

FURUTA INEQUALITY OF INDEFINITE TYPE

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Abstract. In this article, we study matrix inequalities on an (indefinite) inner product space, including a generalization of Furuta inequality: let A, B be J -selfadjoint matrices with non-negative eigenvalues and $I \geq J A \geq J B$. Then for each $r \geq 0$,

$$(A^{\frac{r}{2}} A^p A^{\frac{r}{2}})^{\frac{1}{q}} \geq J (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{1}{q}}$$

holds for $p \geq 0, q \geq 1$ with $(1+r)q \geq p+r$.

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