# CHAPTER 47

## FURTHER RESULTS ON THE DEPOSITION OF COHESIVE SEDIMENTS

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### Abstract

Experimental investigations on the depositional behavior of suspensions of fine cohesive sediments in turbulent flows have been conducted in a special laboratory apparatus. This apparatus consists essentially of an annular channel containing the water-sediment mixture, and an annular ring positioned within the channel and in contact with the water surface. A turbulent shear flow is generated by a simultaneous rotation of the channel and ring in opposite directions at the proper relative speeds to minimize secondary currents in the bottom of the channel, so that the sediment deposits uniformly.

The first reported results indicated that after a short period of rapid deposition the suspended sediment concentration approaches a constant value, known as the "equilibrium" concentration, $C_{eq}$ . The ratio $\frac{C_{eq}}{C_o}$ , where $C_o$ is the initial concentration, is constant for constant flow conditions, and independent of $C_o$ . It was found to depend very strongly on the average shear stress $\tau_{ch}$ over the channel perimeter.

The following are the more recent results of the continuing investigations.

The shear stress $\tau_b$ on the channel bottom has been found to be approximately uniform for flows for which the sediment deposits uniformly. This stress was also found to be a power function of $\tau_{ch}$ . This suggests that $\tau_b$ controls $\frac{C_{eq}}{C_o}$ , i.e., the percentage of the initial sediment which eventually deposits. There exists a well-defined minimum bottom shear stress, $\tau_{b_{min}}$ , below which essentially no sediment stays in suspension. The ratio $\frac{C_{eq}}{C_o}$ is related to the difference $\tau_b - \tau_{b_{min}}$ by a logarithmic-

normal law, whose geometric standard deviation and geometric mean are expected to be functions of the physical and chemical properties of the sediment and the water quality.

Limited time-deposition data indicated that the relative concentration $C' = \frac{C_o - C(t)}{C_o - C_{eq}}$, where $C(t)$ is the instantaneous concentration, varies in proportion to the logarithm of time $t$, and to the logarithm of the $C_o$. The rate of deposition, there, $\frac{dC}{dt}$ varies in inverse proportion to $t$, and in proportion to $C_o$. These conclusions are valid within limited time and concentration intervals.

## INTRODUCTION

The depositional behavior of fine cohesive sediments in turbulent flows controls the shoaling process in estuarial channels. These sediments are predominently composed of silt and clay ranging in size from a small fraction of one micron up to 50 microns. Although they may be abundant in rivers as suspended sediment, they are not normally encountered in the bed of alluvial channels, but are transported as wash load (2), and generally deposit in areas of very low velocities such as estuaries and reservoirs. Because of their small size and their large specific area (surface area per unit volume), the behavior of these particles in a flow field is controlled by a variety of interparticle forces. Some of these forces are attractive, such as the van der Waals atomic forces, and, others are repulsive, such as the surface ionic forces due to charge deficiency of either surface molecules or adsorbed ions. The net effect of these interparticle forces may be repulsion or attraction. In the first case the smaller particles (smaller than about one micron) may stay in suspension even in quiescent water for an extremely long period of time, due to Brownian motion. In fact, even very slight agitations can be adequate to prevent settling of the heavier particles. In the second case particles tend to cling together and form agglomerations known as flocs. Their size and settling velocities may become several orders of magnitude higher than those of the individual particles. This phenomenon, greatly enhanced by dissolved salts, is known as floculation and is the cause,under proper hydraulic conditions, of rapid deposition of fine suspended sediment in estuaries.

A rational approach to the control of shoaling in estuarial waters, generally due to predominantly fine sediments (2), requires a good understanding of the behavior of this type of sediment in a flow field. More specifically, the important flow parameters and soil properties which control the initiation and rates of deposition need to be investigated, and quantitative functional relationships between these variables need to be determined.

The present phase of the investigation described here is concerned with the role of flow parameters.

## SUMMARY OF THE FIRST RESULTS

Fig. 1 shows a picture of the experimental apparatus used. It consists of an annular channel with external and internal diameters of 28-3/8 and 36 inches respectively and with a depth of 12 inches. An annular ring of slightly smaller width than that of the channel is positioned within the channel and can be adjusted vertically to contact the water surface in the channel. A turbulent flow field is generated by rotating the channel and ring in opposite directions. The ring supports are instrumented so that the total tangential shear stress on the ring (and thus on the water surface) can be measured. The relative speeds of channel and ring have been adjusted to minimize the effects of secondary currents on the bottom, so that the sediment deposits almost uniformly across the channel. The details of the equipment are described in (3,6). For depths between 8 cm and 16 cm the average shear stress in the ring varies in proportion to $(\Delta\omega)^2$, where $\Delta\omega$ is relative angular velocity between the channel and ring, and is independent of the depth. This shear stress, $\tau_r$, is given by the equation

$$\tau_r = 4.65 \times 10^{-5} (\Delta\omega)^2 \qquad (1)$$

where $\tau_r$ is given in psf and $\Delta\omega$ in rpm. The first experiments with kaolinite clay-silt suspensions (1,3,6) revealed the following important depositional characteristics:

1. For given geometry, sediment, and flow conditions, the suspended sediment concentration eventually reaches a constant value, herein called "equilibrium concentration", $C_{eq}$.
2. The ratio of the equilibrium concentration to the initial concentration $C_o$, i.e., $\frac{C_{eq}}{C_o}$, is independent of $C_o$, but depends strongly on flow conditions, sediment properties, water chemistry and channel geometry. Hence each flow can maintain in suspension a constant fraction of a particular sediment.
3. For the three different depths used the ratio $\frac{C_{eq}}{C_o}$ appears to be very strongly controlled by the average shear stress, $\tau_{ch}$, around the channel boundary, provided that the speed of the channel and ring are so adjusted that the sediment deposits uniformly. $\tau_{ch}$ was determined by considering the torque of all the shear stress about the axis of rotation and using eq. 1. It is given by the following expression

$$\tau_{ch} = 4.65 \times 10^{-5} \frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}} \qquad (2)$$

where d is the depth and b the width of the channel.

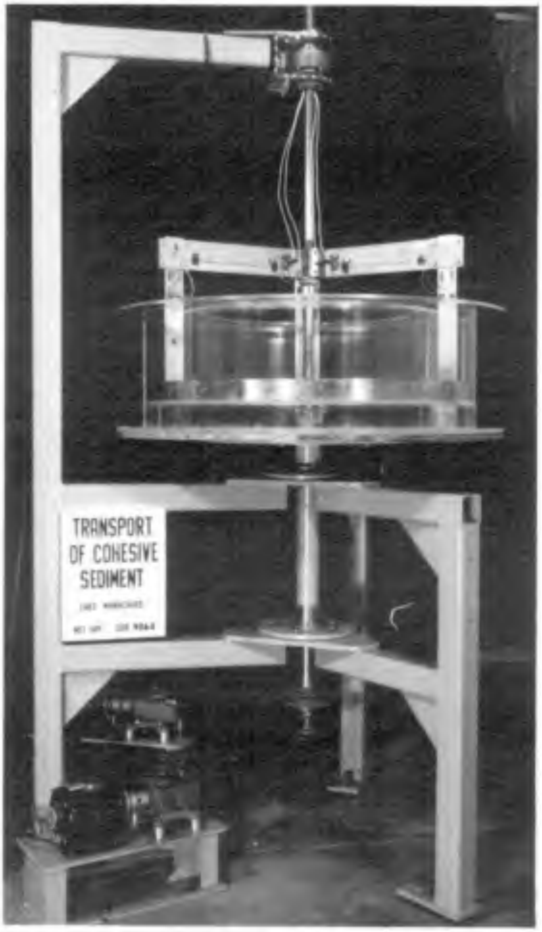

Fig. 1 Experimental Apparatus

4. $\frac{C_{eq}}{C_o}$ varies for different combinations of speeds other than those corresponding to uniform deposition even if $\Delta\omega$ remains constant. Hence the secondary currents generated by the rotational motion have a significant effect on the equilibrium concentrations and the rate of deposition.

5. A size analysis of a sample of suspended material obtained at equilibrium concentration showed that the sediment contained the entire grain size range of the original material although in the average it was somewhat finer. This confirms the fact that the interparticle physico-chemical forces are far more important as a settling agent than the individual particle size and it suggests that there is no strong correlation between these forces and particle size.

## RECENT EXPERIMENTAL RESULTS

### I. Equilibrium Concentrations

The shear stress at the bottom of the channel measured by a Preston tube (in clear water) was found to be almost uniform across the channel as long as the deposition was uniform. Therefore the shear stress $\tau_o$ at the centerline of the channel was considered as a good measure of the average bottom shear stress $\tau_b$ .

The quantity $P_\omega = \frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}}$ , called "shear stress parameter", was shown to be proportional to the average shear stress, $\tau_{ch}$ , over the channel boundary, and therefore, according to the earlier results, controls the relative equilibrium concentration $\frac{C_{eq}}{C_o}$ .

The shear stress at the centerline of the channel, $\tau_o \approx \tau_b$ , was plotted versus the shear stress parameter on logarithmic paper, as shown in Fig. 2. The plot may be approximated by a straight line corresponding to the empirical expression

$$\tau_b = \left[ 4.20 \times 10^{-5} \frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}} \right]^{1.20} \tag{3}$$

or, combining eqs. 2 and 3,

$$\tau_b = (.903\ \tau_{ch})^{1.20} \tag{4}$$

All shear stresses are in psf and $\Delta\omega$ is in rpm.

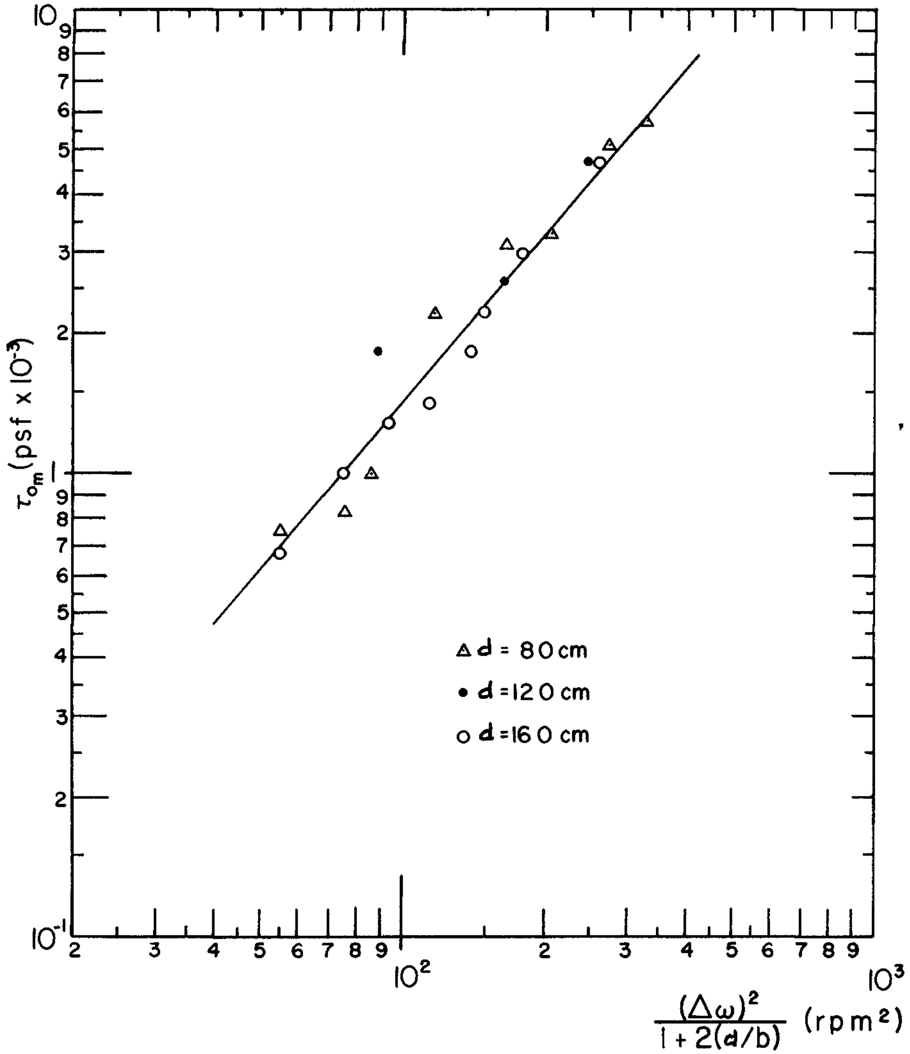

Fig. 2 LOGARITHMIC CORRELATION BETWEEN MEASURED SHEAR STRESS AND SHEAR STRESS PARAMETER

Hence it can be said that the bottom shear stress appears to be the important flow variable which controls deposition. There exists a minimum value of the shear stress parameter, $P_{\omega_{min}}$, below which $C_{eq} = 0$. The ratio $\frac{C_{eq}}{C_o}$ is shown plotted in Fig. 3 versus the difference $\Delta P_\omega = P_\omega - P_{\omega_{min}}$ in a logarithmic normal graph. It is seen that over a wide range of $\Delta P_\omega$ and $\frac{C_{eq}}{C_o}$ the points can be fitted very closely by a straight line described by the equation

$$\frac{C_{eq}}{C_o} = \frac{1}{3.085\sqrt{\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\log \Delta P_\omega - \log M}{\sigma_g}\right)^2\right\} \tag{5}$$

where M is the geometric mean and $\sigma_g$ is the geometric standard deviation of $\Delta P_\omega$. M and $\sigma_g$ are expected to be functions of the physico-chemical properties of the sediment and of the water. For the kaolinite clay used and distilled water, $P_{\omega_{min}} = 65(\text{rpm})^2$, $M = 58(\text{rpm})^2$ and $\sigma_g = 3.085$.

Therefore eq. 5 becomes

$$\frac{C_{eq}}{C_o} = \frac{1}{3.085\sqrt{\pi}} \exp\left\{-\frac{1}{2}\left[\frac{\log^2\left(\frac{(\Delta\omega)^2}{.58(1 + 2\frac{d}{b})} - 1.120\right)}{9.52}\right]\right\} \tag{6}$$

or combining eq. 3 and 6

$$\frac{C_{eq}}{C_o} = \frac{1}{3.085\sqrt{2\pi}} \exp\left\{-\frac{1}{19.04} \log^2\left(410\ \tau_b^{0.834} - 1.120\right)\right\} \tag{7}$$

The dependence of the relative equilibrium concentration can be explained in the following way.

As stated in the introduction, under certain conditions there exist net attractive forces between the particles. As a result, particles may cling to each other, forming flocs. When the flocs grow large enough they start settling towards the bottom of the channel, a region of high velocity gradients. In that region the flocs are subjected to the highest disruptive stresses. These stresses are generated by the velocity difference at two extreme points of the floc, and by the collision with other particles.

The net interparticle attractive forces vary in intensity. In the process of particle collision and floculation some particles will be attracted to each other with stronger bonds than others. Hence, the average shear strength of the flocs, or their resistance to the high boundary disruptive stresses, will vary over a wide range. The flocs with high enough bonds will eventually reach the bed and will become part of it. The flocs with

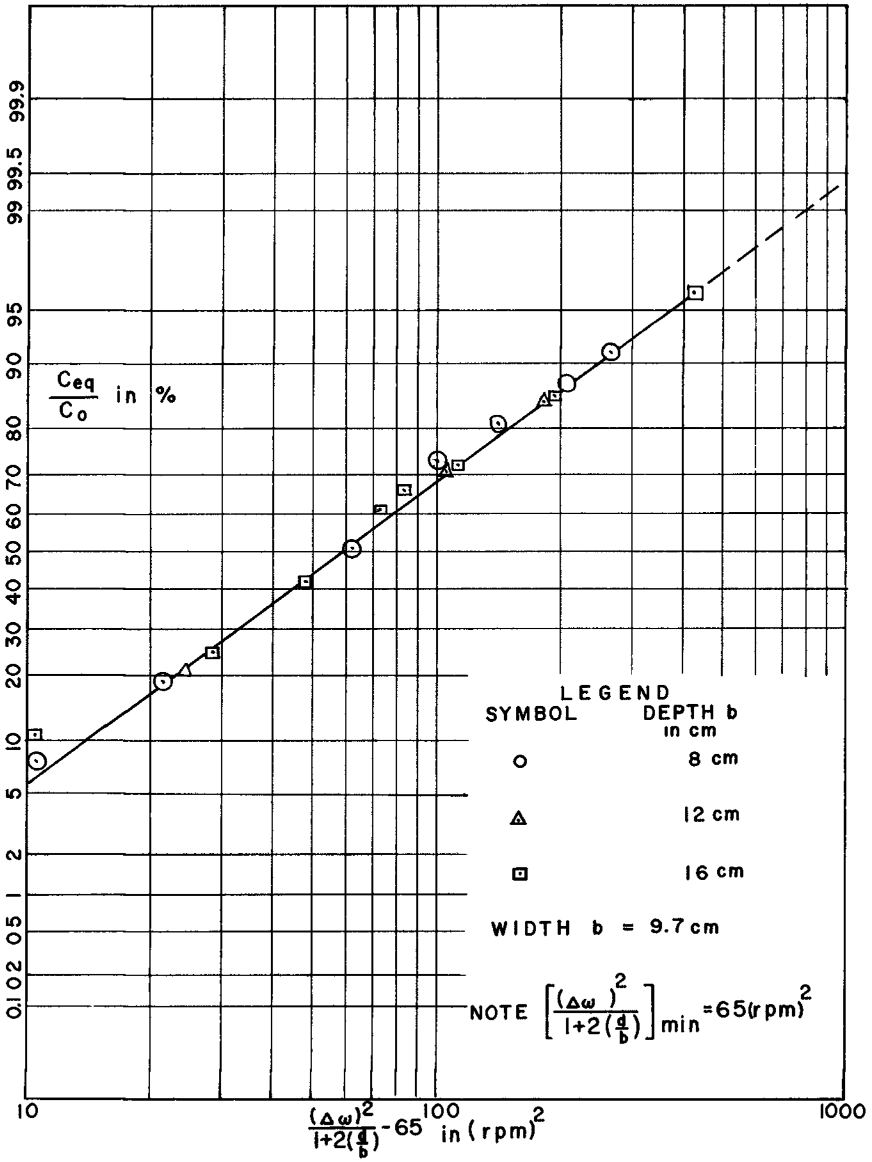

Fig 3 Relation between Relative Equilibrium Concentration and Shear Stress Parameter

weaker bonds will be disrupted and reentrained without reaching the stage of developing permanent bonds with the bed. This study and previous investigations (5,8) have clearly shown that the bed shear stresses for the initiation of erosion of deposited flocs are considerably higher than the shear stresses for which the same type of sediment deposits. The reason, or at least one of the reasons, for this phenomenon is the property of thixotropy, i.e., the gradual increase of the interparticle bonds with time. Hence, the bonds of a deposited floc, as well as its bonds with the bed will increase after deposition considerably above the flow induced shear stresses.

Therefore, the relative equilibrium concentration, $\frac{C_{eq}}{C_o}$ represents in fact the percentage of fine particles which can form flocs with sufficiently strong bonds to resist the flow induced disruptive shear stress near the bed. The remaining particles in suspension will never be able to form such flocs.

At this point a note seems appropriate about the nature of the equilibrium concentration of fine sediment suspensions in contrast to that of coarse sediment. In the latter case and as long as the sediment falls in the category of bed material load (2), a simultaneous entrainment and deposition of particles of a given size occurs. The concentration of such particles in suspension reaches a constant value when the number of particles eroded per unit bed area and unit time is equal to the number of particles deposited per unit area and unit time. At this point it is said that the sediment transport capacity of a given flow for a particular sediment size is saturated.

In the case of fine sediments no such flow saturation exists. If deposited fine particles and flocs could be simultaneously deposited and resuspended, then the rate of their reentrainment should be constant for given flow. However, the rate of deposition does increase with concentration. Hence increasing the overall initial concentration $C_o$ would result in an increase of the deposition rate without increasing the erosion rate. As a result the suspended sediment concentration would reduce to the point when the deposition rate would be made equal to the constant erosion rate; that is the $C_{eq}$ should then be independent of the initial concentration $C_o$.

Another series of experiments was performed to test the validity of the hypothesis that no exchange between the bed and the suspension occurs after the equilibrium concentration has been reached. A suspension of 1835 gm/liter was allowed to deposit to an equilibrium concentration of 983 ppm, giving $\frac{C_{eq}}{C_o} = 0.535$. After equilibrium, the channel was slowly flushed, by simultaneously (and slowly, as not to disturb the bed) draining the suspension and replenishing the tank with clear water at the same rate, while maintaining the channel and ring speeds. If no resuspension occurs, the concentration in the channel should decrease according to $C(t) = C_{eq}\, e^{-(Q/V)t}$, where $Q$ is the flushing rate and $V$ is the volume of water in the tank. Measurements of suspended sediment concentration during the flushing process, given in Table I, show this to be the case.

Table I

Suspended Sediment Concentrations vs. Time

| t<br>min | Calculated C(t)<br>ppm | Measured C(t)<br>ppm |
|---|---|---|
| 0 | 983 | 983 |
| 10 | 796 | 800 |
| 20 | 644 | 650 |
| 30 | 520 | 500 |
| 40 | 420 | 400 |
| 50 | 341 | 300 |
| 60 | 275 | 300 |

The flushing was continued until a concentration of 200 ppm in suspension was reached, when flushing ceased. The channel was still rotating at the same speed, and samples of suspension taken some time later showed no further change in concentration, indicating that the bed was not eroding.

Next, the channel was stopped, and the water agitated with an electric mixer to resuspend the bed, yielding a "second initial concentration", $C'_o$, of 945 ppm. After sampling, the channel was set running at the same speed as before, until again equilibrium was reached, at a "second equilibrium concentration", $C'_{eq}$ of 340 ppm, for a ratio of $\frac{C'_{eq}}{C'_o}$ of 0.36.

It is instructive to account for the quantities of sediment involved in the processes outlined above. Based on a tank volume of 19.9 liters, the weights can be calculated from the concentrations as follows: The initial dose was 36.5 g of clay. At the first equilibrium, 19.6 g was still in suspension; thus 16.9 g had deposited. After flushing to 200 ppm, 4.0 g was left in suspension. When the bed was resuspended, however, only 18.8 g was in the tank, compared with 16.9 + 4.0 = 20.9 g; this indicates that 2.1 g of bed material was eroded during the flushing process. The fact that the concentration didn't change after the flushing stopped, however, suggests that this erosion may have been due to the disturbance of the flushing process. After the second settling period, 6.8 g was left in suspension, with 12.0 g deposited.

The difference between $\frac{C_{eq}}{C_o}$ (0.535) and $\frac{C'_{eq}}{C'_o}$ (0.36) suggest that in the flushing process, much of the clay unable to settle was removed; however, some must have been entrained in the bed, or the bed-forming flocs, since if all the material that had settled initially (16.9 g), (less that eroded during flushing, 2.1 g) had settled again, the value of

$\frac{C'_{eq}}{C'_o}$ would have been $\frac{18.8 - 14.8}{18.8} = 0.21$ instead of 0.36. Alternately, the intense agitation used in the resuspension of the bed may have disrupted the flocs sufficiently to alter their ability to settle.

## II. Deposition Rates

The deposition rates were also studied on the basis of limited time-concentration data. Therefore the presented conclusions should be considered as tentative. Two approaches were used: In the first approach the instantaneous suspended sediment concentration, $C(t)$, was plotted versus the time, $t$, in minutes, on logarithmic paper. Fig. 4 shows the plot for the 8 cm. depth. It is seen that the data falls on straight lines of various slopes. These lines become abruptly horizontal when the point of equilibrium concentration is reached. The data for the 12 cm. and 16 cm. cases give similar plots (1). The equation describing the time deposition relation is of the form:

$$C(t) = K_1 t^{\alpha} \tag{8}$$

where $\alpha$ is the slope of the lines in Fig. 4 and $K_1$ is a reference concentration value which may be obtained for any arbitrary reference time, $t_r$. Since eq. 8 is dimensional, it is valid for $C$ in ppm and $t$ in minutes. Moreover, all the curves were obtained for an initial concentration $C_o = 8{,}020$ ppm.

Fig. 5 shows a plot of concentration-time data for constant speed, 16 cm. depth and variable initial concentration $C_o$. The points again plot very nearly on straight and parallel lines. So do similar data for the 8 cm. depth case. It may be concluded, therefore, that the exponent $\alpha$ is constant for constant flow conditions and independent for the suspended sediment concentration.

The various values of $\alpha$ were next plotted versus the shear stress parameter, $P_\omega = \frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}}$ on a logarithmic paper (Fig. 6). It is seen that the points are concentrated near a straight line which indicates a strong correlation of $\alpha$ with $P_\omega$ and, therefore, with the bed shear stress, $\tau_b$. This relation has the form

$$\alpha = K_2 \left[\frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}}\right]^{\beta} \tag{9}$$

From Fig. 6 it was found that $\beta = -2.21$ and $K_2 = -9{,}760$.

Introducing the value of $P_\omega$ from equation (3) we obtain:

$$\alpha = -2.14 \times 10^{-4}\, \tau_b^{-1.84} \tag{10}$$

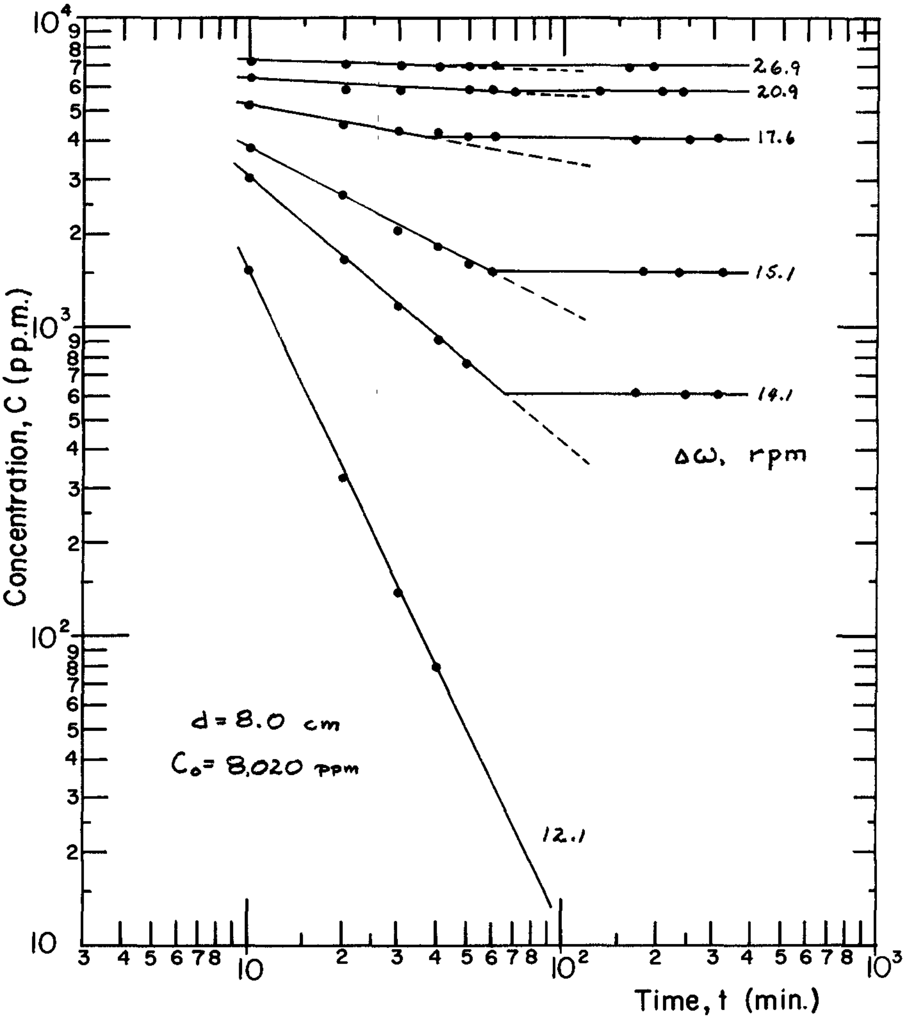

Fig. 4 Concentration-Time Relationships

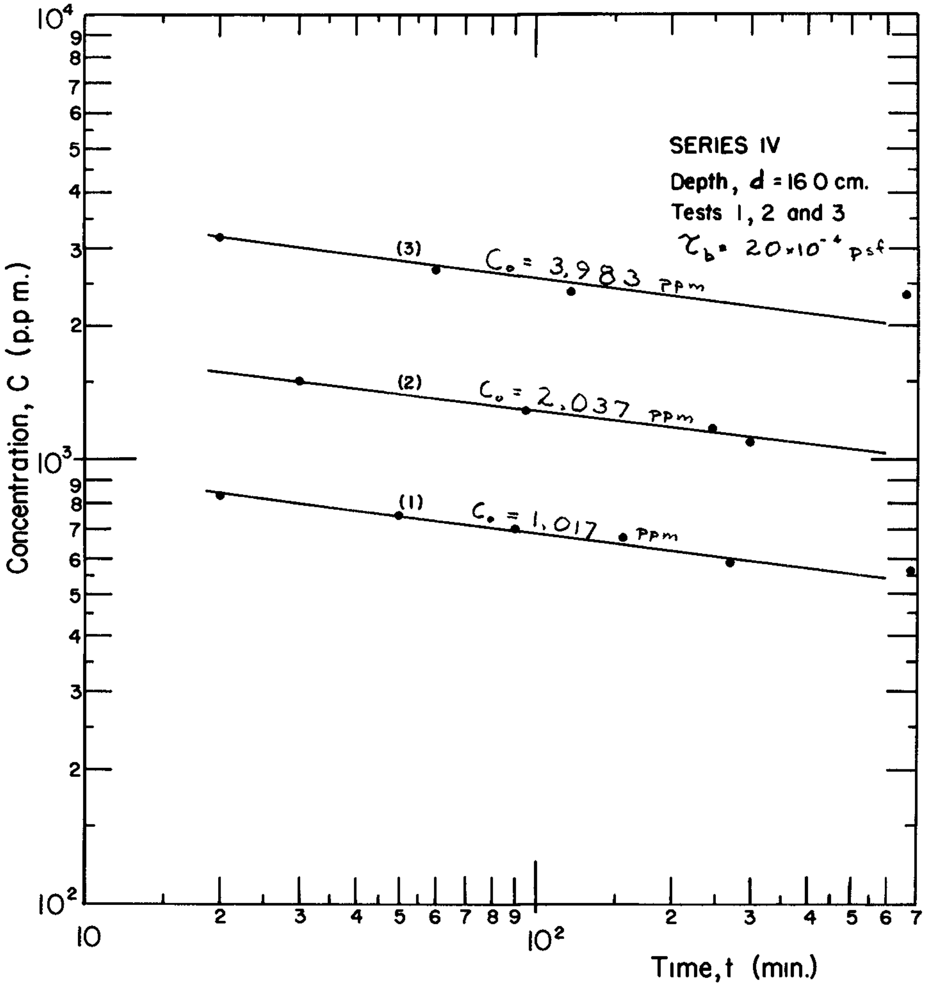

Fig 5 Concentration-Time Relationships

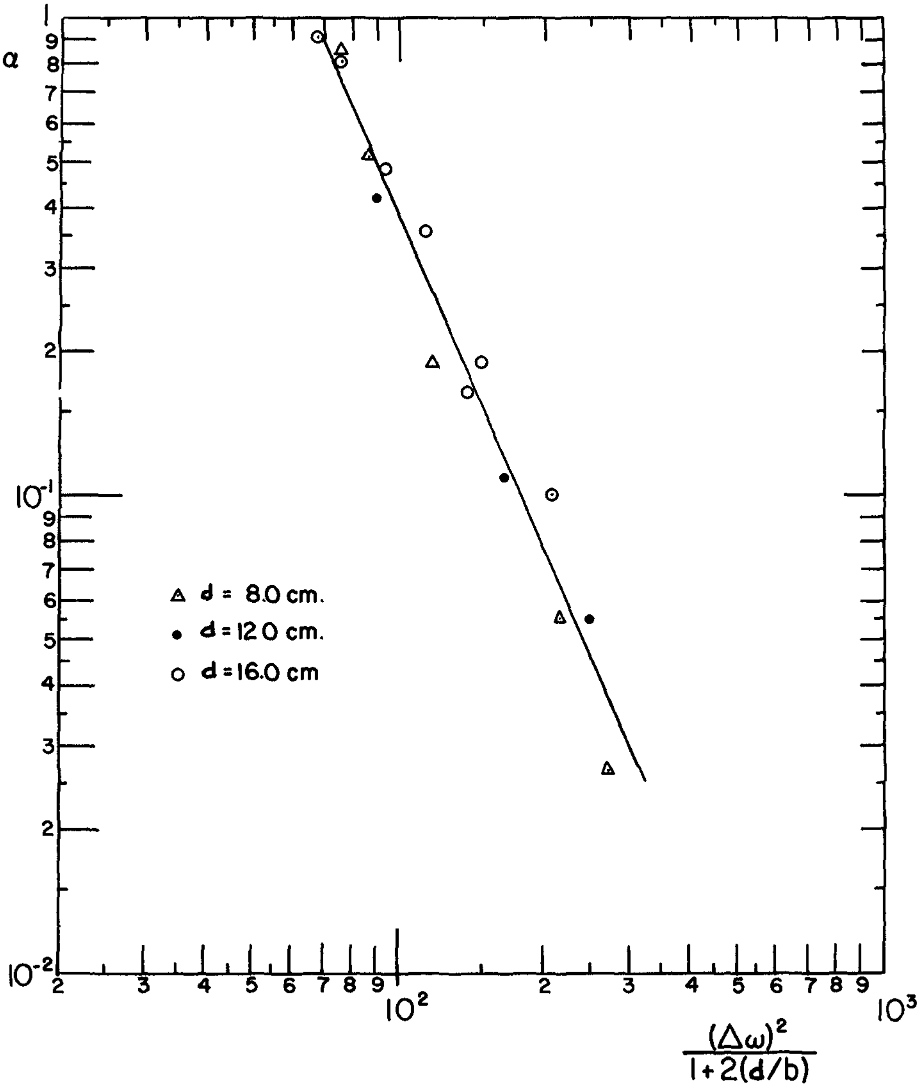

Fig. 6 Variation of Exponent α with Shear Stress Parameter

The coefficient $K_1$ can be expressed in terms of the concentration at 10 minutes, the time at which the first concentration sample was taken i.e., $K_1 = \frac{C_{10}}{(10)^{\alpha}}$. A close correlation esixts between $\frac{K_1}{C_o}$ and the shear stress parameter as shown in Fig. 7. Hence equation (8) becomes

$$C(t) = C_o \, f(\tau_b) \, t^{-2.14 \times 10^{-6} \, \tau_b^{-1.84}} \tag{11}$$

That is at any time $t$, the instantaneous concentration $C(t)$ is proportional to the initial concentration $C_o$ and a function of time and the bed shear stress $\tau_b$.

In the second approach the relative instantaneous concentration $C = \frac{C_o - C(t)}{C_o - C_{eq}}$, that is the fraction of $C_o - C_{eq}$, deposited at time $t$ after the beginning of deposition, was plotted versus log t (Fig. 8) for a depth of 16 cm. and $\tau_b = 0.0020$ psf. The plot gives a set of almost parallel straight lines corresponding to various initial concentrations. The extrapolated values of $C'$ at $t = 10$ min. plotted versus log $C_o$ give another straight line (Fig. 9). From Figs. 8 and 9 the following equation is obtained:

$$C' = \frac{C_o - C(t)}{C_o - C_{eq}} = -0.592 + 0.135 \log C_o + 0.455 \log t \tag{12}$$

The same plot of data for constant $C_o$ but varying speeds for depths 8 cm. and 16 cm. indicated a rather wide scattering but no correlation with $\tau_b$. The average lines through these points are shown in Fig. 8. They also have a slope of 0.455. However, the other constants, which depend also on the water chemistry, differ. These last data were from experiments with tap water, whereas the ones eq. 12 are based upon, were from tests with distilled water.

The rate of deposition is given by:

$$\frac{dC(t)}{dt} = -\frac{0.198}{t} \, C_o \left(1 - \frac{C_{eq}}{C_o}\right) \tag{13}$$

where $\frac{C_{eq}}{C_o}$ is a function of the bed shear stress according to eq. 7.

Hence both approaches give a deposition rate proportional to the initial concentration, $C_o$ and to a function of time and the bed shear stress.

All time-deposition equations are based on data between $t = 10$ and 300 minutes. The lowest measured values of $C'$ were of the order of 0.5. More detailed investigations are needed in this direction.

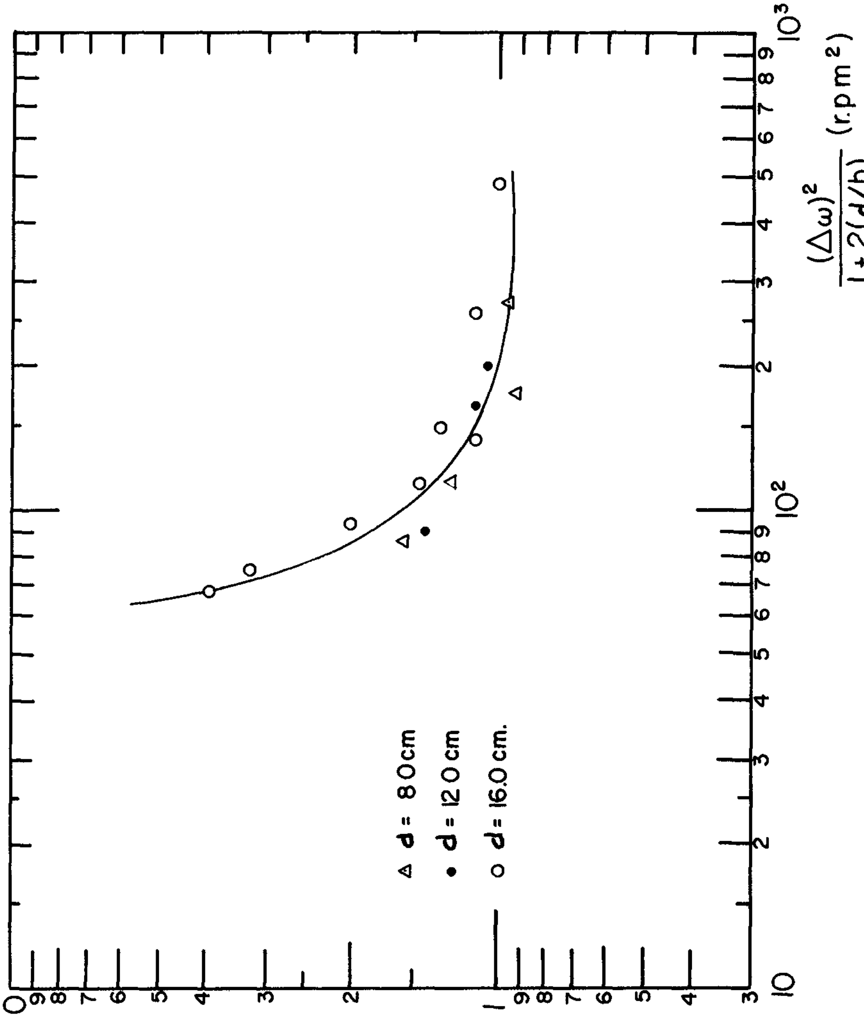

Fig. 7 Variation of $K_1/C_o$ with Shear Stress Parameter

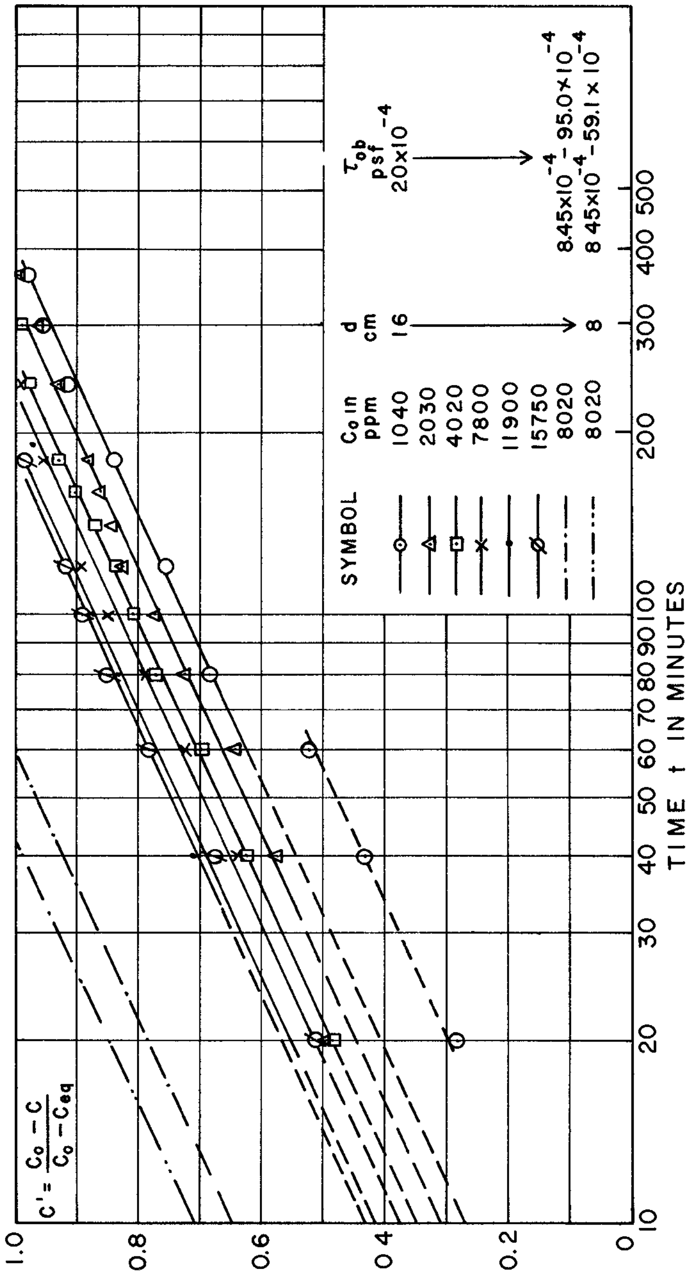

Fig. 8 Variation of Relative Concentration $\frac{C_o - C(t)}{C_o - C_{eq}}$ with Time

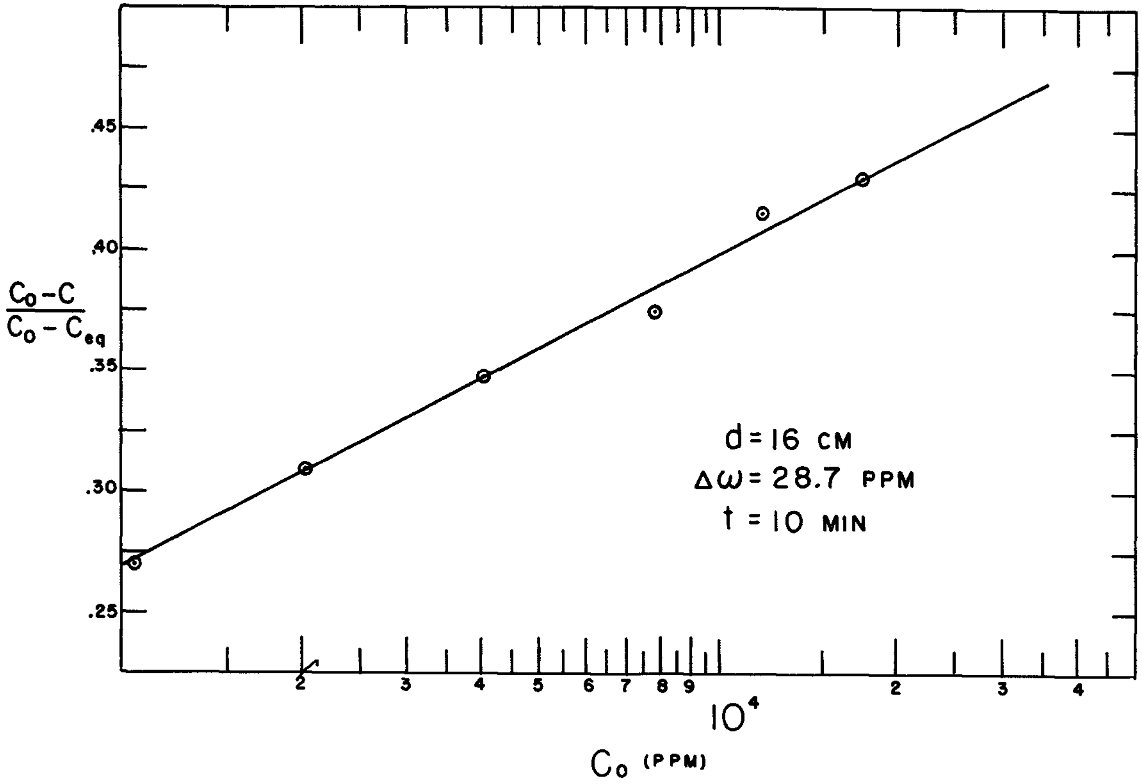

Fig. 9 Variation of Relative Concentration $\frac{C_o - C(t)}{C_o - C_{eq}}$ at 10 Minutes with Initial Concentration $C_o$

## SUMMARY AND CONCLUSIONS

The recent experiments with kaolinite clay suspensions in the rotating annular channel revealed the following new important depositional characteristics of fine sediments in turbulent flows.

1. The shear stress at the bottom of the channel was found to be almost uniform across the channel as long as the deposition was uniform. Hence by a proper regulation of the speeds of the channel and the ring a two-dimensional flow can be well approximated.
2. For the experimental range of the channel depth, the average bed shear stress $\tau_b$ is very nearly proportional to the $\tau_{ch}^{1.20}$. It may be concluded, therefore, that this bottom shear stress is the variable controlling the relative equilibrium concentration, $\frac{C_{eq}}{C_o}$.
3. The relative equilibrium concentration is related to the shear stress parameter $\frac{(\Delta\omega)^2}{1 + 2\frac{d}{b}}$ and to the bed shear stress $\tau_b$ by log-normal relations given by equations (5) and (7). A qualitative argument as to why the bed shear stress is the controlling variable has been discussed.
4. Slow and gradual flushing of the suspended sediment at equilibrium confirmed the earlier deduced fact that after equilibrium has been achieved erosions and deposition of fine sediment cannot take place simultaneously. The nature of the equilibrium concentration of fine material in relation to that for coarse sediment has been discussed.
5. Limited time-concentration data revealed that the instantaneous suspended sediment concentration and the deposition rates depend on the initial concentration and the bed shear stress.

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