

Fuzzy Assignment Problem with Generalized Fuzzy Numbers

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Abstract

In this paper, we present new algorithms in classical and linear programming for fuzzy assignment problem with fuzzy cost based on the ranking method. The fuzzy cost is measured as generalized fuzzy number. We developed the classical algorithm using fundamental theorems of fuzzy assignment problem to obtain minimum fuzzy cost and also for the variations in the fuzzy assignment problem. Proposed algorithms are illustrated with an example. The proposed algorithms are easy to understand and apply to find the optimal fuzzy cost occurring in the real life situations. There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the assignment problems.

Keywords: Fuzzy assignment problem; ranking fuzzy numbers; generalized fuzzy numbers; fuzzy linear programming.

1. Introduction

The assignment problem is to resolve the problem of assigning a number of origins to the equal number of destinations at a minimum cost or maximum profit. It can assign persons to jobs, classes to rooms, operators to machines, drivers to trucks, trucks to delivery routes, or problems to research teams, etc. To

find solutions to assignment problems, various algorithms such as linear programming [1-4], Hungarian algorithm [5], neural network [6], genetic algorithm [7] have been developed. Over the past 50 years, many variations of the classical assignment problems are proposed e.g. bottleneck assignment problem, generalized assignment problem, quadratic assignment problem etc. In recent years, fuzzy transportation and fuzzy assignment problems have received much concentration. Lin and Wen [8] proposed an efficient algorithm based on the labeling method for solving the linear fractional programming case. The elements of the cost matrix of the assignment problem are considered as subnormal fuzzy intervals with increasing linear membership functions, whereas the membership function of the total cost is a fuzzy interval with decreasing linear membership function. Sakawa et al. [9] solved the problems on production and work force assignment in a firm using interactive fuzzy programming for two level linear and linear fractional programming models. Chen [10] projected a fuzzy assignment model that considers all persons to have same skills. Longsheng Huang and Li-pu Zhang [11] developed a mathematical model for the fuzzy assignment problem and transformed the model as certain assignment problem with restriction of qualification. Chen Liang-Hsuan and Lu Hai-Wen [12] developed a procedure for resolving assignments problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model to determine the assignments with the maximum efficiency. Yuan Feng and Lixing Yan [13] developed a constrained goal programming model for two-objective k -cardinality assignment problem. Linzhong Liu and Xin Goa [14] considered the genetic algorithm for solving the fuzzy weighted equilibrium and multi-job assignment problem. Majumdar and Bhunia [15] developed an exclusive genetic algorithm to solve a generalized assignment problem with imprecise cost(s)/time(s). In which the impreciseness of cost(s)/time(s) are represented by interval valued numbers. Xionghui ye and Jiuping Xu [16] developed a priority based genetic algorithm to a fuzzy vehicle routing assignment model with connection network. The total costs which include preparing costs as the objective function and the preparing costs and the commodity flow demand is regarded as fuzzy variables.

Chen [17] pointed out that in many cases it is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. In most of the papers the generalized fuzzy numbers are converted into normal fuzzy numbers through normalization process [18] and then normal fuzzy numbers are used to solve the real life problems. Kaufmann and Gupta [18] pointed out that there is a serious disadvantage of the normalization process. Basically we have transformed a measurement of an objective value to a valuation of a subjective value, which results in the loss of information. Although this procedure is mathematically correct, it decreases the amount of information that is available in the original data, and we should avoid it. There are several papers [19-24] in the literature in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the fuzzy

assignment problems. We apply the ranking method defined on generalized trapezoidal fuzzy numbers [25] to rank the fuzzy cost present in the proposed fuzzy assignment problem because it has more advantages over the existing fuzzy ranking methods.

The rest of the paper is organized as follows: In section 2, we briefly introduce the basic definitions and arithmetic operations of fuzzy numbers. Section 3 presents the ranking method based on incenter of centroids. In Section 4, fuzzy assignment problem, mathematical formulation of fuzzy assignment problem and fundamental theorems of fuzzy assignment problem are reviewed. Section 5 presents fuzzy assignment algorithms in classical form and fuzzy linear programming model. In section 6, numerical example is presented to show the applications of the proposed algorithms and the total optimal fuzzy costs for the proposed algorithms are shown. Finally, the conclusion is given in section 7.

2. Basic definitions and arithmetic operations of fuzzy numbers

In this section some basic definitions and fuzzy arithmetic operations are defined.

2.1 Basic definitions

Definition 2.1: The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the Universal set X fall within a specified range i.e., $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set A .

The function $\mu_{\tilde{A}}$ is called membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2: A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x - a)}{(b - a)}, & a \leq x \leq b \\ 1 & , b \leq x \leq c \\ \frac{(x - d)}{(c - d)}, & c \leq x \leq d \end{cases}$$

Definition 2.3: A generalized fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ w & , b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Definition 2.4: A fuzzy number $\tilde{A} = (a, b, d)$ is said to be triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{(d-x)}{(d-b)}, & b \leq x \leq d \\ 0, & \text{elsewhere} \end{cases}$$

Definition 2.5: A generalized fuzzy number $\tilde{A} = (a, b, d; w)$ is said to be generalized triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{w(d-x)}{(d-b)}, & b \leq x \leq d \\ 0, & \text{elsewhere} \end{cases}$$

2.2 Fuzzy arithmetic operations

In this paper, we use fuzzy arithmetic operators shown in (i) - (iii) to deal with the fuzzy arithmetic operations between generalized fuzzy numbers.

Assume that there are two generalized trapezoidal fuzzy numbers \tilde{A}_1 and \tilde{A}_2 where $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$. The arithmetic operations between the generalized trapezoidal fuzzy numbers \tilde{A}_1 and \tilde{A}_2 are as follows:

(i) Fuzzy number addition \oplus

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= (a_1, b_1, c_1, d_1; w_1) \oplus (a_2, b_2, c_2, d_2; w_2) \\ &= (a_1 \oplus a_2, b_1 \oplus b_2, c_1 \oplus c_2, d_1 \oplus d_2; \min(w_1, w_2)) \end{aligned}$$

where $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are any real numbers.

(ii) Fuzzy number subtraction \ominus

$$\begin{aligned} \tilde{A}_1 \ominus \tilde{A}_2 &= (a_1, b_1, c_1, d_1; w_1) \ominus (a_2, b_2, c_2, d_2; w_2) \\ &= (a_1 \ominus d_2, b_1 \ominus c_2, c_1 \ominus b_2, c_2, d_1 \ominus a_2; \min(w_1, w_2)) \end{aligned}$$

where $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are any real numbers.

(iii) Fuzzy scalar multiplication

$$\begin{aligned} k\tilde{A} &= (ka_1, kb_1, kc_1, kd_1; w_1); k > 0 \\ k\tilde{A} &= (kd_1, kc_1, kb_1, ka_1; w_1); k < 0 \end{aligned}$$

3. Ranking of generalized fuzzy numbers

In this section the ranking of generalized fuzzy numbers using the incenter of the centroids [25] of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is defined:

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left(\frac{\alpha \left(\frac{a+2b}{3} \right) + \beta \left(\frac{b+c}{2} \right) + \gamma \left(\frac{2c+d}{3} \right)}{\alpha + \beta + \gamma}, \frac{\alpha \left(\frac{w}{3} \right) + \beta \left(\frac{w}{2} \right) + \gamma \left(\frac{w}{3} \right)}{\alpha + \beta + \gamma} \right) \quad (1)$$

where

$$\alpha = \frac{\sqrt{(c-3b+2d)^2 + w^2}}{6}, \quad \beta = \frac{\sqrt{(2c+d-a-2b)^2}}{3}, \quad \text{and}$$

$$\gamma = \frac{\sqrt{(3c-2a-b)^2 + w^2}}{6}.$$

As a special case, for triangular fuzzy number $\tilde{A} = (a, b, d; w)$. i.e., $c = b$ the incenter of centroids is given by

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left(\frac{x \left(\frac{a+2b}{3} \right) + yb + z \left(\frac{2b+d}{3} \right)}{x + y + z}, \frac{x \left(\frac{w}{3} \right) + y \left(\frac{w}{2} \right) + z \left(\frac{w}{3} \right)}{x + y + z} \right) \quad (2)$$

where

$$x = \frac{\sqrt{(2d-2b)^2 + w^2}}{6}, \quad y = \frac{\sqrt{(d-a)^2}}{3}, \quad \text{and} \quad z = \frac{\sqrt{(2b-2a)^2 + w^2}}{6}$$

The ranking function [25] of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$. which maps the set of all fuzzy numbers to a set of real numbers is

$$R(\tilde{A}) = (\bar{x}_0 \times \bar{y}_0) = \left(\left(\frac{\alpha \left(\frac{a+2b}{3} \right) + \beta \left(\frac{b+c}{2} \right) + \gamma \left(\frac{2c+d}{3} \right)}{\alpha + \beta + \gamma} \right) \times \left(\frac{\alpha \left(\frac{w}{3} \right) + \beta \left(\frac{w}{2} \right) + \gamma \left(\frac{w}{3} \right)}{\alpha + \beta + \gamma} \right) \right) \quad (3)$$

This is the area between the incenter of the centroids $I_{\tilde{A}}(\bar{x}_0, \bar{y}_0)$ as defined in Eq. (1) and the original point.

Mode, spread, left spread, and right spread of \tilde{A} are defined respectively as

$$m(\tilde{A}) = \frac{1}{2} \int_0^w (b+c) dx = \frac{w}{2}(b+c) \quad (4)$$

$$s(\tilde{A}) = \int_0^w (d-a) dx = w(d-a) \quad (5)$$

$$ls(\tilde{A}) = \int_0^w (b-a) dx = w(b-a) \quad (6)$$

$$rs(\tilde{A}) = \int_0^w (d-c) dx = w(d-c) \quad (7)$$

Rule 1: $R(\tilde{A}) > R(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$, $R(\tilde{A}) < R(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$, If $R(\tilde{A}) = R(\tilde{B})$ then use rule 2.

Rule 2: $m(\tilde{A}) > m(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$, $m(\tilde{A}) < m(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$, If $m(\tilde{A}) = m(\tilde{B})$ then use rule 3.

Rule 3: $s(\tilde{A}) > s(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$, $s(\tilde{A}) < s(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$, If $s(\tilde{A}) = s(\tilde{B})$ then use rule 4.

Rule 4: $ls(\tilde{A}) > ls(\tilde{B}) \Rightarrow \tilde{A} > \tilde{B}$, $ls(\tilde{A}) < ls(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$, If $ls(\tilde{A}) = ls(\tilde{B})$ then use rule 5.

Rule 5: $w_1 > w_2 \Rightarrow \tilde{A} > \tilde{B}$, $w_1 < w_2 \Rightarrow \tilde{A} < \tilde{B}$, If $w_1 = w_2$ then $\tilde{A} \approx \tilde{B}$.

The comparison of two generalized fuzzy numbers \tilde{A} and \tilde{B} can be decided using above rules 1 to 5.

4. Fuzzy Assignment Problem

Suppose there are n works to be performed and n persons are available for doing the works. Assume that each person can do each work at a time, though with unreliable grade of efficiency. Let \tilde{c}_{ij} be the fuzzy cost if the i^{th} person is assigned the j^{th} work, the problem is to find a minimum fuzzy cost with fuzzy assignment. Fuzzy assignment problem with fuzzy cost is represented in table I.

Table I: Fuzzy assignment with fuzzy cost
Works

		Works					
		1	2	...	j	...	n
Persons	1	\tilde{c}_{11}	\tilde{c}_{12}	...	\tilde{c}_{1j}	...	\tilde{c}_{1n}
	2	\tilde{c}_{21}	\tilde{c}_{22}	...	\tilde{c}_{2j}	...	\tilde{c}_{2n}

	i	\tilde{c}_{i1}	\tilde{c}_{i2}	...	\tilde{c}_{ij}	...	\tilde{c}_{in}

	n	\tilde{c}_{n1}	\tilde{c}_{n2}	...	\tilde{c}_{nj}	...	\tilde{c}_{nn}

4.1 Mathematical formulation of fuzzy assignment problem

Mathematically, the fuzzy assignment problem in table I can be stated as:

$$\text{Min } \tilde{z} = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}, i = 1, 2, \dots, n$$

subject to

$$x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^n x_{ij} = 1 \text{ (one work is done by the } i^{th} \text{ person, } i=1, 2, \dots, n) \text{ and}$$

$$\sum_{j=1}^n x_{ij} = 1 \text{ (only one person should be assigned the } j^{th} \text{ work, } j=1, 2, \dots, n)$$

where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

4.2 Fundamental Theorems of Fuzzy Assignment problem

The solution to fuzzy assignment problem is fundamentally based on the following two theorems.

Theorem 1: Fuzzy Reduction Theorem on assignment problem

The fuzzy assignment minimizes the total fuzzy cost for the new fuzzy cost matrix; it also minimizes the total fuzzy cost for the original fuzzy cost matrix. If the addition (subtraction) of a constant fuzzy number to a every fuzzy element of a row (or column) of the fuzzy cost matrix (\tilde{c}_{ij}) , where the fuzzy cost (\tilde{c}_{ij}) is represented either by normal or non-normal triangular or trapezoidal fuzzy number.

Proof: Let $x_{ij} = X_{ij}$ minimizes the total fuzzy cost

$$\tilde{z} = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \quad (8)$$

over all x_{ij} such that $x_{ij} \geq 0$ and

$$\sum_{i=1}^n x_{ij} = \sum_{j=1}^n x_{ij} = 1 \quad (9)$$

It is to be shown that the assignment $x_{ij} = X_{ij}$ also minimizes new total fuzzy cost

$$\tilde{z}' = \sum_{i=1}^n \sum_{j=1}^n (\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) x_{ij}$$

for all $i, j = 1, 2, \dots, n$, where \tilde{u}_i and \tilde{v}_j are fuzzy constants subtracted from i^{th} row and j^{th} column of the cost matrix (\tilde{c}_{ij}) .

To prove this, it may be written as

$$\tilde{z}' = \sum_{i=1}^n \sum_{j=1}^n (\tilde{c}_{ij} x_{ij}) \ominus \left(\sum_{i=1}^n \tilde{u}_i \sum_{i=1}^n x_{ij} \right) \ominus \left(\sum_{j=1}^n \tilde{v}_j \sum_{i=1}^n x_{ij} \right)$$

using Equations (8) and (9), we get

$$\tilde{z}' = \tilde{z} \ominus \sum_{i=1}^n \tilde{u}_i \ominus \sum_{j=1}^n \tilde{v}_j$$

The terms that are subtracted from \tilde{z} to give \tilde{z}' are independent of x_{ij} 's, it follows that \tilde{z}' is minimized whenever \tilde{z} is minimized, and conversely.
//

Theorem 2: If (x_{ij}) , $i = 1, 2, \dots, n$; $j = 1, 2, \dots, n$ is an optimal solution for an assignment problem with cost (\tilde{c}_{ij}) , then it is also optimal for the problem with cost (\tilde{c}'_{ij}) when

$$\begin{aligned} (\tilde{c}'_{ij}) &= (\tilde{c}_{ij}) \text{ for } i, j = 1, 2, \dots, n ; j \neq k \\ (\tilde{c}'_{ij}) &= (\tilde{c}_{ik}) - \tilde{A} \text{ , where } \tilde{A} \text{ is a fuzzy constant} \end{aligned}$$

Proof: we have

$$\begin{aligned} \tilde{z}' &= \sum_i \sum_j \tilde{c}'_{ij} x_{ij} = \sum_i \left(\sum_{j \neq k} \tilde{c}'_{ij} + \tilde{c}'_{ik} \right) x_{ij} \\ &= \sum_i \left(\sum_{j \neq k} \tilde{c}_{ij} + \tilde{c}'_{ik} - \tilde{A} \right) x_{ij} \\ &= \sum_i \sum_j \tilde{c}_{ij} x_{ij} - \tilde{A} \sum_i x_{ij} \\ &= \tilde{z} - \tilde{A} \text{ since } \left(\sum_i x_{ij} = 1 \right) \end{aligned}$$

Thus if (x_{ij}) minimizes \tilde{z} so will it \tilde{z}'

Theorem 3: In an assignment problem with cost (\tilde{c}_{ij}) , if all $(\tilde{c}_{ij}) \geq 0$ then a feasible solution (x_{ij}) which satisfies $\sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} = 0$, is optimal for the problem.

Proof: Since all $(\tilde{c}_{ij}) \geq 0$ and all $(x_{ij}) \geq 0$,
the objective function

$\tilde{z} = \sum_i \sum_j \tilde{c}_{ij} x_{ij}$ cannot be negative. The minimum possible value that \tilde{z} can attain is 0

Thus, any feasible solution (x_{ij}) that satisfies $\sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} = 0$, will be optimal.

5. Fuzzy Assignment Algorithms in classical form and fuzzy linear programming

In this section, new algorithms are proposed both in classical form and fuzzy linear programming, to find minimum total fuzzy cost and variations in the fuzzy assignment problems.

5.1. Classical algorithms for obtaining the total optimal fuzzy cost

In this section, new algorithms are proposed to find total optimal fuzzy cost and variations in the fuzzy assignment problems.

5.1.1. Minimum total fuzzy cost

For obtaining the minimum total fuzzy cost, we use algorithm1.

Algorithm1:

Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced one or not. If it is a balanced one (i.e, number of persons are equal to the number of works) then go to step 3. If it is an unbalanced one (i.e number of persons are not equal to the number of works) then go to step 2.

Step 2: Introduce dummy rows and/or columns with zero fuzzy costs so as to form a balanced one.

Step 3: Find the rank of each cell \tilde{c}_{ij} of the chosen fuzzy cost matrix by using the ranking procedure as mentioned in section 3 and determine the minimum element in each row and its corresponding fuzzy element.

Step 4: For each row in the fuzzy cost matrix of table II, subtract the minimum fuzzy element obtained in step 3 in the row from each fuzzy element in that row to get the reduced fuzzy cost matrix.

Step 5: Find the rank of each cell \tilde{c}_{ij} of the reduced fuzzy cost matrix obtained in step 4 by using the ranking procedure as mentioned in section 3 and determine the minimum element in each column and its corresponding fuzzy element.

Step 6: For each column in the reduced fuzzy cost matrix obtained in step 4, subtract the minimum fuzzy element obtained in step 5 in the column from each fuzzy element of that column to get the first modified fuzzy cost matrix.

Step 7: Find the rank of each cell \tilde{c}_{ij} of the first modified fuzzy cost matrix obtained in step 6 by using the ranking procedure as mentioned in section 3. Thus, the first modified matrix is obtained.

Step 8: If the rank of any cell of the first modified matrix obtained in step 7, i.e., $R(\tilde{c}_{ij}) = 0$, then draw the minimum number of horizontal and vertical lines to cover all such type of cells in the resulting matrix. Let the minimum number of lines be N. Now there may arise two cases:

Case (i) if $N = n$, the number of rows (columns) of given matrix, then an optimal fuzzy assignment can be made. So make the $R(\tilde{c}_{ij}) = 0$ assignment to get the required optimal solution.

Case (ii) if $N < n$, then determine the minimum element in the matrix and its corresponding fuzzy element which is not covered by the N lines.

Step 9: If it falls under step 8 of case (ii) then, subtract this minimum fuzzy element from all uncovered fuzzy elements and add the same fuzzy element at the intersection of horizontal and vertical lines. Thus, the second modified fuzzy cost matrix is obtained.

Step 10: Find the rank of each cell \tilde{c}_{ij} of the second modified fuzzy cost matrix obtained in step 9 by using the ranking procedure as mentioned in section 3. Thus, the second modified matrix is obtained.

Step 11: Again repeat step 8, step 9 and step 10 until minimum number of lines become equal to the number of rows (columns) of the given matrix i.e., $N = n$.

Step 12: (to make $R(\tilde{c}_{ij})=0$ assignment) .Examine the rows successively until a row-wise exactly single $R(\tilde{c}_{ij})=0$ is found, mark this $R(\tilde{c}_{ij})=0$ by 'O' to make the assignment. Then, mark a cross (*) over all $R(\tilde{c}_{ij})=0$ if lying in the column of the marked 'O' ($R(\tilde{c}_{ij})=0$), showing that they cannot be considered for future assignment. Continue in this manner until all the rows have been examined. Repeat the same procedure for columns also.

Step13: Repeat the step 12 successively until one of the following situations arises:

- (i) If no unmarked ($R(\tilde{c}_{ij})=0$) is left, then the process ends; or
- (ii) if there lie more than one of the unmarked ($R(\tilde{c}_{ij})=0$) in any column or row, then mark 'O' one of the unmarked ($R(\tilde{c}_{ij})=0$) arbitrarily and mark a asterisk (*) in the cells of remaining zeros in its rows and columns. Repeat the process until no unmarked ($R(\tilde{c}_{ij})=0$) is left in the matrix.

Step 14: Thus exactly one marked 'O' ($R(\tilde{c}_{ij})=0$) in each row and each column of the matrix is obtained. The assignment corresponding to these marked 'O' ($R(\tilde{c}_{ij})=0$) will give the optimal assignment.

Step 15: Substitute the optimal assignment obtained in step 14 in the original fuzzy cost matrix to get the optimal fuzzy assignment.

Step 16: Add the optimal fuzzy assignment obtained in step 15 using fuzzy number addition mentioned in section 2, to get the total optimal fuzzy cost.

5.1.2. Variations in the Fuzzy Assignment problem

In this section, we discuss two variations of the fuzzy assignment problem.

Case (i) Fuzzy Assignment algorithm to obtain the maximum total fuzzy cost
Sometimes, the assignment problem deals with the maximization of an objective function rather than to minimize it. In such cases we use the algorithm 2.

Algorithm 2:

Step 1: Find the ranks of each cell of the given fuzzy cost matrixes by using the ranking procedure as mentioned in section 3 and determine the maximum element.

Step 2: Convert the given fuzzy cost matrix into a minimization problem by subtracting from the highest fuzzy element, all the fuzzy elements of the given fuzzy cost matrix.

Step 3: Apply the usual procedure as mentioned in algorithm1.

Case (ii) Fuzzy Assignment algorithm to obtain the minimum total fuzzy cost when restrictions are made on the fuzzy assignment problem

Sometimes technical, legal or other restrictions do not permit the assignment of a particular facility to a particular job. In such cases, we use algorithm 3.

Algorithm 3:

Step 1: Assign a very high fuzzy cost to the cells which do not permit the assignment so that the activity will be automatically excluded from the optimal solution.

Step 2: Apply the usual procedure as mentioned in algorithm 1.

5.2. Algorithm using fuzzy assignment linear programming

In this section, a new algorithm is proposed to find minimum total fuzzy cost, maximum total fuzzy cost and to find the minimum total fuzzy cost when restrictions are made in the fuzzy assignment problems using fuzzy assignment linear programming.

Algorithm 4:

Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced one or not. If it is a balanced one (i.e, number of persons are equal to the number of works) then go to step 3. If it is an unbalanced one (i.e number of persons are not equal to the number of works) then go to step 2.

Step 2: Introduce dummy rows and/or columns with zero fuzzy costs so as to form a balanced one.

Step 3: Formulate the fuzzy assignment problem based on the chosen condition into the following fuzzy linear programming problems.

Case (i) to find the minimum total fuzzy cost formulate the fuzzy linear programming as follows:

$$\begin{aligned} \text{Min } \tilde{z} &= \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \\ \text{subject to } \sum_{i=1}^n x_{ij} &= 1 \quad (\text{only one person should be assigned the } j^{\text{th}} \\ &\quad \text{work, } j=1,2,\dots,n) \\ \sum_{j=1}^n x_{ij} &= 1 \quad (\text{one work is done by the } i^{\text{th}} \text{ person,} \\ &\quad i=1,2,\dots,n) \\ x_{ij} &= 1 \quad \forall i, j. \end{aligned}$$

where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

$\tilde{c}_{ij} = (a, b, c, d; w)$: Fuzzy payment to i^{th} person for doing j^{th} work.

$\sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$: Total fuzzy cost for performing all the works.

The fuzzy cost (\tilde{c}_{ij}) denotes a non-negative generalized trapezoidal fuzzy number.

Case (ii) to find the maximum total fuzzy cost; formulate the fuzzy linear programming as follows:

$$\text{Max } \tilde{z} = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$$

$$\begin{aligned} \text{subject to } \sum_{i=1}^n x_{ij} &\leq 1 \quad (\text{only one person should be assigned the } j^{\text{th}} \\ &\quad \text{work, } j=1,2,\dots,n) \\ \sum_{j=1}^n x_{ij} &\leq 1 \quad (\text{one work is done by the } i^{\text{th}} \text{ person,} \\ &\quad i=1,2,\dots,n) \\ x_{ij} &\leq 1 \quad \forall i, j. \end{aligned}$$

where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

$\tilde{c}_{ij} = (a, b, c, d; w)$: Fuzzy payment to i^{th} person for doing j^{th} work.

$\sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$: Total fuzzy cost for performing all the works.

The fuzzy cost (\tilde{c}_{ij}) denotes a non-negative generalized trapezoidal fuzzy number.

Case (iii) to find the minimum total fuzzy cost when restrictions are made, formulate the fuzzy linear programming as follows:

$$\begin{aligned} \text{Min } \tilde{z} &= \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \\ \text{subject to } x_{ij} &= \begin{cases} 1 & \text{if } i^{\text{th}} \text{ person is assigned } j^{\text{th}} \text{ work} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\sum_{i=1}^n x_{ij} = 1 \quad (\text{only one person should be assigned the } j^{\text{th}} \text{ work, } j=1,2,\dots,n)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad (\text{one work is done by the } i^{\text{th}} \text{ person, } i=1,2,\dots,n)$$

$$x_{ij} = 0 \text{ or } 1 \quad \forall i, j.$$

where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

$\tilde{c}_{ij} = (a, b, c, d; w)$: Fuzzy payment to i^{th} person for doing j^{th} work.

$\sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$: Total fuzzy cost for performing all the works.

The fuzzy cost (\tilde{c}_{ij}) denotes a non-negative generalized trapezoidal fuzzy number.

Step 4: Convert the fuzzy linear programming problem (case (i) or case (ii) or case (iii)), obtained in step 3, into the following crisp linear programming problem

$$z = \sum_{i=1}^n \sum_{j=1}^n R(\tilde{c}_{ij}) x_{ij}, \quad i = 1, 2, \dots, n$$

subject to respective constraints.

Step 5: Using fuzzy ranking method in section 3, the values of $R(\tilde{c}_{ij})$, $\forall i, j$ are calculated.

Using the values of $R(\tilde{c}_{ij})$, the crisp linear programming problem obtained in step 4 may be written as

$$z = (R(c_{11}))x_{11} \oplus (R(c_{12}))x_{12} \oplus (R(c_{13}))x_{13} \oplus (R(c_{14}))x_{14} \oplus (R(c_{21}))x_{21} \oplus (R(c_{22}))x_{22} \\ \oplus (R(c_{23}))x_{23} \oplus (R(c_{24}))x_{24} \oplus (R(c_{31}))x_{31} \oplus (R(c_{32}))x_{32} \oplus (R(c_{33}))x_{33} \oplus (R(c_{34}))x_{34}$$

subject to respective constraints.

Step 6: Using TORA software, find the optimal solution of the crisp linear programming problem, obtained in step 5.

Step 7: To get the total optimal fuzzy cost substitute the optimal solution obtained in step 6 in the respective objective function of step 3.

6. Numerical Example

To illustrate the proposed algorithms, consider a fuzzy assignment problem with four persons and four works. Fuzzy costs consider here to be the generalized trapezoidal fuzzy number for allocating each person. The fuzzy cost for each person would take to perform each work is given in the effectiveness fuzzy cost matrix as shown in tables (II, III). The fuzzy assignment costs in Table II and Table III are costs without restrictions and with restrictions respectively.

TABLE II: Fuzzy assignment costs

	I	II	III	IV
A	(4,6,8,10;0.1)	(20,24,26,28;0.2)	(14,16,17,19;0.3)	(6,8,11,13;0.1)
B	(8,11,13,15;0.2)	(24,26,28,30;.3)	(0,2,4,6;0.1)	(23,25,26,28;0.2)
C	(30,34,38,40;0.1)	(15,17,19,20;0.2)	(14,16,18,20;0.4)	(12,14,15,18;0.3)
D	(14,16 19,20;0.2)	(22,24,26,28;0.6)	(20,22,24,26;0.4)	(6,8,10,12;0.2)

TABLE III: Fuzzy assignment costs when restrictions are made

	I	II	III	IV
A	-	(20,24,26,28;0.2)	(14,16,17,19;0.3)	(6,8,11,13;0.1)
B	(8,11,13,15;0.2)	-	(0,2,4,6;0.1)	(23,25,26,28;0.2)
C	(30,34,38,40;0.1)	(15,17,19,20;0.2)	-	(12,14,15,18;0.3)
D	(14,16 19,20;0.2)	(22,24,26,28;0.6)	(20,22,24,26;0.4)	-

6.1. Fuzzy optimal cost using classical algorithm

In this section, applications of the proposed classical algorithms are explained with an example to find minimum total optimal fuzzy cost and variations in the fuzzy assignment problems.

6.1.1. Minimum total fuzzy cost for the numerical example

Step 1: The fuzzy assignment problem chosen in the example, as shown in table II is a balanced one.

Step 2: Using Step 3 of the proposed algorithm mentioned in section 5.1.1. the rank of each cell $R(\tilde{c}_{ij})$ of table II is given in table IV.

TABLE IV: Ranks of table II

	I	II	III	IV
A	0.29	1.04	2.06	0.39
B	0.99	3.37	0.12	2.12
C	1.49	1.50	2.83	1.81
D	1.45	6.24	3.83	0.74

The minimum element in row A is 0.29 and its corresponding fuzzy element is (4, 6, 8, 10; 0.1).
 The minimum element in row B is 0.12 and its corresponding fuzzy element is (0, 2, 4, 6; 0.1).
 The minimum element in row C is 1.49 and its corresponding fuzzy element is (15, 17, 19, 20; 0.2).
 The minimum element in row D is 0.74 and its corresponding fuzzy element is (6, 8, 10, 12; 0.2).

Step 3: Subtract the minimum fuzzy element obtained in step 2 in each row from every fuzzy element in that row of table II and the reduced fuzzy cost matrix is presented in table V.

TABLE V: Reduced fuzzy cost matrix

	I	II	III	IV
A	(-6,-2,2,6;0.1)	(10,16,20,24;0.1)	(4,8,11,15;0.1)	(-4,0,5,9;0.1)
B	(2,7,11,15;0.1)	(18,22,26,30;0.1)	(-6,-2,2,6;0.1)	(17,21,24,28;0.1)
C	(10,15,21,25;0.1)	(-5,-2,2,5;0.2)	(-6,-3,1,5;0.2)	(-8,-5,-2,3;0.2)
D	(2,6,11,14;0.2)	(10,14,18,22;0.2)	(8,12,16,20;0.2)	(-6,-2,2,6;0.2)

Step 4: Using Step 5 of the proposed algorithm mentioned in section 5.1.1., the rank of each cell $R(\tilde{c}_{ij})$ of table V is given in table VI.

TABLE VI: Ranks of table V

	I	II	III	IV
A	0	0.74	0.39	0.10
B	0.37	0.99	0	0.93
C	0.74	0	-0.08	-0.29
D	0.70	1.33	1.16	0

The minimum element in column I is 0 and its corresponding fuzzy element is (-6, -2, 2, 6; 0.1)
 The minimum element in column II is 0 and its corresponding fuzzy element is (-5,-2, 2, 5; 0.1)
 The minimum element in column III is -0.08 and its corresponding fuzzy element is (-6, -3, 1, 5; 0.2)
 The minimum element in column IV is 0 and its corresponding fuzzy element is (-8, -5, -2, 3; 0.2)

Step 5: Subtract the minimum fuzzy element obtained in step 4 in each column from every fuzzy element in that column of table V and the first modified fuzzy cost matrix is presented in table VII.

TABLE VII: First modified fuzzy cost matrix

	I	II	III	IV
A	(-12,-4,4,12;0.1)	(5,14,22,29;0.1)	(-1,7,14,21;0.1)	(-7,2,10,17;0.1)
B	(-4,5,13,21;0.1)	(13,20,28,35;0.1)	(-11,-3,5,12;0.1)	(14,23,29,36;0.1)
C	(4,13,23,31;0.1)	(-10,-4,4,10;0.2)	(-11,-4,4,11;0.2)	(-11,-3,3,11;0.2)
D	(-4,4,13,20;0.1)	(-5,12,20,27;0.2)	(3,11,19,26;0.2)	(-9,0,7,14;0.2)

Step 6: Using Step 7 of the proposed algorithm mentioned in section 5.1.1. the rank of each cell $R(\tilde{c}_{ij})$ of table VII is given in table VIII. Obtain the first modified crisp matrix

TABLE VIII: First modified crisp matrix

	I	II	III	IV
A	0	0.70	0.39	0.20
B	0.37	0.95	0	1.25
C	0.79	0	0	0
D	0.35	0.62	0.58	0.10

Step 7: Using step 8 of the proposed algorithm mentioned in section 5.1.1. for table VIII, it falls under case(ii) i.e., $N < n$.

TABLE IX: Resulting Matrix

	I	II	III	IV
A	0	0.70	0.39	0.20
B	0.37	0.95	0	1.25
C	0.79	0	0	0
D	0.35	0.62	0.58	0.10

The minimum element which is not covered by the lines (table IX) is 0.10 and its corresponding fuzzy element is (-9,0,7,14;0.2).

Step 8: Subtract the minimum fuzzy element obtained in step 7 from all uncovered fuzzy elements and add the same fuzzy element at the intersection of horizontal and vertical lines of table VII. Thus, the second modified fuzzy cost matrix is presented in table X.

TABLE X: Second modified fuzzy cost matrix

	I	II	III	IV
A	(-12,-4,4,12;0.1)	(-9,7,22,38;0.1)	(-1,7,14,21;0.1)	(-21,-5,10,26;0.1)
B	(-4,5,13,21;0.1)	(-1,13,28,44;0.1)	(-11,-3,5,12;0.1)	(0,16,29,45;0.1)
C	(-5,13,30,45;0.1)	(-10,-4,4,10;0.2)	(-20,-4,11,25;0.2)	(-11,-3,3,11;0.2)
D	(-4,4,13,20;0.1)	(-19,5,20,36;0.2)	(3,11,19,26;0.2)	(-23,-7,7,23;0.2)

Step 9: Using step10, of the proposed algorithm mentioned in section 5.1.1. the rank of each cell $R(\tilde{c}_{ij})$ of table X is given in table XI. Obtain the second modified crisp matrix.

TABLE XI: Second modified crisp matrix

	I	II	III	IV
A	0	0.60	0.39	0.10
B	0.37	0.85	0	0.83
C	0.89	0	0.29	0
D	1.35	1.04	0.58	0

Step 10: Using step 8 of the proposed algorithm mentioned in section 5.1.1. for table XI, it falls under case(i) i.e., $N= n$, the resulting matrix is given in table XII.

TABLE XII: Resulting Matrix

	I	II	III	IV
A	0	0.60	0.39	0.10
B	0.37	0.85	0	0.83
C	0.89	0	0.29	0
D	1.35	1.04	0.58	0

Step 11: Using steps12, 13, and 14 of the proposed algorithm mentioned in section5.1.1. Tables XII(a), XII(b) show the necessary steps for reaching the optimal assignment.

TABLE XII (a): assignment

	I	II	III	IV
A	O	0.60	0.39	0.10
B	0.37	0.85	O	0.83
C	0.89	0	0.29	0
D	1.35	1.04	0.58	O

TABLE XII (b): optimal assignment

	I	II	III	IV
A	O	0.60	0.39	0.10
B	0.37	0.85	O	0.83
C	0.89	O	0.29	*
D	1.35	1.04	0.58	O

Step 12: The optimal assignment from table XII (b) is A-I, B-III, C-II, D-IV

Step 13: From the original fuzzy assignment cost matrix presented in table II, the optimal fuzzy cost assignment is calculated and presented in table XIII.

TABLE XIII : Optimal fuzzy cost assignment

Optimal assignment	A-I	B-III	C-II	D-IV
Fuzzy cost	(4,6,8,10;0.1)	(0,2,4,6;0.1)	(15,17,19,20;0.2)	(6,8,10,12;0.2)

Step14: Using step16 of the proposed algorithm as mentioned in section 5.1.1. , the minimum total fuzzy cost is (25, 33, 41, 48; 0.1)

6.1.2. Maximum total fuzzy cost for the numerical example

Step 1: Find the rank of each cell \tilde{c}_{ij} of the chosen fuzzy cost matrix of table II by using the ranking procedure as mentioned in section 3 and determine the maximum element and its corresponding fuzzy element.

TABLE XIV: Ranks of table II

	I	II	III	IV
A	0.29	1.04	2.06	0.39
B	0.99	3.37	0.12	2.12
C	1.49	1.50	2.83	1.81
D	1.45	6.24	3.83	0.74

The maximum element is 6.24 and its corresponding fuzzy element is (22, 24, 26, 28; 0.6)

Step 2: Using step 2 of the proposed algorithm mentioned in section 5.1.2., the resultant fuzzy cost matrix is given in table XV.

TABLE XV: Resultant fuzzy cost matrix

	I	II	III	IV
A	(4,6,8,10;0.1)	(20,24,26,28;0.2)	(14,16,17,19;0.3)	(6,8,11,13;0.1)
B	(8,11,13,15;0.2)	(24,26,28,30;.3)	(0,2,4,6;0.1)	(23,25,26,28;0.2)
C	(30,34,38,40;0.1)	(15,17,19,20;0.2)	(14,16,18,20;0.4)	(12,14,15,18;0.3)
D	(14,16 19,20;0.2)	(22,24,26,28;0.6)	(20,22,24,26;0.4)	(6,8,10,12;0.2)

Step 3: After applying the usual procedure to table XV as mentioned in algorithm1 of section 5.1.1., the optimal fuzzy cost assignment thus obtained is shown in table XVI.

TABLE XVI: Optimal fuzzy cost assignment

Optimal assignment	A-II	B-IV	C-I	D-III
Fuzzy cost	(20,24,26,28;0.2)	(23,25,26,28;0.2)	(30,34,38,40;0.1)	(20,22,24,26;0.4)

Step 4: Using step16 of the proposed algorithm mentioned in section 5.1.1. , the maximum total fuzzy cost is (93, 105, 114, 118; 0.1).

6.1.3. Minimum total fuzzy cost when restrictions are made on the fuzzy assignment problem for the example

Step 1: Assign a very high fuzzy cost to table III, to the cells which do not permit the assignment so that the activity will be automatically excluded from the optimal solution.

Step 2: After applying the usual procedure as mentioned in algorithm1 of section 5.1.1.for the table III, the optimal fuzzy cost assignment thus obtained is shown in table XVII

TABLE XVII: Optimal fuzzy cost assignment

Optimal assignment	A-IV	B-III	C-II	D-I
Fuzzy cost	(6,8,11,13;0.1)	(0,2,4,6;0.1)	(15,17,19,20;0.2)	(14,16,19,20;0.2)

Step 3: Using step16 of the proposed algorithm mentioned in section 5.1.1. , the minimum total fuzzy cost is (35, 43, 53, 59; 0.1).

6.2. Fuzzy optimal solution using fuzzy linear programming algorithm

In this section, applications of the proposed fuzzy linear programming algorithm are explained with an example to find minimum total fuzzy cost, maximum total fuzzy cost and to find the minimum total fuzzy cost when restrictions are made in the fuzzy assignment problems.

6.2.1. Minimum total fuzzy cost for the numerical example

Step 1: The fuzzy assignment problem chosen in the example, as shown in table II is a balanced one.

Step 2: Using case (i) of step3 of the proposed algorithm mentioned in section 5.2., the fuzzy linear programming formulation of the fuzzy assignment problem, as shown in table (II) is:

Minimize $(4,6,8,10;0.1)x_{11} \oplus (20,24,26,28;0.2)x_{12} \oplus (14,16,17,19;0.3)x_{13}$
 $\oplus (6,8,11,13;0.1)x_{14} \oplus (8,11,13,15;0.2)x_{21} \oplus (24,26,28,30;.3)x_{22} \oplus (0,2,4,6;0.1)x_{23}$
 $\oplus (23,25,26,28;0.2)x_{24} \oplus (30,34,38,40;0.1)x_{31} \oplus (15,17,19,20;0.2)x_{32}$
 $\oplus (14,16,18,20;0.4)x_{33} \oplus (12,14,15,18;0.3)x_{34} \oplus (14,16,19,20;0.2)x_{41}$
 $\oplus (22,24,26,28;0.6)x_{42} \oplus (20,22,24,26;0.4)x_{43} \oplus (6,8,10,12;0.2)x_{44}$

subject to

$$\begin{aligned} x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} &= 1, & x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} &= 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} &= 1, & x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} &= 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} &= 1, & x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} &= 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} &= 1, & x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} &= 1, \\ x_{ij} &= 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4 \end{aligned}$$

Step 3: Using step 4 of the proposed algorithm mentioned in section 5.2., the formulated fuzzy linear programming problem is converted into the following crisp linear programming problem:

Minimize $(4,6,8,10;0.1)x_{11} \oplus R(20,24,26,28;0.2)x_{12} \oplus R(14,16,17,19;0.3)x_{13}$
 $\oplus R(6,8,11,13;0.1)x_{14} \oplus R(8,11,13,15;0.2)x_{21} \oplus R(24,26,28,30;.3)x_{22}$
 $\oplus R(0,2,4,6;0.1)x_{23} \oplus R(23,25,26,28;0.2)x_{24} \oplus R(30,34,38,40;0.1)x_{31}$
 $\oplus R(15,17,19,20;0.2)x_{32} \oplus R(14,16,18,20;0.4)x_{33} \oplus R(12,14,15,18;0.3)x_{34}$
 $\oplus R(14,16,19,20;0.2)x_{41} \oplus R(22,24,26,28;0.6)x_{42} \oplus R(20,22,24,26;0.4)x_{43}$
 $\oplus R(6,8,10,12;0.2)x_{44}$

subject to

$$\begin{aligned} x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} &= 1, & x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} &= 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} &= 1, & x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} &= 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} &= 1, & x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} &= 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} &= 1, & x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} &= 1, \\ x_{ij} &= 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4 \end{aligned}$$

Step 4: Using Section 3, the values of $R(\tilde{c}_{ij})$, $\forall i, j$ are

$$R(\tilde{c}_{11})=0.29, R(\tilde{c}_{12})= 1.04, R(\tilde{c}_{13})=2.06, R(\tilde{c}_{14})= 0.39, R(\tilde{c}_{21})=0.99, \\ R(\tilde{c}_{22})=3.37, R(\tilde{c}_{23})=0.12, R(\tilde{c}_{24})= 2.12, R(\tilde{c}_{31})=1.49, R(\tilde{c}_{32})=1.50, \\ R(\tilde{c}_{33})=2.83, R(\tilde{c}_{34})= 1.81 , R(\tilde{c}_{41})= 1.45, R(\tilde{c}_{42})= 6.24, R(\tilde{c}_{43})= 3.83, \\ R(\tilde{c}_{44})= 0.74.$$

Using the values of $R(\tilde{c}_{ij})$, the crisp linear programming problem obtained in step 3 may be written as:

$$\text{Minimize } (0.29)x_{11} \oplus (1.04)x_{12} \oplus (2.06)x_{13} \oplus (0.39)x_{14} \oplus (0.99)x_{21} \oplus (3.37)x_{22} \\ \oplus (0.12)x_{23} \oplus (2.12)x_{24} \oplus (1.49)x_{31} \oplus (1.50)x_{32} \oplus (2.83)x_{33} \oplus (1.81)x_{34} \oplus (1.45)x_{41} \\ \oplus 6.24)x_{42} \oplus (3.833)x_{43} \oplus (0.74)x_{44}$$

subject to

$$x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} = 1, \quad x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} = 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} = 1, \quad x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} = 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} = 1, \quad x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} = 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} = 1, \quad x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} = 1, \\ x_{ij} = 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4$$

Step 5: Solving the crisp linear programming problem, obtained in step 4, the optimal solution is $x_{11} = 1, x_{22} = 1, x_{32} = 1, x_{44} = 1$.

Step 6: The minimum total fuzzy cost is obtained by substituting the optimal solution obtained in step 5 in the objective function of step 2 is

$$(25, 33, 41, 48; 0.1).$$

6.2.1. Maximum total fuzzy cost for the numerical example

Step 1: The fuzzy assignment problem chosen in the example, as shown in table II is a balanced one.

Step 2: Using case (ii) of step 3 of the proposed algorithm mentioned in section 5.2., the fuzzy linear programming formulation of the fuzzy assignment problem, as shown in table II is:

$$\text{Maximize } (4,6,8,10;0.1)x_{11} \oplus (20,24,26,28;0.2)x_{12} \oplus (14,16,17,19;0.3)x_{13} \\ \oplus (6,8,11,13;0.1)x_{14} \oplus (8,11,13,15;0.2)x_{21} \oplus (24,26,28,30;.3)x_{22} \oplus (0,2,4,6;0.1)x_{23} \\ \oplus (23,25,26,28;0.2)x_{24} \oplus (30,34,38,40;0.1)x_{31} \oplus (15,17,19,20;0.2)x_{32} \\ \oplus (14,16,18,20;0.4)x_{33} \oplus (12,14,15,18;0.3)x_{34} \oplus (14,16,19,20;0.2)x_{41} \oplus \\ (22,24,26,28;0.6)x_{42} \oplus (20,22,24,26;0.4)x_{43} \oplus (6,8,10,12;0.2)x_{44}$$

subject to

$$x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} \leq 1, \quad x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} \leq 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} \leq 1, \quad x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} \leq 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} \leq 1, \quad x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} \leq 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} \leq 1, \quad x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} \leq 1, \\ x_{ij} = 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4$$

Step 3: Using step 4 of the proposed algorithm mentioned in section 5.2., the formulated fuzzy linear programming problem is converted into the following crisp linear programming problem:

Maximize

$$\begin{aligned} & R(4,6,8,10;0.1)x_{11} \oplus R(20,24,26,28;0.2)x_{12} \oplus R(14,16,17,19;0.3)x_{13} \\ & \oplus R(6,8,11,13;0.1)x_{14} \oplus R(8,11,13,15;0.2)x_{21} \oplus R(24,26,28,30;.3)x_{22} \\ & \oplus R(0,2,4,6;0.1)x_{23} \oplus R(23,25,26,28;0.2)x_{24} \oplus R(30,34,38,40;0.1)x_{31} \\ & \oplus R(15,17,19,20;0.2)x_{32} \oplus R(14,16,18,20;0.4)x_{33} \oplus R(12,14,15,18;0.3)x_{34} \\ & \oplus R(14,16,19,20;0.2)x_{41} \oplus R(22,24,26,28;0.6)x_{42} \oplus R(20,22,24,26;0.4)x_{43} \\ & \oplus R(6,8,10,12;0.2)x_{44} \end{aligned}$$

Subject to

$$\begin{aligned} x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} &\leq 1, & x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} &\leq 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} &\leq 1, & x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} &\leq 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} &\leq 1, & x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} &\leq 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} &\leq 1, & x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} &\leq 1, \\ x_{ij} &= 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4 \end{aligned}$$

Step 4: Using Section 3, the values of $R(\tilde{c}_{ij})$, $\forall i, j$ are $R(\tilde{c}_{11})=0.29$,

$$\begin{aligned} R(\tilde{c}_{12}) &= 1.04, & R(\tilde{c}_{13}) &= 2.06, & R(\tilde{c}_{14}) &= 0.39, & R(\tilde{c}_{21}) &= 0.99, & R(\tilde{c}_{22}) &= 3.37, \\ R(\tilde{c}_{23}) &= 0.12, & R(\tilde{c}_{24}) &= 2.12, & R(\tilde{c}_{31}) &= 1.49, & R(\tilde{c}_{32}) &= 1.50, & R(\tilde{c}_{33}) &= 2.83, & R(\tilde{c}_{34}) &= \\ & 1.81, & R(\tilde{c}_{41}) &= 1.45, & R(\tilde{c}_{42}) &= 6.24, & R(\tilde{c}_{43}) &= 3.83, & R(\tilde{c}_{44}) &= 0.74. \end{aligned}$$

Using the values of $R(\tilde{c}_{ij})$, the crisp linear programming problem obtained in step3 may be written as:

Maximize

$$\begin{aligned} & (0.29)x_{11} \oplus (1.04)x_{12} \oplus (2.06)x_{13} \oplus (0.39)x_{14} \oplus (0.99)x_{21} \oplus (3.37)x_{22} \oplus (0.12)x_{23} \\ & \oplus (2.12)x_{24} \oplus (1.49)x_{31} \oplus (1.50)x_{32} \oplus (2.83)x_{33} \oplus (1.81)x_{34} \oplus (1.45)x_{41} \oplus (6.24)x_{42} \\ & \oplus (3.833)x_{43} \oplus (0.74)x_{44} \end{aligned}$$

subject to

$$\begin{aligned} x_{11} \oplus x_{12} \oplus x_{13} \oplus x_{14} &\leq 1, & x_{21} \oplus x_{22} \oplus x_{23} \oplus x_{24} &\leq 1, \\ x_{31} \oplus x_{32} \oplus x_{33} \oplus x_{34} &\leq 1, & x_{41} \oplus x_{42} \oplus x_{43} \oplus x_{44} &\leq 1, \\ x_{11} \oplus x_{21} \oplus x_{31} \oplus x_{41} &\leq 1, & x_{12} \oplus x_{22} \oplus x_{32} \oplus x_{42} &\leq 1, \\ x_{13} \oplus x_{23} \oplus x_{33} \oplus x_{43} &\leq 1, & x_{14} \oplus x_{24} \oplus x_{34} \oplus x_{44} &\leq 1, \\ x_{ij} &= 1 \quad \forall i = 1,2,3,4 \text{ and } j=1,2,3,4 \end{aligned}$$

Step 5: Solving the crisp linear programming problem, obtained in step4, the optimal solution is

$$x_{13} = 1, \quad x_{24} = 1, \quad x_{31} = 1, \quad x_{42} = 1.$$

Step 6: The maximum total fuzzy cost is obtained by substituting the optimal solution obtained in step 5 in the objective function of step2 is (89, 99, 107, 115; 0.1).

6.2.3. Minimum total fuzzy cost when restrictions are made for the example.

Step 1: The fuzzy assignment problem chosen in the example, as shown in table (III) is a balanced one.

Step 2: Using case (iii) of step3 of the proposed algorithm mentioned in section 5.2., the fuzzy linear programming formulation of the fuzzy assignment problem, as shown in table(III) is:

Minimize $(20,24,26,28;0.2)x_{12} \oplus (14,16,17,19;0.3)x_{13} \oplus (6,8,11,13;0.1)x_{14}$
 $\oplus (8,11,13,15;0.2)x_{21} \oplus (0,2,4,6;0.1)x_{23} \oplus (23,25,26,28;0.2)x_{24}$
 $\oplus (30,34,38,40;0.1)x_{31} \oplus (15,17,19,20;0.2)x_{32} \oplus (12,14,15,18;0.3)x_{34}$
 $\oplus (14,16,19,20;0.2)x_{41} \oplus (22,24,26,28;0.6)x_{42} \oplus (20,22,24,26;0.4)x_{43}$
 subject to

$$\begin{aligned} x_{12} \oplus x_{13} \oplus x_{14} &= 1, & x_{21} \oplus x_{23} \oplus x_{24} &= 1, \\ x_{31} \oplus x_{32} \oplus x_{34} &= 1, & x_{41} \oplus x_{42} \oplus x_{43} &= 1, \\ x_{21} \oplus x_{31} \oplus x_{41} &= 1, & x_{12} \oplus x_{32} \oplus x_{42} &= 1, \\ x_{13} \oplus x_{23} \oplus x_{43} &= 1, & x_{14} \oplus x_{24} \oplus x_{34} &= 1, \\ x_{ij} &= 0 \text{ or } 1 \quad \forall i = 1,2,3,4 \text{ and } j = 1,2,3,4 \end{aligned}$$

Step 3: Using step 4 of the proposed algorithm mentioned in section 5.2., the formulated fuzzy linear programming problem is converted into the following crisp linear programming problem:

Minimize
 $R(20,24,26,28;0.2)x_{12} \oplus R(14,16,17,19;0.3)x_{13} \oplus R(6,8,11,13;0.1)x_{14} \oplus$
 $R(8,11,13,15;0.2)x_{21} \oplus R(0,2,4,6;0.1)x_{23} \oplus R(23,25,26,28;0.2)x_{24} \oplus$
 $R(30,34,38,40;0.1)x_{31} \oplus R(15,17,19,20;0.2)x_{32} \oplus R(12,14,15,18;0.3)x_{34} \oplus$
 $R(14,16,19,20;0.2)x_{41} \oplus R(22,24,26,28;0.6)x_{42} \oplus R(20,22,24,26;0.4)x_{43}.$
 subject to

$$\begin{aligned} x_{12} \oplus x_{13} \oplus x_{14} &= 1, & x_{21} \oplus x_{23} \oplus x_{24} &= 1, \\ x_{31} \oplus x_{32} \oplus x_{34} &= 1, & x_{41} \oplus x_{42} \oplus x_{43} &= 1, \\ x_{21} \oplus x_{31} \oplus x_{41} &= 1, & x_{12} \oplus x_{32} \oplus x_{42} &= 1, \\ x_{13} \oplus x_{23} \oplus x_{43} &= 1, & x_{14} \oplus x_{24} \oplus x_{34} &= 1, \\ x_{ij} &= 0 \text{ or } 1 \quad \forall i = 1,2,3,4 \text{ and } j = 1,2,3,4 \end{aligned}$$

Step 4: Using Section 3, the values of $R(\tilde{c}_{ij})$, $\forall i, j$ are

$$\begin{aligned} R(\tilde{c}_{12}) &= 1.04, & R(\tilde{c}_{13}) &= 2.06, & R(\tilde{c}_{14}) &= 0.39, & R(\tilde{c}_{21}) &= 0.99, & R(\tilde{c}_{23}) &= 0.12, \\ R(\tilde{c}_{24}) &= 2.12, & R(\tilde{c}_{31}) &= 1.49, & R(\tilde{c}_{32}) &= 1.50, & R(\tilde{c}_{34}) &= 1.81, & R(\tilde{c}_{41}) &= 1.45, \\ R(\tilde{c}_{42}) &= 6.24, \\ R(\tilde{c}_{43}) &= 3.83. \end{aligned}$$

Using the values of $R(\tilde{c}_{ij})$, the crisp linear programming problem obtained in step 3 may be written as:

Minimize $(1.04)x_{12} \oplus (2.06)x_{13} \oplus (0.39)x_{14} \oplus (0.99)x_{21} \oplus (0.12)x_{23} \oplus (2.12)x_{24} \oplus$
 $(1.49)x_{31} \oplus (1.50)x_{32} \oplus (1.81)x_{34} \oplus (1.45)x_{41} \oplus (6.24)x_{42} \oplus (3.833)x_{43}.$
 subject to

$$\begin{aligned} x_{12} \oplus x_{13} \oplus x_{14} &= 1, & x_{21} \oplus x_{23} \oplus x_{24} &= 1, \\ x_{31} \oplus x_{32} \oplus x_{34} &= 1, & x_{41} \oplus x_{42} \oplus x_{43} &= 1, \\ x_{21} \oplus x_{31} \oplus x_{41} &= 1, & x_{12} \oplus x_{32} \oplus x_{42} &= 1, \end{aligned}$$

$$x_{13} \oplus x_{23} \oplus x_{43} = 1, \quad x_{14} \oplus x_{24} \oplus x_{34} = 1,$$

$$x_{ij} = 0 \text{ or } 1 \quad \forall i = 1,2,3,4 \text{ and } j = 1,2,3,4$$

Step 5: Solving the crisp linear programming problem, obtained in step4, the optimal solution is

$$x_{14} = 1, \quad x_{23} = 1, \quad x_{32} = 1, \quad x_{41} = 1.$$

Step 6: The minimum total fuzzy cost is obtained by substituting the optimal solution obtained in step 5 in the objective function of step 2 is (35, 43, 53, 59; 0.1).

TABLE XVIII: Total Optimal fuzzy costs of the proposed algorithms

Fuzzy assignment algorithms	Minimum total fuzzy cost	Maximum total fuzzy cost	Minimum fuzzy cost when restrictions are made
Proposed classical algorithm	(25, 33, 41, 48; 0.1)	(93, 105, 114, 118; 0.1)	(35, 43, 53, 59; 0.1)
Proposed fuzzy linear programming algorithm	(25, 33, 41, 48; 0.1)	(89, 99, 107, 115; 0.1)	(35, 43, 53, 59; 0.1)

7. Conclusion

In this paper, new algorithms have been developed for fuzzy assignment problem with fuzzy cost based on the ranking method. The algorithms have proposed in this paper are easy to understand and apply to find the optimal fuzzy cost occurring in the real life situations. There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the assignment problems.

References

1. M. L. Balinski, A Competitive (dual) simplex method for the assignment problem, *Math. Program*, 34(2) (1986), 125-141.
2. R.S. Barr, F. Glover, D. Klingman, The alternating basis algorithm for assignment problems, *Math. Program*, 13(1) (1977), 1-13.
3. M.S. Hung, W.O. Rom, Solving the assignment problem by relaxation, *Oper. Res.*, 28(4) (1980), 969-982.
4. L.F. McGinnis, Implementation and testing of a primal-dual algorithm for the assignment problem, *Oper. Res.*, 31(2) (1983), 277-291.
5. H.W. Kuhn, The Hungarian method for the assignment problem, *Naval Research Logistics Quarterly*, 2 (1955), 83-97.
6. S.P. Eberhardt, T. Duad, A. Kerns, T.X. Brown, A.P. Thakoor, Competitive neural architecture for hardware solution to the assignment problem, *Neural Networks*, 4(4) (1991), 431-442.
7. D. Avis, L. Devroye, An analysis of a decomposition heuristic for the assignment problem, *Oper. Res. Lett.*, 3(6) (1985), 279-283.
8. Lin Chi-Jen, Wen Ue-Pyng, A labelling algorithm for the fuzzy assignment problem, *Fuzzy Sets and Systems* 142 (2004), 373-391.
9. M. Sakawa, I. Nishizaki, Y. Uemura, Interactive fuzzy programming for two-level linear and linear fractional production and assignment problems: a case study, *European Journal on operations Research* 135 (2001), 142-157.
10. M.S. Chen, On a fuzzy assignment problem, *Tamkang Journal* 22 (1985), 407-411.
11. Long-Sheng Huang, Li-pu Zhang, solution method for fuzzy assignment problem with Restriction of Qualification, *Proceedings of the Sixth International Conference on Intelligent Systems Design and Applications (ISDA'06)*, 2006.
12. Chen Liang-Hsuan, Lu Hai-Wen, An extended assignment problem considering multiple and outputs, *Applied Mathematical Modelling* 31 (2007), 2239-2248.

13. Feng.Y, Yang L A two-objective fuzzy k-cardinality assignment problem, *Journal of computational and Applied Mathematics* 197 (2006), 233-244.
14. Liu. L, Gao X Fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm, *Applied Mathematical Modelling* 33(2009), 3926-3935.
15. Majumdar .J, Bhunia A K, Elitist genetic algorithm for assignment problem with imprecise goal, *European Journal of Operational Research* 177 (2007), 684-692.
16. Ye. X, Xu J, A fuzzy vehicle routing assignment model with connection network based on priority-based genetic algorithm, *World Journal of Modelling and simulation* 4 (2008), 257- 268.
17. S.H. Chen, operations on fuzzy numbers with function principal, *Tamkang J. Management Sci.* 6(1985), 13-25.
18. A. Kaufmann, M.M. Gupta, *Introduction to Fuzzy Arithmetics : Theory and Applications*, Van Nostrand Reinhold, New York, 1985.
19. S.J. Chen, S.M. Chen, A new method for handling multicriteria fuzzy decision making problems using FN-IOWA operators, *Cybernet. Syst.* 34(2003), 109-137.
20. C.S. Mahapatara, T.K. Roy, Fuzzy multi-objective mathematical programming on reliability optimization model, *Appl.Math.Comput.* 174 (2006), 643-659.
21. S.J. Chen, S.M. Chen, Fuzzy risk analysis on the ranking of generalized trapezoidal fuzzy numbers, *Appl. Intell.* 26 (2007), 1-11.
22. S.M. Chen, J.H. Chen, Fuzzy risk analysis based on the ranking generalized fuzzy numbers with different heights and different spreads, *Expert syst. Appl.* 36 (2009), 6833-6842.
23. S.M. Chen, C.H. Wang, Fuzzy risk analysis based on ranking fuzzy numbers using α -cuts, belief features and signal/noise ratios, *Expert syst. Appl.* 36 (2009), 5576-5581.
24. D.Yong, S. Wenkang, D.Feng, L Qi, A new similarity measure of generalized fuzzy numbers and its application to pattern recognition, *Pattern Recognit. Lett.* 25 (2004), 875-883.

25. Y.L.P. Thorani, P.Phani Bushan Rao and N. Ravi Shankar, Ordering Generalized Trapezoidal Fuzzy Numbers, Int. J. Contemp. Math. Sciences, 7(12) (2012), 555-573.

Received: April 24, 2013