Fuzzifying Allen's Temporal Interval Relations

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***Abstract*—When the time span of an event is imprecise, it can be represented by a fuzzy set, called a fuzzy time interval. In this paper, we propose a framework to represent, compute, and reason about temporal relationships between such events. Since our model is based on fuzzy orderings of time points, it is not only suitable to express precise relationships between imprecise events ("Roosevelt died *before* the beginning of the Cold War") but also imprecise relationships ("Roosevelt died *just* before the beginning of the Cold War"). We show that, unlike previous models, our model is a generalization that preserves many of the properties of the 13 relations Allen introduced for crisp time intervals. Furthermore, we show how our model can be used for efficient fuzzy temporal reasoning by means of a transitivity table. Finally, we illustrate its use in the context of question answering systems.**

***Index Terms*—Fuzzy ordering, fuzzy relation, interval algebra, question answering, temporal reasoning.**

I. Introduction

TEMPORAL representation and reasoning is an important facet in the design of many intelligent systems. For example, question answering systems require at least some basic temporal representation scheme to answer simple temporal questions such as "When was Franklin Roosevelt born?" To enable question answering systems to answer more complex temporal questions, considerable effort has been made to extract temporal information from natural language texts (e.g., [1], [12], [15], [16], and [23]–[25]) and to analyze complex temporal questions (e.g., [22]). However, temporal relationships expressed in natural language are often vague, e.g., "Roosevelt died *just* before the end of the Second World War." Moreover, historic time periods are more often than not characterized by a gradual beginning and/or ending [17]. The traditional temporal reasoning formalisms need to be extended to cope with this kind of vagueness, which is inherently associated with real–world temporal information.

One of those well-known formalisms is Allen's temporal interval algebra [3]. Allen defined a set of 13 qualitative relations that may hold between two compact intervals $A = [a^-, a^+]$ and $B = [b^-, b^+]$. Table I shows how Allen expressed these precise relations by means of constraints on the boundaries of the crisp intervals involved. In this paper, we extend Allen's work to a more general formalism that can handle precise as well as imprecise relationships between crisp and fuzzy intervals.

TABLE I
Allen's Temporal Interval Relations Between Intervals $A = [a^-, a^+]$ and $B = [b^-, b^+]$

Name	Definition		
before	$b(A,B)$	$\equiv$	$a^+ < b^-$
overlaps	$o(A,B)$	$\equiv$	$a^- < b^-$ and $b^- < a^+$ and $a^+ < b^+$
during	$d(A,B)$	$\equiv$	$b^- < a^-$ and $a^+ < b^+$
meets	$m(A,B)$	$\equiv$	$a^+ = b^-$
starts	$s(A,B)$	$\equiv$	$a^- = b^-$ and $a^+ < b^+$
finishes	$f(A,B)$	$\equiv$	$a^+ = b^+$ and $b^- < a^-$
equals	$e(A,B)$	$\equiv$	$a^- = b^-$ and $a^+ = b^+$
after	$bi(A,B)$	$\equiv$	$b(B,A)$
overlapped-by	$oi(A,B)$	$\equiv$	$o(B,A)$
contains	$di(A,B)$	$\equiv$	$d(B,A)$
met–by	$mi(A,B)$	$\equiv$	$m(B,A)$
started–by	$si(A,B)$	$\equiv$	$s(B,A)$
finished–by	$fi(A,B)$	$\equiv$	$f(B,A)$

Our first concern is generalizing the definitions of the qualitative relations of Table I to make them applicable to fuzzy intervals as opposed to only crisp intervals. Indeed, when an event is characterized by a gradual beginning and/or ending, it is natural to represent the corresponding time span as a fuzzy set, which we call a fuzzy (time) interval. Depending on the intended application, this fuzzy set can either be defined by an expert (e.g., [17] and [20]) or constructed automatically (e.g., [27]). Since we cannot refer to the gradual beginning and endings of a fuzzy interval in the same way we refer to the boundaries of a crisp interval, we first have to provide a way to express that, for instance, the beginning of a fuzzy interval A is before the beginning of a fuzzy interval B (as needed in the definition of the qualitative relation "overlaps"). We suggest to do this by measuring the highest extent to which there exists a time point in A that occurs before all the time points in B. In general, in our approach, qualitative relations between fuzzy intervals are defined in terms of the ordering of the gradual beginning and endings of these intervals, which in turn are defined in terms of the ordering of the time points belonging to these intervals. The resulting qualitative relations between the fuzzy intervals are gradual, i.e., they may hold to some degree only; hence the name fuzzy temporal interval relations. When A and B are crisp, our approach reduces to Allen's work.

Our second goal is providing a means to model *imprecise* relations to be able to express that, for instance, event A took place *just* before event B, or that A occurred *long* after B. Although these kind of relations are not considered in Allen's original model, in our approach we arrive at them quite elegantly by using imprecise orderings of time points in the model sketched above. The resulting approach is applicable again to both crisp and fuzzy time intervals.

This paper is organized as follows. In the next section, we review related work concerning fuzzifications of Allen's interval relations. In Section III, we show how imprecise relationships

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