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Further Observations of Sunrise and Sunset Fading of Very-Low-Frequency Signals[1]

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An earlier explanation of sunrise and sunset fading on long VLF paths is confirmed by examining data taken over a wider range of frequency and directions of propagation. It is found, however, that there is an appreciable dependence of the fading period on the magnetic direction of propagation. It is shown that this dependence is qualitatively in agreement with what would be expected from a sharply bounded ionosphere and a transverse magnetic field.

1. Introduction

In an earlier paper [Crombie, 1964] (subsequently referred to as paper 1), some of the characteristic features of the remarkable periodic fading of VLF signals observed over two long paths were described. The directions of these two paths (Canal Zone to Germany, 18 kc/s, Hawaii to Boulder, Colo., 19.8 kc/s) were both mainly to the east. On the basis of the data obtained from them it was concluded that, at sunrise, the major features of the observed fading could be interpreted as being essentially due to interference between the first-order modes propagating in the daytime earth-ionosphere waveguide. It was suggested that these two modes were the result of conversion of the first- and second-order modes propagating in the nighttime portion of the waveguide. Since the latter modes had different phase velocities, the relative phase, and hence the combined amplitude and phase of the two resulting first-order modes, depended on the distance of the converting region (possibly the sunrise line) from the source of the first- and second-order modes, and not on the distance of the receiver from the sunrise line. It seemed probable that the transmitting antenna was the source of the two initial modes.

At sunset, however, for the particular geography involved, it was necessary that the (assumed single) mode incident in the sunset boundary be converted at the boundary into two modes of the first and second order. These subsequently interfere in the nighttime region to the east of the sunset line, causing the sunset fading. In this case the relative amplitude of the two modes depends on the distance of the receiver from the boundary.

The feature common to both the sunrise and sunset effects was the presence of two significant modes in the nighttime portion of the guide, and that the second-order modes was weaker than the first, at the frequencies used. Furthermore, in both cases the characteristics of the fading were determined by the interference of first- and second-order modes and the movement of the sunrise or sunset line (the "terminator") along the path. As the terminator moved from one position where the two modes were in antiphase to the next, a distance D, the received signal passed through one complete cycle of the interference pattern. The distance D is thus given by

$$D=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} \tag{1}$$

where λ_1 and λ_2 were the guide wavelengths of the first- and second-order modes in the nighttime portion of the earth-ionosphere waveguide. By using a simple sharply bounded isotropic model of the ionosphere to determine λ_1 and λ_2 it was found that the appropriate ionosphere height was about 85 km, confirming that the mode interference was occurring in the nighttime portion of the path. Other earth-ionosphere waveguide models would of course have given slightly different heights.

Although the data used in the above work yielded a consistent picture, they pertained to a rather limited situation since, as mentioned above, the two frequencies were close together and the paths were in similar directions. Nevertheless, for each path about 1 year of data was available and used.

Since the earlier paper was written, the author has had an opportunity to examine more data of limited duration but covering a much wider range of frequencies and directions of propagation. This paper will be concerned therefore in demonstrating the extent to which the new data confirm the earlier conclusions, and in accounting for the difference.

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2. Experimental Data

Figure 1 shows much of the new data. For each path examined, the distance D was determined from geometry [Crary, 1965] and the observed time between signal minima (see paper 1). The value of D was then plotted against the appropriate frequency. The points NPM-Bo at 19.8 kc/s and NBA-Fr [Brady et al., 1963] at 18 kc/s are the mean of the observations given in paper 1. The data at 18 kc/s on the NBA-Bo and NBA-Maui paths have been taken from previously published papers [Brady et al., 1964a, b]. The 24-kc/s points have been taken from 1 month's observations on NBA at Frankfurt [Eitzenberger, private communication], Maui [Katahara, private communication], and Boulder [Steele, private communication]. The WWVL-Maui data are from Morgan and Blair [1965],

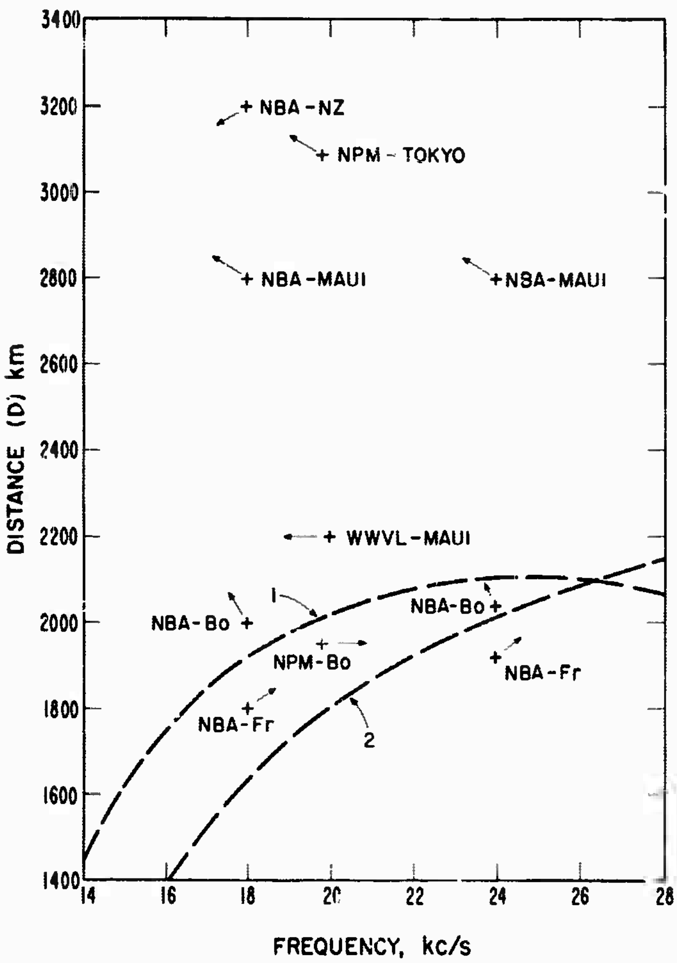

FIGURE 1. *Effect of frequency and direction of propagation on the distance, D, between minima at sunrise.*

The small arrows at each point show qualitatively the effective direction of propagation with respect to the horizontal component of the earth's magnetic field. (Magnetic North is at the top of the page.) The dashed lines show the values of D which would be expected if the ionosphere conductivity gradient was 0.5/km and for reference heights of 90 km (curve 1) and 70 km (curve 2).

the NPM-Tokyo point from Nakajima et al. [1963], while the NBA-New Zealand point is from the paper by Burtt [1963]. Observations at Stanford University on NSS (21.4 kc/s) also are in conformity with the data shown in figure 1, although they are not shown.

The small arrows attached to each point in figure 1 are intended to show qualitatively the average bearing of the path with respect to the direction of the horizontal component of the earth's magnetic field. This average bearing is weighted in favor of the lower latitude portions of the paths where the horizontal component of the field is greatest.

Figure 1 shows two main effects, that of frequency on D and that of the direction of propagation on D. These will now be discussed.

3. Effect of Frequency

The data used in paper 1 were obtained for frequencies of 18 and 19.8 kc/s. The range of frequencies shown in figure 1 is from 18 to 24 kc/s, the increase resulting mainly from the change in frequency of NBA from 18 to 24 kc/s. This change produced relatively little effect, the observations at Frankfurt and Boulder showing a small increase in D with the increase in frequency, while at Maui there was essentially no change. It should be noted that although D was relatively unchanged by the change in frequency, the shape of the sunrise phase curves was considerably altered. This is due to the much increased amplitude of the second-order mode which at times exceeds that of the first-order mode. As a result (see paper 1) the direction of the rapid phase change at the signal minima is reversed. If we exclude the four points for which $D > 2400$ km, shown in figure 1, which will be discussed later, it is clear that the remainder are in general agreement with the behavior shown by the dashed lines. These lines show the values of D computed from phase velocities given by Wait and Spies [1964] for isotropic ionosphere models which can be represented in the form

$$\omega_r = \omega_r(h) \exp [\beta(h - z)],$$

where $\omega_r = \omega_0^2/\nu$ at the height z and h is the height at which $\omega_r(h) = 2.5 \times 10^5$; ν and ω_0 are the collision frequency and angular plasma frequency at the height z. The lines in figure 1 are for $h = 80$ and 90 km and $\beta = 0.5$. As noted above, they are in general agreement with the experimental points below $D = 2400$ km.

4. Effect of Direction of Propagation

Figure 1 apparently shows a very pronounced dependence of D on the direction of propagation with respect to the direction of the earth's magnetic field. If we divide the observations into three groups of direction, namely, the quadrants centered on the east (NPM-Boulder, NBA-Frankfurt), on the north or south (NBA-Boulder), and on the west (WWVL-Maui, NBA to Maui, NPM-Tokyo and NBA-New Zealand), and take the mean value of D for each group irrespective of frequency, the results can be represented as in table 1. Despite the conflicting WWVL-Maui result, it can be seen that there is a strong dependence on the direction of propagation. This one discordant value, WWVL (in Colorado) to Maui, shown in figure 1, is for a path which has a much greater proportion (about one third) of land along it than any of the other paths. This does not explain the anomalous value, however. Although there is little difference in the values of D for propagation to the east and to the north or south, the value of D for propagation to the west is appreciably greater.

Table 1

Direction	Mean D (km)
To the east	1900
To the north or south	2025
To the west	2800

5. Source of Second-Order Mode

Figures 2 and 3 show the average rates of change of phase of the 18-kc/s signals from NBA as observed at Maui [Brady et al., 1946b] at sunrise and sunset. The geographical relation of the transmitter, receiver, and terminator is also shown. At sunrise on this path (which occurs between 10 and 16 UT for the months shown in fig. 2), the depth of fading increases as the terminator approaches the receiver. It is evident that any second-order mode excited by the transmitter is being propagated towards the receiver over increasing lengths of illuminated path in which the second mode is highly attenuated. Thus the increased depth of fading observed as the terminator approaches the receiver is a result of the second-order mode generated in the vicinity of the terminator. Figure 3 for the same path at sunset (which commences at the transmitter between 00 UT and 02 UT) shows the effect of the second-order mode generated at the transmitter. The depth of fading is greatest when the terminator is closest to the transmitter and thus represents the effect of the second-order mode, excited by the transmitter, which is converted into a first-order mode at the terminator.

These observations clarify an issue which could not be decided in paper 1, that is, whether or not it was necessary for the second-order mode to be excited only by the transmitter. It is clear from the observations at Maui that this is not necessary, and that the second-order mode which is present in the nighttime

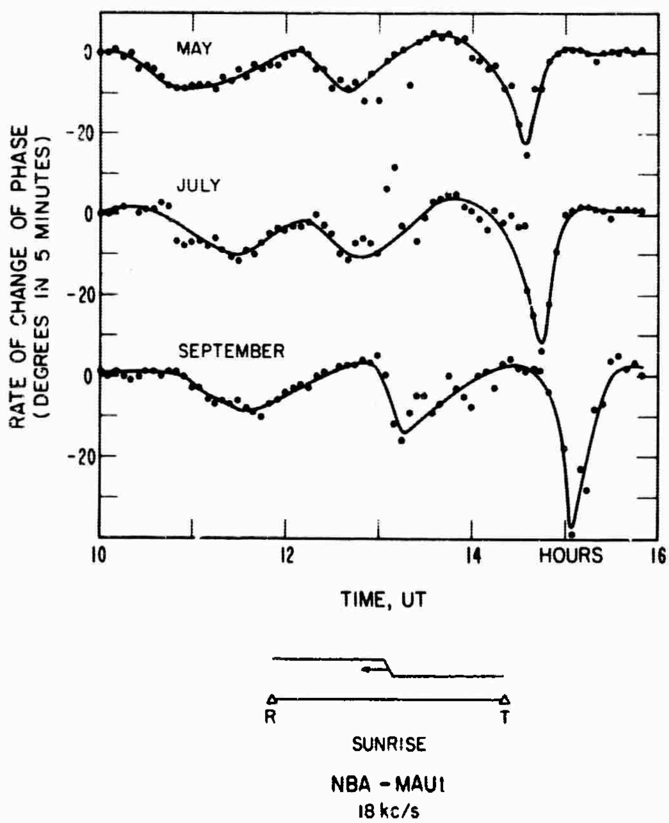

Figure 2. *Rate of change of phase observed at Maui on the 18-kc/s signals from NBA at sunrise.*

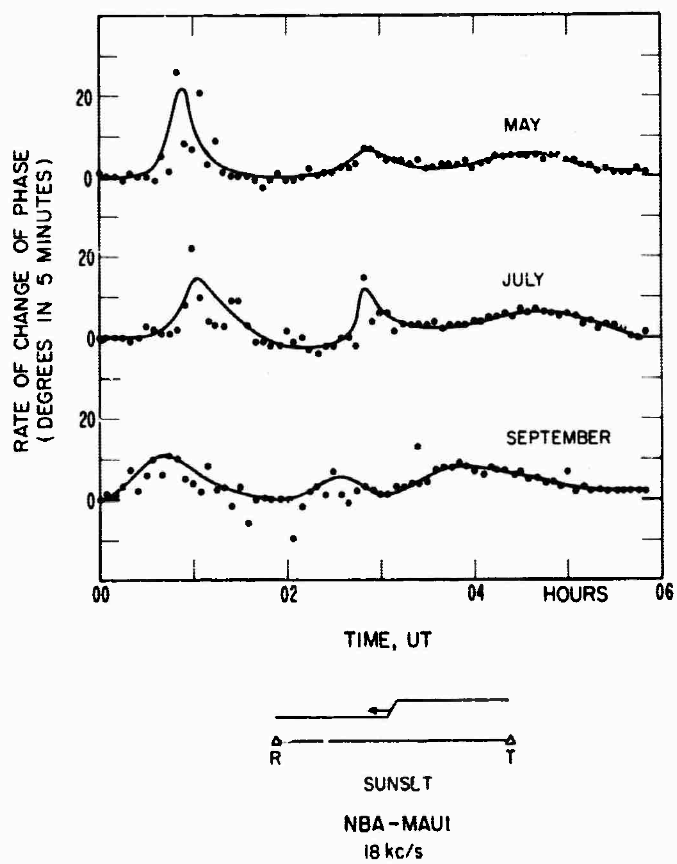

Figure 3. *Rate of change of phase observed at Maui on the 18-kc/s signals from NBS at sunset.*

portion of the path can also be produced at or near the terminator.

6. Discussion: Effect of Direction of Propagation

The most striking feature of figure 1 is the dependence of D on the direction of propagation. This suggests rather strongly that it may be due to a nonreciprocal effect on the difference in phase velocities of the first- and second-order modes.

Let us consider an idealized earth-ionosphere waveguide in which the earth and ionosphere are flat and perfectly reflecting, but the ionosphere changes the phase of the reflected wave by θ radians. The guide wavelength, λ_n, of the nth mode of a wave whose free-space wavelength is λ_0 is

$$\lambda_n = \lambda_0 / \sqrt{1-C_n^2}, \tag{2}$$

where

$$C_n = (n - \theta/2\pi)(\lambda_0/2h), \tag{3}$$

and h is the height of the uniform ionosphere which is assumed to have a sharp lower boundary. The distance $D = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} \cong \frac{\lambda_0^2}{\lambda_2-\lambda_1}$ is then given by

$$D = \lambda_0^2/(C_2^2 - C_1^2)(\lambda_0/2),$$

since

$$1/\sqrt{1-C_n^2} \cong 1 + \tfrac{1}{2} C_n^2.$$

Thus

$$D = (8h^2/\lambda_0)/(3 - \theta/\pi). \tag{4}$$

It is worth noting that when $\theta = \pi$, a reasonable idealization for a sharply bounded isotropic ionosphere,

$$D = 4h^2/\lambda_0,$$

a result obtained earlier [Crombie, 1962]. Equation (4) shows that if D is to be increased by variations in θ, then θ must increase. To the extent to which this idealized model represents the actual case, the dependence of D on the direction of propagation must result from a value of θ, the phase shift on reflection, which is greater for east-to-west propagation than for propagation in the opposite direction. This is just the behavior found by Barber and Crombie [1959] for a sharply bounded ionosphere, and thus qualitatively accounts for the observations.

Equation (3) is a rather crude approximation since losses in the earth and ionosphere have been neglected. As a result the guide wavelength λ_n as calculated from (2), and the corresponding phase velocity which is proportional to λ_n, might be in error. More exact calculations for a sharply bounded ionosphere have shown [Wait, 1962], however, that the phase velocity of the first-order mode is less for east-to-west propagation than for the opposite direction. This supports the conclusion reached above since (2) and (3) show that λ_n, and hence the phase velocity, is decreased when θ is increased.

Wait and Spies [1964] have considered the case of an ionosphere in which the electron density increases upwards in an exponential model. They find, in this case, that the effect of the magnetic field is to cause a reduction in the phase shift on reflection for propagation to the west. Clearly, for the admittedly crude model used here, this should produce a decrease in in D for propagation to the west. (This is confirmed by the second-order mode calculation for an exponential ionosphere in a supplement to Wait and Spies [1964].) The difference in the effect of the earth's magnetic field on the reflection phase shift for the two models confirms the views expressed by Wait and Spies [1964] that at night the ionization gradient is quite sharp. Thus the behavior is perhaps more like that of a sharp rather than an exponential boundary.

Further evidence concerning the dependence of the phase velocity on direction of propagation is contained in a paper by Swanson, et al. [1965], in which it is shown that at 10.2 kc/s the diurnal variation on the Hawaii-Canal Zone path is about 10 percent greater for east-to-west propagation than for the opposite direction of propagation. Swanson, et al. [1965] indicate that the difference occurs at night. Thus the nighttime phase velocity for east-to-west propagation is less than for propagation in the opposite direction. Provided that this difference is due to a change in phase shift on reflection, (2) and (3) show that the phase change on reflection, θ, must be greater for east-to-west propagation than for propagation in the opposite direction. This is also in agreement with the conclusions reached above.

7. Conclusions

The explanation of sunrise and sunset fading observed on long VLF paths given in an earlier paper is confirmed by data for propagation in the opposite direction to that initially used. Other data for differing frequencies and propagation direction are also considered. It is found that the difference in the phase velocities of the first- and second-order modes under nighttime conditions is less for propagation towards the west than for propagation towards the east. This is found to be qualitatively consistent with the results expected for a sharply bounded ionosphere and a horizontal terrestrial magnetic field.

Observations on signals received in darkness from the east show that it is not necessary that the second-order mode, which plays an important part in the fading, be excited by the transmitting antenna; the evidence is that it can be adequately excited at or near the sunrise terminator.

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