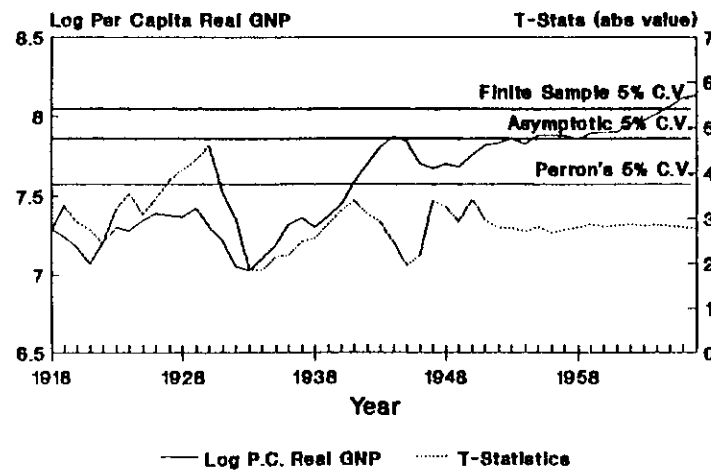
Real GNP



Nominal GNP



Per Capita Real GNP



Industrial Production

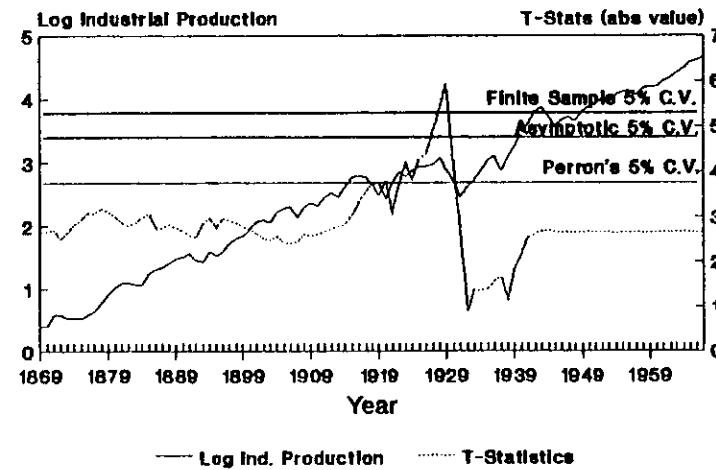
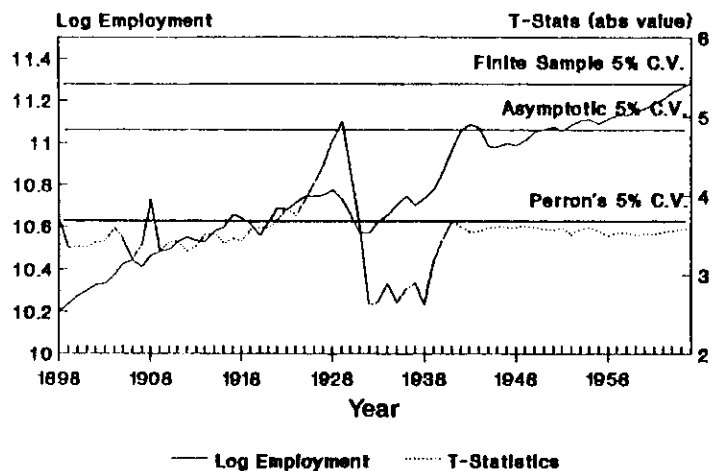
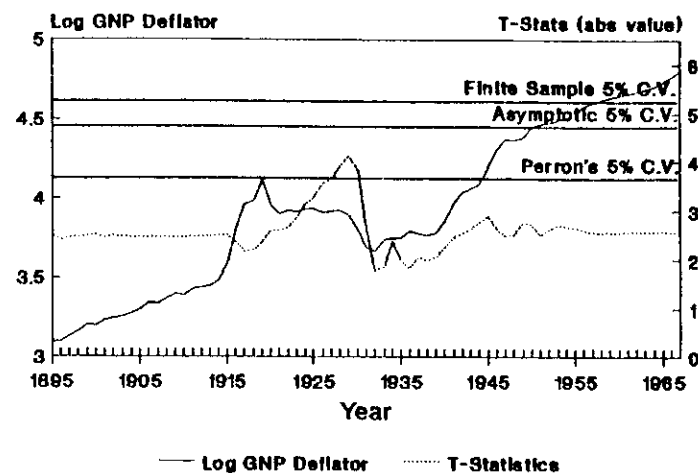


Figure 2 (cont'd)

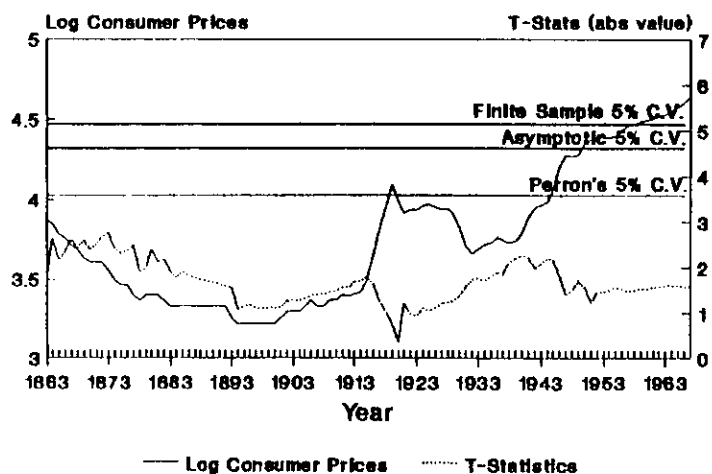
Employment



GNP Deflator



Consumer Prices



Nominal Wages

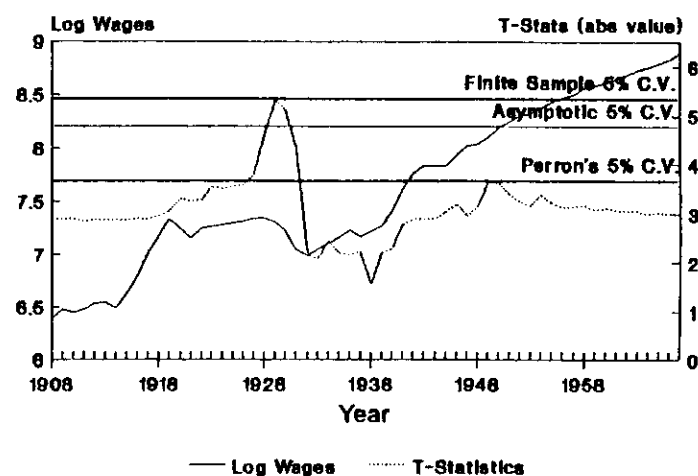
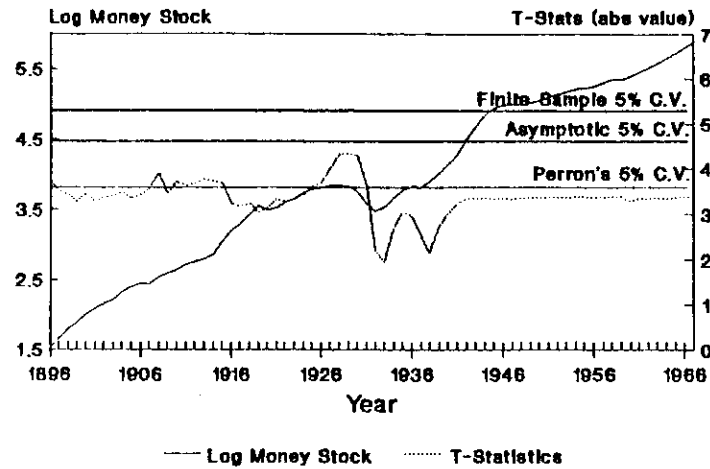
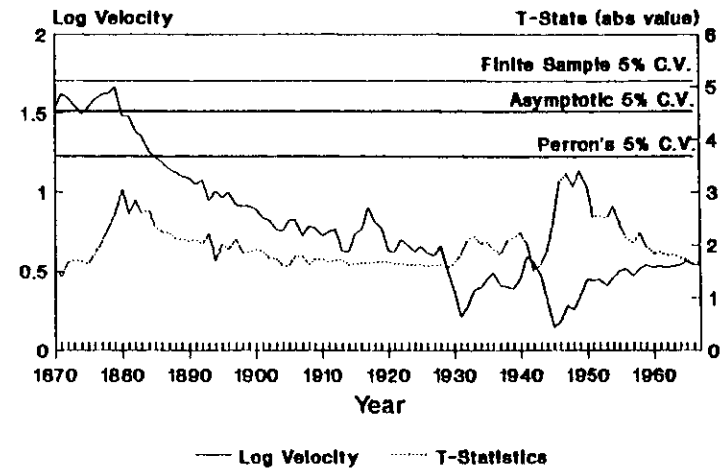


Figure 2 (cont'd)

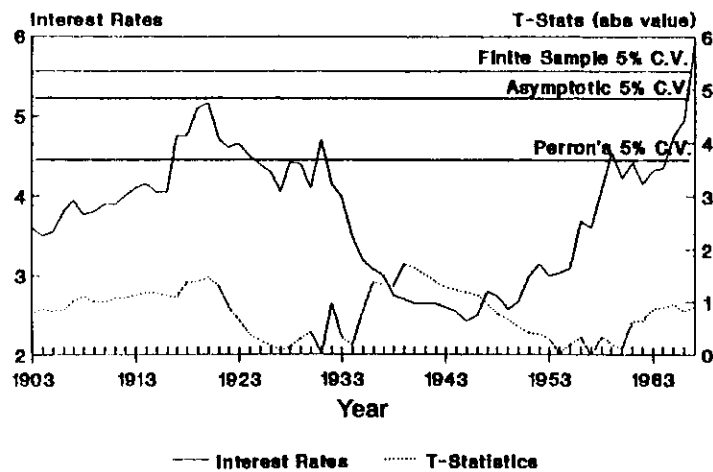
Money Stock



Velocity



Interest Rates



Quarterly Real GNP

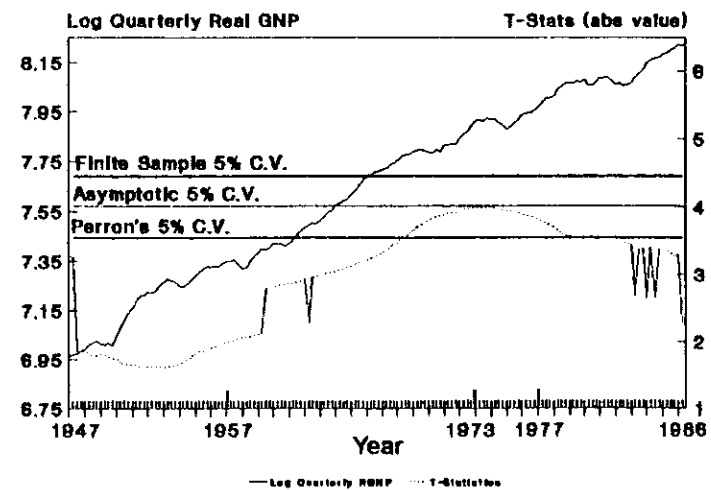
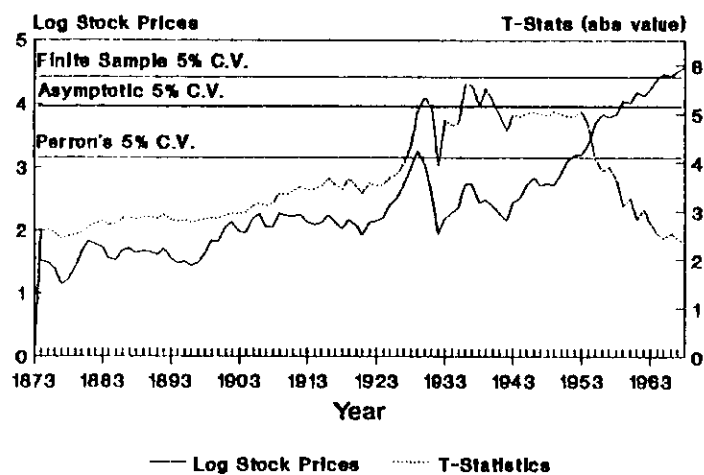


Figure 2 (cont'd)

Common Stock Prices



Real Wages

