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Fusion-Reactor Plasmas with Polarized Nuclei*

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Nuclear fusion rates can be enhanced or suppressed by polarization of the reacting nuclei. In a magnetic fusion reactor, the depolarization time is estimated to be longer than the reaction time.

The dependence of nuclear fusion reactions on nuclear spin[1] suggests that polarization of the reacting particles may provide some control of the reaction rates and the angular distribution of the reaction products. The large cross section for the reaction $D(T,n)He^4$ at low energy arises primarily from a $J = 3/2^+$ resonant level of He^5 at 107 keV above the energy of the free D and T nuclei.[2] At low energies, the reaction occurs only in the $\ell = 0$ state, so that the angular momentum must be supplied by the spin of the D and T nuclei. Since D has spin 1 and T spin 1/2, their possible combined spin states are $S = 3/2$ and $1/2$. The reaction is due almost entirely to interacting pairs of D and T nuclei with $S = 3/2$. The statistical weight of this state is 4 while that of the $S = 1/2$ state is 2. Thus, for a plasma of unpolarized nuclei, effectively only 2/3 of the interactions contribute to the reaction rate.

We consider now the case of a magnetic D-T reactor where the fractions of D nuclei polarized parallel, transverse and antiparallel to $\vec{B}$ are d_+, d_o, and d_-, respectively, while the corresponding fractions of the T nuclei are t_+ and t_-. Then the total cross section is

$$\sigma = \left(a + \frac{2}{3}b + \frac{1}{3}c\right) f\sigma_o + \left(\frac{2}{3}b + \frac{4}{3}c\right)(1 - f)\sigma_o \qquad (1)$$

where $a = d_+t_+ + d_-t_-$, $b = d_o$, $c = d_+t_- + d_-t_+$, and $f\sigma_o$ is the cross section for the $3/2^+$ state. The magnitude of f has been estimated at about 0.95[3], but may be greater than 0.99.[4] (The remainder of the cross section is ascribed to a $1/2^+$ state that lies 3 MeV above the $3/2^+$ state.) For an unpolarized plasma, $a = b = c = 1/3$ so that $\sigma = 2/3\sigma_o$. On the other hand, if all the nuclei are polarized along $\vec{B}$, then $a = 1$, $b = c = 0$, and $\sigma = f\sigma_o$, so that the enhancement of reactivity is $(3/2)f$.

The resultant angular distributions of the neutrons and alpha particles are:

$$\frac{d\sigma}{d\Omega} = \frac{f\sigma_o}{2\pi}\left[\frac{3}{4}a \sin^2\theta + \left(\frac{2}{3}b + \frac{1}{3}c\right)\left(\frac{(4/f) - 3 + 3\cos^2\theta}{4}\right)\right] \qquad (2)$$

where θ is the pitch angle relative to $\vec{B}$. If all the nuclei are polarized parallel to $\vec{B}$, the angular distribution of the neutrons and alphas is $\sin^2\theta$; if the D nuclei are polarized transverse to $\vec{B}$, then the distribution is $(4/f) - 3 + 3\cos^2\theta$. The polarization of the neutrons also varies with θ. At $\theta = 90°$, it is given by

$$n_+ - n_- = \frac{3}{4}\left(d_-t_- - d_+t_+\right) + \frac{1}{6}d_o\left(t_+ - t_-\right) + \frac{1}{12}\left(d_+t_- - d_-t_+\right) \qquad (3)$$

where n_+ and n_- are the fractions of neutrons polarized parallel and antiparallel to $\vec{B}$. (We have set $f = 1$.)

The D-D reaction is more complex than the D-T reaction and its properties are less well known; therefore, we can give only an indication of the potential effects of polarization. For this purpose, we will follow Ref. 5, which points out that the possible initial states for the reacting pairs of deuterons are ^1S, ^5S, ^3P, ^1D, ^5D, etc., and attributes the majority of the reactions to the ^1S and ^3P states. Fitting this theory to observed total and differential cross sections for D-D, Ref. 5 obtains a total reaction cross section that is proportional to $9f_1P_o + 21f_3P_1$, where P_o and P_1 are probabilities for penetration of the barrier (Coulomb and centrifugal), f_1 is the fraction of pairs in the ^1S singlet state and f_3 is the fraction in the ^3P triplet state. For unpolarized D nuclei, $f_1 = 1/9$ and $f_3 = 1/3$; the enhancement factor due to polarization can be written in the form

$$\varepsilon = \left(9f_1P_o + 21f_3P_1\right)/\left(P_o + 7P_1\right). \qquad (4)$$

Table I uses values of P_1/P_o given in Ref. 5 to calculate enhancement factors for three different types of polarization: (a) The deuterons are oriented parallel and antiparallel to $\vec{B}$ (as would be appropriate in a colliding beam reactor) so that $f_1 = 1/3$, $f_3 = 1/2$. (b) They are oriented transverse to $\vec{B}$, so that $f_1 = 1/3$, $f_3 = 0$. (c) They are all oriented parallel, so that $f_1 = f_3 = 0$. In the present rudimentary estimate, enhancement factors of about 2.5 are found for cases (a) and (b), while case (c) suppresses the reaction. A more accurate treatment of the ^3P state, taking the angular momentum J into account, would tend to reduce ε for cases (a) and (b).

There would be little practical value in polarizing nuclei if the depolarization rates were rapid compared with the fusion reaction rate. At first sight, it would appear that, because of the small energy difference between the two polarization states ($\Delta E \simeq 10^{-7} - 10^{-6}$ eV << kT), an unpolarized equilibrium would be rapidly established. However, as far as we can see, the mechanisms for depolarization of nuclei in a magnetic fusion reactor are suprisingly weak. We will consider four such mechanisms:

(1) Inhomogeneous static magnetic fields. Let $\omega_2 = eB_o/2m_pc$ be the deuteron cyclotron frequency, and let $\Omega_2 \equiv \Delta E/\hbar = g_2eB_o/2m_pc$ be the deuteron precession frequency, where ΔE is the Zeeman energy for a change of spin orientation $\Delta m = 1$, and g_2 is the magnetic moment of the deuteron in nuclear magnetons. Similarly, let Ω_3 and g_3 be the precession frequency and magnetic moment of the triton. Then $\Omega_2 = 0.86\ \omega_2$, and $\Omega_3 = 5.96\ \omega_2$. If a nucleus with velocity v passes through static magnetic-field inhomogeneities of scale s, it sees them at a frequency v/s. As in the case of the

adiabaticity of the ordinary magnetic moment of the particle gyromotion, frequencies below the nuclear precession frequency -- i.e., static inhomogenities on a scale that is large compared with the ion gyroradius ($s >> \rho_i$) -- cannot change the polarization.

(2) Binary collisions. Simple electrostatic Coulomb scattering does not affect the nuclear spins, but there are many other potential depolarization mechanisms: The triton can interact with electrons, deuterons and other tritons by spin-orbit and spin-spin interactions; the deuteron can also interact by means of its quadrupole moment. Fortunately, the associated depolarization rates turn out to be quite small.[6] During each collision, the change in polarization from state α to state β is small and of random sign. We have calculated the cross section of σ_i for the rate of increase

$$\frac{d\beta^2}{dt} = n\sigma_i v_{rel} \tag{5}$$

by process i, where n is the particle density and v_{rel} is the relative velocity. The cross sections for interaction with electrons are found to be of the same order as for ions; because of the factor v_{rel} in eq. (5), depolarization by electrons therefore predominates. For spin-orbit depolarization of T, we have $\sigma_i = (4\pi/3)g_3^2 r_p^2 \ell n(c/\omega_p ƛ) = 1.7 \quad 10^{-29}$ cm^2, where $r_p \equiv (e^2/m_p c^2$, $\omega_p^2 = (4\pi n e^2)/m_e$ and $ƛ = \hbar/m_e v$. For spin-spin depolarization, $\sigma_i = (11/9)\pi g_3^2 r_p^2 = 8 \times 10^{-31}$ cm^2. For the d_o state of D, σ_i is smaller by $(g_2/g_3)^2 = 0.083$ than for T; for the d_+ or d_- states, it is smaller by $1/2(g_2/g_3)^2 = 0.042$. Interaction with the quadrupole moment is negligible for electrons. Using typical reactor parameters, $n = 2 \times 10^{14}$ cm^{-3}, $T = 10^4$eV, we find the rate of depolarization to be 2.1×10^{-5} s^{-1} for T, 1.75×10^{-6} s^{-1} for the d_o state of D, and 0.9×10^{-6} s^{-1} for the d_+ or d_- state of D. These rates are small compared with the typical 1 s^{-1} rate for fusion energy multiplication or the 10^{-2} s^{-1} rate for complete fuel burn-up. There is also a contribution from elastic nuclear scattering, which we estimate at $\Delta\beta^2 \lesssim 10^{-4}$ per fusion event.

(3) Magnetic fluctuations. A polarized moving nucleus will tend to be depolarized by those harmonics of the fluctuating fields which are left-circularly polarized with respect to $\vec{B}$, if the Doppler-shifted frequency in the frame of the nucleus is equal to its precession frequency. Defining the intensity of magnetic fluctuations as I_ω, where $(\delta\tilde{B})^2 \equiv \int I_\omega d\omega$, then

$$\frac{d\beta^2}{dt} = \left(\frac{ge}{2m_p c}\right)^2 I_\omega(\Omega) = \frac{(ge\tilde{B}/2m_p c)^2}{\Delta\omega} \tag{6}$$

where $\Delta\omega$ is the band width around Ω over which $\tilde{B}^2$ extends in the frame of the nucleus.[7] The resonant frequency in the laboratory frame is $\omega = \Omega_i - k_z v_z - n\omega_i$, where k_z is the component along $\vec{B}$ of the wave number of the fluctuation. The cyclotron frequency term $n\omega_i$ in this equation ($n = 0, \pm 1, \pm 2$, etc.) is produced by the gyromotion of the nucleus, with the amplitude of the higher harmonics seen by the nucleus reduced by $J_n(k_\perp \rho_i)$. In thermal equilibrium, plasma fluctuations are very small: For a 10^4 eV Planck spectrum of e-m waves, we find that $d\beta^2/dt \sim 10^{-14}$s^{-1}.

A depolarization rate sufficiently large so as to prevent reactor operation (i.e., $d\beta^2/dt \gtrsim 1\ s^{-1}$) would imply $\tilde{B} \gtrsim 3(\Delta\omega/\Omega)^{1/2}$ gauss in the case of either D or T. For a highly non-Maxwellian plasma velocity distribution, microinstabilities around the deuteron cyclotron frequency could indeed give rise to significant depolarization through direct interaction (n = o) with the precession of the deuteron ($\Omega_2 = 0.86\omega_2$). In a roughly Maxwellian plasma, however, such waves are strongly damped, so that their amplitude should be small. Spatial gradients of plasma temperature and density tend to excite lower-frequency field-perturbations with longer wavelengths, which could interact through higher-n resonances. For example, with n = -1, the Ω_2-resonance can occur for a transverse Alfvén wave at $\omega = 0.15\omega_2$, while the higher-frequency triton precession ($\Omega_3 = 5.96\omega_2$) could resonate with a whistler mode propagating at an angle to $\vec{B}$. Because of the complexity of the plasma wave spectrum, it is difficult to place detailed upper limits on "anomalous" depolarization in a magnetic fusion reactor, but for a moderately close approach to thermal equilibrium (i.e., avoidance of steep gradients, especially in velocity space), the desired degree of quiescence seems likely to be attainable.

(4) Atomic effects. The polarized nucleus of a hydrogenic atom is not depolarized by ionization, but if recombination (or charge-exchange) couples the nucleus to an electron of opposite spin, it can be depolarized with 50% probability. This process, however, is inhibited by an external magnetic field B sufficiently strong compared with the critical field B_c at which the Zeeman splitting equals the hyperfine splitting: The probability of spin exchange is then reduced[8] by the factor $(B_c/2B_o)^2$. Since B_c is only of order

3×10^2 gauss for D and 10^3 gauss for T, multiple processes of recombination into atomic hydrogen, followed by reionization, could take place in a 5×10^4 gauss field without significant depolarization. (Recombination into molecular hydrogen, however, could expose the nucleus to more rapid depolarization by spin-orbit coupling associated with the molecular tumbling.)

One obvious economic advantage of polarizing the nuclear fuel of a reactor is the enhancement of fusion power (1.5 for D-T, 2-2.5 for D-D). This enhancement would be particularly helpful for small-sized reactors with intrinsically low power multiplication.[9] The ability to suppress reactions is also of practical value: For example, if the nuclei of a $D\text{-}He^3$ fuel mixture are all polarized parallel to $\vec{B}$, the D-D reaction rate will be suppressed, while the $D\text{-}He^3$ rate is enhanced by 1.5 (similar to D-T). In this way, it is possible to approximate a neutron-free fusion reactor without resorting to high-temperature, low-power processes such as p-Li.

In the case of the D-T reaction, the ability to control the anisotropy of the emitted alpha particles allows enhancement of the fraction trapped into well-confined orbits ($d_+ t_+$ being favorable for mirror machines and d_o for tori) and improvement of MHD stability properties (d_o being favorable for tori). Control of alpha-driven plasma currents and microinstabilities may also be possible. Reactor shielding and blanket design would benefit: e.g., in tori, tangential emission (the d_o case) could minimize the neutron load on the constricted small-major-radius side of the vessel. The polarization of the neutrons should prove useful in research.

A fusion reactor could be fueled with polarized atomic hydrogen gas (e.g., using the polarizing method of Ref. 10), or with polarized neutral beams (produced by charge-exchange of polarized ion beams in a longitudinal magnetic field). Injection of polarized frozen hydrogen pellets would be attractive, but appears problematical -- as does the use of polarized targets for inertial fusion.

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TABLE I

Enhancement Factors for the D-D Reaction

E(keV)	P_1/P_o	ε(a)	ε(b)	ε(c)
0	0.008	2.92	2.83	0
100	0.021	2.81	2.62	0
200	0.033	2.72	2.43	0
300	0.054	2.59	2.18	0

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