PHYSICAL REVIEW D VOLUME 49, NUMBER 9 1 MAY 1994

Further investigation on the precise formulation of the equivalence theorem

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(Received 26 May 1993)

Based on a systematic analysis of the renormalization schemes in the general R_ξ gauge, the precise formulation of the equivalence theorem for longitudinal weak boson scatterings is given both in the $SU(2)_L$ Higgs theory and in the realistic $SU(2) \times U(1)$ electroweak theory to all orders in the perturbation for an arbitrary Higgs boson mass m_H. It is shown that there is generally a renormalization-scheme- and ξ dependent modification factor $C_{\rm mod}$ and a simple formula for $C_{\rm mod}$ is obtained. Furthermore, a convenient particular renormalization scheme is proposed in which $C_{\rm mod}$ is exactly unity. Results of $C_{\rm mod}$ in other currently used schemes are also discussed especially on their ξ and m_H dependence through explicit one-loop calculations. It is shown that in some currently used schemes the deviation of $C_{\rm mod}$ from unity and the ξ dependence of $C_{\rm mod}$ are significant even in the large-m_H limit. Therefore care should be taken when applying the equivalence theorem.

PACS number(s): 12.15.Ji, 11.10.Gh

I. INTRODUCTION

The mechanism of electroweak symmetry breaking is the most unclear issue in the standard model, and it will be one of the most investigated problems in the future study of high energy physics. At the Superconducting Super Collider (SSC) and the CERN Large Hadron Collider (LHC), the electroweak symmetry-breaking mechanism can be probed through longitudinal weak boson scatterings. Since the longitudinal component of the weak boson V_L^a (V^a stands for $W^\pm$ or Z^0) arises from absorbing the would-be Goldstone boson ϕ^a through the Higgs mechanism [1], one may intuitively believe that the scattering of V_L^a's is related to the scattering of ϕ^a's. The quantitative relation between the two scattering amplitudes at energy $E \gg M_W$ is described by the well-known equivalence theorem (ET) which states that

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi) = T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)+O(M_W/E)\ , \tag{1}$$

where Φ denotes other possible on-shell physical particles. This simple relation was first pointed out by Cornwall, Levin, and Tiktopoulos, and by Vayonakis [2] at the tree level. A sketch of the proof in the 't Hooft–Feynman gauge for the case of $n = 1$ was then given by Lee, Quigg, and Thacker [3]. Chanowitz and Gaillard [4], followed by Gounaris, Kögerler, and Neufeld [5], studied the general proof in the R_ξ gauge and they claimed that the simple relation (1) holds to all orders in perturbation for arbitrary values of the Higgs boson mass m_H. As an important and useful tool for studying the electroweak symmetry breaking mechanism, this naive formulation of the ET has been widely used by various authors [6]. However, it was pointed out recently by Yao and Yuan [7] and Bagger and Schmidt [8] from a more careful examination of loop contributions that there should, in general, be a modification factor $C_{\rm mod}$ associated with each external Goldstone boson field ϕ^{a_i}, and $C_{\rm mod} \neq 1$ beyond the tree level; i.e., (1) should be modified as

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi) = C_{\rm mod}^n T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi) +O(M_W/E)\ . \tag{2}$$

In Ref. [8], a complicated expression for $C_{\rm mod}$ in the $SU(2)_L$ theory is given and it is argued that $C_{\rm mod}$ can be formally defined to be exactly unity by a suitable choice of a Goldstone-boson wave function renormalization constant, but no clue was found as to which renormalization scheme will ensure $C_{\rm mod} = 1$. They then performed an approximate simplification of $C_{\rm mod}$ in the heavy Higgs limit under certain subtraction conditions. In the realis-

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tic SU(2)×U(1) theory, we find that the general expressions of $C_{\rm mod}$ are much more complicated (cf. Sec. III). Since the ET is so useful, it is of special importance to make this issue clearer and to exactly simplify the expressions for $C_{\rm mod}$.

In this paper we shall present a systematic study of the general proof of the precise formulation of the ET. We first give a systematic analysis of the renormalization schemes in the general R_ξ gauge for both the $SU(2)_L$ theory and SU(2)×U(1) electroweak theory with special attention to the freedom of adjusting the renormalization constants in the unphysical sector restricted by the Ward-Takahashi (WT) identities. Two particular renormalization schemes, *scheme I* and *scheme II*, with special and convenient determinations of the unphysical renormalization constants are proposed for the sake of simplifying the formulation of the ET. We then give a general proof of the precise formulation of the ET which is generally of the form of Eq. (2) to all orders in the perturbation and for arbitrary value of m_H with $C_{\rm mod}$ specifically given. The precise formulation is given both in the $SU(2)_L$ theory and the realistic SU(2)×U(1) theory which has not been systematically studied in the literatures. In (2), $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ and $C^n_{\rm mod}$ are in general unphysical quantities which depend on the renormalization scheme and the gauge parameter ξ, while the product $C^n_{\rm mod}T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ is physical with an uncertainty of $O(M_W/E)$; i.e., the leading order unphysical parts in $C^n_{\rm mod}$ and $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ cancel each other. We shall see that in *scheme I* the expression for $C_{\rm mod}$ is simplified as *a single quantity already determined in this renormalization scheme itself* and in *scheme II* $C_{\rm mod}$ is *exactly unity*; i.e., *the original simple form (1) of the ET holds in scheme II.* The realization of these schemes is irrelevant to the explicit calculation of $C_{\rm mod}$, so that they are convenient in practical calculations. Finally, we present several applications with explicit calculations up to the one-loop level. The comparison of *scheme II* with other currently used schemes is shown in the explicit results, and the ξ and m_H dependence of $C^n_{\rm mod}$ and $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ in schemes other than *scheme II* are specially examined. It is shown that in some currently used schemes the deviation of $C_{\rm mod}$ from unity and its ξ and m_H dependence are significant even in the large-m_H limit. Therefore care should be taken when applying the ET. A brief sketch of this study has been published in a previous Letter [9] and in this paper we present the complete and detailed investigations.

This paper is organized as follows. Section II presents the systematic analysis of the renormalization schemes in the R_ξ gauge for the $SU(2)_L$ theory and the SU(2)×U(1) electroweak theory in which *scheme I* and *scheme II* are defined. The general proof of the precise formulation of the ET is given in Sec. III. The ξ and m_H dependence of $C_{\rm mod}$ and $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ in some currently used renormalization schemes other than *scheme II* are examined in Sec. IV through explicit calculations up to one loop for large m_H. Section V is an explicit illustration of the up to one-loop results in *scheme I* and *scheme II* for arbitrary m_H in the simple U(1) Higgs theory. A summary of this study and conclusions are given in Sec. VI. In Appendix A, we present a simple derivation of the Slavnov-Taylor identity used in Sec. III for the proof of ET. Some technical details in the text are given in Appendixes B and C.

II. ANALYSIS OF THE RENORMALIZATION SCHEMES IN THE GENERAL R_ξ GAUGE

Consider the standard model. The weak boson, Higgs boson, Goldstones boson, ghost, and antighost fields are denoted by $V^a_\mu, H,\ \phi^a,\ c^a$, and $\bar{c}^a$, respectively. We take the general R_ξ gauge with the gauge-fixing term of the form

$$\begin{aligned}\mathcal{L}_{\rm GF} &= -\tfrac{1}{2}(F^a_0)^2\ ,\\ F^a_0 &\equiv (\xi^a_0)^{-\frac{1}{2}}\partial_\mu V^{a\mu}_0 - (\xi^a_0)^{\frac{1}{2}}\kappa^a_0\phi^a_0\ ,\end{aligned}\tag{3}$$

where the subscript 0 denotes the bare quantities. Here we put in (3) a free parameter κ^a_0 instead of taking it to be the mass of $V^{a\mu}_0$ for generality. Let χ^i_0 be a general symbol denoting the fields except c^a_0 and $\bar{c}^a_0$, g_0 be the bare gauge coupling constant, and T^a be the generator of the gauge group. Following Ref. [10] we define

$$\begin{aligned}&D^a_i(\chi_0) \equiv R^a_i + T^a_{ij}\chi^j_0\ ,\\ &R^a_i = \begin{cases} -g_0^{-1}\delta_{ia}\partial_\mu & \text{if } \chi^i_0 \text{ is the gauge field } V^a_\mu,\\ 0 \text{ otherwise.}\end{cases}\end{aligned}\tag{4}$$

The Faddeev-Popov term $\mathcal{L}_{\rm FP}$ can be written as

$$\begin{aligned}\mathcal{L}_{\rm FP} &= \textstyle\int d^4y\ \bar{c}^a_0(x)\mathcal{M}_{ab}(x,y)c^b_0(y)\ ,\\ \mathcal{M}_{ab} &\equiv \underline{\mathbf{K}}^{ai}_0 D^b_i(\chi_0)\ ,\end{aligned}\tag{5}$$

where

$$\underline{\mathbf{K}}^a_0 \equiv \begin{pmatrix}(\xi^a_0)^{-\frac{1}{2}}\partial_\mu\\ -(\xi^a_0)^{\frac{1}{2}}\kappa^a_0\end{pmatrix}\ .\tag{6}$$

When doing renormalization we determine the multiplicative renormalization constants for the physical sector in the same way as in Ref. [11], i.e., taking the usual on-shell scheme. In what follows we concentrate our attention to the renormalization of the *unphysical sector.* In order to examine the freedom of adjusting the renormalization constants for the unphysical sector, we look at the Ward-Takahashi (WT) identities for the inverse propagators, which put constraints on the renormalization constants. Consider the generating functional

$$\begin{aligned}Z[J,I,\bar{I},K,L] &\equiv \exp(iW[J,I,\bar{I},K,L])\\ &= \int \mathcal{D}\chi_0\mathcal{D}c_0\mathcal{D}\bar{c}_0 \exp\left[i\left(\mathcal{S}[\chi_0,c_0,\bar{c}_0] + \int d^4x(J_i\chi^i_0 + \bar{I}^a c^a_0 + \bar{c}^a_0 I^a + K^i D^a_i c^a_0 + \tfrac{1}{2}g_0 f^{abc}L^a c^b_0 c^c_0)\right)\right]\ ,\end{aligned}\tag{7}$$

where f^{abc} is the structure constant of the gauge group, $\mathcal{S}$ is the action of the fields including the gauge fixing and the Faddeev-Popov terms, and $J_i, I^a, \bar{I}^a, K^i, L^a$ are external sources. Let ω be an infinitesimal Grassmann parameter. The invariance of $\mathcal{S}$ under the Becchi-Rouet-Stora-Tyutin (BRST) transformations [12]

$$\begin{aligned} \chi_0^i &\to \chi_0^i + D_a^i(\chi_0)C_0^a\omega \ , \\ c_0^a &\to c_0^a - \tfrac{1}{2}g_0 f^{abc}c_0^b c_0^c\omega \ , \\ \bar{c}_0^a &\to \bar{c}_0^a - F_0^a(\chi_0)\omega \ , \end{aligned} \tag{8}$$

leads to the following generating equation for the WT identities [10]:

$$\begin{aligned} &\frac{\delta\tilde\Gamma}{\delta K_i}\frac{\delta\tilde\Gamma}{\delta\chi_{\rm cl}^i} + \frac{\delta\tilde\Gamma}{\delta L_a}\frac{\delta\tilde\Gamma}{\delta c_{\rm cl}^a} = 0 \ , \\ &\underline{\mathbf{K}}_0^{ai}\frac{\delta\tilde\Gamma}{\delta K_i} = \frac{\delta\tilde\Gamma}{\delta\bar{c}_{\rm cl}^a} \ , \end{aligned} \tag{9}$$

where $\chi_{\rm cl}^i, c_{\rm cl}^a, \bar{c}_{\rm cl}^a$ are classical fields defined by

$$\chi_{\rm cl}^i = \frac{\delta W}{\delta J_i}, \quad c_{\rm cl}^a = \frac{\delta W}{\delta \bar{I}_a}, \quad \bar{c}_{\rm cl}^a = -\frac{\delta W}{\delta I_a} \ , \tag{10}$$

$\tilde\Gamma$ is

$$\tilde\Gamma \equiv \Gamma + \int d^4x \tfrac{1}{2}(F_0^a)^2$$

with

$$\Gamma[\chi_{\rm cl}, c_{\rm cl}, \bar{c}_{\rm cl}, K, L] = W[J, I, \bar{I}, K, L] - \int d^4x (J_i\chi_{\rm cl}^i + \bar{I}_a c_{\rm cl}^a + \bar{c}_{\rm cl}^a I_a).$$

Taking the functional derivatives of (9) with respect to $\chi_{\rm cl}^i$, $c_{\rm cl}^a$, $\bar{c}_{\rm cl}^a$, we obtain the following WT indentities for the inverse progagators:

$$\begin{aligned} &\int d^4z \ i\tilde{\mathcal{D}}_{0,ij}^{-1}(z,y)\tilde{X}_{ai}(x,z) = 0, \\ &iS_{0,ab}^{-1}(x,y) = \underline{\mathbf{K}}_0^{ai}\tilde{X}_{bi}(y,x), \end{aligned} \tag{11}$$

where $S_0(x,y)$ is the ghost propagator, and

$$\begin{aligned} i\tilde{\mathcal{D}}_{0,ij}^{-1}(z,y) &\equiv \frac{\delta^2\tilde\Gamma}{\delta\chi_{\rm cl}^j(y)\delta\chi_{\rm cl}^i(z)}, \\ \tilde{X}_{ai}(x,z) &\equiv \langle 0|TD_i^b(\chi_0)(z)c_0^b(z)|\bar{c}_0^a(x)\rangle \ . \end{aligned} \tag{12}$$

In the following we analyze the renormalization schemes for the unphysical sectors in the SU(2)$_L$ theory and the SU(2)$\times$U(1) electroweak theory separately based on the above WT identities.

A. The SU(2)$_L$ theory

This is the case of neglecting the Weinberg angle in the SU(2)$\times$U(1) electroweak theory. In this case $V_\mu^a = W_\mu^a$. We simply take $\xi_0^a = \xi_0$, $\kappa_0^a = \kappa_0$ for $a = 1, 2, 3$. We define the renormalization constants in the unphysical sector as

$$\phi_0^a = Z_\phi^{\frac{1}{2}}\phi^a \ , c_0^a = Z_c c^a \ , \bar{c}_0^a = \bar{c}^a \ , \xi_0 = Z_\xi\xi \ , \kappa_0 = Z_\kappa\kappa \ . \tag{13}$$

In the present case, the specific form of WT identities (11) (in the momentum representation) which give relations between renormalization constants reads

$$\begin{aligned} &ik^\mu[i\mathcal{D}_{0,\mu\nu}^{-1}(k) + \xi_0^{-1}k_\mu k_\nu] + M_{W0}\hat{C}_0(k^2)[i\mathcal{D}_{0,\phi\nu}^{-1}(k) - i\kappa_0 k_\nu] = 0 \ , \\ &ik^\mu[-i\mathcal{D}_{0,\phi\mu}^{-1}(k) + i\kappa_0 k_\mu] + M_{W0}\hat{C}_0(k^2)[i\mathcal{D}_{0,\phi\phi}^{-1}(k) + \xi_0\kappa_0^2] = 0 \ , \\ &iS_{0,ab}^{-1}(k) = [1 + \Delta_3(k^2)][k^2 - \xi_0\kappa_0 M_{W0}\hat{C}_0(k^2)]\delta_{ab} \ , \end{aligned} \tag{14}$$

where

$$\hat{C}_0(k^2) = \frac{1 + \Delta_1(k^2) + \Delta_2(k^2)}{1 + \Delta_3(k^2)} \tag{15}$$

and

$$\begin{aligned} \Delta_1(k^2)\delta^{ab} &\equiv \frac{g_0}{2M_{W0}}\int_q \langle 0|H_0(-k-q)c_0^b(q)|\bar{c}_0^a(k)\rangle \ , \\ \Delta_2(k^2)\delta^{ab} &\equiv -\frac{g_0}{2M_{W0}}\epsilon^{bcd}\int_q \langle 0|\phi_0^c(-k-q)c_0^d(q)|\bar{c}_0^a(k)\rangle \ , \\ ik_\mu\Delta_3(k^2)\delta^{ab} &\equiv -g_0\epsilon^{bcd}\int_q \langle 0|W_{\mu 0}^c(-k-q)c_0^d(q)|\bar{c}_0^a(k)\rangle \ , \end{aligned} \tag{16}$$

in which $\int_q$ is short for $\int \frac{d^4q}{(2\pi)^4}$. After renormalization, (14) becomes

$$
\begin{aligned}
&ik^\mu[i\mathcal{D}^{-1}_{\mu\nu}(k) + \tfrac{Z_W}{Z_\xi}\xi^{-1}k_\mu k_\nu] + Z_{M_W}\left(\tfrac{Z_W}{Z_\phi}\right)^{\frac{1}{2}}\hat{C}_0(k^2)M_W[i\mathcal{D}^{-1}_{\phi\nu}(k) - Z_\kappa Z_W^{\frac{1}{2}} Z_\phi^{\frac{1}{2}} ik_\nu\kappa] = 0 \ , \\
&ik^\mu[-i\mathcal{D}^{-1}_{\phi\mu}(k) + Z_\kappa Z_W^{\frac{1}{2}} Z_\phi^{\frac{1}{2}} ik_\mu\kappa] + Z_{M_W}\left(\tfrac{Z_W}{Z_\phi}\right)^{\frac{1}{2}}\hat{C}_0(k^2)M_W[i\mathcal{D}^{-1}_{\phi\phi}(k) + Z_\kappa^2 Z_\xi Z_\phi \xi\kappa^2] = 0 \ , \\
&iS^{-1}_{ab}(k) = Z_c[1+\Delta_3(k^2)][k^2 - \xi\kappa M_W Z_\xi Z_\kappa Z_{M_W}\hat{C}_0(k^2)]\delta_{ab} \ ,
\end{aligned}
\tag{17}
$$

where $\mathcal{D}_{0,\mu\nu} = Z_W\mathcal{D}_{\mu\nu}$, $\mathcal{D}_{0,\phi\nu} = Z_\phi^{\frac{1}{2}} Z_W^{\frac{1}{2}}\mathcal{D}_{\phi\nu}$, etc. Since all renormalized quantities are finite, the divergences in (17) must cancel each other. This puts constraints on the renormalization constants:

$$
\begin{aligned}
&Z_\xi = \Omega_\xi Z_W \ , \qquad\qquad Z_\kappa = \Omega_\kappa Z_W^{\frac{1}{2}} Z_\phi^{-\frac{1}{2}} Z_\xi^{-1} \ , \\
&Z_\phi = \Omega_\phi Z_W Z_{M_W}^2 \hat{C}_0(\text{sub. point}) \ , \ Z_c = \Omega_c[1+\Delta_3(\text{sub. point})]^{-1} \ ,
\end{aligned}
\tag{18}
$$

where Ω_ξ, Ω_κ, Ω_ϕ, and Ω_c are finite constants to be determined by the subtraction conditions. With (18), Eq. (17) can be written as

$$
\begin{aligned}
&ik^\mu[i\mathcal{D}^{-1}_{\mu\nu}(k) + \Omega_\xi^{-1}\xi^{-1}k_\mu k_\nu] + M_W\hat{C}(k^2)[i\mathcal{D}^{-1}_{\phi\nu}(k) - \Omega_\xi^{-1}\Omega_\kappa ik_\nu\kappa] = 0 \ , \\
&ik^\mu[-i\mathcal{D}^{-1}_{\phi\mu}(k) + \Omega_\xi^{-1}\Omega_\kappa ik_\mu\kappa] + M_W\hat{C}(k^2)[i\mathcal{D}^{-1}_{\phi\phi}(k) + \Omega_\xi^{-1}\Omega_\kappa^2\xi\kappa^2] = 0 \ , \\
&iS^{-1}_{ab}(k) = \Omega_c R_3(k^2)[k^2 - \xi\kappa M_W\Omega_\kappa\hat{C}(k^2)]\delta_{ab} \ ,
\end{aligned}
\tag{19}
$$

where $R_3(k^2) \equiv [1+\Delta_3(k^2)][1+\Delta_3(\text{sub. point})]^{-1}$ is a finite function of k^2, and

$$
\hat{C}(k^2) = \left(\frac{Z_W}{Z_\phi}\right)^{\frac{1}{2}} Z_{M_W}\hat{C}_0(k^2) \ . \tag{20}
$$

We shall see in Sec. III that *this $\hat{C}(k^2)$ is directly related to the modification factor appearing in (2).*

On the other hand, the inverse physical propagators can be expressed in terms of the proper self-energies as

$$
\begin{aligned}
&i\mathcal{D}^{-1}_{0,\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)[-k^2 + M_{W0}^2 - \Pi_{0,WW}(k^2)] + \frac{k_\mu k_\nu}{k^2}[-\xi_0^{-1}k^2 + M_{W0}^2 - \tilde{\Pi}_{0,WW}(k^2)] \ , \\
&i\mathcal{D}^{-1}_{0,\phi\mu}(k) = -ik_\mu[M_{W0} - \kappa_0 + \tilde{\Pi}_{0,W\phi}(k^2)] \ , \\
&i\mathcal{D}^{-1}_{0,\phi\phi}(k) = k^2 - \xi_0\kappa_0^2 - \tilde{\Pi}_{0,\phi\phi}(k^2) \ , \\
&iS_0^{-1}(k) \ = k^2 - \xi_0\kappa_0 M_{W0} - \tilde{\Pi}_{0,c\bar{c}}(k^2) \ ,
\end{aligned}
\tag{21}
$$

and

$$
\begin{aligned}
&i\mathcal{D}^{-1}_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)[-k^2 + M_W^2 - \Pi_{WW}(k^2)] + \frac{k_\mu k_\nu}{k^2}[-\xi^{-1}k^2 + M_W^2 - \tilde{\Pi}_{WW}(k^2)] \ , \\
&i\mathcal{D}^{-1}_{\phi\mu}(k) = -ik_\mu[M_W - \kappa + \tilde{\Pi}_{W\phi}(k^2)] \ , \\
&i\mathcal{D}^{-1}_{\phi\phi}(k) = k^2 - \xi\kappa^2 - \tilde{\Pi}_{\phi\phi}(k^2) \ , \\
&iS^{-1}(k) = k^2 - \xi\kappa M_W - \tilde{\Pi}_{c\bar{c}}(k^2) \ .
\end{aligned}
\tag{22}
$$

Taking the inverse of (22) we can see that all the unphysical parts of the full propagators manifest the same tree-level pole at

$$
k^2 = \xi\kappa M_W \ . \tag{23}
$$

Substituting (21) and (22) into the WT identities (14) and (19), respectively, we obtain

$$
\begin{aligned}
&[\tilde{\Pi}_{0,WW}(k^2) - M_{W0}^2][\tilde{\Pi}_{0,\phi\phi}(k^2) - k^2] = k^2[\tilde{\Pi}_{0,W\phi}(k^2) + M_{W0}]^2 \ , \\
&\hat{C}_0(k^2) = \frac{M_{W0}^2 - \tilde{\Pi}_{0,WW}(k^2)}{M_{W0}[M_{W0} + \tilde{\Pi}_{0,W\phi}(k^2)]} = \frac{k^2[M_{W0} + \tilde{\Pi}_{0,W\phi}(k^2)]}{M_{W0}[k^2 - \tilde{\Pi}_{0,\phi\phi}(k^2)]} \ , \\
&\tilde{\Pi}_{0,c\bar{c}}(k^2) = -k^2\Delta_3(k^2) + \xi_0\kappa_0 M_{W0}[\Delta_1(k^2) + \Delta_2(k^2)] \ ,
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
(\tilde{\Pi}_{WW} - M_W^2)(\tilde{\Pi}_{\phi\phi} - k^2) - k^2(\tilde{\Pi}_{W\phi} + M_W)^2 = {} & \xi^{-1}(1-\Omega_\xi^{-1})[(k^2 - \xi\kappa M_W)^2 - k^2(\tilde{\Pi}_{\phi\phi} + 2\xi\kappa\tilde{\Pi}_{W\phi} - \xi^2\kappa^2\tilde{\Pi}_{WW}] \\
& + 2\kappa\Omega_\xi^{-1}(\Omega_\kappa - 1)[(k^2 - \xi\kappa M_W)M_W + k^2\tilde{\Pi}_{W\phi} + \xi\kappa\tilde{\Pi}_{WW}] \\
& + \kappa^2\Omega_\xi^{-1}(\Omega_\kappa - 1)^2[k^2 - \xi M_W^2 + \xi\tilde{\Pi}_{WW}] \ ,
\end{aligned}
$$

$$\hat{C}(k^2) = \frac{M_W^2 - \tilde{\Pi}_{WW} + (\Omega_\xi^{-1} - 1)\xi^{-1}k^2}{M_W^2 + M_W\tilde{\Pi}_{W\phi}(k^2) + M_W\kappa(\Omega_\xi^{-1}\Omega_\kappa - 1)}$$
$$= \frac{k^2}{M_W}\frac{M_W + \tilde{\Pi}_{W\phi} + (\Omega_\xi^{-1}\Omega_\kappa - 1)\kappa}{k^2 - \tilde{\Pi}_{\phi\phi} + (\Omega_\xi^{-1}\Omega_\kappa^2 - 1)\xi\kappa^2}\ , \tag{25}$$

$$\tilde{\Pi}_{c\bar{c}} = (k^2 - \xi\kappa M_W) - \Omega_c R_3(k^2)[k^2 - \xi\kappa M_W \Omega_\kappa \hat{C}(k^2)]\ .$$

The first and third identities in (25) give constraints on the unphysical proper self-energies, and the second identity gives constraint on $\hat{C}(k^2)$. *Equation (25) is of special importance in constructing renormalization schemes which can simplify the expression for $\hat{C}(\xi\kappa M_W)$.*

Now we construct renormalization schemes for the unphysical sector in the general R_ξ gauge. We first consider the case of $\xi \neq 0$. For the four renormalization constants we need four subtraction conditions to fix them. From (18) we see that this means the fixing of the four finite constants Ω_ξ, Ω_κ, Ω_ϕ, and Ω_c. In order to make our formulation and its application to explicit loop calculations as simple as possible, we choose the subtraction conditions such that all the unphysical mass poles coincide with the tree-level pole (23), i.e., our subtraction conditions are chosen to be

$$\tilde{\Pi}_{WW}(\xi\kappa M_W) = \tilde{\Pi}_{W\phi}(\xi\kappa M_W) = \tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = 0\ . \tag{26}$$

To see how these conditions fix the constants Ω_ξ, Ω_κ, Ω_ϕ, and Ω_c, we look at the proper self-energy counterterms defined by

$$\begin{aligned}
\Pi_{0,WW}(k^2) &= \Pi_{WW}(k^2) - \delta\Pi_{WW}\ ,\\
\tilde{\Pi}_{0,WW}(k^2) &= \tilde{\Pi}_{WW}(k^2) - \delta\tilde{\Pi}_{WW}\ ,\\
\tilde{\Pi}_{0,W\phi}(k^2) &= \tilde{\Pi}_{W\phi}(k^2) - \delta\tilde{\Pi}_{W\phi}\ ,\\
\tilde{\Pi}_{0,\phi\phi}(k^2) &= \tilde{\Pi}_{\phi\phi}(k^2) - \delta\tilde{\Pi}_{\phi\phi}\ ,\\
\tilde{\Pi}_{0,c\bar{c}}(k^2) &= \tilde{\Pi}_{c\bar{c}}(k^2) - \delta\tilde{\Pi}_{c\bar{c}}\ .
\end{aligned} \tag{27}$$

From (21) and (22) we can obtain the following exact expressions for the counterterms which hold to all orders in perturbation:

$$\begin{aligned}
\delta\Pi_{WW} &= (1 - Z_W^{-1})(k^2 - M_W^2) + (1 - Z_{M_W}^2)M_W^2 + (1 - Z_W^{-1})\Pi_{WW}(k^2)\ ,\\
\delta\tilde{\Pi}_{WW} &= \xi^{-1}(\Omega_\xi^{-1} - 1)Z_W^{-1}(k^2 - \xi\kappa M_W) + [(\Omega_\xi^{-1} - 1)Z_W^{-1}M_W\kappa + (Z_W^{-1} - Z_{M_W}^2)M_W^2] + (1 - Z_W^{-1})\tilde{\Pi}_{WW}(k^2)\ ,\\
\delta\tilde{\Pi}_{W\phi} &= [Z_{M_W} - (Z_W Z_\phi)^{-\frac{1}{2}}]M_W - [\Omega_\xi^{-1}\Omega_\kappa - 1](Z_W Z_\phi)^{-\frac{1}{2}}\kappa + [1 - (Z_W Z_\phi)^{-\frac{1}{2}}]\tilde{\Pi}_{W\phi}(k^2)\ ,\\
\delta\tilde{\Pi}_{\phi\phi} &= (Z_\phi^{-1} - 1)(k^2 - \xi\kappa M_W) + \xi\kappa[(Z_\phi^{-1} - 1)M_W + (\Omega_\xi^{-1}\Omega_\kappa^2 - 1)Z_\phi^{-1}\kappa] + (1 - Z_\phi^{-1})\tilde{\Pi}_{\phi\phi}(k^2)\ ,\\
\delta\tilde{\Pi}_{c\bar{c}} &= (Z_c^{-1} - 1)(k^2 - \xi\kappa M_W) + \left[\Omega\kappa(Z_W/Z_\phi)^{\frac{1}{2}}Z_{M_W} - 1\right]\xi\kappa M_W + (1 - Z_c^{-1})\tilde{\Pi}_{c\bar{c}}(k^2)\ .
\end{aligned} \tag{28}$$

We also give here a simpler expression of (28) when we keep the accuracy only up to one-loop level:

$$\begin{aligned}
\delta\Pi_{WW} &= \delta Z_W(k^2 - M_W^2) - 2\delta Z_{M_W}M_W^2\ ,\\
\delta\tilde{\Pi}_{WW} &= -\delta\Omega_\xi\xi^{-1}(k^2 - \xi\kappa M_W) - M_W[\delta\Omega_\xi\kappa + (\delta Z_W + 2\delta Z_{M_W})M_W]\ ,\\
\delta\tilde{\Pi}_{W\phi} &= [\tfrac{1}{2}(\delta Z_W + \delta Z_\phi) + \delta Z_{M_W}]M_W + [\delta\Omega_\xi - \delta\Omega_\kappa]\kappa\ ,\\
\delta\tilde{\Pi}_{\phi\phi} &= -\delta Z_\phi(k^2 - \xi\kappa M_W) - \xi\kappa[\delta Z_\phi M_W + (\delta\Omega_\xi - 2\delta\Omega_\kappa)\kappa]\ ,\\
\delta\tilde{\Pi}_{c\bar{c}} &= -\delta Z_c(k^2 - \xi\kappa M_W) + [\delta\Omega_\kappa - \tfrac{1}{2}\delta Z_\phi + \tfrac{1}{2}\delta Z_W + \delta Z_{M_W}]\xi\kappa M_W\ ,
\end{aligned} \tag{29}$$

where $\delta Z \equiv Z - 1$, $\delta\Omega \equiv \Omega - 1$.

From (28) or (29) we see that $\tilde{\Pi}_{WW}(\xi\kappa M_W)$ and $\tilde{\Pi}_{\phi\phi}(\xi\kappa M_W)$ can be made vanishing by adjusting the constants Ω_ξ and Ω_κ (or Ω_ϕ), respectively: i.e.,

$$\begin{aligned}
&\tilde{\Pi}_{WW}(\xi\kappa M_W) = 0 \text{ by adjusting } \Omega_\xi\ ,\\
&\tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = 0 \text{ by adjusting } \Omega_\kappa \text{ or } \Omega_\phi(Z_\phi)\ .
\end{aligned} \tag{30}$$

After doing this, the first identity in (25) gives

$$\tilde{\Pi}_{W\phi}(\xi\kappa M_W)[\tilde{\Pi}_{W\phi}(\xi\kappa M_W) + 2M_W + 2\kappa(\Omega_\kappa\Omega_\xi^{-1} - 1)] = \Omega_\xi^{-1}(\Omega_\kappa - 1)^2\kappa(M_W - \kappa)\ , \tag{31}$$

and the right-hand side (RHS) vanishes if we take $\kappa = M_W$ or $\Omega_\kappa = 1$: i.e.,

$$\tilde{\Pi}_{W\phi}(\xi\kappa M_W) = 0 \quad \text{if } \kappa = M_W \text{ or } \Omega_\kappa = 1\ . \tag{32}$$

Note that if we keep the accuracy only up to one-loop level, the RHS of (31) vanishes automatically, so that the requirement $\kappa = M_W$ or $\Omega_\kappa = 1$ is needed only beyond one loop. Having these, the second identity in (25) gives

$$\hat{C}(\xi\kappa M_W) = \frac{M_W + (\Omega_\xi^{-1} - 1)\kappa}{M_W + \tilde{\Pi}_{W\phi}(\xi\kappa M_W) + (\Omega_\xi^{-1}\Omega_\kappa - 1)\kappa} = \begin{cases} \Omega_\kappa^{-1} & \text{if } \kappa = M_W\ , \\ 1 & \text{if } \Omega_\kappa = 1\ , \end{cases} \tag{33}$$

and the third identity in (25) implies

$$\tilde{\Pi}_{c\bar{c}}(\xi\kappa M_W) = 0 \quad \text{if } \kappa = M_W \text{ or } \Omega_\kappa = 1\ . \tag{34}$$

From the above analysis we see that we can construct two convenient renormalization schemes.

Scheme I. $\kappa = M_W$, Ω_ξ and Ω_κ are determined from (30), $\Omega_\phi(Z_\phi)$ and $\Omega_c(Z_c)$ are determined by the usual normalization conditions requiring the residues of $\mathcal{D}_{\phi\phi}$ and S_{ab} to be unity at $k^2 = \xi\kappa M_W$.

Scheme II. κ is arbitrary, $\Omega_\kappa = 1$, Ω_ξ and $\Omega_\phi(Z_\phi)$ are determined from (30), $\Omega_c(Z_c)$ is determined by the usual normalization condition requiring the residue of S_{ab} to be unity at $k^2 = \xi\kappa M_W$.

Note that in *scheme II* the residue of $\mathcal{D}_{\phi\phi}$ at $k^2 = \xi\kappa M_W$ is not normalized in the conventional way, but this does not affect the physics. Furthermore, the determination of Ω_ϕ (Z_ϕ) in *scheme II* concerns only the calculation of the renormalized proper self-energy $\tilde{\Pi}_{\phi\phi}$, so that it is easy to implement. In these two schemes, the expressions for $\hat{C}(\xi\kappa M_W)$ are very simple: i.e.,

$$\hat{C}(\xi\kappa M_W) = \begin{cases} \Omega_\kappa^{-1} & \text{in scheme I}\ , \\ 1 & \text{in scheme II}\ . \end{cases} \tag{35}$$

Next we consider the case of $\xi = 0$ (Landau gauge) in which some of the formulas in (25) are not clearly defined. In the Landau gauge we have the following well-known relations [13,14]: (a) there is no W_μ^a-ϕ^a mixing and $\mathcal{D}_{\mu\nu}(k) \propto g_{\mu\nu} - k_\mu k_\nu/k^2$; (b) the poles of $D_{\phi\phi}(k)$ and $S_{ab}(k)$ are all at $k^2 = 0$; (c) the ghost fields c^a and $\bar{c}^a$ do not couple to the Higgs and Goldstone boson fields. Relation (a) means that the longitudinal components of $\mathcal{D}_{\mu\nu}$ and $\mathcal{D}_{\mu\phi}$ containing $\tilde{\Pi}_{WW}$ and $\tilde{\Pi}_{W\phi}$ vanish, so that the only relevant unphysical proper self-energies are $\tilde{\Pi}_{\phi\phi}$ and $\tilde{\Pi}_{c\bar{c}}$. Relation (b) implies that $\tilde{\Pi}_{\phi\phi}$ and $\tilde{\Pi}_{c\bar{c}}$ automatically satisfy the tree-level mass-shell conditions $\tilde{\Pi}_{\phi\phi}(0) = \tilde{\Pi}_{c\bar{c}}(0) = 0$. With relation (c), the expression (20) for $\hat{C}(k^2)$ reduces to

$$\hat{C}(k^2) = \left(\frac{Z_W}{Z_\phi}\right)^{\frac{1}{2}} \frac{Z_{M_W}}{1 + \Delta_3(k^2)}. \tag{36}$$

Hence $\hat{C}(0)$ or $\hat{C}(M_W^2)$ is not so much simplified as in the case of $\xi \neq 0$ [cf. (35)]. In (36) Z_W and Z_{M_W} are well fixed, so that the only possibility of making $\hat{C}(M_W^2)$ unity is to adjust the unphysical Z_ϕ but this must rely on the detailed explicit calculation on $\Delta_3(k^2)$ order by order in loop expansion. The Landau gauge has been studied by many authors, see, for example, Refs. [13,14]. In the scheme taken by Marciano and Willenbrock (MW) [14], Z_ϕ is determined by the usual condition normalizing the residue of $\mathcal{D}_{\phi\phi}$ at $k^2 = 0$; therefore, in that scheme, neither $\hat{C}(0)$ nor $\hat{C}(M_W^2)$ is unity.

Our conclusions in the $SU(2)_L$ theory are summarized in Table I.

B. The SU(2)×U(1) electroweak theory

The realistic SU(2)×U(1) electroweak theory is more complicated than the simple $SU(2)_L$ theory due to the various mixings in the neutral sector. For convenience we introduce the matrix notation

$$\bar{W}_0^\pm \equiv \begin{pmatrix} W_0^{\pm\mu} \\ \phi_0^\pm \end{pmatrix},\quad N_0^\mu \equiv \begin{pmatrix} Z_0^\mu \\ A_0^\mu \end{pmatrix},\quad \bar{N}_0 \equiv \begin{pmatrix} N_0^\mu \\ \phi_0^Z \end{pmatrix}, \tag{37}$$

$$\mathcal{C}_0 \equiv \begin{pmatrix} c_0^Z \\ c_0^A \end{pmatrix},\quad \bar{\mathcal{C}}_0 \equiv (\bar{c}_0^Z, \bar{c}_0^A)\ ,$$

where A_0^μ is the photon field, ϕ_0^Z is the would-be Goldstone boson absorbed by Z_0^μ. The matrix notations for the propagators in the neutral sector are

$$\mathbf{D}_{0,NN}^{\mu\nu} = \langle 0|TN_0^\mu N_0^{\nu T}|0\rangle = \begin{bmatrix} \mathcal{D}_{ZZ,0}^{\mu\nu} & \mathcal{D}_{ZA,0}^{\mu\nu} \\ \mathcal{D}_{AZ,0}^{\mu\nu} & \mathcal{D}_{AA,0}^{\mu\nu} \end{bmatrix},$$

$$\mathbf{D}_{0,\bar{N}\bar{N}} = \langle 0|T\bar{N}_0\bar{N}_0^T|0\rangle = \begin{bmatrix} \mathcal{D}_{NN,0}^{\mu\nu} & \mathcal{D}_{N\phi^Z,0}^{\mu} \\ \mathcal{D}_{\phi^Z N,0}^{\nu} & \mathcal{D}_{\phi^Z\phi^Z,0} \end{bmatrix} = \begin{bmatrix} \mathcal{D}_{ZZ,0}^{\mu\nu} & \mathcal{D}_{ZA,0}^{\mu\nu} & \mathcal{D}_{Z\phi^Z,0}^{\mu} \\ \mathcal{D}_{AZ,0}^{\mu\nu} & \mathcal{D}_{AA,0}^{\mu\nu} & \mathcal{D}_{A\phi^Z,0}^{\mu} \\ \mathcal{D}_{\phi^Z Z,0}^{\nu} & \mathcal{D}_{\phi^Z A,0}^{\nu} & \mathcal{D}_{\phi^Z\phi^Z,0} \end{bmatrix}, \tag{38}$$

$$\mathbf{S}_{0,N} = \langle 0|T\mathcal{C}_0\bar{\mathcal{C}}_0|0\rangle = \begin{bmatrix} S_{ZZ,0} & S_{ZA,0} \\ S_{AZ,0} & S_{AA,0} \end{bmatrix}.$$

TABLE I. Main features of schemes I, II, and the MW scheme in $SU(2)_L$ theory.

Renormalization	On-shell conditions in Eq. (26)				$\hat{C}(\xi\kappa M_W)$
schemes	$\tilde{\Pi}_{WW} = 0$	$\tilde{\Pi}_{W\phi} = 0$	$\tilde{\Pi}_{\phi\phi} = 0$	$\tilde{\Pi}_{c\bar{c}} = 0$	
Scheme I $\{\Omega_\xi, \Omega_\kappa, Z_\phi^{\text{on}}, Z_c^{\text{on}}\}_{\xi\neq 0}$	Adjust $\Omega_\xi - 1$	WT with $\kappa = M_W$	Adjust $\Omega_\kappa - 1$	WT with $\kappa = M_W$	Ω_κ^{-1} $(\kappa = M_W)$
Scheme II $\{\Omega_\xi, \Omega_\kappa = 1, Z_\phi, Z_c^{\text{on}}\}_{\xi\neq 0}$	Adjust $\Omega_\xi - 1$	WT	Adjust $Z_\phi - 1$	WT	1
MW scheme $\{\Omega_\xi = \Omega_\kappa = 1, Z_\phi^{\text{on}}, Z_c^{\text{on}}\}_{\xi=0}$	\	\	WT	WT	$\left(\frac{Z_W}{Z_\phi}\right)^{1/2} \frac{Z_{M_W}}{1 + \Delta_3(0)}$

The gauge fixing term (3) is now

$$\mathcal{L}_{\rm GF} = -\tfrac{1}{2}(F_0^+F_0^- + F_0^-F_0^+) - \tfrac{1}{2}(F_0^N)^T F_0^N \ , \qquad (39)$$

where

$$F_0^{\pm} = (\xi_0^W)^{-\frac{1}{2}}\partial_\mu W_0^{\pm\mu} - (\xi_0^W)^{\frac{1}{2}}\kappa_0^W \phi_0^{\pm} \equiv (\underline{\mathbf{K}}_0^{\pm})^T \bar{W}_0^{\pm} \ ,$$
$$F_0^N = \begin{pmatrix} F_0^Z \\ F_0^A \end{pmatrix} = (\xi_0^N)^{-\frac{1}{2}}\partial_\mu N_0^\mu - \bar{\kappa}_0 \phi_0^Z \equiv (\underline{\mathbf{K}}_0^N)^T \bar{N}_0 \ , \qquad (40)$$

and

$$\underline{\mathbf{K}}_0^{\pm} \equiv \begin{pmatrix} (\xi_0^W)^{-\frac{1}{2}}\partial_\mu \\ -(\xi_0^W)^{\frac{1}{2}}\kappa_0^W \end{pmatrix} \ , \qquad \underline{\mathbf{K}}_0^N = \begin{pmatrix} (\xi_0^N)^{-\frac{1}{2}}\partial_\mu \\ -\bar{\kappa}_0 \end{pmatrix} \ ,$$
$$(\xi_0^N)^{-\frac{1}{2}} \equiv \begin{bmatrix} (\xi_0^Z)^{-\frac{1}{2}} & (\xi_0^{ZA})^{-\frac{1}{2}} \\ (\xi_0^{AZ})^{-\frac{1}{2}} & (\xi_0^A)^{-\frac{1}{2}} \end{bmatrix} \ , \quad \bar{\kappa}_0 = \begin{pmatrix} (\xi_0^Z)^{\frac{1}{2}}\kappa_0^Z \\ (\xi_0^A)^{\frac{1}{2}}\kappa_0^A \end{pmatrix} \ . \qquad (41)$$

Here we have distinguished the gauge parameters $\xi_0^W, \xi_0^Z, \xi_0^A, \kappa_0^W, \kappa_0^Z, \kappa_0^A$, etc.

Now the specific forms of WT identities (11) are

$$\left.\begin{array}{l} \int d^4z\ i\tilde{\mathcal{D}}_{ij,0}^{-1}(z,y)\tilde{X}_{ai}(x,z) = 0 \ , \\ iS_0^{\pm^{-1}}(x,y) = (\underline{\mathbf{K}}_0^{\pm})^T \langle 0|TD_{\bar{W}^\pm}^a(x)c_0^a(x)|\bar{c}_0^{\pm}(y)\rangle \ , \end{array}\right\} \text{for the charged sector} \ , \qquad (42)$$

and

$$\left.\begin{array}{l} \int d^4z\ i\tilde{\mathbf{D}}_{\bar{N}\bar{N},0}^{-1^T}(z,y)\tilde{\mathbf{X}}_{\bar{N}}(x,z) = 0 \ , \\ i\mathbf{S}_{0,N}^{-1}(x,y) = (\underline{\mathbf{K}}_0^N)^T \tilde{\mathbf{X}}_{\bar{N}}(y,x) \ , \end{array}\right\} \text{for the neutral sector} \ , \qquad (43)$$

where

$$\begin{aligned} \tilde{\mathbf{X}}_{\bar{N}}(x,y) &\equiv \frac{\delta^2\tilde{\Gamma}}{\delta\mathcal{C}_0(x)\delta K_{\bar{N}}^T(y)} = \langle 0|TD_{\bar{N}}^b(y)c_0^b(y)|\bar{\mathcal{C}}_0(x)\rangle \ , \\ i\mathbf{S}_{0,N}^{-1}(x,y) &\equiv \frac{\delta^2\Gamma}{\delta\mathcal{C}_0(y)\delta\bar{\mathcal{C}}_0(x)} \ , \\ i\tilde{\mathbf{D}}_{\bar{N}\bar{N},0}^{-1}(x,y) &\equiv \frac{\delta^2\tilde{\Gamma}}{\delta\bar{N}_0(y)\delta\bar{N}_0^T(x)} = i\mathbf{D}_{\bar{N}\bar{N},0}^{-1}(x,y) + \underline{\mathbf{K}}_0^N(x)[\underline{\mathbf{K}}_0^N(y)]^T \ . \end{aligned} \qquad (44)$$

The renormalization constants in the physical and unphysical sectors are defined as

$$\begin{aligned} &\alpha_0 = Z_\alpha \alpha \quad (\text{or } e_0 = Z_e e, Z_\alpha = Z_e^2) \ , \\ &M_{W0} = Z_{M_W} M_W \ , \quad M_{Z0} = Z_{M_Z} M_Z \ , \quad m_{H0} = Z_{m_H} m_H \ , \quad m_{f_i 0} = Z_{m_{f_i}} m_{f_i} \ , \\ &W_0^{\pm\mu} = Z_W^{\frac{1}{2}} W^{\pm\mu} \ , \quad H_0 = Z_H^{\frac{1}{2}} H \ , \quad \psi_{f_i 0} = Z_{f_i}^{\frac{1}{2}} \psi_{f_i} \ , \\ &N_0^\mu = Z_N^{\frac{1}{2}} N^\mu, \quad \mathbf{Z}_N^{\frac{1}{2}} = \begin{bmatrix} Z_{ZZ}^{\frac{1}{2}} & Z_{ZA}^{\frac{1}{2}} \\ Z_{AZ}^{\frac{1}{2}} & Z_{AA}^{\frac{1}{2}} \end{bmatrix} \equiv \begin{bmatrix} Z_{ZZ}^{\frac{1}{2}} \ , & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} \ , & Z_{AA}^{\frac{1}{2}} \end{bmatrix} \ , \end{aligned} \qquad (45)$$

and

$$\begin{aligned} &\xi_0^W = Z_{\xi^W}\xi^W, \quad \kappa_0^W = Z_{\kappa^W}\kappa^W \ , \\ &(\xi_0^N)^{-\frac{1}{2}} = (\xi^N)^{-\frac{1}{2}}\mathbf{Z}_{\xi^N}^{-\frac{1}{2}}, \quad (\xi^N)^{-\frac{1}{2}} \equiv \begin{bmatrix} (\xi^Z)^{-\frac{1}{2}} & 0 \\ 0 & (\xi^A)^{-\frac{1}{2}} \end{bmatrix} \ , \\ &\bar{\kappa}_0 = \mathbf{Z}_{\bar{\kappa}}\bar{\kappa} \ , \quad \bar{\kappa} = \begin{pmatrix} (\xi^Z)^{\frac{1}{2}}\kappa^Z \\ 0 \end{pmatrix} \ , \quad \phi_0^{\pm} = Z_{\phi^\pm}^{\frac{1}{2}}\phi^{\pm} \ , \quad \phi_0^Z = Z_{\phi^Z}^{\frac{1}{2}}\phi^Z \ , \\ &c_0^{\pm} = Z_c^W c^{\pm} \ , \quad \bar{c}_0^{\pm} = \bar{c}^{\pm} \ , \quad \mathcal{C}_0 = \mathbf{Z}_c^N \mathcal{C} \ , \quad \bar{\mathcal{C}}_0 = \bar{\mathcal{C}} \ , \quad \mathbf{Z}_c^N \equiv \begin{bmatrix} Z_c^{ZZ} & Z_c^{ZA} \\ Z_c^{AZ} & Z_c^{AA} \end{bmatrix} \ , \end{aligned} \qquad (46)$$

where $\alpha \equiv \dfrac{e^2}{4\pi}$ is the electromagnetic fine structure constant, ψ_f's are the fermion fields, and m_f's are the fermion masses.

The analysis of the renormalization schemes in the charged sector is completely similar to what has been done in the $SU(2)_L$ theory. The factor $\hat{C}_0^W(k^2)$ corresponding to (15) is now

$$\hat{C}_0^W(k^2) = \frac{1 + \Delta_1^W(k^2) + \Delta_2^W(k^2)}{1 + \Delta_3^W(k^2)} \ , \qquad (47)$$

where

$$\Delta_1^W \equiv \Delta_{11}^W(k^2) ,$$
$$\Delta_2^W \equiv \Delta_{21}^W(k^2) + \Delta_{22}^W(k^2) + \Delta_{23}^W(k^2) , \quad (48)$$
$$\Delta_3^W \equiv \Delta_{31}^W(k^2) + \Delta_{32}^W(k^2) + \Delta_{33}^W(k^2) + \Delta_{34}^W(k^2) ,$$

and the lengthy expressions for the Δ_{ij}^W's are given in Appendix B. Note that the formulas for Δ_1^W, Δ_2^W, and Δ_3^W are more complicated than those in (16) due to the presence of the additional U(1) gauge group. Repeating the same steps shown in the case of the $SU(2)_L$ theory we can obtain all the formulas corresponding to (17)–(36) with the substitutions $Z_\phi \to Z_{\phi^\pm}, Z_c \to Z_c^W, \xi \to \xi^W, \kappa \to \kappa^W, \Omega_\xi \to \Omega_\xi^W, \Omega_\kappa \to \Omega_\kappa^W, \Omega_\phi \to \Omega_\phi^W, \Omega_c \to \Omega_c^W, \hat{C}(k^2) \to \hat{C}^W(k^2)$, etc. Table I with the above substitutions shows also the conclusions for the charged sector in the SU(2)×U(1) theory.

Next we show explicitly the analysis for the neutral sector in the SU(2)×U(1) theory. The specific form of the WT identities (11) in the momentum representation is now

$$ik_\mu i\tilde{\mathbf{D}}_{0,NN}^{-1\mu\nu}(k) + M_{Z0}\hat{\mathbf{C}}_0^N(k^2)i\tilde{\mathbf{D}}_{0,\phi^z N}^{-1\ \nu}(k) = 0 ,$$
$$-ik_\mu i\tilde{\mathbf{D}}_{0,\phi^z N}^{-1\ \mu}(k) + M_{Z0}\hat{\mathbf{C}}_0^N(k^2)i\tilde{\mathbf{D}}_{0,\phi^z\phi^z}^{-1}(k) = 0 ,$$
$$i\mathbf{S}_{0,N}^{-1}(k) = [k^2 - M_{Z0}\bar{\kappa}_0\hat{\mathbf{C}}_0^N(k^2)^T\xi_{N0}^{\frac{1}{2}}]\xi_{N0}^{-\frac{1}{2}}\hat{\mathbf{X}}_{N\bar{C}}(k^2)\xi_{N0}^{\frac{1}{2}} , \quad (49)$$

where

$$\hat{\mathbf{C}}_0^N(k^2) \equiv [\hat{\mathbf{X}}_{N\bar{C}}(k^2)^T]^{-1}\hat{\mathbf{X}}_{\phi^z\bar{C}}(k^2)^T \equiv \begin{pmatrix} \hat{C}_0^Z(k^2) \\ \hat{C}_0^A(k^2) \end{pmatrix} \quad (50)$$

with $\hat{\mathbf{X}}_{N\bar{C}}$ and $\hat{\mathbf{X}}_{\phi^z\bar{C}}$ defined by

$$\tilde{\mathbf{X}}_{\bar{N}}(k) = \begin{pmatrix} \tilde{\mathbf{X}}_{N_\nu\bar{C}}(k) \\ \tilde{\mathbf{X}}_{\phi^z\bar{C}}(k) \end{pmatrix} \equiv \begin{pmatrix} ik_\nu\hat{\mathbf{X}}_{N\bar{C}}(k^2) \\ M_{Z0}\hat{\mathbf{X}}_{\phi^z\bar{C}}(k^2) \end{pmatrix} ,$$
$$ik_\nu\hat{\mathbf{X}}_{N\bar{C}}(k^2) = \int_q\langle 0|D_{N_\nu}^b(-k-q)c_0^b(q)|\bar{C}_0(k)\rangle$$
$$\equiv ik_\nu[I + \Delta_3^N(k^2)] \equiv ik_\nu \begin{bmatrix} 1+\Delta_3^{ZZ}(k^2) & \Delta_3^{ZA}(k^2) \\ \Delta_3^{AZ}(k^2) & 1+\Delta_3^{AA}(k^2) \end{bmatrix} , \quad (51)$$
$$\hat{\mathbf{X}}_{\phi^z\bar{C}}(k^2)^T = M_{Z0}^{-1}\int_q\langle 0|D_{\phi^z}^b(-k-q)c_0^b(q)|\bar{C}_0(k)\rangle^T \equiv \begin{pmatrix} 1+\Delta_1^{ZZ}(k^2)+\Delta_2^{ZZ}(k^2) \\ \Delta_1^{ZA}(k^2)+\Delta_2^{ZA}(k^2) \end{pmatrix} .$$

The lengthy expressions for Δ_i^{ab}'s are given in Appendix B. After renormalization, (49) becomes

$$ik_\mu[i\mathbf{D}_{NN}^{-1\mu\nu}(k) + k^\mu k^\nu(\mathbf{Z}_{\xi_N}^{-\frac{1}{2}}\mathbf{Z}_N^{\frac{1}{2}})^T\xi_N^{-1}(\mathbf{Z}_{\xi_N}^{-\frac{1}{2}}\mathbf{Z}_N^{\frac{1}{2}})] + M_Z\hat{\mathbf{C}}^N(k^2)[i\mathbf{D}_{\phi^z N}^{-1\nu}(k) - ik^\nu Z_{\phi^z}^{\frac{1}{2}}\bar{\kappa}^T\mathbf{Z}_{\bar{\kappa}}^T\xi_N^{-\frac{1}{2}}(\mathbf{Z}_{\xi_N}^{-\frac{1}{2}}\mathbf{Z}_N^{\frac{1}{2}})] = 0 ,$$
$$ik_\mu[-i\mathbf{D}_{\phi^z N}^{-1\mu}(k) + ik^\mu(\mathbf{Z}_{\xi_N}^{-\frac{1}{2}}\mathbf{Z}_N^{\frac{1}{2}})^T\xi_N^{-\frac{1}{2}^T}\mathbf{Z}_{\bar{\kappa}}\bar{\kappa}\mathbf{Z}_\phi^{\frac{1}{2}}] + M_Z\hat{\mathbf{C}}^N(k^2)[i\mathbf{D}_{\phi^z\phi^z}^{-1}(k) + \bar{\kappa}^T Z_{\phi^z}^{\frac{1}{2}}\mathbf{Z}_{\bar{\kappa}}^T\mathbf{Z}_{\bar{\kappa}}Z_{\phi^z}^{\frac{1}{2}}\bar{\kappa}] = 0 ,$$
$$i\mathbf{S}_N^{-1}(k) = [k^2 - M_Z Z_{\phi^z}^{\frac{1}{2}}Z_{\bar{\kappa}}\bar{\kappa}\hat{\mathbf{C}}^N(k^2)^T\mathbf{Z}_N^{-\frac{1}{2}}\mathbf{Z}_{\xi_N}^{\frac{1}{2}}\xi_N^{\frac{1}{2}}][\xi_N^{-\frac{1}{2}}\mathbf{Z}_{\xi_N}^{-\frac{1}{2}}\hat{\mathbf{X}}_{NC}(k^2)\mathbf{Z}_{\xi_N}^{\frac{1}{2}}\xi_N^{\frac{1}{2}}\mathbf{Z}_c^N] , \quad (52)$$

where

$$\hat{\mathbf{C}}^N(k^2) \equiv (\mathbf{Z}_N^{\frac{1}{2}^T} Z_{\phi^z}^{-\frac{1}{2}})Z_{M_Z}\hat{\mathbf{C}}_0^N(k^2) \equiv \begin{pmatrix} \hat{C}^Z(k^2) \\ \hat{C}^A(k^2) \end{pmatrix} . \quad (53)$$

The finiteness of the renormalized quantities implies that the renormalization constants satisfy the relations

$$\mathbf{Z}_{\xi_N}^{-\frac{1}{2}} \equiv \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}}\mathbf{Z}_N^{-\frac{1}{2}} , \quad \mathbf{Z}_{\bar{\kappa}} \equiv \xi_N^{\frac{1}{2}^T}\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}^T}\xi_N^{-\frac{1}{2}^T}\mathbf{\Omega}_{\bar{\kappa}}Z_{\phi^z}^{-\frac{1}{2}} ,$$
$$Z_{\phi^z}^{\frac{1}{2}} \equiv \Omega_{\phi^z}^{\frac{1}{2}}Z_{M_Z}[Z_{ZZ}^{\frac{1}{2}}\hat{C}_0^Z(\text{sub. point}) + Z_{AZ}^{\frac{1}{2}}\hat{C}_0^A(\text{sub. point})] , \quad (54)$$
$$\equiv \tilde{\Omega}_{\phi^z}^{\frac{1}{2}}Z_{M_Z}[Z_{ZA}^{\frac{1}{2}}\hat{C}_0^Z(\text{sub. point}) + Z_{AA}^{\frac{1}{2}}\hat{C}_0^A(\text{sub. point})] ,$$

where

$$\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}} \equiv \begin{bmatrix} (\Omega_\xi^{ZZ})^{-\frac{1}{2}} & (\Omega_\xi^{ZA})^{-\frac{1}{2}} \\ (\Omega_\xi^{AZ})^{-\frac{1}{2}} & (\Omega_\xi^{AA})^{-\frac{1}{2}} \end{bmatrix} \equiv \begin{bmatrix} (1+\delta\Omega_\xi^{ZZ})^{-\frac{1}{2}} & -\frac{1}{2}\delta\Omega_\xi^{ZA} \\ -\frac{1}{2}\delta\Omega_\xi^{AZ} & (1+\delta\Omega_\xi^{AA})^{-\frac{1}{2}} \end{bmatrix} ,$$
$$\mathbf{\Omega}_{\bar{\kappa}} \equiv \begin{bmatrix} \Omega_\kappa^{ZZ} & 0 \\ \Omega_\kappa^{AZ} & 0 \end{bmatrix} \equiv \begin{bmatrix} 1+\delta\Omega_\kappa^{ZZ} & 0 \\ \delta\Omega_\kappa^{AZ} & 0 \end{bmatrix} \quad (55)$$

are finite constants. In (54) we have not presented the matrix expression for $\mathbf{Z}_c^N$ which is very complicated because it ensures the finiteness of the product of several matrices on the right-hand side of the third identity in (52). Actually, the explicit expression for $\mathbf{Z}_c^N$ is not really needed in the following analysis. With (54), the first two WT identities in (52) can be written as

$$\begin{aligned} &ik_\mu[i\mathbf{D}_{NN}^{-1\mu\nu}(k) + k^\mu k^\nu \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}^T} \xi_N^{-1} \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}}] + M_Z \hat{\mathbf{C}}^N(k^2)[i\mathbf{D}_{\phi^Z N}^{-1\nu}(k) - ik^\nu \bar{\kappa}^T \mathbf{\Omega}_{\bar{\kappa}}^T \xi_N^{-\frac{1}{2}} \mathbf{\Omega}_{\xi_N}^{-1}] = 0 \ , \\ &ik_\mu[-i\mathbf{D}_{\phi^Z N}^{-1\mu}(k) + ik^\mu \mathbf{\Omega}_{\xi_N}^{-1^T} \xi_N^{-\frac{1}{2}^T} \mathbf{\Omega}_{\bar{\kappa}} \bar{\kappa}] + M_Z \hat{\mathbf{C}}^N(k^2)[i\mathbf{D}_{\phi^Z\phi^Z}^{-1}(k) + \bar{\kappa}^T \mathbf{\Omega}_{\bar{\kappa}}^T \xi_N^{-\frac{1}{2}} \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}} \xi_N \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}^T} \xi_N^{-\frac{1}{2}^T} \mathbf{\Omega}_{\bar{\kappa}} \bar{\kappa}] = 0. \end{aligned} \tag{56}$$

We then introduce the matrix notation for the bare proper self-energies and masses,

$$\begin{aligned} &\mathbf{\Pi}_0^{NN}(k^2) = \begin{bmatrix} \Pi_0^{ZZ} \ \Pi_0^{ZA} \\ \Pi_0^{AZ} \ \Pi_0^{AA} \end{bmatrix}_{(k^2)} , \quad \tilde{\mathbf{\Pi}}_0^{NN}(k) = \begin{bmatrix} \tilde{\Pi}_0^{ZZ} \ \tilde{\Pi}_0^{ZA} \\ \tilde{\Pi}_0^{AZ} \ \tilde{\Pi}_0^{AA} \end{bmatrix}_{(k^2)} , \\ &\tilde{\mathbf{\Pi}}_0^{N\phi^Z}(k^2) = \begin{pmatrix} \tilde{\Pi}_0^{Z\phi^Z} \\ \tilde{\Pi}_0^{A\phi^Z} \end{pmatrix}_{(k^2)} , \quad \mathbf{M}_{N0}^2 = \begin{bmatrix} M_{Z0}^2 & 0 \\ 0 & 0 \end{bmatrix} , \quad \tilde{\mathbf{M}}_{N0}^2 = M_{Z0}\bar{\kappa}_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \xi_{N0}^{\frac{1}{2}} , \\ &\tilde{\mathbf{\Pi}}_0^{\mathcal{C}\bar{\mathcal{C}}}(k^2) = \begin{bmatrix} \tilde{\Pi}_0^{c^Z\bar{c}^Z} \ \tilde{\Pi}_0^{c^Z\bar{c}^A} \\ \tilde{\Pi}_0^{c^A\bar{c}^Z} \ \tilde{\Pi}_0^{c^A\bar{c}^A} \end{bmatrix}_{(k^2)} , \end{aligned} \tag{57}$$

and in terms of which the bare inverse propagators can be expressed as

$$\begin{aligned} &i\mathbf{D}_{0,NN}^{-1\mu\nu}(k) = [g^{\mu\nu} - \tfrac{k^\mu k^\nu}{k^2}][-k^2 I + \mathbf{M}_{N0}^2 - \mathbf{\Pi}_0^{NN}(k^2)] + \tfrac{k^\mu k^\nu}{k^2}\left[\mathbf{M}_{N0}^2 - k^2 \xi_{N0}^{-\frac{1}{2}^T} \xi_{N0}^{-\frac{1}{2}} - \tilde{\mathbf{\Pi}}_0^{NN}(k^2)\right] , \\ &i\mathbf{D}_{0,N\phi^Z}^{-1\mu}(k) = ik^\mu\left[\mathbf{M}_{N0}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \xi_{N0}^{-\frac{1}{2}^T} \bar{\kappa}_0 + \tilde{\mathbf{\Pi}}_0^{N\phi^Z}(k^2)\right] , \\ &i\mathbf{D}_{0,\phi^Z\phi^Z}^{-1}(k) = k^2 - \bar{\kappa}_0^T \bar{\kappa}_0 - \tilde{\mathbf{\Pi}}_0^{\phi^Z\phi^Z}(k^2) , \\ &i\mathbf{S}_{0,N}^{-1}(k) = k^2 I - \tilde{\mathbf{M}}_N^2 - \tilde{\mathbf{\Pi}}_0^{\mathcal{C}\bar{\mathcal{C}}}(k^2) . \end{aligned} \tag{58}$$

Substituting (58) into (49) we obtain the WT identities for the bare unphysical proper self-energies:

$$\begin{aligned} &(\tilde{\Pi}_0^{ZZ} - M_{Z0}^2)(\tilde{\Pi}_0^{\phi^Z\phi^Z} - k^2) = k^2(M_{Z0} + \tilde{\Pi}_0^{Z\phi^Z})^2 , \\ &(\tilde{\Pi}_0^{ZZ} - M_{Z0}^2)\tilde{\Pi}_0^{AA} = (\tilde{\Pi}_0^{ZA})^2 , \\ &\tilde{\Pi}_0^{ZA} = -M_{Z0}\tilde{\Pi}_0^{A\phi^Z} \hat{C}_0^Z(k^2) , \\ &\hat{C}_0^Z(k^2) = \frac{M_{Z0}^2 - \tilde{\Pi}_0^{ZZ}}{M_{Z0}(M_{Z0} + \tilde{\Pi}_0^{Z\phi^Z})} , \\ &\hat{C}_0^A(k^2) = \frac{-\tilde{\Pi}_0^{ZA}}{M_{Z0}(M_{Z0} + \tilde{\Pi}_0^{Z\phi^Z})} , \\ &\tilde{\mathbf{\Pi}}_0^{\mathcal{C}\bar{\mathcal{C}}}(k^2) = k^2 I - \tilde{\mathbf{M}}_{N0}^2 - [k^2 - M_{Z0}\bar{\kappa}_0 \hat{\mathbf{C}}_0^N(k^2)^T \xi_{N0}^{\frac{1}{2}}]\xi_{N0}^{-\frac{1}{2}} \hat{\mathbf{X}}_{N\bar{\mathcal{C}}}(k^2)\xi_{N0}^{\frac{1}{2}} . \end{aligned} \tag{59}$$

This form is equivalent to that given in Ref. [15]. The second identity in (59) means that only two of $\tilde{\Pi}_0^{ZZ}$, $\tilde{\Pi}_0^{AA}$, and $\tilde{\Pi}_0^{ZA}$ are independent. The matrix notation for the renormalized proper self-energies and their relations to the renormalized propagators are of the same form as (57) and (58) with the subscript "0" removed. The renormalized mass matrix $\tilde{\mathbf{M}}_N^2$ is of the simple form

$$\tilde{\mathbf{M}}_N^2 = M_Z\bar{\kappa}\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \xi_N^{\frac{1}{2}} = \begin{bmatrix} \xi_Z\kappa_Z M_Z & 0 \\ 0 & 0 \end{bmatrix} .$$

Then from (56) and the third identity in (52) we can derive our general WT identities for the renormalized proper self-energies which are

$$
\begin{aligned}
&(\tilde{\Pi}_{ZZ} - M_Z^2)(\tilde{\Pi}_{\phi^z\phi^z} - k^2) - k^2(\tilde{\Pi}_{Z\phi^z} + M_Z)^2 \\
&\quad = [1 - (\Omega_\xi^{ZZ})^{-1}]\xi_Z^{-1}[(k^2 - \xi_Z\kappa_Z M_Z)^2 - k^2(\tilde{\Pi}_{\phi^z\phi^z} + 2\xi_Z\kappa_Z\tilde{\Pi}_{Z\phi^z}) - (\xi_Z\kappa_Z)^2\tilde{\Pi}_{ZZ}] \\
&\qquad +(\Omega_\kappa^{ZZ} - 1)(\Omega_\xi^{ZZ})^{-1}2\kappa_Z[(k^2 - \xi_Z\kappa_Z M_Z)M_Z + k^2\tilde{\Pi}_{Z\phi^z} + \xi_Z\kappa_Z\tilde{\Pi}_{ZZ}] \\
&\qquad +(\Omega_\kappa^{ZZ} - 1)^2(\Omega_\xi^{ZZ})^{-1}\kappa_Z^2[k^2 - \xi_Z M_Z^2 + \xi_Z\tilde{\Pi}_{ZZ}] \ ,
\end{aligned}
$$

$$
\tilde{\Pi}_{ZA} - k^2(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\xi_Z^{-1} = \frac{\tilde{\Pi}_{ZZ} - M_Z^2 + k^2[1 - (\Omega_\xi^{ZZ})^{-1}]\xi_Z^{-1}}{\tilde{\Pi}_{Z\phi^z} + M_Z + \kappa_Z(\Omega_\kappa^{ZZ}/\Omega_\xi^{ZZ} - 1)}[\tilde{\Pi}_{A\phi^z} + \kappa_Z(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\Omega_\kappa^{ZZ}] \ ,
$$

$$
\begin{aligned}
&\tilde{\Pi}_{AA} + k^2[-(\Omega_\xi^{ZA})^{-1}\xi_Z^{-1} + (1 - (\Omega_\xi^{ZZ})^{-1})\xi_A^{-1}] \\
&\qquad = \frac{\tilde{\Pi}_{ZA} - k^2(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\xi_Z^{-1}}{\tilde{\Pi}_{Z\phi^z} + M_Z + \kappa_Z(\Omega_\kappa^{ZZ}/\Omega_\xi^{ZZ} - 1)}[\tilde{\Pi}_{A\phi^z} + \kappa_Z(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\Omega_\kappa^{ZZ}] \ , \qquad (60)
\end{aligned}
$$

$$
\hat{C}^Z(k^2) = \frac{M_Z^2 - \tilde{\Pi}_{ZZ} + k^2[(\Omega_\xi^{ZZ})^{-1} - 1]\xi_Z^{-1}}{M_Z[\tilde{\Pi}_{Z\phi^z} + M_Z + \kappa_Z(\Omega_\kappa^{ZZ}/\Omega_\xi^{ZZ} - 1)]} \ ,
$$

$$
\hat{C}^A(k^2) = \frac{\tilde{\Pi}_{ZA} - k^2(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\xi_Z^{-1}}{\tilde{\Pi}_{Z\phi^z} + M_Z + \kappa_Z(\Omega_\kappa^{ZZ}/\Omega_\xi^{ZZ} - 1)} \ ,
$$

$$
\tilde{\mathbf{\Pi}}^{c\bar{c}}(k^2) = k^2 I - \mathbf{M}_N^2 - [k^2 - M_Z \xi_N^{\frac{1}{2}^T} \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}^T} \xi_N^{-\frac{1}{2}^T} \mathbf{\Omega}_{\bar{\kappa}\bar{\kappa}} \hat{\mathbf{C}}^N(k^2)^T \mathbf{\Omega}_{\xi_N}^{\frac{1}{2}} \xi_N^{\frac{1}{2}}] \xi_N^{-\frac{1}{2}} \mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}} \mathbf{Z}_N^{-\frac{1}{2}} \hat{\mathbf{X}}_{N\bar{c}}(k^2) \mathbf{Z}_N^{\frac{1}{2}} \mathbf{\Omega}_{\xi_N}^{\frac{1}{2}} \xi_N^{\frac{1}{2}} \mathbf{Z}_c^N \ ,
$$

in which we have chosen

$$
(\Omega_\xi^{AZ})^{-\frac{1}{2}} = 0, \quad \Omega_\kappa^{AZ} = -\xi_Z^{-\frac{1}{2}}\xi_A^{\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}(\Omega_\xi^{AA})^{\frac{1}{2}}\Omega_\kappa^{ZZ}, \qquad (61)
$$

for simplifying the expression (60). Equation (60) is similar to (25) but is much more complicated. The first three and the last equations in (60) give constraints on the unphysical proper self-energies, and the fourth and fifth equations give constraints on $\hat{C}^Z(k^2)$ and $\hat{C}^A(k^2)$, *which are very useful in constructing renormalization schemes simplifying the expressions for* $\hat{C}^Z(\xi_Z\kappa_Z M_Z)$ *and* $\hat{C}^A(0)$.

We first analyze the renormalization schemes in the case of $\xi_N \neq 0$. For gauge fields in the physical sector, there are five independent renormalization constants, namely, Z_{M_Z}, Z_{ZZ}, Z_{ZA}, Z_{AZ}, and Z_{AA}. The standard on-shell subtraction conditions are

$$
\begin{aligned}
&\Pi_{ZZ}(M_Z^2) = 0, \quad \Pi'_{ZZ}(M_Z^2) = 0 \ , \\
&\Pi_{AA}(0) = 0, \qquad \Pi'_{AA}(0) = 0 \ , \\
&\Pi_{ZA}(M_Z^2) = \Pi_{AZ}(M_Z^2) = 0 \ , \\
&\Pi_{ZA}(0) = \Pi_{AZ}(0) = 0 \ ,
\end{aligned} \qquad (62)
$$

which contain six equations. However, from (58) we see that the nonsingular requirement of $i\mathbf{D}_{0NN}^{-1\mu\nu}(k)$ at $k^2 = 0$ [15,13] implies that

$$
\mathbf{\Pi}_0^{NN}(0) = \tilde{\mathbf{\Pi}}_0^{NN}(0) \ . \qquad (63)
$$

Together with the second identity in (59), we see that there are actually only five independent conditions in (62) which are just sufficient to determine the five independent renormalization constants. For the unphysical neutral sector, there are altogether *eleven independent renormalization constants* [cf. (46), (54), and (55)], namely, four elements in $\mathbf{\Omega}_{\xi_N}^{-1/2}$, two elements in $\mathbf{\Omega}_{\bar{\kappa}}$, one $\mathbf{\Omega}_{\phi^z}^{-1/2}$ and four elements in $\mathbf{Z}_c^N$. We have already chosen $(\mathbf{\Omega}_\xi^{AZ})^{-1/2}$ and $\mathbf{\Omega}_\kappa^{AZ}$ to satisfy (61), so that there are *nine remaining arbitrary independent constants to be determined by the subtraction conditions.* Similar to what we have done in the $SU(2)_L$ theory, we may choose the subtraction conditions to make the nine unphysical mass poles coincide with the tree level poles to simplify the loop calculations and also the expression for $\hat{\mathbf{C}}^N$, but this needs more considerations. First of all, not all such conditions are relevant to the determination of the nine renormalization constants. For example, the three conditions

$$
\tilde{\Pi}_{ZA}(0) = \tilde{\Pi}_{AZ}(0) = \tilde{\Pi}_{AA}(0) = 0 \qquad (64)
$$

are related to the corresponding conditions in (62) in the physical sector through the nonsingular requirement (63), so that (64) do not give any restrictions to the above nine constants. Moreover, we can see from the last identity in (60) and the choice (61) that $\tilde{\Pi}^{c^A\bar{c}^Z}(k^2)$ and $\tilde{\Pi}^{c^A\bar{c}^A}(k^2)$

are all proportional to the k^2, so that

$$\tilde{\Pi}_{c^A\bar{c}^Z}(0) = \tilde{\Pi}_{c^A\bar{c}^A}(0) = 0 \; ; \tag{65}$$

i.e., they are also irrelevant to the determination of the nine constants. Therefore we can at most write down six relevant subtraction conditions such as (26). However, from (60) we see that $\tilde{\Pi}^{A\phi^Z}(k^2)$ does not appear in the expressions for $\hat{C}^Z(k^2)$ and $\hat{C}^A(k^2)$, therefore the value of $\tilde{\Pi}^{A\phi^Z}(0)$ may not be taken to be zero for the purpose of simplifying $\hat{C}^Z$ and $\hat{C}^A$. Thus we take the following five subtraction conditions:

$$\begin{aligned} &\tilde{\Pi}_{ZZ}(\xi_Z\kappa_Z M_Z) = 0 \; , \quad \tilde{\Pi}_{\phi^Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0, \\ &\tilde{\Pi}_{Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0 \; , \\ &\tilde{\Pi}_{c^Z\bar{c}^Z}(\xi_Z\kappa_Z M_Z) = 0, \quad \tilde{\Pi}_{c^Z\bar{c}^A}(0) = 0 \; . \end{aligned} \tag{66}$$

We can further see from the first identity in (60) that if the first two conditions ($\tilde{\Pi}_{ZZ} = 0, \tilde{\Pi}_{\phi^Z\phi^Z} = 0$ at $k^2 = \xi_Z\kappa_Z M_Z$) in (66) are satisfied, we have

$$\tilde{\Pi}_{Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0, \quad \text{if } \kappa_Z = M_Z \text{ or } \Omega_\kappa^{ZZ} = 1 \; . \tag{67}$$

Therefore the third conditions in (66) may be a relevant condition determining Ω_κ^{ZZ} if κ_Z is arbitrary, but it may not be a relevant condition if $\kappa_Z = M_Z$. Hence we still need four more conditions (if κ_Z is arbitrary) or five more conditions (if $\kappa_Z = M_Z$) for the determination of the nine renormalization constants. These can be taken to be the usual normalization conditions [11,15]

$$\begin{aligned} &k^{-2}\tilde{\Pi}_{AA}(k^2)|_{k^2=0} = 0, \qquad k^{-2}\tilde{\Pi}_{ZA}(k^2)|_{k^2=0} = 0, \\ &\left.\frac{d\tilde{\Pi}_{c^Z\bar{c}^Z}}{dk^2}\right|_{k^2=\xi_Z\kappa_Z M_Z} = 0 \; , \quad \left.\frac{d\tilde{\Pi}_{c^A\bar{c}^A}}{dk^2}\right|_{k^2=0} = 0, \end{aligned} \tag{68}$$

for the case with κ_Z being arbitrary and when choosing $\kappa_Z = M_Z$ we include one more condition:

$$\left.\frac{d\tilde{\Pi}_{\phi^Z\phi^Z}}{dk^2}\right|_{k^2=\xi_Z\kappa_Z M_Z} = 0 \; . \tag{69}$$

To see how these conditions determine the nine constants, let us look at the propor self-energy counterterms defined by $\delta\tilde{\Pi} \equiv \tilde{\Pi} - \tilde{\Pi}_0$. From the definitions of bare proper self-energies (58) and similar definitions for the renormalized ones, and the general relations $\mathbf{D}_{0,\bar{N}\bar{N}} = \mathbf{Z}_{\bar{N}}^{1/2}\mathbf{D}_{\bar{N}\bar{N}}(\mathbf{Z}_{\bar{N}}^{1/2})^T$ and $\mathbf{S}_{0,N} = \mathbf{Z}_c^N\mathbf{S_N}$, we obtain the following exact expressions for the $\delta\Pi$'s:

$$\begin{aligned} \delta\mathbf{\Pi}_{NN} &= k^2(I - \mathbf{Z}_N^{-\frac{1}{2}T}\mathbf{Z}_N^{-\frac{1}{2}}) - Z_{M_Z}\mathbf{M}_N^2 + \mathbf{Z}_N^{-\frac{1}{2}T}\mathbf{M}_N^2\mathbf{Z}_N^{-\frac{1}{2}} + [\mathbf{\Pi}_{NN} - \mathbf{Z}_N^{-\frac{1}{2}T}\mathbf{\Pi}_{NN}\mathbf{Z}_N^{-\frac{1}{2}}] \; , \\ \delta\tilde{\mathbf{\Pi}}_{NN} &= k^2\mathbf{Z}_N^{-\frac{1}{2}T}[\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}T}\xi_N^{-1}\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}} - \xi_N^{-1}]\mathbf{Z}_N^{-\frac{1}{2}} + [\mathbf{Z}_N^{-\frac{1}{2}T}\mathbf{M}_N^2\mathbf{Z}_N^{-\frac{1}{2}} - \mathbf{M}_N^2 Z_{M_Z}^2] + [\tilde{\mathbf{\Pi}}_{NN} - \mathbf{Z}_N^{-\frac{1}{2}T}\tilde{\mathbf{\Pi}}_{NN}\mathbf{Z}_N^{-\frac{1}{2}}] \; , \\ \delta\tilde{\mathbf{\Pi}}_{N\phi^Z} &= (Z_{M_Z} - Z_{\phi^Z}^{-\frac{1}{2}}\mathbf{Z}_N^{-\frac{1}{2}T})M_Z\begin{pmatrix}1\\0\end{pmatrix} + Z_{\phi^Z}^{-\frac{1}{2}}\mathbf{Z}_N^{-\frac{1}{2}T}[\xi_N^{-\frac{1}{2}T} - \mathbf{\Omega}_{\xi_N}^{-1T}\xi_N^{-\frac{1}{2}T}\mathbf{\Omega}_{\bar{\kappa}}]\bar{\kappa} + [I - Z_{\phi^Z}^{-\frac{1}{2}}\mathbf{Z}_N^{-\frac{1}{2}T}]\tilde{\mathbf{\Pi}}_{N\phi^Z} \; , \\ \delta\tilde{\mathbf{\Pi}}_{\phi^Z\phi^Z} &= (Z_{\phi^Z}^{-1} - 1)k^2 + \xi_Z\kappa_Z^2\begin{pmatrix}1\\0\end{pmatrix}^T[\mathbf{\Omega}_{\bar{\kappa}}^T\xi_N^{-\frac{1}{2}}\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}}\xi_N\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}T}\xi_N^{-\frac{1}{2}T}\mathbf{\Omega}_{\bar{\kappa}} - I]\begin{pmatrix}1\\0\end{pmatrix}Z_{\phi^Z}^{-1} + (1 - Z_{\phi^Z}^{-1})\tilde{\mathbf{\Pi}}_{\phi^Z\phi^Z} \; , \\ \delta\tilde{\mathbf{\Pi}}_{\mathcal{C}\bar{\mathcal{C}}} &= (k^2 I - \tilde{\mathbf{M}}_N^2)((\mathbf{Z}_c^N)^{-1} - I) + \xi_Z\kappa_Z M_Z \\ &\quad \times \left(\xi_N^{-\frac{1}{2}T}\mathbf{\Omega}_{\xi_N}^{-\frac{1}{2}T}\xi_N^{-\frac{1}{2}T}\mathbf{\Omega}_{\bar{\kappa}}\begin{bmatrix}1&0\\0&0\end{bmatrix}(Z_{M_Z}Z_{\phi^Z}^{-\frac{1}{2}}\mathbf{Z}_N^{\frac{1}{2}})\mathbf{\Omega}_{\xi_N}^{\frac{1}{2}}\xi_N^{\frac{1}{2}}\xi_Z^{-\frac{1}{2}} - \begin{bmatrix}1&0\\0&0\end{bmatrix}\right) + \tilde{\mathbf{\Pi}}_{\mathcal{C}\bar{\mathcal{C}}}(I - (\mathbf{Z}_c^N)^{-1}) \; . \end{aligned} \tag{70}$$

To one loop, (70) takes a simpler form

$$\begin{aligned} \delta\mathbf{\Pi}_{NN} &= \begin{bmatrix} \delta Z_{ZZ}(k^2 - M_Z^2) - 2\delta Z_{M_Z}M_Z^2 & \frac{1}{2}[\delta Z_{ZA}(k^2 - M_Z^2) + \delta Z_{AZ}k^2] \\ \frac{1}{2}[\delta Z_{ZA}(k^2 - M_Z^2) + \delta Z_{AZ}k^2] & \delta Z_{AA}k^2 \end{bmatrix} \; , \\ \delta\tilde{\mathbf{\Pi}}_{NN} &= \begin{bmatrix} -\xi_Z^{-1}\delta\Omega_\xi^{ZZ}k^2 - (\delta Z_{ZZ} + 2\delta Z_{M_Z})M_Z^2 & -\frac{1}{2}[\xi_Z^{-1}\delta\Omega_\xi^{ZA}k^2 + \delta Z_{ZA}M_Z^2] \\ -\frac{1}{2}[\xi_Z^{-1}\delta\Omega_\xi^{ZA}k^2 + \delta Z_{ZA}M_Z^2] & -\xi_A^{-1}\delta\Omega_\xi^{AA}k^2 \end{bmatrix} \; , \\ \delta\tilde{\mathbf{\Pi}}_{N\phi^Z} &= \begin{pmatrix} [\frac{1}{2}(\delta Z_{ZZ} + \delta Z_{\phi^Z}) + \delta Z_{M_Z}]M_Z + [\delta\Omega_\xi^{ZZ} - \delta\Omega_\kappa^{ZZ}]\kappa_Z \\ \frac{1}{2}\delta Z_{ZA}M_Z + (\delta\Omega_\xi^{ZA} - \delta\Omega_\kappa^{AZ}\xi_Z^{\frac{1}{2}}\xi_A^{-\frac{1}{2}})\kappa_Z \end{pmatrix} \; , \\ \delta\tilde{\Pi}_{\phi^Z\phi^Z} &= -\delta Z_{\phi^Z}k^2 + \xi_Z\kappa_Z^2(2\delta\Omega_\kappa^{ZZ} - \delta\Omega_\xi^{ZZ}) \; , \\ \delta\tilde{\mathbf{\Pi}}_{\mathcal{C}\bar{c}} &= \begin{bmatrix} \delta\tilde{\Pi}_{c^Z\bar{c}^Z} & \delta\tilde{\Pi}_{c^Z\bar{c}^A} \\ \delta\tilde{\Pi}_{c^A\bar{c}^Z} & \delta\tilde{\Pi}_{c^A\bar{c}^A} \end{bmatrix} \; , \\ \delta\tilde{\Pi}_{c^Z\bar{c}^Z} &= -\delta Z_c^{ZZ}(k^2 - \xi_Z\kappa_Z M_Z) + \xi_Z\kappa_Z M_Z[\delta\Omega_\kappa^{ZZ} + \tfrac{1}{2}(\delta Z_{ZZ} - \delta Z_{\phi^Z}) + \delta Z_{M_Z}] \; , \\ \delta\tilde{\Pi}_{c^Z\bar{c}^A} &= -\delta Z_c^{ZA}(k^2 - \xi_Z\kappa_Z M_Z) + \tfrac{1}{2}\xi_Z^{\frac{1}{2}}\xi_A^{\frac{1}{2}}\kappa_Z M_Z(\delta\Omega_\xi^{ZA} + \delta Z_{ZA}) \; , \\ \delta\tilde{\Pi}_{c^A\bar{c}^Z} &= -\delta Z_c^{AZ}k^2 + \xi_Z\kappa_Z M_Z(\delta\Omega_\kappa^{AZ} - \tfrac{1}{2}\xi_Z^{-\frac{1}{2}}\xi_A^{\frac{1}{2}}\delta\Omega_\xi^{ZA}) \; , \\ \delta\tilde{\Pi}_{c^A\bar{c}^A} &= -\delta Z_c^{AA}k^2 \; , \end{aligned} \tag{71}$$

where $\delta Z \equiv Z - 1$, $\delta\Omega \equiv \Omega - 1$. From these expressions we see that, for arbitrary κ_Z, we can have

$$\begin{aligned}
&\tilde{\Pi}_{ZZ}(\xi_Z\kappa_Z M_Z) = 0 \ \text{by adjusting } \Omega_\xi^{ZZ} , \\
&k^{-2}\tilde{\Pi}_{AA}(k^2)\,|_{k^2=0} = 0 \ \text{by adjusting } \Omega_\xi^{AA} , \\
&k^{-2}\tilde{\Pi}_{ZA}(k^2)\,|_{k^2=0} = 0 \ \text{by adjusting } \Omega_\xi^{ZA} , \\
&\tilde{\Pi}_{\phi^Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0 \ \text{by adjusting } \Omega_\kappa^{ZZ} \text{ or } Z_{\phi^Z} , \\
&\tilde{\Pi}_{c^Z\bar{c}^Z}(\xi_Z\kappa_Z M_Z) = 0 \ \text{by adjusting } Z_c^{AZ} , \\
&\tilde{\Pi}_{c^Z\bar{c}^A}(0) = 0 \ \text{by adjusting } Z_c^{ZA} , \\
&d\tilde{\Pi}_{c^Z\bar{c}^Z}/dk^2\,|_{k^2=\xi_Z\kappa_Z M_Z} = 0 \ \text{by adjusting } Z_c^{ZZ}, \\
&d\tilde{\Pi}_{c^A\bar{c}^A}/dk^2\,|_{k^2=0} = 0 \ \text{by adjusting } Z_c^{AA} .
\end{aligned} \tag{72}$$

For the case $\kappa_Z = M_Z$, Eq. (67) is irrelevant, we need one more condition (69), and from (70) or (71) we see that we have

$$d\tilde{\Pi}_{\phi^Z\phi^Z}/dk^2\,|_{k^2=\xi_Z\kappa_Z M_Z} = 0 \ \text{by adjusting } Z_{\phi^Z} . \tag{73}$$

Thus we can have two different convenient schemes. The first one is to take $\kappa_Z = M_Z$ with Ω_κ^{ZZ} determined by (72). This is just a generalization of our *scheme I* to the case of the SU(2)×U(1) theory. The second one is to take κ_Z arbitrary but $\Omega_\kappa^{ZZ} = 1$ with Z_{ϕ^Z} determined by the fourth condition in (72). This is just a generalization of our *scheme II* to the present case. It is easy to see from the fourth and fifith identities in (60) that in these two schemes $\hat{C}^Z(\xi_Z\kappa_Z M_Z)$ and $\hat{C}^A(0)$ are simplified to

$$\hat{C}^Z(\xi_Z\kappa_Z M_Z) = \begin{cases} 1/\Omega_\kappa^{ZZ} & \text{in scheme I ,} \\ 1 & \text{in scheme II ,} \end{cases}$$
$$\hat{C}^A(0) = 0 , \quad \text{in both scheme I and scheme II .} \tag{74}$$

In these schemes

$$\tilde{\Pi}_{A\phi^Z}(0) = -\kappa_Z(\Omega_\xi^{ZZ})^{-\frac{1}{2}}(\Omega_\xi^{ZA})^{-\frac{1}{2}}\Omega_\kappa^{ZZ} . \tag{75}$$

Finally we consider the case $\xi^Z = \xi^A = 0$ (Landau gauge) in which some of the formulas in (60) are not

TABLE II. Main features of scheme I, II, and MW scheme in the neutral sector.

	Scheme I	Scheme II	MW-scheme
On-shell conditions	$\{\mathbf{\Omega}_{\xi_N}, \mathbf{\Omega}_{\bar{\kappa}}, Z_{\phi^Z}, \mathbf{Z}_c^N\}$ $\xi_Z, \xi_A \neq 0$, $\Omega_\kappa^{ZZ} \neq 1$	$\{\mathbf{\Omega}_{\xi_N}, \mathbf{\Omega}_{\bar{\kappa}}, Z_{\phi^Z}, \mathbf{Z}_c^N\}$ $\xi_Z, \xi_A \neq 0$, $\Omega_\kappa^{ZZ} = 1$	$\{Z_{\phi^Z}, \mathbf{Z}_c^N\}$ $\xi_Z = \xi_A = 0$, $Z_c^{ZA} = Z_c^{AZ} = 0$
$\tilde{\Pi}_{ZZ}(\xi_Z\kappa_Z M_Z) = 0$	Adjust $\delta\Omega_\xi^{ZZ}$	Adjust $\delta\Omega_\xi^{ZZ}$	\
$\tilde{\Pi}_{ZA}(0) = 0$	Nonsingular condition	Nonsingular condition	\
$\tilde{\Pi}_{AA}(0) = 0$	Nonsingular condition	Nonsingular condition	\
$\frac{\tilde{\Pi}_{ZA}(k^2)}{k^2}\,\|_{k^2=0} = 0$	Adjust $\delta\Omega_\xi^{ZA}$	Adjust $\delta\Omega_\xi^{ZA}$	\
$\frac{\tilde{\Pi}_{AA}(k^2)}{k^2}\,\|_{k^2=0} = 0$	Adjust $\delta\Omega_\xi^{AA}$	Adjust $\delta\Omega_\xi^{AA}$	\
$\tilde{\Pi}_{Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0$	WT (When beyond one loop we must set $\kappa_Z = M_Z$)	WT	\
$\tilde{\Pi}_{A\phi^Z}(0) =$	$\frac{1}{2}\kappa_Z(\Omega_\xi^{ZZ})^{-\frac{1}{2}}\delta\Omega_\xi^{ZA}\Omega_\kappa^{ZZ}$	$\frac{1}{2}\kappa_Z(\Omega_\xi^{ZZ})^{-\frac{1}{2}}\delta\Omega_\xi^{ZA}\Omega_\kappa^{ZZ}$	\
$\tilde{\Pi}_{\phi^Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0$	Adjust $\delta\Omega_\kappa^{ZZ}$	Adjust δZ_{ϕ^Z}	WT
$\tilde{\Pi}'_{\phi^Z\phi^Z}(\xi_Z\kappa_Z M_Z) = 0$	Adjust δZ_{ϕ^Z}	\	Adjust δZ_{ϕ^Z}
$\tilde{\Pi}_{c^Z\bar{c}^Z}(\xi_Z\kappa_Z M_Z) = 0$	WT (When beyond one loop we must adjust δZ_c^{AZ})	WT (When beyond one loop we must adjust δZ_c^{AZ})	WT
$\tilde{\Pi}_{c^Z\bar{c}^A}(0) = 0$	Adjust δZ_c^{ZA}	Adjust δZ_c^{ZA}	WT
$\tilde{\Pi}_{c^A\bar{c}^Z}(0) = 0$	WT	WT	WT
$\tilde{\Pi}_{c^A\bar{c}^A}(0) = 0$	WT	WT	WT
$\tilde{\Pi}'_{c^Z\bar{c}^Z}(\xi_Z\kappa_Z M_Z) = 0$	Adjust δZ_c^{ZZ}	Adjust δZ_c^{ZZ}	Adjust δZ_c^{ZZ}
$\tilde{\Pi}'_{c^A\bar{c}^A}(0) = 0$	Adjust δZ_c^{AA}	Adjust δZ_c^{AA}	Adjust δZ_c^{AA}
$\hat{C}^Z(\xi_Z\kappa_Z M_Z)$	$(\Omega_\kappa^{ZZ})^{-1}$ (if $\kappa_Z = M_Z$)	1	See Eq. (76)
$\hat{C}^A(0)$	0	0	See Eq. (76)

clearly defined. Now we have the following well-known relations in the Landau gauge [13,14]: (a) there are no Z_μ-ϕ^Z and A_μ-ϕ^Z mixings and the longitudinal components of $\mathbf{D}^{\mu\nu}_{NN}$ and $\mathbf{D}^{\mu}_{N\phi^Z}$ (containing $\tilde{\mathbf{\Pi}}_{NN}$ and $\tilde{\mathbf{\Pi}}_{N\phi^Z}$) vanish; (b) the poles of $\mathcal{D}_{\phi^Z\phi^Z}$ and $\mathbf{S}_N$ are all at $k^2=0$; (c) the neutral ghost fields $\mathcal{C}$ and $\bar{\mathcal{C}}$ do not couple to the Higgs and Goldstone boson fields. With these relations we get

$$\hat{\mathbf{C}}_N(k^2) = \frac{1}{Z^{\frac{1}{2}}_{\phi^Z}} \frac{Z_{M_Z}}{(1+\Delta_3^{ZZ})(1+\Delta_3^{AA})-\Delta_3^{ZA}\Delta_3^{AZ}} \begin{bmatrix} Z^{\frac{1}{2}}_{ZZ}(1+\Delta_3^{AA}) - Z^{\frac{1}{2}}_{AZ}\Delta_3^{ZA} \\ Z^{\frac{1}{2}}_{ZA}(1+\Delta_3^{AA}) - Z^{\frac{1}{2}}_{AA}\Delta_3^{ZA} \end{bmatrix} \equiv \begin{pmatrix} \hat{C}^Z(k^2) \\ \hat{C}^A(k^2) \end{pmatrix} \tag{76}$$

which is not so much simplified as in (74). In (76) the renormalization constants Z_{ZZ}, Z_{AZ}, Z_{AA}, and Z_{M_Z} in the physical sector are all well fixed. The only adjustable parameter for simplifying $\hat{\mathbf{C}}_N$ is Z_{ϕ^Z} and the choice of which depends on the detailed explicit calculation of Δ_3^{ZZ}, Δ_3^{ZA}, Δ_3^{AZ}, and Δ_3^{AA} order by order in loop expansions. In the conventional schemes used in the Landau gauge, e.g., the MW scheme [14], Z_{ϕ^Z} is determined by the usual condition normalizing the residue of $\mathcal{D}_{\phi^Z\phi^Z}$ at $k^2=0$, thus in that scheme $\hat{\mathbf{C}}_N(k^2)$ cannot be simplifed.

For clarity we summarize our results for *scheme I*, *scheme II*, and the MW scheme (for Landau gauge) in Table II.

III. PRECISE FORMULATION OF THE EQUIVALENCE THEOREM

A. A physical analysis of the equivalence theorem

Intuitively, we expect that the amplitude of physical longitudinal weak boson is related to that of the unphysical Goldstone boson due to the Higgs mechanism. However, it is unlikely that the physical amplitude can be equal to the unphysical one up to loop-level even we neglect the $O(M_W/E)$ terms since the wave function renormalizations for the physical and unphysical fields are different and the latter is arbitrary. So we expect that *generally there should be multiplicative modification factors in (2) which ensure the renormalization scheme and ξ independence of the RHS of (2).*

We start from considering some Slavnov-Taylor (ST) identities for the Green functions, which are useful in the proof of the ET. Turning off the external source K^i and L^a in (7), the invariance of the action S under the BRST transformation (8) leads to the generating equation

$$\int \mathcal{D}\chi_0\mathcal{D}c_0\mathcal{D}\bar{c}_0\{J_iD_i^a(\chi_0)c_0^a - \tfrac{1}{2}g_0f^{abc}\bar{I}^ac_0^bc_0^c - F_0^a(\chi_0)I^a\} \times \exp i\{S[\chi_0,c_0,\bar{c}_0] + \int d^4\,\chi(J_i\chi_0^i + \bar{I}^ac_0^a + \bar{c}_0^aI^a)\} = 0\ . \tag{77}$$

Taking functional derivatives with respect to the external sources of (77) we can obtain the ST identities

$$\langle 0|TF_0^a(x)F_0^b(y)|0\rangle = -i\delta^{ab}\delta^4(x-y),$$
$$\langle 0|TF_0^a(x)\chi_0^i(y)|0\rangle = -\langle 0|TD_i^c(y)c_0^c(y)\bar{c}_0^a(x)|0\rangle\ , \tag{78}$$

which will be used later in the proof of the precise formulation of the ET. One can further obtain the following general ST identity in the momentum representation [5]:

$$\langle 0|F_0^{a_1}(k_1)\cdots F_0^{a_n}(k_n)\Phi|0\rangle = 0\ , \tag{79}$$

where $F_0^a(k) = i(\xi_0^a)^{-1/2}k_\mu V_0^{a\mu}(k) - (\xi_0^a)^{1/2}\kappa_0^a\phi_0^a$, and Φ denotes possible on-shell physical fields. In (79) external Φ legs have been amputated. In Appendix A, we present a simpler proof of (79) in the current path integral formalism. With (79) we can give a physical analysis of the ET.

Consider first a gauge theory without spontaneous symmetry breaking (SSB). The two transverse components of massless gauge field are physical, while the unphysical longitudinal and scalar components are constrained by the gauge fixing condition. Let us take the covariant gauge

$$F_0^a(k) = i(\xi_0^a)^{-\frac{1}{2}}k_\mu V_0^{a\mu}(k)\ . \tag{80}$$

The longitudinal and scalar polarization vectors of $V_0^{a\mu}$ can be written as

$$\epsilon_L^\mu(k) = (0,\vec{k}/k^0),\ \epsilon_S^\mu(k) = (1,\vec{0})\ . \tag{81}$$

We then have

$$\epsilon_L^\mu(k) + \epsilon_S^\mu(k) = \frac{k^\mu}{k^0}\ , \tag{82}$$

so that

$$F_0^a(k) = i(\xi_0^a)^{-\frac{1}{2}}k^0[V_{0L}^a(k) + V_{0S}^a(k)]\ . \tag{83}$$

Substituting (83) into (79) and doing renormalization and F^a-leg amputation, we directly get the scattering amplitude

$$T(V_L^{a_1}(k_1)+V_S^{a_1}(k_1),\ldots,V_L^{a_n}(k_n)+V_S^{a_n}(k_n),\Phi)=0\,, \qquad (84)$$

which is just a quantitative formulation of the V_L^a-V_S^a constraint mechanism in the physical in/out states for a massless gauge theory.

If SSB takes place, the gauge fields become massive and the longitudinal component V_L^a is "released" to be physical. We shall see that in the constraint (84) V_L^a will now be replaced by the unphysical would-be Goldstone boson field. Let us take the R_ξ gauge (3),

$$F_0^a(k)=i(\xi_0^a)^{-\frac{1}{2}}k_\mu V_0^{a\mu}(k)-(\xi_0^a)^{\frac{1}{2}}\kappa_0^a\phi_0^a(k)\,. \qquad (85)$$

Now the longitudinal and scalar polarization vectors for a massive vector field with a physical mass M_a can be written as

$$\epsilon_L^\mu(k)=\frac{1}{M_a}(|\vec{k}|,k^0\vec{k}/|\vec{k}|),\ \epsilon_S^\mu(k)=k^\mu/M_a\,. \qquad (86)$$

Thus

$$F_0^a(k)=i(\xi_0^a)^{-\frac{1}{2}}M_aV_{0S}^a(k)-(\xi_0^a)^{\frac{1}{2}}\kappa_0^a\phi_0^a(k)\,. \qquad (87)$$

Repeating the above procedures with care on the V_{0S}^a-ϕ_0^a mixing, we get, corresponding to (84),

$$T(V_S^{a_1}(k_1)-iC^{a_1}(k_1^2)\phi^{a_1}(k_1),\ldots,V_S^{a_n}(k_n)-iC^{a_n}(k_n^2)\phi^{a_n}(k_n),\Phi)=0\,, \qquad (88)$$

where the exact expression for $C^a(k^2)$ will be derived in the following subsections. Since M_a is the characteristic of the SSB we infer that (82) holds as $M_a\to 0$. Therefore with $M_a\neq 0$, $\epsilon_L^\mu(k)+\epsilon_S^\mu(k)$ must be of the form

$$\epsilon_L^\mu(k)+\epsilon_S^\mu(k)=\frac{k^\mu}{A}+O(M_a/E)\,, \qquad (89)$$

with A being a certain normalization factor. Therefore, at high energy, $\epsilon_L^\mu(k)$ and $\epsilon_S^\mu(k)$ are related up to an $O(M_a/E)$ term. Indeed, if we take the $M_a\neq 0$ expressions (86) we see that $A=\frac{1}{2}M_a$, and thus

$$\epsilon_L^\mu(k)=\epsilon_S^\mu(k)+O(M_a/E)\,. \qquad (90)$$

So we can define

$$V_L^a(k)\equiv V_S^a(k)+v^a(k)\,,\qquad v^a=O(M_a/E)\,, \qquad (91)$$

and

$$\begin{aligned}\bar{F}^a&\equiv V_S^a-iC^a\phi^a\equiv V_L^a-\bar{Q}^a\,,\\ \bar{Q}^a&\equiv iC^a\phi^a+v^a=iC^a\phi^a+O(M_a/E)\,.\end{aligned} \qquad (92)$$

Then (88) becomes

$$0=T(\bar{F}^{a_1},\ldots,\bar{F}^{a_n},\Phi)\ \ (n\geq 1)\,, \qquad (93)$$

i.e.,

$$\begin{aligned}0=&\,T(V_L^{a_1}-\bar{Q}^{a_1},\ldots,V_L^{a_n}-\bar{Q}^{a_n},\Phi)\\ =&\,T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)+(-)^nT(\bar{Q}^{a_1},\ldots,\bar{Q}^{a_n},\Phi)\\ &+\sum_{1\leq j\leq n-1}^{P_j}T(V_L^{a_{l_1}},\ldots,V_L^{a_{l_j}},\bar{F}^{a_{l_{j+1}}}-V_L^{a_{l_{j+1}}},\ldots,\bar{F}^{a_{l_n}}-V_L^{a_{l_n}},\Phi)\ \ [\text{cf. }(92)]\\ =&\,T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)+(-)^nT(\bar{Q}^{a_1},\ldots,\bar{Q}^{a_n},\Phi)+\sum_{1\leq j\leq n-1}^{P_j}T(V_L^{a_{l_1}},\ldots,V_L^{a_{l_j}},-V_L^{a_{l_{j+1}}},\ldots,-V_L^{a_{l_n}},\Phi)\ \ [\text{cf. }(93)]\\ =&\,T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)+(-)^nT(\bar{Q}^{a_1},\ldots,\bar{Q}^{a_n},\Phi)+\sum_{j=1}^{n-1}C_n^j(-)^{n-j}T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)\,.\end{aligned}$$

Using the identity $0=(1-1)^n=1+(-)^n+\sum_{j=1}^{n-1}C_n^j(-)^{n-j}$ we have

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)=T(\bar{Q}^{a_1},\ldots,\bar{Q}^{a_n},\Phi)\,. \qquad (94)$$

Substituting (92) into (94) we get the general formula

$$T(V_L^{a_1}(k_1),\ldots,V_L^{a_n}(k_n),\Phi)=C_{\rm mod}^{a_1}\cdots C_{\rm mod}^{a_n}T(i\phi^{a_1}(k_1),\ldots,i\phi^{a_n}(k_n),\Phi)+O(M_a/E)\,, \qquad (95)$$

where $C_{\rm mod}^a\equiv C^a(k^2)|_{k^2=M_a^2}$. This is just the general precise formulation of the ET. Therefore *the ET is just a direct consequence of the V_S^a-ϕ^a constraint mechanism in gauge theories with SSB [cf. (88)] and the high energy relation (90)* . The only task remained for proving the precise formulaton of the ET is to derive the quantitative expression for $C_{\rm mod}^a$ and then try to simplify it in a rigorous way. We shall do it for the $SU(2)_L$ theory and the realistic $SU(2)\times U(1)$ theory separately in the following two subsections.

B. General proof of the precise formulation of the equivalence theorem in the $SU(2)_L$ theory

Now we give a general proof of the form (95) in the $SU(2)_L$ theory with the modification factor $C_{\rm mod}^a$ precisely specified to all orders in the perturbation. In the following proof we distingush the renormalized M_W from the physically observed mass $M_W^{\rm phys}$ of the W_μ^a field (pole of the physical propagator of W_μ^a). Such a distinction is necessary in some currently used renormalization schemes. For convenience we define the five-component matrix notation

$$\bar{W}_0^a = \begin{pmatrix} W_{0\mu}^a \\ \phi_0^a \end{pmatrix} .$$

In the $SU(2)_L$ theory $\xi_0^a = \xi_0, \kappa_0^a = \kappa_0$. With the symbol $\underline{\mathbf{K}}_0$ defined in (6), the gauge fixing function (3) can now be written as

$$F_0^a = \underline{\mathbf{K}}_0^T \bar{W}_0^a. \tag{96}$$

The second ST identity in (78) can then be written as

$$\begin{aligned} \underline{\mathbf{K}}_0^M \langle 0|T\bar{W}_{0_M}^a(x)W_{0\nu}^b(y)|0\rangle &= -\langle 0|TD_{\nu b}^d(y)c_0^d(y)\bar{c}_0^a(x)|0\rangle \ , \\ \underline{\mathbf{K}}_0^M \langle 0|T\bar{W}_{0_M}^a(x)\phi_0^b(y)|0\rangle &= -\langle 0|TD_{\phi b}^d(y)c_0^d(y)\bar{c}_0^a(x)|0\rangle \ , \end{aligned} \tag{97}$$

where $M = 0,1,2,3,5$ ($M = 5$ denotes the Goldstone boson field). In the momentum representation, (97) can be further written as

$$\underline{\mathbf{K}}_0^M \mathbf{D}_{0MM'}^{ab}(k) = -\mathbf{X}_{M'}^{ab}(k) \ , \tag{98}$$

where $\mathbf{D}_{0MM'}^{ab}(x,y) \equiv \langle 0|T\bar{W}_{0M}^a(x)\bar{W}_{0M'}^b(y)|0\rangle$ is the $\bar{W}_0^a$ propagator,

$$\mathbf{X}^{ab}(k) \equiv \begin{pmatrix} X_\nu^{ab}(k) \\ X_\phi^{ab}(k) \end{pmatrix} = \begin{pmatrix} \tilde{X}_\nu^{ab}(k)S_0(k) \\ \tilde{X}_\phi^{ab}(k)S_0(k) \end{pmatrix} \tag{99}$$

in which $S_0(x,y)\delta^{ab} \equiv \langle 0|Tc_0^a(x)\bar{c}_0^b(y)|0\rangle$ is the ghost propagator, $\tilde{X}_\nu^{ab}$ and $\tilde{X}_\phi^{ab}$ are defined in (12), specifically

$$\begin{aligned} \tilde{X}_\nu^{ab}(k) &= ik_\nu[1+\Delta_3(k^2)]\delta^{ab}, \\ \tilde{X}_\phi^{ab}(k) &= M_{W0}[1+\Delta_1(k^2)+\Delta_2(k^2)]\delta^{ab} \end{aligned} \tag{100}$$

with the Δ's given in (16).

To derive the modification factor in (88) and (95) from the identity (79) we only need to consider the $n = 1$ case for simplicity. In this case, (79), (96), and (98) give

$$0 = \underline{\mathbf{K}}_0^M G[\bar{W}_{0M}^a(k), \Phi] = -\mathbf{X}_{M'}^{ab}(k)T[\bar{W}_{0b}^{M'}(k), \Phi] \ , \tag{101}$$

where $G[\cdots]$ and $T[\cdots]$ denote the Green function and S-matrix element, respectively. Thus (101) leads to

$$k^\mu T[W_{0\mu}^a(k), \Phi] = \hat{C}_0(k^2)M_{W0}T[i\phi_0^a, \Phi], \tag{102}$$

where $\hat{C}_0(k^2) \equiv [1+\Delta_1(k^2)+\Delta_2(k^2)]/[1+\Delta_3(k^2)]$ is just the function defined in (15). After renormalization (102) becomes

$$k^\mu T[W_\mu^a(k), \Phi] = \hat{C}(k^2)M_W T[i\phi_0^a, \Phi] \ , \tag{103}$$

where the renormalized

$$\hat{C}(k^2) = (Z_W/Z_\phi)^{1/2} Z_{M_W} \hat{C}_0(k^2)$$

has been given in (20).

Let $M_W^{\rm phys}$ be the physically observed mass of W_μ^a. The longitudinal and scalar polarization vectors are then given by (86) with $M_a = M_W^{\rm phys}$. Using (86) and (90), Eq. (103) at $k^2 = (M_W^{\rm phys})^2$ can be written as

$$T[W_L^a(k), \Phi] = C_{\rm mod} T[i\phi^a(k), \Phi] + O(M_W^{\rm phys}/E) \ , \tag{104}$$

where

$$C_{\rm mod} = \frac{M_W}{M_W^{\rm phys}} \hat{C}[(M_W^{\rm phys})^2] \ . \tag{105}$$

Equation (104) with (105) is just the precise formulation of the ET when $n = 1$. Substituting (105) into (95) we get the general precise form of ET. In general $M_W^{\rm phys}$ and M_W are not equal. $M_W^{\rm phys} = M_W$ only in the on-shell subtraction scheme, and then $C_{\rm mod} = \hat{C}(M_W^2)$ which is just the expression for $C_{\rm mod}$ in Ref. [8]. In our renormalization *schemes I* and *II*, the simplified expressions for $\hat{C}(\xi\kappa M_W)$ are given in (33). Thus we have

$$C_{\rm mod} = \begin{cases} \Omega_\kappa^{-1} \ , & \text{in scheme I with } \kappa = M_W \text{ and } \xi = 1 \ , \\ 1 \ , & \text{in scheme II with } \kappa = \xi^{-1}M_W \ . \end{cases} \tag{106}$$

In the Landau gauge, if we take the MW scheme [14], the modification factor is [cf. (36)]

$$C_{\rm mod} = (Z_W/Z_\phi)^{\frac{1}{2}} \frac{Z_{M_W}}{1+\Delta_3(M_W^2)} \ . \tag{107}$$

We shall show in following sections that (107) is not unity at the one-loop level.

There is also another commonly used renormalization scheme in which the gauge fixing function F^a is unchanged after renormalization, i.e., $F^a = F_0^a$ [13,16]. This scheme corresponds to $\Omega_\xi = \Omega_\kappa = 1$ in our formalism. From (105) and the second identity in (25) we get in this scheme

$$C_{\rm mod} = \frac{M_W^2 - \tilde{\Pi}_{WW}(M_W^2)}{M_W^2 + M_W\tilde{\Pi}_{W\phi}(M_W^2)} \tag{108}$$

which is simplified to include only two unphysical proper self-energies $\tilde{\Pi}_{WW}$ and $\tilde{\Pi}_{W\phi}$. We shall present in Sec. IV an up to the one-loop calculation of (108) which is *not* unity.

We see from the above results that $C_{\rm mod}$ is scheme dependent. *Our scheme II with $\kappa = \xi^{-1}M_W$ is the scheme in which the ET takes its naive simple form (1).* Therefore *scheme II* is the most convenient scheme for applying the ET. In other renormalization schemes, the ET takes the general form (2), so that $C_{\rm mod}$ should be calculated when applying the ET. *In our scheme I with $\kappa = M_W$ and $\xi = 1$, $C_{\rm mod}$ reduces exactly to a single quantity Ω_κ^{-1} which has already been determined by the subtraction condtition $\tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = 0$ in this renormalization scheme itself* (cf. Table I). Therefore our *scheme I* is also convenient for practical applications.

C. General proof of the precise formulation of the equivalence theorem in the SU(2)×U(1) theory

For the charged sector in SU(2)×U(1) theory, the proof is completely similar to what we have done in the $SU(2)_L$ theory, and we have

$$C^W_{\rm mod} = \frac{M_W}{M_W^{\rm phys}}\hat{C}^W[(M_W^{\rm phys})^2] = \frac{M_W}{M_W^{\rm phys}}\left(\frac{Z_W}{Z_{\phi^\pm}}\right)^{1/2} Z_{M_W}\hat{C}^W_0[(M_W^{\rm phys})^2]$$
$$= \frac{M_W}{M_W^{\rm phys}}\left(\frac{Z_W}{Z_{\phi^\pm}}\right)^{1/2} Z_{M_W}\frac{1+\Delta^W_{11}(k^2)+[\Delta^W_{21}(k^2)+\Delta^W_{22}(k^2)+\Delta^W_{23}(k^2)]}{1+[\Delta^W_{31}(k^2)+\Delta^W_{32}(k^2)+\Delta^W_{33}(k^2)+\Delta^W_{34}(k^2)]}\Bigg|_{k^2=(M_W^{\rm phys})^2}, \tag{109}$$

where $\hat{C}^W_0(k^2)$ is given in (47) and (48).

For the neutral sector, using the notation $\bar{N}_0$ and F^N_0 defined in (37) and (40), the second ST identity in (78) can be written as

$$(\underline{\mathbf{K}}^N_0)^T\mathcal{D}_{0,\bar{N}\bar{N}}(x,y) = -\mathbf{X}^T_{\bar{N}}(x,y)\ , \tag{110}$$

with

$$\mathbf{X}_{\bar{N}}(x,y) \equiv \langle 0|T\mathbf{D}^b_{\bar{N}}\mathcal{C}^b_0(y)\bar{\mathcal{C}}_0(x)|0\rangle. \tag{111}$$

$\underline{\mathbf{K}}^N_0$ and $\mathcal{D}_{0\bar{N}\bar{N}}$ are defined in (41) and (38), respectively. In the case of $n=1$, the ST identity (79) reads

$$0 = G[F^N_0,\Phi] = (\underline{\mathbf{K}}^N_0)^T G[\bar{N}_0,\Phi]$$
$$= (\underline{\mathbf{K}}^N_0)^T\mathcal{D}_{0\bar{N}\bar{N}}T[\bar{N}_0,\Phi]\ . \tag{112}$$

Using (110) and transforming into the momentum representation, we get

$$\mathbf{X}^T_{\bar{N}}(k)T[\bar{N}_0(k),\Phi] = 0\ , \tag{113}$$

where

$$\mathbf{X}_{\bar{N}}(k) \equiv \begin{pmatrix}\mathbf{X}_{N_\nu\bar{\mathcal{C}}}(k)\\ \mathbf{X}_{\phi^Z\bar{\mathcal{C}}}(k)\end{pmatrix} \equiv \begin{pmatrix}\tilde{\mathbf{X}}_{N_\nu\bar{\mathcal{C}}}(k)S^N_0(k)\\ \tilde{\mathbf{X}}_{\phi^Z\bar{\mathcal{C}}}(k)S^N_0(k)\end{pmatrix}$$
$$\equiv \begin{pmatrix}ik_\nu\hat{\mathbf{X}}_{N\bar{\mathcal{C}}}(k^2)S^N_0(k^2)\\ M_{Z0}\hat{\mathbf{X}}_{\phi^Z\bar{\mathcal{C}}}(k^2)S^N_0(k^2)\end{pmatrix} \tag{114}$$

with $\hat{\mathbf{X}}_{N\bar{\mathcal{C}}}(k^2)$ and $\hat{\mathbf{X}}_{\phi^Z\bar{\mathcal{C}}}(k^2)$ are defined in (51). Thus (112) gives

$$k_\mu T[N^\mu_0(k),\Phi] = M_{Z0}\hat{\mathbf{C}}^N_0(k^2)T[i\phi^Z_0(k),\Phi]\ , \tag{115}$$

in which $\hat{\mathbf{C}}^N_0(k^2)$ is just the function defined in (50). After renormalization, (114) becomes

$$k_\mu T[N^\mu(k),\Phi] = M_Z\hat{\mathbf{C}}^N(k^2)T[i\phi^Z(k),\Phi]\ , \tag{116}$$

where $\hat{\mathbf{C}}^N(k^2)$ is given in (53). We can then relate the amplitude $k_\mu T[N^\mu(k),\Phi]$ to $T[Z_L(k),\Phi]$ by using (86) and (90) with $M_a = M_Z^{\rm phys}$, and obtain, from (115),

$$T[Z_L(k),\Phi] = C^Z_{\rm mod}T[i\phi^Z(k),\Phi] + O(M_Z^{\rm phys}/E)\ , \tag{117}$$

where

$$C^Z_{\rm mod} = \frac{M_Z}{M_Z^{\rm phys}}\hat{C}^Z[(M_Z^{\rm phys})^2]$$
$$= \frac{M_Z}{M_Z^{\rm phys}}\frac{Z_{M_Z}}{Z^{1/2}_{\phi^Z}}$$
$$\times\frac{Z^{1/2}_{ZZ}[(1+\Delta^{AA}_3)(1+\Delta^{ZZ}_1+\Delta^{ZZ}_2)-\Delta^{AZ}_3(\Delta^{ZA}_1+\Delta^{ZA}_2)]+Z^{1/2}_{AZ}[(1+\Delta^{ZZ}_3)(1+\Delta^{ZA}_1+\Delta^{ZA}_2)-\Delta^{ZA}_3(\Delta^{ZZ}_1+\Delta^{ZZ}_2)]}{(1+\Delta^{ZZ}_3)(1+\Delta^{AA}_3)-\Delta^{ZA}_3\Delta^{AZ}_3}\Bigg|_{k^2=(M_Z^{\rm phys})^2}. \tag{118}$$

Substituting (109) and (118) into (95) we obtain the general precise formulation of the ET in the SU(2)×U(1) theory. Equations (109) and (118) show that the modification factors ($C^W_{\rm mod}$, $C^Z_{\rm mod}$) are much complicated. Thus a rigorous simplification for these factors is certainly necessary for practical applications. From our results in Sec. II B we obtain the *exactly simplified* expressions for the modification factors in our *scheme I* and *scheme II* as follows:

$$C^W_{\rm mod} = \begin{cases}(\Omega^W_\kappa)^{-1}\ , & \text{in scheme I with } \kappa_W = M_W \text{ and } \xi_W = 1\ ,\\ 1, & \text{in scheme II with } \kappa_W = \xi_W^{-1}M_W\ ;\end{cases}$$
$$C^Z_{\rm mod} = \begin{cases}(\Omega^{ZZ}_\kappa)^{-1}\ , & \text{in scheme I with } \kappa_Z = M_Z \text{ and } \xi_Z = 1\ ,\\ 1, & \text{in scheme II with } \kappa_Z = \xi_Z^{-1}M_Z\ ,\end{cases} \tag{119}$$
$$C^A_{\rm mod} = 0\quad,\qquad \text{both in scheme I and scheme II}\ .$$

In the Landau gauge, if we take the MW scheme [14], the modification factors are [cf. (36) and (76)]

$$C^W_{\rm mod} = \left(\frac{Z_W}{Z_{\phi^\pm}}\right)^{1/2} \frac{Z_{M_W}}{1+\Delta^W_{31}(M_W^2)+\Delta^W_{32}(M_W^2)+\Delta^W_{33}(M_W^2)+\Delta^W_{34}(M_W^2)} ,$$

$$C^Z_{\rm mod} = \frac{Z_{M_Z}}{Z^{1/2}_{\phi^Z}} \frac{Z^{1/2}_{ZZ}[1+\Delta^{AA}_3(M_Z^2)] - Z^{1/2}_{AZ}\Delta^{ZA}_3(M_Z^2)}{[1+\Delta^{ZZ}_3(M_W^2)][1+\Delta^{AA}_3(M_Z^2)] - \Delta^{ZA}_3(M_Z^2)\Delta^{AZ}_3(M_Z^2)} , \tag{120}$$

which are rather complicated. Here we also present our simplified expressions for the modification factors in the $F^a = F^a_0$ scheme [13,16]:

$$C^W_{\rm mod} = \frac{M_W^2 - \tilde{\Pi}_{WW}(M_W^2)}{M_W^2 + M_W\tilde{\Pi}_{W\phi^\pm}(M_W^2)} ,$$

$$C^Z_{\rm mod} = \frac{M_Z^2 - \tilde{\Pi}_{ZZ}(M_Z^2)}{M_Z^2 + M_Z\tilde{\Pi}_{Z\phi^Z}(M_Z^2)} . \tag{121}$$

In summary, we have proved that, in the SU(2)×U(1) theory, the general formulation of ET is (95) with the modification factors given in (109) and (118). The modification factors and the Goldstone boson scattering amplitude are renormalization scheme and ξ dependent. In our *scheme I* and *scheme II*, the formulas for the modification factors are greatly simplified as given in (119). Especially in *scheme II* (95) reduces to the naive simple form (1), so that this scheme is the most convenient scheme for applying the ET.

IV. ONE-LOOP CALCULATIONS IN THE SU(2)×U(1) THEORY IN THE HEAVY HIGGS LIMIT

A. The modification factors

We present here explicit one-loop calculations of modification factors $C^W_{\rm mod}$ and $C^Z_{\rm mod}$ in the SU(2)×U(1) theory for very large m_H. The calculations will be given in various currently used renormalization schemes other than *scheme II*, from which we can compare the ξ and m_H dependence of $C^W_{\rm mod}$ and $C^Z_{\rm mod}$ in various schemes. In the heavy Higgs limit we keep only terms containing positive powers of m_H or $\ln m_H$, and neglect all terms which are m_H independent or vanishing as $m_H \to \infty$. In this approach, the quantities Δ_2's and Δ_3's in (47) and (51) are negligible relative to the Δ_1's. Furthermore, the difference between $M_W^{\rm phys}$ ($M_Z^{\rm phys}$) and M_W (M_Z) is of the loop order, so that at one-loop level and for the heavy Higgs case, $C^W_{\rm mod}$ and $C^Z_{\rm mod}$ given in (109) and (118) reduce to

$$C^W_{\rm mod} = 1 + \tfrac{1}{2}(\delta Z_W - \delta Z_{\phi^\pm} + \delta Z_{M_W^2}) + \Delta^W_1(M_W^2) + (M_W/M_W^{\rm phys} - 1) ,$$

$$C^Z_{\rm mod} = 1 + \tfrac{1}{2}(\delta Z_{ZZ} - \delta Z_{\phi^Z} + \delta Z_{M_Z^2}) + \Delta^{ZZ}_1(M_Z^2) + (M_Z/M_Z^{\rm phys} - 1) . \tag{122}$$

We thus only need to calculate the one-loop contributions to the renormalization constants Z_i's, Δ^W_1, Δ^{ZZ}_1, $M_W/M_W^{\rm phys}$, and $M_Z/M_Z^{\rm phys}$. The Lagrangian for the Higgs sector is

$$\mathcal{L}_H = (D_\mu s_0)^\dagger (D^\mu s_0) - V(s_0) ,$$

$$D^\mu = \partial^\mu - \frac{ig_0'}{2}B_0^\mu - \frac{ig_0}{2}\tau^a W_0^{a\mu} , \qquad s_0 = \begin{pmatrix} -i\phi_0^+ \\ \frac{1}{\sqrt{2}}(v_0 + H_0 + i\phi_0^Z) \end{pmatrix} ,$$

$$-V(s_0) = -\lambda_0[s_0^\dagger s_0 - \frac{\mu_0^2}{2\lambda}]^2$$

$$= -\frac{1}{2}(m_{H0}^2 + \frac{\delta T}{v_0})H_0^2 - \frac{1}{2}\frac{\delta T}{v_0}[2\phi_0^+\phi_0^- + (\phi_0^Z)^2] - \delta T H_0 - \lambda_0 v_0[H_0(2\phi_0^+\phi_0^- + (\phi_0^Z)^2) + H_0^3]$$

$$-\frac{\lambda_0}{4}[(2\phi_0^+\phi_0^- + (\phi_0^Z)^2)^2 + 2{H_0}^2(2\phi_0^+\phi_0^- + (\phi_0^Z)^2) + H_0^4] , \tag{123}$$

where $m_{H0}^2 = 2\lambda_0 v_0^2$, $\delta T = (\lambda_0 v_0^2 - \mu_0^2)v_0$. The loop-order quantity δT is prescribed to cancel the complete H-tadpole contributions to ensure $\langle H\rangle = 0$ as it should be. To one loop, this requires

$$-i\delta T/v = -\frac{g^2}{4}\frac{M_H^2}{M_W^2}[\,I_1(\xi_W M_W^2) + \tfrac{1}{2}I_1(\xi_Z M_Z^2) + \tfrac{3}{2}I_1(m_H^2)] \,, \tag{124}$$

where the function I_1 is defined in Appendix C. The renormalization constants in (122) are related to the proper self-energy counterterms by (29) and (71). The Higgs renormalization constants δZ_H and $\delta Z_{m_H^2}$ are related to the proper self-energy counterterm by

$$\delta\Pi_H = -\delta Z_H(k^2 - m_H^2) + \delta Z_{m_H^2} m_H^2 \,. \tag{125}$$

What we should calculate is the one-loop contributions to the bare proper self-energies in the heavy Higgs limit. The results are

$$\begin{aligned}
\Pi_{H0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{m_H^4}{M_W^2}\frac{1}{8}\Bigg[12(I_0(M_W^2)+2) - 9\ln\frac{m_H^2}{M_W^2} - \ln\frac{\xi_W^2\xi_Z}{\cos^2\theta_W} - 9I_{20}(k^2;m_H^2,m_H^2)\\
&\qquad -2I_{20}(k^2;\xi_W M_W^2,\xi_W M_W^2) - I_{20}(k^2;\xi_Z M_Z^2,\xi_Z M_Z^2)\Bigg]\,,\\
\Pi_{WW,0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{1}{4}\left[\frac{1}{2}m_H^2 + \left(\frac{1}{3}k^2 + 3M_W^2\right)\ln\frac{m_H^2}{M_W^2}\right]\,,\\
\tilde{\Pi}_{WW,0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{1}{4}\left[\frac{1}{2}m_H^2 + 3M_W^2\ln\frac{m_H^2}{M_W^2}\right]\,,\\
\Pi_{ZZ,0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{1}{4\cos^2\theta_W}\left[\frac{1}{2}m_H^2 + \left(\frac{1}{3}k^2 + 3M_Z^2\right)\ln\frac{m_H^2}{M_Z^2}\right]\,,\\
\tilde{\Pi}_{ZZ,0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{1}{4\cos^2\theta_W}\left[\frac{1}{2}m_H^2 + 3M_Z^2\ln\frac{m_H^2}{M_Z^2}\right]\,,\\
\tilde{\Pi}_{\phi^\pm\phi^\pm,0}(k^2) &= -\frac{g^2}{16\pi^2}\left[\frac{1}{8}\frac{m_H^2}{M_W^2} + \left(\frac{3}{4} - \frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right]k^2\,,\\
\tilde{\Pi}_{\phi^z\phi^z,0}(k^2) &= -\frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\frac{1}{8}\frac{m_H^2}{M_Z^2} + \left(\frac{3}{4} - \frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right]k^2\,,\\
\tilde{\Pi}_{W\phi^\pm,0}(k^2) &= +\frac{g^2}{16\pi^2}M_W\left[\frac{1}{8}\frac{m_H^2}{M_W^2} + \left(\frac{3}{4} - \frac{\xi_W}{4}\right)\ln\frac{m_H^2}{M_W^2}\right]\,,\\
\tilde{\Pi}_{Z\phi^z,0}(k^2) &= +\frac{g^2}{16\pi^2}\frac{M_Z}{\cos^2\theta_W}\left[\frac{1}{8}\frac{m_H^2}{M_Z^2} + \left(\frac{3}{4} - \frac{\xi_Z}{4}\right)\ln\frac{m_H^2}{M_Z^2}\right]\,,\\
\tilde{\Pi}_{c^\pm\bar{c}^\pm,0}(k^2) &= \frac{g^2}{16\pi^2}\frac{1}{4}\xi_W^2 M_W^2\ln\frac{m_H^2}{M_W^2}\,,\\
\tilde{\Pi}_{c^z\bar{c}^z,0}(k^2) &= \frac{g^2}{16\pi^2}\frac{1}{4}\frac{\xi_Z^2 M_Z^2}{\cos^2\theta_W}\ln\frac{m_H^2}{M_Z^2}\,,
\end{aligned} \tag{126}$$

where the functions I_0 and I_{20} are defined in Appendix C. With these and Eqs. (125), (29), and (71), we can determine the relevant renormalization constants by imposing certain subtraction conditions constraining the renormalized proper self-energies.

In the present approximation, the calculated Δ_1^W and Δ_1^{ZZ} are

$$\begin{aligned}
\Delta_1^W(k^2) &= \frac{g^2}{16\pi^2}\frac{\xi_W}{4}\ln\frac{m_H^2}{M_W^2}\,,\\
\Delta_1^{ZZ}(k^2) &= \frac{g^2}{16\pi^2}\frac{\xi_Z}{4\cos^2\theta_W}\ln\frac{m_H^2}{M_Z^2}\,.
\end{aligned} \tag{127}$$

Now we present the calculated $C_{\rm mod}^W$ and $C_{\rm mod}^Z$ in various renormalization schemes as follows.

1. Scheme I

The subtraction conditions in our *scheme I* have described in details in Sec. II. In this scheme $M_W^{\rm phys} = M_W$, $M_Z^{\rm phys} = M_Z$. The determined renormalization constants are

$$
\begin{aligned}
\delta Z_H &= \frac{g^2}{16\pi^2}\frac{m_H^2}{M_W^2}\left[\frac{3}{2}-\frac{\sqrt{3}\pi}{4}\right] , \\
\delta Z_{m_H} &= -\frac{g^2}{16\pi^2}\left\{\frac{m_H^2}{M_W^2}\left[-\frac{3}{4}\left(\frac{1}{\epsilon}-\gamma-\ln\frac{m_H^2}{4\pi\mu^2}+2\right)+\frac{9\pi}{16\sqrt{3}}\right]-\left(2\xi_W+\frac{\xi_Z}{\cos^2\theta_W}\right)\ln\frac{m_H}{M_W}\right\} , \\
\delta Z_W &= \frac{g^2}{16\pi^2}\frac{1}{12}\ln\frac{m_H^2}{M_W^2} , \\
\delta Z_{M_W} &= -\frac{g^2}{16\pi^2}\left[\frac{1}{16}\frac{m_H^2}{M_W^2}+\frac{5}{12}\ln\frac{m_H^2}{M_W^2}\right] , \qquad \delta\Omega_\xi^W = 0 , \\
\delta Z_{ZZ} &= \frac{g^2}{16\pi^2}\frac{1}{12\cos^2\theta_W}\ln\frac{m_H^2}{M_W^2} , \\
\delta Z_{M_Z} &= -\frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\frac{1}{16}\frac{m_H^2}{M_W^2}+\frac{5}{12}\ln\frac{m_H^2}{M_W^2}\right] , \qquad \delta\Omega_\xi^{ZZ} = 0 , \\
\delta Z_{\phi^\pm} &= \frac{g^2}{16\pi^2}\left[-\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(-\frac{3}{4}+\frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right] , \qquad \delta\Omega_\kappa^W = 0 , \\
\delta Z_{\phi^Z} &= \frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[-\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(-\frac{3}{4}+\frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right] , \qquad \delta\Omega_\kappa^{ZZ} = 0 , \\
\delta Z_c^W &= \delta Z_c^{ZZ} = 0 .
\end{aligned}
\tag{128}
$$

From (126) and (128) we find that $\tilde{\Pi}_{W\phi^\pm}(k^2)$ and $\tilde{\Pi}_{Z\phi^Z}(k^2)$ all vanish in the heavy Higgs limit. This and the last equation in (128) are all consistent with the consequences of WT identities listed in Tables I and II, so that these results may be regarded as an explicit check of the general WT identities (25) and (59). Note that in the present approximation $\delta\Omega_\xi^W$, $\delta\Omega_\xi^{ZZ}$, $\delta\Omega_\kappa^W$, $\delta\Omega_\kappa^{ZZ}$ are negligibly small, and also from (122) explicit calculations give

$$
C_{\rm mod}^W \simeq 1, \quad C_{\rm mod}^Z \simeq 1 , \tag{129}
$$

which coincide with the exact result (119). Thus, *scheme I* behaves approximately like *scheme II* in the heavy Higgs case.

2. The on-shell scheme by Böhm et al. and Hollik [11]

In this scheme $\kappa_W = M_W$, $\kappa_Z = M_Z$, $M_W^{\rm phys} = M_W$, $M_Z^{\rm phys} = M_Z$. The Goldstone wave function renormalization constants are taken to be $Z_{\phi^\pm} = Z_{\phi^Z} = Z_H$. This is different from our *scheme I* and the calculated $\delta\Omega_\kappa^W$ and $\delta\Omega_\kappa^{ZZ}$ are

$$
\begin{aligned}
\delta\Omega_\kappa^W &= \frac{g^2}{16\pi^2}\left[\left(\frac{13}{16}-\frac{\sqrt{3}}{8}\pi\right)\frac{m_W^2}{M_H^2}+\left(\frac{3}{8}-\frac{\xi_W}{4}\right)\ln\frac{m_H^2}{M_W^2}\right] , \\
\delta\Omega_\kappa^{ZZ} &= \frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\left(\frac{13}{16}-\frac{\sqrt{3}}{8}\pi\right)\frac{m_H^2}{M_Z^2}+\left(\frac{3}{8}-\frac{\xi_Z}{4}\right)\ln\frac{m_H^2}{M_Z^2}\right] .
\end{aligned}
\tag{130}
$$

Using (128) (for δZ_i's) and (127) we evaluate the modification factors in (122) as

$$
\begin{aligned}
C_{\rm mod}^W &= 1+\frac{g^2}{16\pi^2}\left[\left(-\frac{13}{16}+\frac{\sqrt{3}}{8}\pi\right)\frac{m_H^2}{M_W^2}-\frac{3}{8}\ln\frac{m_H^2}{M_W^2}+\frac{\xi_W}{4}\ln\frac{m_H^2}{M_W^2}\right] \simeq (\Omega_\kappa^W)^{-1} , \\
C_{\rm mod}^Z &= 1+\frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\left(-\frac{13}{16}+\frac{\sqrt{3}}{8}\pi\right)\frac{m_H^2}{M_Z^2}-\frac{3}{8}\ln\frac{m_H^2}{M_Z^2}+\frac{\xi_Z}{4}\ln\frac{m_H^2}{M_Z^2}\right] \simeq (\Omega_\kappa^{ZZ})^{-1} .
\end{aligned}
\tag{131}
$$

This also coincides with the exact results in (119) since in *scheme I* our simplified forms for $(C_{\rm mod}^W, C_{\rm mod}^Z)$ are generally valid for any choice of $(Z_{\phi^\pm}, Z_{\phi^Z})$. We see from (131) that in this scheme $C_{\rm mod}^W$ and $C_{\rm mod}^Z$ all acquire non-negligible loop corrections which contain both m_H^2 and $\ln m_H$ terms and depend on ξ_W, ξ_Z. Numerically, $g^2 = 0.422$ [14]. Taking $m_H = 1$ TeV to estimate the size of $C_{\rm mod}^W - 1$ and $C_{\rm mod}^Z - 1$, we obtain [17]

$$
\begin{aligned}
\delta C_{\rm mod}^W &\equiv C_{\rm mod}^W - 1 = -0.060 + 0.003\xi_W , \\
\delta C_{\rm mod}^Z &\equiv C_{\rm mod}^Z - 1 = -0.062 + 0.004\xi_Z .
\end{aligned}
\tag{132}
$$

In W_L-W_L or Z_L-Z_L scatterings, the total modification factor in (95) is $(C_{\rm mod}^W)^4 \approx 1 + 4\delta C_{\rm mod}^W$ or $(C_{\rm mod}^Z)^4 \approx 1 + 4\delta C_{\rm mod}^Z$ which deviates from unity by about 23% in the 't Hooft–Feynman gauge. Therefore the precise form of ET in this scheme is significantly different from the naive form (1).

3. The MW scheme in the Landau gauge [14]

In this scheme $Z_{\phi^\pm}$ and Z_{ϕ^Z} are determined by $d\Pi_{\phi^\pm\phi^\pm}/dk^2|_{k^2=0} = d\Pi_{\phi^Z\phi^Z}/dk^2|_{k^2=0} = 0$, $M_W^{\rm phys} = M_W$, $M_Z^{\rm phys} = M_Z$, and $\xi_W = \xi_Z = \xi_A = 0$. Thus (122) reduces to

$$\begin{aligned} C_{\rm mod}^W &= 1 + \tfrac{1}{2}(\delta Z_W - \delta Z_{\phi^\pm} + \delta Z_{M_W^2}) \ , \\ C_{\rm mod}^Z &= 1 + \tfrac{1}{2}(\delta Z_{ZZ} - \delta Z_{\phi^Z} + \delta Z_{M_Z^2}) \ . \end{aligned} \tag{133}$$

From (128) we obtain

$$C_{\rm mod}^W \simeq 1 \ , \qquad C_{\rm mod}^Z \simeq 1 \ ; \tag{134}$$

i.e., up to one-loop there is neither an m_H^2 term nor a $\ln m_H$ term in $C_{\rm mod}^W$ and $C_{\rm mod}^Z$, so that this scheme is convenient for applying the ET in the heavy Higgs limit.

4. The on-shell scheme by Aoki et al. [15]

In this scheme, the subtraction condition, for the physical sector is the on-shell condition so that $M_W^{\rm phys} = M_W$ and $M_Z^{\rm phys} = M_Z$, while that for the unphysical sector is the minimal subtraction which, in the heavy Higgs limit, corresponds to $\delta Z_{\phi^\pm} = \delta Z_{\phi^Z} = 0$. We then obtain, from (122) and (128),

$$\begin{aligned} C_{\rm mod}^W &= 1 + \frac{g^2}{16\pi^2}\left[-\frac{1}{16}\frac{m_H^2}{M_W^2} + \left(-\frac{3}{8} + \frac{\xi_W}{4}\right)\ln\frac{m_H^2}{M_W^2}\right] \ , \\ C_{\rm mod}^Z &= 1 + \frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[-\frac{1}{16}\frac{m_H^2}{M_Z^2} + \left(-\frac{3}{8} + \frac{\xi_Z}{4}\right)\ln\frac{m_H^2}{M_Z^2}\right] \ . \end{aligned} \tag{135}$$

Therefore the modification factors also contain large m_H and ξ dependence in this scheme. Numerically, for $m_H = 1$ TeV,

$$\begin{aligned} \delta C_{\rm mod}^W &= -0.031 + 0.003\xi_W \ , \\ \delta C_{\rm mod}^Z &= -0.032 + 0.004\xi_Z \ . \end{aligned} \tag{136}$$

5. The complete minimal subtraction scheme

The result is the same as that in the on-shell scheme by Aoki *et al.* [15] since $C_{\rm mod}^W$ and $C_{\rm mod}^Z$ are related only to the unphysical sector.

6. The intermediate scheme [18]

This is a widely used scheme with the Fermi constant G_μ taken as input instead of M_W. In this scheme $M^{\rm phys} \neq M_W$. The renormalization scheme for the unphysical sector is not specified. If we take the scheme in Ref. [11] or Ref. [15] for the unphysical sector we get *large* $C_{\rm mod}^W - 1$ and $C_{\rm mod}^Z - 1$ shown in (131) or (135). If we take our *scheme I* for the unphysical sector we get $C_{\rm mod}^W \simeq 1$ and $C_{\rm mod}^Z \simeq 1$ [cf. (129)] in the heavy Higgs case.

7. The $F^a = F_0^a$ scheme [13,16]

In this scheme $\delta\Omega_\kappa^W = \delta\mathbf{\Omega}_{\xi_N} = \delta\mathbf{\Omega}_{\bar\kappa} = 0$. The Goldstone boson wave function renormalization constants are normalized by $Z_{\phi^\pm} = Z_{\phi^Z} = Z_H$. In Refs. [13] and [16], the 't Hooft-Feynman gauge is taken, i.e., $\xi_W = \xi_Z = \xi_A = 1$. Explict one-loop calculation in the heavy Higgs limit gives

$$\begin{aligned} &\tilde\Pi_{WW}(k^2) = 0 \ , \quad \tilde\Pi_{ZZ}(k^2) = 0 \ , \\ &\tilde\Pi_{\phi^\pm\phi^\pm}(k^2) = \frac{g^2}{16\pi^2}\left[\left(-\frac{13}{8} + \frac{\sqrt{3}}{4}\pi\right)\frac{m_H^2}{M_W^2} + \left(-\frac{3}{4} + \frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right]k^2 \ , \\ &\tilde\Pi_{\phi^Z\phi^Z}(k^2) = \frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\left(-\frac{13}{8} + \frac{\sqrt{3}}{4}\pi\right)\frac{m_H^2}{M_Z^2} + \left(-\frac{3}{4} + \frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right]k^2 \ , \\ &\tilde\Pi_{W\phi^\pm}(k^2) = \frac{g^2}{16\pi^2}\left[\left(\frac{13}{16} - \frac{\sqrt{3}}{8}\pi\right)\frac{m_H^2}{M_W^2} + \left(\frac{3}{8} - \frac{\xi_W}{4}\right)\ln\frac{m_H^2}{M_W^2}\right]M_W \ , \\ &\tilde\Pi_{Z\phi^Z}(k^2) = \frac{g^2}{16\pi^2}\frac{1}{\cos^2\theta_W}\left[\left(\frac{13}{16} - \frac{\sqrt{3}}{8}\pi\right)\frac{m_H^2}{M_Z^2} + \left(\frac{3}{8} - \frac{\xi_Z}{4}\right)\ln\frac{m_H^2}{M_Z^2}\right]M_Z \ . \end{aligned} \tag{137}$$

From these and (122) we obtain the results of $C^W_{\rm mod}$ and $C^Z_{\rm mod}$ which are just the same as (131) but here $\delta\Omega^W_\kappa = \delta\Omega^{ZZ}_\kappa \equiv 0$. Thus in the $F^a = F^a_0$ scheme $\delta C^W_{\rm mod}$ and $\delta C^Z_{\rm mod}$ cannot be ignored. We can also calculate $C^W_{\rm mod}$ and $C^Z_{\rm mod}$ by using (121), and we get the same results. This can be regarded as a check of the WT identities (25) and (59) up to one loop in the heavy Higgs limit.

B. An example of the equivalence theorem: Heavy Higgs decay $H \to W^+_L W^-_L$

We take the heavy Higgs decay $H \to W^+_L W^-_L$ as an example to illustrate the precise formulation (95) of the ET up to one loop in the heavy Higgs limit. The specific form of (95) is now

$$T[H \to W^+_L W^-_L] = (iC^W_{\rm mod})^2 T[H \to \phi^+\phi^-] + O(M^2_W/m^2_H) \ . \tag{138}$$

The left-hand side of (138) is physical, independent of the renormalization scheme and the gauge parameter. In the heavy Higgs limit, up to one loop, the calculated result is

$$T(H \to W^+_L W^-_L) = \left[1 + \frac{g^2}{16\pi^2}\frac{m^2_H}{M^2_W}\left(\frac{19}{16} - \frac{3\sqrt{3}}{8}\pi + \frac{5\pi^2}{48}\right)\right] T_0 \ , \tag{139}$$

$$T_0 = \frac{ig}{2}\frac{{m_H}^2}{M^{\rm phys}_W} \ ,$$

where T_0 is the tree-level amplitude. In (139) we have kept only the terms with positive power of m_H. Numerically for $m_H = 1$ TeV,

$$T(H \to W^+_L W^-_L) = (1 + 0.0731) T_0 \ . \tag{140}$$

Next we calculate the right-hand side of (138) in various renormalization schemes other than *scheme II* to the same accuracy.

1. *Scheme I*

The calculated $T(H \to \phi^+\phi^-)$ is the same as the right-hand side of (139). Together with (129) and (139) we get the ET (138).

2. *The on-shell scheme by Böhm et al. and Hollik [11]*

The calculated $T(H \to \phi^+\phi^-)$ is

$$T[H \to \phi^+\phi^-] = -\left\{1 + \frac{g^2}{16\pi^2}\left[\frac{m^2_H}{M^2_W}\left(\frac{45}{16} - \frac{5\sqrt{3}}{8}\pi + \frac{5\pi^2}{48}\right) - \frac{\xi_W}{2}\ln\frac{m^2_H}{M^2_W}\right]\right\} T_0 \ . \tag{141}$$

In (141) we have kept a $\xi_W \ln \frac{m^2_H}{M^2_W}$ term as well for examining the total ξ_W dependence of the RHS of (138). We see that this ξ_W-dependent term is *exactly canceled* by that in $(C^W_{\rm mod})^2 \approx 1 + 2\delta C^W_{\rm mod}$ given in (131), so that the product $(C^W_{\rm mod})^2 T(H \to \phi^+\phi^-)$ is ξ_W-independent as it should be in (138). Numerically, for $m_H = 1$ TeV, we have

$$\begin{aligned}(C^W_{\rm mod})^2 &= 1 - 0.111 + 0.007\xi_W \ , \\ T[H \to \phi^+\phi^-] &= -[1 + 0.184 - 0.007\xi_W] T_0 \ , \\ (iC^W_{\rm mod})^2 T[H \to \phi^+\phi^-] &= [1 + 0.0731] T_0 \ ,\end{aligned} \tag{142}$$

which, together with (140), realizes the ET (138). We see that there are *significant cancellations* between $(C^W_{\rm mod})^2$ and $T(H \to \phi^+\phi^-)$ in (142), so that *it is important to notice that we should use the precise formulation (138) of the ET rather than the naive simple formulation (1) in this renormalization scheme.*

3. *The MW scheme in the Landau gauge [14]*

The situation is the same as that in *scheme I* in the present approximation.

4. *The on-shell scheme by Aoki et al. [15]*

Explicit calculation gives

$$\begin{aligned}T[H \to \phi^+\phi^-] &= -\left\{1 + \frac{g^2}{16\pi^2}\frac{m^2_H}{M^2_W}\left[\frac{21}{16} - \frac{3\sqrt{3}}{8}\pi + \frac{5\pi^2}{48}\right] - \frac{g^2}{16\pi^2}\frac{\xi_W}{2}\ln\frac{m^2_H}{M^2_W}\right\} T_0 \\ &= (1 + 0.125 - 0.007\xi_W) T_0 \quad (\text{for } m_H = 1 \text{ TeV}) \ .\end{aligned} \tag{143}$$

We see again from (135) and (143) that the ξ_W-dependent terms in $(C^W_{\rm mod})^2$ and $T(H \to \phi^+\phi^-)$ just cancel each other. There are also large cancellations of the ξ_W-independent terms between $(C^W_{\rm mod})^2$ and $T(H \to \phi^+\phi^-)$. Hence, *distinguishing the precise and the naive formulations of the ET is also important in this scheme.*

5. The complete minimal subtraction scheme

The result is the same as that in the on-shell scheme by Aoki *et al.*

6. The $F^a = F^a_0$ scheme [13,16]

The result is the same as that in the on-shell scheme by Böhm *et al.* and Hollik.

We have seen from the above explicit calculations that the form of the ET depends significantly on the renormalization scheme. In some schemes, the ξ dependence of the Goldstone boson scattering amplitude and the modification factors exists even to leading order in M_W/E. Also, to leading order in M_W/E, the difference between the ξ-independent parts in the V_L amplitude and the ϕ^a amplitude can be quite significant in some schemes. Therefore *be sure to use the precise formulation is important in the applications of the ET.*

V. EQUIVALENCE THEOREM IN THE U(1) HIGGS THEORY AND THE COMPLETE ONE-LOOP CALCULATIONS

As an illustration of our general formulation we first give the precise formulation of the ET in the simple U(1) Higgs model and then present the complete one-loop calculations. Consider a scalar field $s_0 = \frac{1}{\sqrt{2}}(v_0 + H_0 + i\phi_0)$ interacting with U(1) gauge field A^μ_0, in which v_0 is the vacuum expectation value (VEV), H_0 is the Higgs field, and ϕ_0 is the Goldstone boson field. The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F_{0\mu\nu}F_0^{\mu\nu} + (D_\mu s_0)^\dagger (D^\mu s_0) - V(s_0) \ , \qquad (144)$$

where $F_0^{\mu\nu} \equiv \partial^\mu A_0^\nu - \partial^\nu A_0^\mu$, $D^\mu \equiv \partial^\mu + i g_0 A_0^\mu$, and $V(s_0)$ is

$$\begin{aligned} V(s_0) &= \lambda_0\left[s_0^\dagger s_0 - \frac{{\mu_0}^2}{2\lambda_0}\right]^2 \\ &= \frac{1}{2}(m_{H0}^2 + \frac{\delta T}{v_0})H_0^2 + \frac{1}{2}\frac{\delta T}{v_0}\phi_0^2 + \delta T H_0 + \lambda_0 v_0 (H_0\phi_0^2 + H_0^3) + \frac{\lambda_0}{4}(\phi_0^4 + H_0^4 + 2\phi_0^2 H_0^2) \ , \end{aligned} \qquad (145)$$

in which $m_{H0}^2 = 2\lambda_0 v_0^2$, $\delta T = v_0(\lambda_0 v_0^2 - \mu_0^2)$. We take the R_ξ gauge. The gauge fixing and Faddeev-Popov terms are

$$\begin{aligned} \mathcal{L}_{\rm GF} &= -\frac{1}{2}F_0^2 \ , \qquad F_0 = \xi_0^{-\frac{1}{2}}\partial_\mu A_0^\mu - \xi_0^{\frac{1}{2}}\kappa_0\phi_0 \ , \\ \mathcal{L}_{\rm FP} &= \bar{c}_0(-\partial^2 - \xi_0\kappa_0 M_0 - g_0\xi_0\kappa_0 H_0)c_0 \ , \end{aligned} \qquad (146)$$

where c_0 ($\bar{c}_0$) is the ghost (antighost) field and $M_0 = g_0 v_0$ is the bare mass of gauge field. Now the WT identities in (11) read

$$\begin{aligned} & ik^\mu[i\mathcal{D}^{-1}_{0,\mu\nu}(k) + \xi_0^{-1}k_\mu k_\nu] + M_0\hat{C}_0(k^2)[i\mathcal{D}^{-1}_{0,\phi\nu}(k) - i\kappa_0 k_\nu] = 0 \ , \\ & ik^\mu[-i\mathcal{D}^{-1}_{0,\phi\mu}(k) + i\kappa_0 k_\mu] + M_0\hat{C}_0(k^2)[i\mathcal{D}^{-1}_{0,\phi\phi}(k) + \xi_0\kappa_0^2] = 0 \ , \\ & iS_0^{-1}(k) = k^2 - \xi_0\kappa_0 M_0\hat{C}_0(k^2) \ , \end{aligned} \qquad (147)$$

where

$$\begin{aligned} \hat{C}_0(k^2) &= 1 + \Delta_1(k^2) \ , \\ \Delta_1(k^2) &= \frac{g_0}{M_0}\int_q \langle 0 \mid H_0(-k-q)c_0(q) \mid \bar{c}_0(k)\rangle \ . \end{aligned} \qquad (148)$$

The renormalization constants for the unphysical sector are defined as

$$\phi_0 = Z_\phi^{\frac{1}{2}}\phi \ , \quad c_0 = Z_c c \ , \quad \bar{c}_0 = \bar{c} \ , \quad \xi_0 = Z_\xi \xi \ , \quad \kappa_0 = Z_\kappa \kappa \ . \qquad (149)$$

After renormalization the finiteness of the renormalized WT identities gives the constraints

$$\begin{aligned} & Z_\xi = \Omega_\xi Z_A \ , \qquad Z_\kappa = \Omega_\kappa Z_A^{\frac{1}{2}} Z_\phi^{-\frac{1}{2}} Z_\xi^{-1} \ , \\ & Z_\phi = \Omega_\phi Z_A Z_M^2 \hat{C}_0(\text{sub. point}) \ , \quad Z_c = \Omega_c \ , \end{aligned} \qquad (150)$$

where the $\Omega_i - 1 = \delta\Omega_i$ is arbitrary finite loop-order constant. The renormalized $\hat{C}(k^2)$ is

$$\hat{C}(k^2) = \left(\frac{Z_A}{Z_\phi}\right)^{1/2} Z_M \hat{C}_0(k^2) \ . \qquad (151)$$

We can derive the WT identities similar to (24) and (25) in which $\hat{C}(k^2)$ is expressed in terms of the proper self-energies as

$$\hat{C}(k^2) = \frac{M^2 - \tilde{\Pi}_{AA}(k^2) + (\Omega_\xi^{-1} - 1)\xi^{-1}k^2}{M^2 + M\Pi_{A\phi}(k^2) + M\kappa(\Omega_\xi^{-1}\Omega_\kappa - 1)} . \tag{152}$$

Similar to the derivation in Sec. III B we get, in the present theory,

$$C_{\rm mod} = \frac{M}{M_{\rm phys}}\hat{C}[(M^{\rm phys})^2] . \tag{153}$$

Also in our *scheme I* and *scheme II* we have

$$C_{\rm mod} = \begin{cases} \Omega_\kappa^{-1} \text{, in scheme I with } \xi = 1 \text{ and } \kappa = M \text{ ,} \\ 1 \text{ ,} \quad \text{in scheme II with } \kappa = \xi^{-1}M \text{ .} \end{cases} \tag{154}$$

In the MW scheme ($\xi = 0$) [14] we have

$$C_{\rm mod} = (Z_A/Z_\phi)^{\frac{1}{2}} Z_M , \tag{155}$$

and in the $F = F_0$ scheme [13,16] we have

$$C_{\rm mod} = \frac{M^2 - \tilde{\Pi}_{AA}(M^2)}{M^2 + M\tilde{\Pi}_{A\phi}(M^2)} . \tag{156}$$

To one-loop level, the original expression for $C_{\rm mod}$ given in (153) and (151) reduces to

$$\begin{aligned} C_{\rm mod} = 1 + \tfrac{1}{2}(\delta Z_A - \delta Z_\phi + \delta Z_{M^2}) + \Delta_1(M^2) \\ +(M/M^{\rm phys} - 1) . \end{aligned} \tag{157}$$

In *scheme I* and *scheme II*, we can calculate $C_{\rm mod}$ both from (154) and from (157), and this serves as an explicit one-loop level check on the general WT identities which lead to (154).

Now we present the complete one-loop calculations for arbitrary value of m_H. The renormalization constants are related to the proper self-energies by

$$\begin{aligned} \Pi_{AA}(k^2) &= \Pi_{0AA}(k^2) + \delta Z_A(k^2 - M^2) - \delta Z_{M^2}M^2 , \\ \tilde{\Pi}_{AA}(k^2) &= \tilde{\Pi}_{0AA}(k^2) - \xi^{-1}\delta\Omega_\xi k^2 - (\delta Z_A + \delta Z_{M^2})M^2 , \\ \tilde{\Pi}_{A\phi}(k^2) &= \tilde{\Pi}_{0A\phi}(k^2) + [\frac{1}{2}(\delta Z_A + \delta Z_\phi + \delta Z_{M^2}) + (\delta\Omega_\xi - \delta\Omega_\kappa)]M , \\ \tilde{\Pi}_{\phi\phi}(k^2) &= \tilde{\Pi}_{0\phi\phi}(k^2) - \delta Z_\phi(k^2 - \xi M^2) - (\delta Z_\phi + \delta\Omega_\xi - 2\delta\Omega_\kappa)\xi M^2 , \\ \tilde{\Pi}_{c\bar{c}}(k^2) &= \tilde{\Pi}_{0c\bar{c}}(k^2) - \delta Z_c(k^2 - \xi M^2) + [\frac{1}{2}(\delta Z_A - \delta Z_\phi + \delta Z_{M^2}) + \delta\Omega_\kappa)]\xi M^2 , \end{aligned} \tag{158}$$

where we have chosen $\kappa = M$ for convenience. We give separately the calculated results in the $\xi = 1$ gauge (which is related to *scheme I*, *scheme II*, and the $F = F_0$ scheme) and the $\xi = 0$ gauge (which is related to the MW-scheme).

A. The 't Hooft–Feynman gauge ($\xi = 1$)

We first determine the bare quantity δT in (145). By definition, the VEV of the H field should vanish. This requires that δT should cancel the total H-tadpole contributions completely. This requirement fixes δT. Up to one loop we have

$$-i\delta T/v = -3\lambda I_1(m_H^2) - [\lambda + (D-1)g^2]I_1(M^2) \text{ for } \xi = 1 \text{ and } \kappa = M , \tag{159}$$

where I_1 and D are given in Appendix C. The one-loop results of the bare proper self-energies are

$$\begin{aligned} \Pi_{0AA}(k^2) &= -ig^2[I_1(m_H^2) + I_1(M^2) + 4M^2I_2 - 4I_{41}] , \\ \tilde{\Pi}_{0AA}(k^2) &= -ig^2[I_1(m_H^2) + I_1(M^2) + 4M^2I_2 - k^2I_2 - 4k^2I_3 - 4(I_{41} + k^2I_{42})] , \\ \tilde{\Pi}_{0\phi\phi}(k^2) &= -i[(2\lambda + g^2)I_1(m_H^2) - (2\lambda + g^2)I_1(M^2) - (4\lambda^2v^2 - 4g^2k^2 - g^2M^2)I_2 + 4g^2k^2I_3] , \\ \tilde{\Pi}_{0A\phi}(k^2) &= -ig^2M^{-1}[(2m_H^2 - 2M^2)I_3 + (m_H^2 - 4M^2)I_2] , \\ \tilde{\Pi}_{0c\bar{c}}(k^2) &= ig^2M^2I_2 , \end{aligned} \tag{160}$$

where $I_i \equiv I_i(k^2; M^2, m_H^2)$ for $i \geq 2$ are given in Appendix C. With these results we can determine the renormalization constants in various renormalization scheme.

1. Scheme I

The on-shell conditions are

$$\mathrm{Re}\Pi_{AA}(M^2)=0\ ,\ \left.\frac{d\,\mathrm{Re}\Pi_{AA}(k^2)}{dk^2}\right|_{k^2=M^2}=0\ ,\mathrm{Re}\tilde{\Pi}_{AA}(M^2)=0\ ,$$
$$\mathrm{Re}\tilde{\Pi}_{\phi\phi}(M^2)=0\ ,\ \left.\frac{d\,\mathrm{Re}\tilde{\Pi}_{\phi\phi}(k^2)}{dk^2}\right|_{k^2=M^2}=0\ ,\ \left.\frac{d\,\mathrm{Re}\tilde{\Pi}_{c\bar{c}}(k^2)}{dk^2}\right|_{k^2=M^2}=0\ . \tag{161}$$

Hence $M^{\mathrm{phys}}=M$. Equation (161) determines

$$\delta Z_A=4ig^2[M^2I_2'-I_{41}'],\ \delta Z_{M^2}=-\frac{ig^2}{M^2}[I_1(m_H^2)+I_1(M^2)+4M^2I_2-4I_{41}]\ ,$$
$$\delta\Omega_\xi=ig^2[I_2-4M_2I_2'+4I_3+4I_{42}+4I_{41}']\ ,$$
$$\delta Z_\phi=ig^2\left[\left(\frac{m_H^4}{M^2}-5M^2\right)I_2'-4I_2-4I_3-4M^2I_3'\right]\ , \tag{162}$$
$$\delta\Omega_\kappa=\frac{ig^2}{2}\left[\left(\frac{m_H^2}{M^2}+1\right)M^{-2}[I_1(m_H^2)-I_1(M^2)]+\left(2-\frac{m_H^4}{M^4}\right)I_2+\left(\frac{m_H^4}{M^4}-9\right)M^2I_2'\right.$$
$$\left.-4M^2I_3'+4I_3+4I_{42}+4I_{41}'\right]\ ,$$
$$\delta Z_c=ig^2M^2I_2'\ ,$$

where $I_i\equiv I_i(M^2;M^2,m_H^2)$ and $I_i'\equiv dI_i(k^2;M^2,m_H^2)/dk^2|_{k^2=M^2}$ for $i\geq 2$. The calculated result of $\Delta_1(M^2)$ is

$$\Delta_1(M^2)=ig^2I_2\ . \tag{163}$$

With all these we explicitly calculate (157) and get

$$C_{\mathrm{mod}}=1-\frac{ig^2}{2}\left[\left(\frac{m_H^2}{M^2}+1\right)M^{-2}[I_1(m_H^2)-I_1(M^2)]+\left(2-\frac{m_H^4}{M^4}\right)I_2\right.$$
$$\left.+\left(\frac{m_H^4}{M^4}-9\right)M^2I_2'-4M^2I_3'+4I_3+4I_{42}+4I_{41}'\right]$$
$$=1-\delta\Omega_\kappa\simeq\Omega_\kappa^{-1}\ , \tag{164}$$

which coincides with the exact result (154) from the WT identities.

2. Scheme II

In this scheme we take $\Omega_\kappa=1$ instead of imposing the condition $d\,\mathrm{Re}\tilde{\Pi}_{\phi\phi}/dk^2|_{k^2=M^2}=0$. This changes the value of δZ_ϕ. We have now

$$\delta Z_\phi=ig^2\left[\left(\frac{m_H^2}{M^2}+1\right)M^{-2}[I_1(M^2)-I_1(m_H^2)]+\left(\frac{m_H^4}{M^4}-6\right)I_2+4M^2I_2'-8I_3-4I_{41}'-4I_{42}\right]\ . \tag{165}$$

Our explicit one-loop calculation of (157) gives

$$C_{\mathrm{mod}}=1+O(2\ \mathrm{loop}) \tag{166}$$

which is consistent with the rigorous result (154) from WT identities.

3. The $F=F_0$ scheme [13,16]

In this scheme $\xi=1$ and $\kappa=M$, so that the tree-level Goldstone boson mass is M [cf. (23)]. Now this scheme corresponds to $\Omega_\xi=\Omega_\kappa=1$ and $Z_\phi=Z_H$; therefore, there is no freedom of adjustment to make $\tilde{\Pi}_{\phi\phi}(M^2)=0$ (cf.

Table I). Indeed the one-loop calculation gives

$$\tilde{\Pi}_{\phi\phi}(M^2) = \frac{g^2}{16\pi^2}M^2\left(4x^2 - 2x^4 + (1+4x^2-7x^4+2x^6)\ln x + \frac{1-14x^2+11x^4-2x^6}{\sqrt{x^2-4}}x\ln\frac{x+\sqrt{x^2-4}}{2}\right) \quad \text{for } x>2, \tag{167}$$

where $x \equiv m_H/M$. Thus, in this scheme, the total Goldstone boson mass $m_\phi^2 = M^2 + \tilde{\Pi}_{\phi\phi}(M^2)$ is *not* equal to the tree-level value M^2. This coincides with the analysis in Ref. [13]. Considering this fact, the precise formula for $C_{\rm mod}$ is complicated. The one-loop result calculated from (157) is

$$\begin{aligned} C_{\rm mod} &= 1 + \frac{g^2}{16\pi^2}\left[\left(\frac{13}{6} - x^2\right) + \left(\frac{5}{2} - \frac{7}{2}x^2 + x^4\right)\ln x - \frac{1}{2}\frac{2x^2-5}{\sqrt{x^2-4}}x(x^2-3)\ln\frac{x+\sqrt{x^2-4}}{2}\right] \\ &\neq 1 \quad (\text{for } x \equiv m_H/M > 2\ ) . \end{aligned} \tag{168}$$

B. The Landau gauge ($\xi = 0$)

In this gauge, the requirement that δT should completely cancel the total H-tadpole contributions leads to, at the one-loop level,

$$i\delta T/v = g^2\left[\frac{3}{2}\frac{M_H^2}{M^2}I_1(m_H^2) + (D-1)I_1(M^2)\right] , \tag{169}$$

where I_1 and D are given in Appendix C. This fixes δT completely. The calculated one-loop results of the bare proper self-energies are

$$\begin{aligned} \Pi_{0AA}(k^2) &= -ig^2[I_1(M_H^2) + 4M^2I_2(k^2;M^2,m_H^2) - 4I_{41}(k^2;M^2,m_H^2)] , \\ \tilde{\Pi}_{0\phi\phi}(k^2) &= -ig^2\left[\frac{m_H^2}{m^2}I_1(M_H^2) - \frac{m_H^4}{M^2}I_2(k^2;0,m_H^2) + 4k^2I_2(k^2;M^2,m_H^2)\right. \\ &\quad \left. +4\frac{k^2}{M^2}[I_{41}(k^2;0,m_H^2) - I_{41}(k^2;M^2,m_H^2)] + 4\frac{k^4}{M^2}[I_{42}(k^2;0,m_H^2) - I_{42}(k^2;M^2,m_H^2)]\right] , \end{aligned} \tag{170}$$

where I_1, I_2, I_{41}, and I_{42} are given in Appendix C. It is easy to prove from (170) that $\tilde{\Pi}_{\phi\phi}(0) = \tilde{\Pi}_{0\phi\phi}(0) = 0$ which is well-known in the Landau gauge. We then take the MW scheme [14] to determine the renormalization constants. The on-shell conditions are

$$\begin{aligned} &\mathrm{Re}\Pi_{AA}(M^2) = 0 , \quad \frac{d}{dk^2}\mathrm{Re}\Pi_{AA}(k^2)\Big|_{k^2=M^2} = 0 , \\ &\mathrm{Re}\tilde{\Pi}_{\phi\phi}(0) = 0 , \quad \frac{d}{dk^2}\mathrm{Re}\tilde{\Pi}_{\phi\phi}(k^2)\Big|_{k^2=0} = 0 . \end{aligned} \tag{171}$$

These lead to

$$\begin{aligned} \delta Z_A &= ig^2[4M^2I_2'(M^2;M^2,m_H^2) - 4I_{41}'(M^2;M^2,m_H^2)] , \\ \delta Z_{M^2} &= ig^2M^{-2}[-I_1(m_H^2) - 4M^2I_2'(M^2;M^2,m_H^2) + 4I_{41}(M^2;M^2,m_H^2)] , \\ \delta Z_\phi &= ig^2\left[\frac{m_H^4}{M^2}I_2'(0;0,m_H^2) - 4I_2(0;M^2,m_H^2) + \frac{4}{M^2}[I_{41}(0;M^2,m_H^2) - I_{41}(0;0,m_H^2)]\right] . \end{aligned} \tag{172}$$

With (172) we can calculate $C_{\rm mod}$ from (155). The result is

$$\begin{aligned} C_{\rm mod} &= 1 + \tfrac{1}{2}(\delta Z_A - \delta Z_\phi + \delta Z_{M^2}) \\ &= 1 + \frac{g^2}{32\pi^2}\left[\left(\frac{15}{2} - \frac{7}{2}x^2 + x^4\right) + \left(\frac{26}{3} - 12x^2 + 7x^4 - \frac{5}{3}x^6 - 4\frac{1+x^2}{1-x^2} + \frac{2x^4}{1-x^2}\right)\ln x\right. \\ &\quad \left. + \left(-12 - 16x + 8x^2 + \frac{20}{3}x^3 - \frac{7}{3}x^4 - \frac{2}{3}x^5 + \frac{1}{3}x^6\right)\frac{1}{\sqrt{x^2-4}}\ln\frac{x+\sqrt{x^2-4}}{2}\right] \quad \text{for } x>2 , \end{aligned} \tag{173}$$

where $x \equiv m_H/M$. We see that in this scheme with $\xi = 0$, $C_{\rm mod}$ is *not* unity. In non-Abelian theories, the interactions are much more complicated. It is quite unlikely that $C_{\rm mod} = 1$ for arbitrary m_H in the MW scheme.

However, in the Landau gauge, $\Delta_1 = 0$ since the Higgs boson decouples from the ghost fields [cf. (148)], and thus in the U(1) case $\hat{C}(k^2)$ in (151) reduces to $\hat{C}(k^2) = (Z_A/Z_\phi)^{\frac{1}{2}} Z_M = C_{\rm mod}$. Therefore we can always choose $Z_\phi = Z_A Z_M^2$ to make $C_{\rm mod} = 1$ [19]. In the non-Abelian case $C_{\rm mod}$ is still complicated in the Landau gauge due to the non-Abelian coupling between the gauge fields and ghost fields [cf. (107),(120)]. So the above choice of Z_ϕ to make $C_{\rm mod} = 1$ concerns the detailed explicit calculations of Δ_3's and is thus *inconvenient* in practical applications of the ET.

VI. CONCLUSIONS

In this paper we have given a general proof of the precise formulation of the ET both in the $SU(2)_L$ theory and the SU(2)×U(1) theory to all orders in the perturbation for arbitrary value of m_H based on the general Slavnov-Taylor identity (79). The precise form of the ET is (95) with the modification factors $C^a_{\rm mod}$ given by (105) for the $SU(2)_L$ theory and (109),(118) for the realistic SU(2)×U(1) theory which has not been given in the literature. The modification factor is proportional to a function $\hat{C}(k^2)$ related to the matrix elements of certain products of field between the vacuum and the antighost states [cf. (15),(20) for the $SU(2)_L$ theory, and (47),(50),(53) for the SU(2)×U(1) theory]. At tree level, the matrix elements vanish and the renormalization constants are unity which lead to $\hat{C}(k^2) = 1$. With loop contributions, the nonfactorized parts of the matrix elements and nontrivial renormalization constants emerge which cause $\hat{C}(k^2) \neq 1$ and this generally makes $C^a_{\rm mod}$ different from unity. Therefore (95) is in general different from the naive simple form (1). Both $C^a_{\rm mod}$ and the Goldstone boson amplitude $T(i\phi^{a_1}, \ldots, i\phi^{a_n}, \Phi)$ in (95) are related to unphysical fields, so that they both depend on the gauge and the renormalization scheme. Our explicit calculation in Sec. IV shows that these dependence are in general *not* M_W/E suppressed. Our calculation shows that the leading gauge- and renormalization-scheme-dependent parts in $C^{a_1}_{\rm mod} \cdots C^{a_n}_{\rm mod}$ and $T(i\phi^{a_1}, \ldots, i\phi^{a_n}, \Phi)$ just cancel each other, so that the RHS of (95) can be equal to the LHS $T(V_L^{a_1}, \ldots, V_L^{a_n}, \Phi)$ which is physical, independent of the gauge and the renormalization scheme. In the naive formalism (1), the leading part on the RHS is generally not gauge and renormalization scheme independent; therefore, (1) cannot be generally valid.

In Sec. II we have given a systematic analysis of the renormalization schemes in the general R_ξ gauge defined in (3). We have considered the Ward-Takahashi identities for the inverse propagators which give constraints on the renormalization constants. Special attention has been paid to the examination of the freedom of adjusting the renormalization constants in the *unphysical sector* for simplifying the expression for $\hat{C}(k^2)$. Based on this analysis we have proposed two convenient renormalization schemes, namely, *scheme I* and *scheme II* defined in Sec. II, in which $\hat{C}(\xi\kappa M_W)$ takes the simple form (35) for the $SU(2)_L$ theory and the charged sector in the SU(2)×U(1) theory, and $\hat{C}^Z(\xi_Z\kappa_Z M_Z)$ and $\hat{C}^A(0)$ take the simple forms in (74) for the neutral sector in the SU(2)×U(1) theory. The details of these two schemes are summarized in Tables I and II. Of special importance is *scheme II* in which the naive simple form (1) of the ET holds exactly. The subtraction conditions chosen in *scheme II* are irrelevant to the explicit calculation of the complicated expressions for $C^a_{\rm mod}$'s. Therefore *scheme II is the most convenient scheme for applying the ET.* Examples of exactly realizing these two schemes are given in the U(1) Higgs theory in Sec. V.

We have also examined the modification factors in various currently used renormalization schemes other than *scheme II* in the $SU(2)_L$ and SU(2)×U(1) theories up to one loop in the heavy Higgs limit. Our calculation shows that in some currently used schemes such as the on-shell scheme by Böhm *et al.* and Hollik [11], the on-shell scheme by Aoki *et al.* [15], the minimal subtraction scheme and the $F^a = F^a_0$ scheme [13,16], the modification factors are significantly different from unity even in the heavy Higgs limit. In these schemes, calculation of W_L-W_L or Z_L-Z_L scattering amplitudes by using the naive form (1) of the ET may cause an error as large as 20% . In the MW scheme [14] and the intermediate scheme [18] with *scheme I* for the unphysical sector, the modification factors are approximately unity in the heavy Higgs limit, so that the application of the naive form (1) of the ET are safe in this limit.

We conclude that care should be taken in the application of the ET if the renormalization is not taken to be *scheme II.* In general, be sure to use the precise form (95) instead of using the naive form (1). Only in *scheme II* the use of the form (1) is always correct.

In our forthcoming paper [20] the above precise formulation of the ET will be generalized to the effective chiral Lagrangian formalism where the electroweak symmetry breaking sector is nonlinearly realized in the derivative expansion.

ACKNOWLEDGMENTS

We are grateful to W.J. Marciano and R.N. Cahn for interesting discussions at the International Symposium on Ultra High Energy Physics (October 1991, CCAST, Beijing, China). This work was supported by the National Natural Science Foundation of China.

APPENDIX A

We give here an alternative proof of the general Slavnov Taylor identity (79) in the text in the path-integral formalism, which is simpler and more direct than the proof in Ref. [5].

Consider the generating functional (7) in the text with the external sources $\bar{I}_a, I_a, K^i, L^a = 0$. The gauge fixing term and the Faddeev-Popov term in the Lagrangian are given in (3) and (5) in the text, but we need not specify the function F^a_0 here. It is well known that the renormalized S-matrix element for physical particles is gauge-independent, i.e., independent of the change of the gauge fixing function $F_0^a \to \tilde{F}_0^a = F_0^a + \Delta F_0^a$ [10]. The only effect of $F_0^a \to \tilde{F}_0^a$ is the change of the wave function renormalization constants which do not affect the physics. Let us take ΔF_0^a to be an arbitrary local function $I^a(x)$, independent of the fields χ_0^i, c_0^a, and $\bar{c}_0^a$, i.e.,

$$\tilde{F}_0^a(x) = F_0^a(x) - I_0^a(x). \tag{A1}$$

In the theory with the gauge fixing function $\tilde{F}_0^a(x)$, the renormalized fields $\chi_0^i, c_0^a, \bar{c}_0^a$ depend on I^a through the I^a dependence of the wave function renormalization constants since $\chi_0^i, c_0^a, \bar{c}_0^a$ are independent of I_0^a. Let the generating functional in the theory with the gauge fixing function $\tilde{F}_0^a(x)$ be

$$\tilde{Z}[J] \equiv \exp(i\tilde{W}[J]) = \int \mathcal{D}\chi_0 \mathcal{D}c_0 \mathcal{D}\bar{c}_0 \exp\left[i\left(\tilde{S}[\chi_0, c_0, \bar{c}_0] + \int d^4x J_i \chi_0^i\right)\right] , \tag{A2}$$

where in $\tilde{S}[\chi_0^i, c_0^a, \bar{c}_0^a]$ the gauge fixing function is $\tilde{F}_0^a(x)$. According to the above reason, the two generating functionals $Z[J]$ and $\tilde{Z}[J]$ must lead to the same renormalized S-matrix element for physical fields. Symbolically, we write this relation as

$$X_{(\mathrm{LSZ})} \frac{\delta^m W[J]}{\delta J_{i_1}(y_1) \cdots \delta J_{i_m}(y_m)}\bigg|_{J=0} = X_{(\mathrm{LSZ})} \frac{\delta^m \tilde{W}[J]}{\delta J_{i_1}(y_1) \cdots \delta J_{i_m}(y_m)}\bigg|_{J=0} , \quad m \geq 0 . \tag{A3}$$

The symbol $X_{(\mathrm{LSZ})}$ means the application of the Lehmann-Symanzik-Zimmermann (LSZ) reduction projector to the connected m-point Green function. We further write (A3) as

$$\langle 0|\underline{\chi}_{i_1}(p_1) \cdots \underline{\chi}_{i_m}(p_m)|0\rangle_{\text{with } F_0^a(x)} = \langle 0|\underline{\chi}_{i_1}(p_1) \cdots \underline{\chi}_{i_m}(p_m)|0\rangle_{\text{with } \tilde{F}_0^a(x)} , \tag{A4}$$

where $\underline{\chi}_{i_l}(p_l)$ $(l = 1, \ldots, m)$ denotes the LSZ amputated asymptotic on-shell physical field which are gauge independent and thus I_a independent. Now we take the functional derivative $\delta^n/\delta I_{a_1}(x_1) \cdots \delta I_{a_n}(x_n)$ on both sides of (A4) and then turn off the I^a's. Since there is no $I^a(x)$ on the LHS of (A4) we get

$$0 = \langle 0|T F_0^{a_1}(x_1) \cdots F_0^{a_n}(x_n)\Phi|0\rangle_{\text{with } \tilde{F}_0^a = F_0^a} , \quad n \geq 1 , \tag{A5}$$

or, in the momentum representation,

$$0 = \langle 0|F_0^{a_1}(k_1) \cdots F_0^{a_n}(k_n)\Phi|0\rangle_{\text{with } \tilde{F}_0^a = F_0^a} , \quad n \geq 1 , \tag{A6}$$

where $\Phi \equiv \prod_{l=1}^m \underline{\chi}_{i_l}$, for $m \geq 1$ and $\Phi \equiv 1$ for $m = 0$. On the RHS of (A6) [(A5)] we have ignored a term of the form $\delta^4(k_1 - k_2)\delta_{a_1 a_2}\delta_{n2}\delta_{m0}$ $(\delta^4(x_1 - x_2)\delta_{a_1 a_2}\delta_{n2}\delta_{m0})$ which does not contribute to the connected S-matrix element. (A6) is just the identity (79) in the text.

APPENDIX B

We give here the quanitities Δ's in the SU(2)×U(1) theory.

In the charged sector

$$\Delta_1^W(k^2) = \frac{g_0}{2M_{W0}} \int_q \langle 0 \mid H_0(-k-q)c_0^-(q) \mid \bar{c}_0^+(k)\rangle$$
$$= Z_g Z_H^{\frac{1}{2}} Z_{M_W}^{-1} \frac{g}{2M_W} \int_q \langle 0 \mid H(-k-q)c^-(q) \mid \bar{c}^+(k)\rangle , \tag{B1}$$

where $\int_q = \int \frac{d^4q}{(2\pi)^4}$, and in the dimensional regularization we have $\int_q \to \mu^\epsilon \int \frac{d^Dq}{(2\pi)^D}$,

$$\Delta_2^W(k^2) = \Delta_{21}^W(k^2) + \Delta_{22}^W(k^2) + \Delta_{23}^W(k^2) , \tag{B2}$$

$$\Delta_{21}^W(k^2) = \frac{-ig_0}{2M_{W0}} \int_q \langle 0 \mid \phi_0^Z(-k-q)c_0^-(q) \mid \bar{c}_0^+(k)\rangle$$
$$= Z_g Z_{\phi^Z}^{\frac{1}{2}} Z_{M_W}^{-1} \frac{-ig}{2M_W} \int_q \langle 0 \mid \phi^Z(-k-q)c^-(q) \mid \bar{c}^+(k)\rangle , \tag{B3}$$

$$\Delta_{22}^W(k^2) = \frac{ig_0}{2M_{Z0}}(1-\tan^2\theta_{W0})\int_q \langle 0 \mid \phi_0^-(-k-q)c_0^Z(q) \mid \bar{c}_0^+(k)\rangle$$
$$= Z_g Z_{\phi^\pm}^{\frac{1}{2}} Z_{M_Z}^{-1}\frac{ig}{2M_Z}(1-Z_{\tan^2\theta_W}\tan^2\theta_W)$$
$$\times\left[\frac{Z_c^{ZZ}}{Z_c^W}\int_q\langle 0 \mid \phi^-(-k-q)c^Z(q) \mid \bar{c}^+(k)\rangle + \frac{Z_c^{ZA}}{Z_c^W}\int_q\langle 0 \mid \phi^-(-k-q)c^A(q) \mid \bar{c}^+(k)\rangle\right] , \quad \text{(B4)}$$

$$\Delta_{23}^W(k^2) = \frac{ie_0}{M_{W0}}\int_q\langle 0 \mid \phi_0^-(-k-q)c_0^A(q) \mid \bar{c}_0^+(k)\rangle$$
$$= Z_e Z_{\phi^\pm}^{\frac{1}{2}} Z_{M_W}^{-1}\frac{ie}{M_W}\left[\frac{Z_c^{AZ}}{Z_c^W}\int_q\langle 0 \mid \phi^-(-k-q)c^Z(q) \mid \bar{c}^+(k)\rangle + \frac{Z_c^{AA}}{Z_c^W}\int_q\langle 0 \mid \phi^-(-k-q)c^A(q) \mid \bar{c}^+(k)\rangle\right], \quad \text{(B5)}$$

$$\Delta_3^W(k^2) = \Delta_{31}^W(k^2) + \Delta_{32}^W(k^2) + \Delta_{33}^W(k^2) + \Delta_{34}^W(k^2) , \quad \text{(B6)}$$

$$ik^\mu\Delta_{31}^W(k^2) = -ie_0\int_q\langle|A_0^\mu(-k-q)c_0^-(q)|\bar{c}_0^+(k)\rangle$$
$$= -Z_e Z_{AZ}^{\frac{1}{2}} ie\int_q\langle 0|Z^\mu(-k-q)c^-(q)|\bar{c}^+(k)\rangle - Z_e Z_{AA}^{\frac{1}{2}} ie\int_q\langle 0|A^\mu(-k-q)c^-(q)0|\bar{c}^+(k)\rangle , \quad \text{(B7)}$$

$$ik^\mu\Delta_{32}^W(k^2) = ie_0\int_q\langle 0|W_0^{-\mu}(-k-q)c_0^A(q)|\bar{c}_0^+(k)\rangle$$
$$= Z_e Z_W^{\frac{1}{2}}\frac{Z_c^{AZ}}{Z_c^W} ie\int_q\langle 0|W^{-\mu}(-k-q)c^Z(q)|\bar{c}^+(k)\rangle + Z_e Z_W^{\frac{1}{2}}\frac{Z_c^{AA}}{Z_c^W} ie\int_q\langle 0|W^{-\mu}(-k-q)c^A(q)|\bar{c}^+(k)\rangle , \quad \text{(B8)}$$

$$ik^\mu\Delta_{33}^W(k^2) = ig_0\cos\theta_{W0}\int_q\langle 0|Z_0^\mu(-k-q)c_0^-(q)|\bar{c}_0^+(k)\rangle$$
$$= -Z_g Z_{\cos\theta_W} Z_{ZZ}^{\frac{1}{2}} ig\cos\theta_W\int_q\langle 0|Z^\mu(-k-q)c^-(q)|\bar{c}^+(k)\rangle$$
$$-Z_g Z_{\cos\theta_W} Z_{ZA}^{\frac{1}{2}} ig\cos\theta_W\int_q\langle 0|A^\mu(-k-q)c^-(q)|\bar{c}^+(k)\rangle , \quad \text{(B9)}$$

$$ik^\mu\Delta_{34}^W(k^2) = ig_0\cos\theta_{W0}\int_q\langle 0|W_0^{-\mu}(-k-q)c_0^Z(q)|\bar{c}_0^+(k)\rangle$$
$$= Z_g Z_{\cos\theta_W} Z_W^{\frac{1}{2}}\frac{Z_c^{ZZ}}{Z_c^W} ig\cos\theta_W\int_q\langle 0|W^{-\mu}(-k-q)c^Z(q)|\bar{c}^+(k)\rangle$$
$$+Z_g Z_{\cos\theta_W} Z_W^{\frac{1}{2}}\frac{Z_c^{ZA}}{Z_c^W} ig\cos\theta_W\int_q\langle 0|W^{-\mu}(-k-q)c^A(q)|\bar{c}^+(k)\rangle . \quad \text{(B10)}$$

In the neutral sector,

$$ik_\nu\Delta_3^{ZZ}(k^2) = \int_q\langle|D_{Z_\nu}^b(-k-q)c_0^b(q)|\bar{c}_0^Z(k^2)\rangle - ik_\nu$$
$$= ig_0\cos\theta_{W0}\left[\int_q\langle 0|W_{0\nu}^+(-k-q)c_0^-(q)|\bar{c}_0^Z(k)\rangle - \int_q\langle 0|W_{0\nu}^-(-k-q)c_0^+(q)|\bar{c}_0^Z(k)\rangle\right]$$
$$= Z_g Z_{\cos\theta_W} Z_W^{\frac{1}{2}} ig\cos\theta_W$$
$$\times\left[\int_q\langle 0|W_\nu^+(-k-q)c^-(q)|\bar{c}^Z(k)\rangle\frac{Z_c^W Z_c^{AA}}{\det \mathbf{Z}_c^N} - \int_q\langle 0|W_\nu^+(-k-q)c^-(q)|\bar{c}^A(k)\rangle\frac{Z_c^W Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right.$$
$$\left. - \int_q\langle 0|W_\nu^-(-k-q)c^+(q)|\bar{c}^Z(k)\rangle\frac{Z_c^W Z_c^{AA}}{\det \mathbf{Z}_c^N} + \int_q\langle 0|W_\nu^-(-k-q)c^+(q)|\bar{c}^A(k)\rangle\frac{Z_c^W Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right] , \quad \text{(B11)}$$

$$
\begin{aligned}
ik_\nu \Delta_3^{AA}(k^2) &= \int_q \langle 0|D^b_{A_\nu}(-k-q)c_0^b(q)|\bar{c}_0^A(k)\rangle - ik_\nu \\
&= ie_0\left[\int_q \langle 0|W^+_{0\nu}(-k-q)c_0^-(q)|\bar{c}_0^A(k)\rangle - \int_q \langle 0|W^-_{0\nu}(-k-q)c_0^+(q)|\bar{c}_0^A(k)\rangle\right] \\
&= Z_e Z_W^{\frac{1}{2}} Z_c^W ie\left[\int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right. \\
&\quad \left. - \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right], \qquad \text{(B12)}
\end{aligned}
$$

$$
\begin{aligned}
ik_\nu \Delta_3^{ZA}(k^2) &= \int_q \langle 0|\hat{D}^b_{Z_\nu}(-k-q)c_0^b(q)|\bar{c}_0^A(k)\rangle \\
&= ig_0\cos\theta_{W0}\left[\int_q \langle 0|W^+_{0\nu}(-k-q)c_0^-(q)|\bar{c}_0^A(k)\rangle - \int_q \langle 0|W^-_{0\nu}(-k-q)c_0^+(q)|\bar{c}_0^A(k)\rangle\right] \\
&= Z_g Z_{\cos\theta_{W0}} Z_W^{\frac{1}{2}} Z_c^W ig\,\cos\theta_W \\
&\quad \times\left[\int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right. \\
&\quad \left. - \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right], \qquad \text{(B13)}
\end{aligned}
$$

$$
\begin{aligned}
ik_\nu \Delta_3^{AZ}(k^2) &= \int_q \langle 0|\hat{D}^b_{A_\nu}(-k-q)c_0^b(q)|\bar{c}_0^Z(k)\rangle \\
&= ie_0\left[\int_q \langle 0|W^+_{0\nu}(-k-q)c_0^-(q)|\bar{c}_0^Z(k)\rangle - \int_q \langle 0|W^-_{0\nu}(-k-q)c_0^+(q)|\bar{c}_0^Z(k)\rangle\right. \\
&= Z_e Z_W^{\frac{1}{2}} Z_c^W ie\left[\int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{AA}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|W^+_\nu(-k-q)c^-(q)|\bar{c}^A(k)\rangle \frac{Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right. \\
&\quad \left. - \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{AA}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|W^-_\nu(-k-q)c^+(q)|\bar{c}^A(k)\rangle \frac{Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right], \qquad \text{(B14)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta_1^{ZZ}(k^2) &= \frac{g_0}{2M_{W0}}\int_q \langle 0|H_0(-k-q)c_0^Z(q)|\bar{c}_0^Z(k)\rangle \\
&= \frac{Z_g Z_H^{\frac{1}{2}}}{Z_{M_W}}\frac{g}{2M_W}\left[\int_q \langle 0|H(-k-q)c^Z(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZZ}Z_c^{AA}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|H(-k-q)c^A(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}Z_c^{AA}}{\det \mathbf{Z}_c^N}\right. \\
&\quad \left. - \int_q \langle 0|H(-k-q)c^Z(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}Z_c^{AZ}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|H(-k-q)c^A(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZA}Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right], \qquad \text{(B15)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta_1^{ZA}(k^2) &= \frac{g_0}{2M_{W0}}\int_q \langle 0|H_0(-k-q)c_0^Z(q)|\bar{c}_0^A(k)\rangle \\
&= \frac{Z_g Z_H^{\frac{1}{2}}}{Z_{M_W}}\frac{g}{2M_W}\left[\int_q \langle 0|H(-k-q)c^Z(q)|\bar{c}^A(k)\rangle \frac{(Z_c^{ZZ})^2}{\det \mathbf{Z}_c^N} + \int_q \langle 0|H(-k-q)c^A(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZA}Z_c^{ZZ}}{\det \mathbf{Z}_c^N}\right. \\
&\quad \left. - \int_q \langle 0|H(-k-q)c^Z(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZZ}Z_c^{ZA}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|H(-k-q)c^A(q)|\bar{c}^Z(k)\rangle \frac{(Z_c^{ZA})^2}{\det \mathbf{Z}_c^N}\right], \qquad \text{(B16)}
\end{aligned}
$$

$$\Delta_2^{ZZ}(k^2) = \frac{ig_0}{2M_{Z0}}\left[\int_q \langle 0|\phi_0^+(-k-q)c_0^-(q)|\bar{c}_0^Z(k)\rangle - \int_q \langle 0|\phi_0^-(-k-q)c_0^+(q)|\bar{c}_0^Z(k)\rangle\right]$$
$$= \frac{Z_g Z_{\phi^\pm}^{\frac{1}{2}} Z_c^W}{Z_{M_Z}}\frac{ig}{2M_Z}\left[\int_q \langle 0|\phi^+(-k-q)c^-(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{AA}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|\phi^+(-k-q)c^-(q)|\bar{c}^A(k)\rangle \frac{Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right.$$
$$\left. - \int_q \langle 0|\phi^-(-k-q)c^+(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{AA}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|\phi^-(-k-q)c^+(q)|\bar{c}^A(k)\rangle \frac{Z_c^{AZ}}{\det \mathbf{Z}_c^N}\right], \tag{B17}$$

$$\Delta_2^{ZA}(k^2) = \frac{ig_0}{2M_{Z0}}\left[\int_q \langle 0|\phi_0^+(-k-q)c_0^-(q)|\bar{c}_0^A(k)\rangle - \int_q \langle 0|\phi_0^-(-k-q)c_0^+(q)|\bar{c}_0^A(k)\rangle\right]$$
$$= \frac{Z_g Z_{\phi^\pm}^{\frac{1}{2}} Z_c^W}{Z_{M_Z}}\frac{ig}{2M_Z}\left[\int_q \langle 0|\phi^+(-k-q)c^-(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} - \int_q \langle 0|\phi^+(-k-q)c^-(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right.$$
$$\left. - \int_q \langle 0|\phi^-(-k-q)c^+(q)|\bar{c}^A(k)\rangle \frac{Z_c^{ZZ}}{\det \mathbf{Z}_c^N} + \int_q \langle 0|\phi^-(-k-q)c^+(q)|\bar{c}^Z(k)\rangle \frac{Z_c^{ZA}}{\det \mathbf{Z}_c^N}\right]. \tag{B18}$$

APPENDIX C

We give here the definitions of momentum integrations appearing in the one-loop calculations in the text:

$$I_1(a^2) \equiv \mu^{2\epsilon}\int_p \frac{1}{p^2-a^2} = \frac{i}{16\pi^2}a^2[I_0(a^2)+1],$$
$$I_0(a^2) \equiv \frac{1}{\epsilon} - \gamma - \ln\frac{a^2}{4\pi\mu^2}, \quad \int_p \equiv \int \frac{d^Dp}{(2\pi)^D}, \quad D = 4-2\epsilon, \tag{C1}$$

$$I_2(k^2;a^2,b^2) \equiv \mu^{2\epsilon}\int_p \frac{1}{(p^2-a^2)[(p+k)^2-b^2]},$$
$$= \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} - \gamma - \ln\frac{ab}{4\pi\mu^2} + 2 - I_{20}(k^2;a^2,b^2)\right], \tag{C2}$$

$$I_{20}(k^2;a^2,b^2) \equiv -\frac{a^2-b^2}{k^2}\ln\frac{b}{a} + \bar{I}_{20}(k^2;a^2,b^2), \tag{C3}$$

$$\bar{I}_{20}(k^2;a^2,b^2) \equiv \begin{cases} -\dfrac{\sqrt{AB}}{k^2}\ln\dfrac{\sqrt{-A}+\sqrt{-B}}{\sqrt{-A}-\sqrt{-B}} & (k^2 \le (a-b)^2), \\ 2\dfrac{\sqrt{-AB}}{k^2}\arctan\sqrt{\dfrac{B}{-A}} & ((a-b)^2 < k^2 < (a+b)^2), \\ +\dfrac{\sqrt{AB}}{k^2}\left[\ln\dfrac{\sqrt{B}+\sqrt{A}}{\sqrt{B}-\sqrt{A}} - i\pi\right] & (k^2 \ge (a+b)^2), \end{cases} \tag{C4}$$

where $A \equiv k^2-(a+b)^2, B \equiv k^2-(a-b)^2$,

$$I_3^\mu(k^2;a^2,b^2) \equiv \mu^{2\epsilon}\int_p \frac{p^\mu}{(p^2-a^2)[(p+k)^2-b^2]} \equiv k^\mu I_3(k^2;a^2,b^2), \tag{C5}$$

$$I_4^{\mu\nu}(k^2;a^2,b^2) \equiv \mu^{2\epsilon}\int_p \frac{p^\mu p^\nu}{(p^2-a^2)[(p+k)^2-b^2]}$$
$$\equiv g^{\mu\nu}I_{41}(k^2;a^2,b^2) + k^\mu k^\nu I_{42}(k^2;a^2,b^2). \tag{C6}$$

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