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Keywords

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Fuzzy Control for Nonlinear Uncertain Electrohydraulic Active Suspensions With Input Constraint

Haiping Du and Nong Zhang

***Abstract*—This paper presents a Takagi–Sugeno (T–S) model-based fuzzy control design approach for electrohydraulic active vehicle suspensions considering nonlinear dynamics of the actuator, sprung mass variation, and constraints on the control input. The T–S fuzzy model is first applied to represent the nonlinear uncertain electrohydraulic suspension. Then, a fuzzy state feedback controller is designed for the obtained T–S fuzzy model with optimized H_∞ performance for ride comfort by using the parallel-distributed compensation (PDC) scheme. The sufficient conditions for the existence of such a controller are derived in terms of linear matrix inequalities (LMIs). Numerical simulations on a full-car suspension model are performed to validate the effectiveness of the proposed approach. The obtained results show that the designed controller can achieve good suspension performance despite the existence of nonlinear actuator dynamics, sprung mass variation, and control input constraints.**

***Index Terms*—Electrohydraulic actuator, input constraint, nonlinear dynamic system, Takagi–Sugeno (T–S) fuzzy modeling, uncertainty, vehicle active suspension.**

I. Introduction

ACTIVE suspensions are currently attracting a great deal of interest in both academia and industry for improving vehicle ride comfort and road holding performance [1], [2]. Since active suspensions need actuators to provide the required forces, one practical consideration in real-world applications involves choosing appropriate actuators that can fit into the suspension packaging space, and satisfy the practical power and bandwidth requirements. It has been noted that electrohydraulic actuators are regarded as one of the most viable choices for an active suspension due to their high power-to-weight ratio and low cost. Therefore, in recent years, many studies have focused on electrohydraulic active suspensions, and various control algorithms have been proposed to deal with the involved highly nonlinear dynamics of electrohydraulic actuators [3]–[10].

It is still a challenge to develop an appropriate control strategy for dealing with the highly nonlinear dynamics of electrohydraulic actuators in active suspensions. Generally speaking, the currently proposed control algorithms for electrohydraulic active suspensions can be divided into two main groups: one is the two-loop control strategy, in which the outer loop is used to provide the desired forces and the inner loop is used to make the electrohydraulic actuators track the desired forces; the other is the sliding-mode-based control strategy. As proved in [11], pure proportional-integral differential (PID)-like controllers are not capable of giving satisfactory performance in the actuator force tracking problem, and more sophisticated control schemes should be employed. Hence, some attempts have been made to compensate for this shortcoming through advanced inner loop force control algorithms, for example [11]–[14]. Nevertheless, due to their highly nonlinear dynamics, using electrohydraulic actuators to track the desired forces is fundamentally limited when interacting with a dynamic environment [11]. On the other hand, the chattering phenomenon is inevitable in sliding mode control, and it may excite unmodeled high-frequency dynamics, which degrades the performance of the system and may even lead to instability. Techniques such as adaptive fuzzy sliding control [5] and self-organizing fuzzy sliding control [15] were then proposed to smooth the chattering phenomenon. However, these approaches need a complicated learning mechanism or a specific performance decision table, which is designed by a trial and error process, and presents certain difficulties in application.

In practice, the vehicle sprung mass varies with the loading conditions, such as the payload and number of vehicle occupants. The control performance of a vehicle suspension will be affected if the sprung mass variation is not considered in the controller design process. In spite of its importance, this problem has not been explicitly dealt with in any previous studies on electrohydraulic suspensions. Furthermore, the constraint on control input voltage sent to the actuator servo-valve has not yet been considered for controller design, although the input power provided to an electrohydraulic actuator is, in practice, limited. Thus, it is surely necessary to develop a new controller design approach that aims at improving the performance of electrohydraulic active suspensions while considering the actuator nonlinear dynamics, sprung mass uncertainty, and control input voltage limitations.

Following the earlier discussion on electrohydraulic active suspensions, in this paper, a fuzzy state feedback controller design method is presented to improve the ride comfort performance of vehicles with electrohydraulic active suspensions through a Takagi–Sugeno (T–S) fuzzy model approach. In recent decades, fuzzy logic control has been proposed as an alternative

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approach to conventional control techniques for complex nonlinear systems. It was originally introduced and developed as a model-free control design approach, and it has been applied to active suspensions [5], [8], [16] to deal with the nonlinearities associated with the actuator dynamics, shock absorbers, suspension springs, etc. However, the model-free fuzzy logic control suffers from a number of criticisms, such as the lack of systematic stability analysis and controller design. It also faces a challenge in the development of fuzzy rules. Recent research on fuzzy logic control has, therefore, been devoted to model-based fuzzy control systems that guarantee not only stability, but also performance of closed-loop fuzzy control systems [17]. The T–S fuzzy system is one of the most popular systems in model-based fuzzy control. It is described by fuzzy IF-THEN rules that represent local linear input–output relations of a nonlinear system. The overall fuzzy model of the nonlinear system is obtained by fuzzy "blending" of the linear models. The T–S model is capable of approximating many real nonlinear systems, e.g., mechanical systems and chaotic systems. Since it employs linear models in the consequent part, linear control theory can be applied for system analysis and synthesis accordingly, based on the parallel-distributed compensation (PDC) scheme [18]. The T–S fuzzy models are therefore becoming powerful engineering tools for the modeling and control of complex dynamic systems.

To apply the T–S model-based fuzzy control strategy to electrohydraulic active suspensions, in this study, the nonlinear uncertain suspension is first represented by a T–S fuzzy model. Then, a fuzzy state feedback controller is designed for the fuzzy T–S model to improve the ride comfort performance by optimizing the H_∞ performance of the transfer function from the road disturbance to the sprung mass accelerations. To avoid the problem of having a large number of inequalities when the input saturation constraint is characterized in terms of the convex hull of some linear combination of linear functions and saturation functions [19], the norm-bounded approach [20], [21] is used here to handle the saturation nonlinearity. The sufficient conditions for the existence of such a controller are derived as linear matrix inequalities (LMIs) that can be solved very efficiently by means of the most powerful tools available to date, e.g., MATLAB LMI Toolbox. The proposed fuzzy state feedback controller design approach is validated by simulations on a full-car electrohydraulic suspension model. A comparison of the results shows that the designed controller can achieve good suspension performance regardless of the actuator nonlinear dynamics, sprung mass variation, and control input constraints.

The rest of this paper is organized as follows. Section II presents the model of a full-car electrohydraulic suspension. The T–S fuzzy model of the nonlinear uncertain suspension is given in Section III. In Section IV, the computational algorithm for the fuzzy state feedback controller is provided. Section V presents the design results and simulations. Finally, the study's findings are summarized in Section VI.

The notation used throughout the paper is reasonably standard. For a real symmetric matrix W, the notation of $W > 0$ ($W < 0$) is used to denote its positive (negative) definiteness, $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm, I is used to denote the identity matrix of appropriate dimensions, and to simplify notation, $*$ is used to represent a block matrix that is readily inferred by symmetry.

II. Electrohydraulic Suspension Model

A full-car electrohydraulic suspension model, as shown in Fig. 1, is considered in this paper. This is a 7-DOF model where the sprung mass is assumed to be a rigid body with freedoms of motion in the vertical, pitch, and roll directions, and each unsprung mass has freedom of motion in the vertical direction. In Fig. 1, z_s is the vertical displacement at the center of gravity, θ and ϕ are the pitch and roll angles of the sprung mass, m_s, m_{uf}, and m_{ur} denote the sprung and unsprung masses, respectively, and I_θ and I_ϕ are pitch and roll moments of inertia. The front and rear displacements of the sprung mass on the left and right sides are denoted by z_{1fl}, z_{1rl}, z_{1fr}, and z_{1rr}. The front and rear displacements of the unsprung masses on the left and right sides are denoted by z_{2fl}, z_{2rl}, z_{2fr}, and z_{2rr}. The disturbances, which are caused by road irregularities, are denoted by w_{fl}, w_{rl}, w_{fr}, and w_{rr}. The front and rear suspension stiffnesses and the front and rear tyre stiffnesses are denoted by k_{sf}, k_{sr}, and k_{tf}, k_{tr}, respectively. The front and rear suspension damping coefficients are c_{sf} and c_{sr}. Four electrohydraulic actuators are placed between the sprung mass and the unsprung masses to generate pushing forces, denoted by F_{fl}, F_{rl}, F_{fr}, and F_{rr}.

Assuming that the pitch angle θ and the roll angle ϕ are small enough, the following linear approximations are applied

$$
\begin{aligned}
z_{1fl}(t) &= z_s(t) + l_f\theta(t) + t_f\phi(t)\\
z_{1fr}(t) &= z_s(t) + l_f\theta(t) - t_f\phi(t)\\
z_{1rl}(t) &= z_s(t) - l_r\theta(t) + t_r\phi(t)\\
z_{1rr}(t) &= z_s(t) - l_r\theta(t) - t_r\phi(t)
\end{aligned} \tag{1}
$$

and a kinematic relationship between $x_s(t)$ and $q(t)$ can be established as

$$x_s(t) = L^T q(t) \tag{2}$$

where $q(t) = [\, z_s(t) \quad \theta(t) \quad \phi(t)\,]^T$, $x_s(t) = [z_{1fl}(t) \quad z_{1fr}(t) \quad z_{1rl}(t) \quad z_{1rr}(t)]^T$, and

$$L = \begin{bmatrix} 1 & 1 & 1 & 1\\ l_f & l_f & -l_r & -l_r\\ t_f & -t_f & t_r & -t_r \end{bmatrix}.$$

In terms of mass, damping, and stiffness matrices, the motion equations of the full-car suspension model can be formalized as

$$
\begin{aligned}
M_s\ddot{q}(t) &= LB_s(\dot{x}_u(t) - \dot{x}_s(t))\\
&\quad + LK_s(x_u(t) - x_s(t)) - LF(t)\\
M_u\ddot{x}_u(t) &= B_s(\dot{x}_s(t) - \dot{x}_u(t)) + K_s(x_s(t) - x_u(t))\\
&\quad + K_t(w(t) - x_u(t)) + F(t)
\end{aligned} \tag{3}
$$

where $x_u(t) = [\, z_{2fl}(t) \quad z_{2fr}(t) \quad z_{2rl}(t) \quad z_{2rr}(t)\,]^T$, $w(t) = [\, w_{fl}(t) \quad w_{fr}(t) \quad w_{rl}(t) \quad w_{rr}(t)\,]^T$, $F(t) = [F_{fl}(t) \quad F_{fr}(t)$

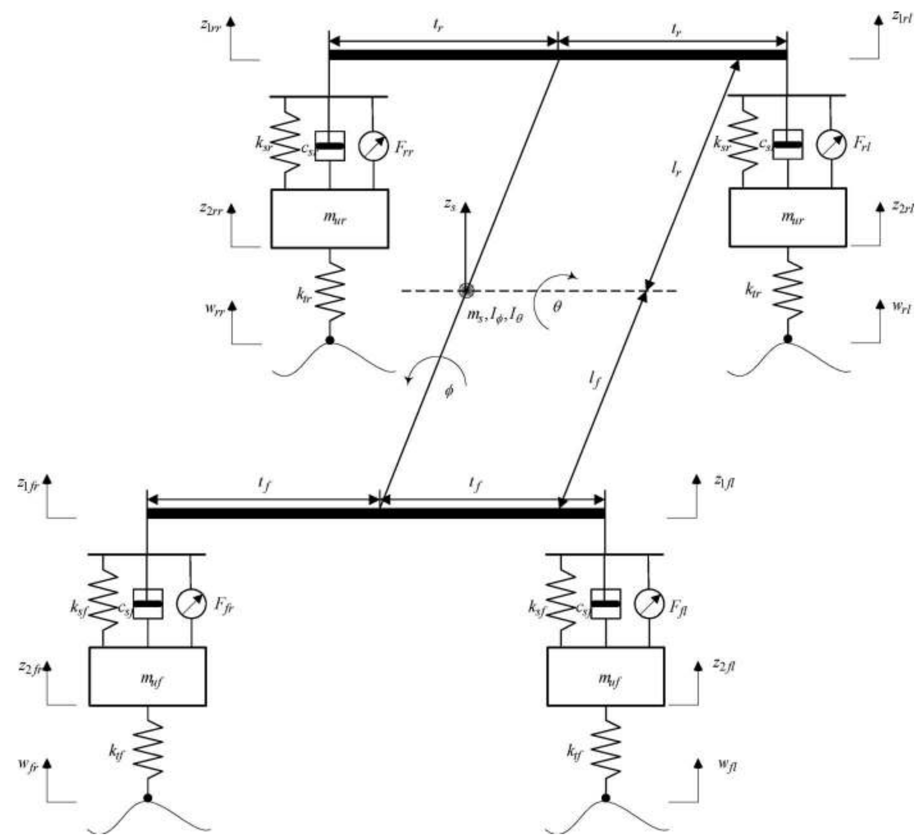

Fig. 1. Full-car suspension model.

$F_{rl}(t) \quad F_{rr}(t)]^T$, and the matrices are given as

$$M_s = \begin{bmatrix} m_s & 0 & 0 \\ 0 & I_\theta & 0 \\ 0 & 0 & I_\phi \end{bmatrix} \quad M_u = \begin{bmatrix} m_{uf} & 0 & 0 & 0 \\ 0 & m_{uf} & 0 & 0 \\ 0 & 0 & m_{ur} & 0 \\ 0 & 0 & 0 & m_{ur} \end{bmatrix}$$

$$B_s = \begin{bmatrix} c_{sf} & 0 & 0 & 0 \\ 0 & c_{sf} & 0 & 0 \\ 0 & 0 & c_{sr} & 0 \\ 0 & 0 & 0 & c_{sr} \end{bmatrix} \quad K_s = \begin{bmatrix} k_{sf} & 0 & 0 & 0 \\ 0 & k_{sf} & 0 & 0 \\ 0 & 0 & k_{sr} & 0 \\ 0 & 0 & 0 & k_{sr} \end{bmatrix}$$

$$K_t = \begin{bmatrix} k_{tf} & 0 & 0 & 0 \\ 0 & k_{tf} & 0 & 0 \\ 0 & 0 & k_{tr} & 0 \\ 0 & 0 & 0 & k_{tr} \end{bmatrix}.$$

Substituting (2) into (3), we obtain

$$M_m \ddot{z}_m(t) + B_m \dot{z}_m(t) + K_m z_m(t) = K_{mt} w(t) + L_m F(t) \tag{4}$$

where $z_m(t) = [\, q^T(t) \quad x_u^T(t)\,]^T$ and

$$M_m = \begin{bmatrix} M_s & 0 \\ 0 & M_u \end{bmatrix} \qquad B_m = \begin{bmatrix} LB_sL^T & -LB_s \\ -B_sL^T & B_s \end{bmatrix}$$

$$K_m = \begin{bmatrix} LK_sL^T & -LK_s \\ -K_sL^T & K_s + K_t \end{bmatrix} \quad K_{mt} = \begin{bmatrix} 0 \\ K_t \end{bmatrix} \quad L_m = \begin{bmatrix} -L \\ I \end{bmatrix}.$$

The state-space form of (4) can be expressed as

$$\dot{x}_g(t) = \hat{A} x_g(t) + \hat{B}_1 w(t) + \hat{B}_2 F(t) \tag{5}$$

where

$$x_g(t) = [z_m^T(t) \quad \dot{z}_m^T(t)]^T \quad \hat{A} = \begin{bmatrix} 0 & I \\ -M_m^{-1}K_m & -M_m^{-1}B_m \end{bmatrix},$$

$$\hat{B}_1 = \begin{bmatrix} 0 \\ M_m^{-1}K_{mt} \end{bmatrix} \quad \hat{B}_2 = \begin{bmatrix} 0 \\ M_m^{-1}L_m \end{bmatrix}.$$

The electrohydraulic actuator dynamics can be expressed as [4], [9], [11]–[13]

$$\dot{F}_i(t) = -\beta F_i(t) - \alpha A_s^2(\dot{z}_{1i}(t) - \dot{z}_{2i}(t)) + \gamma_a A_s \sqrt{P_s - \frac{\mathrm{sgn}(x_{vi}(t))F_i(t)}{A_s}}\, x_{vi}(t),$$

$$\dot{x}_{vi}(t) = \frac{1}{\tau}(-x_{vi}(t) + K_v u_i(t)) \tag{6}$$

where $x_{vi}(t)$ is the spool valve displacement, $u_i(t)$ is the control input voltage to the servo valve, i denotes fl, fr, rl, and rr, respectively, A_s is the actuator ram area, P_s is the hydraulic supply pressure, $\alpha = 4\beta_e/V_t$, $\beta = \alpha C_{tm}$, and $\gamma_a = \alpha C_d \omega_a \sqrt{1/\rho_a}$, where β_e is the effective bulk modulus, V_t is the total actuator volume, C_{tm} is the coefficient of total leakage due to pressure, C_d is the discharge coefficient, ω_a is the spool valve area gradient, and ρ_a is the hydraulic fluid density. τ is the time constant of the spool valve dynamics and K_v is the conversion gain.

The dynamics equation (6) of the electrohydraulic actuator can be further modeled with the state-space form as

$$\dot{x}_{ai}(t) = A_{ai}(t)x_{ai}(t) + \hat{A}_{ai}x_g(t) + B_{ai}u_i(t),$$
$$F_i(t) = C_{ai}x_{ai}(t) \tag{7}$$

where

$$x_{ai}(t) = [\, F_i(t) \quad x_{vi}(t)\,]^T \qquad A_{ai}(t) = \begin{bmatrix} -\beta & \gamma_a A_s f_i(t) \\ 0 & -\dfrac{1}{\tau} \end{bmatrix}$$

$$f_i(t) = \sqrt{P_s - \frac{\text{sgn}(x_{vi}(t))F_i(t)}{A_s}} \qquad B_{ai} = \begin{bmatrix} 0 \\ \dfrac{K_v}{\tau} \end{bmatrix}$$

$$C_{ai} = [1 \quad 0]$$

and $\hat{A}_{ai}x_g(t)$ describes the term of $-\alpha A_s^2(\dot{z}_{1i}(t) - \dot{z}_{2i}(t))$.

Combining the actuator dynamics equation (7) with the suspension model (5), we obtain the electrohydraulic suspension model in state-space form as

$$\dot{x}(t) = A(t)x(t) + B_1 w(t) + B_2 \bar{u}(t) \tag{8}$$

where $x(t) = [\, x_g^T(t) \quad x_{afl}(t) \quad x_{afr}(t) \quad x_{arl}(t) \quad x_{arr}(t)\,]^T$ is the state vector and $\bar{u}(t)$ is the bounded input voltage to the actuator servo valve. In real applications, the input voltage to the servo valve can be bounded as $\bar{u}(t)$ =sat$(u(t))$, where sat$(u(t))$ is a saturation function of control input $u(t)$, defined as

$$\text{sat}(u(t)) = \begin{cases} -u_{\lim}, & \text{if } u(t) < -u_{\lim} \\ u(t), & \text{if } -u_{\lim} \le u(t) \le u_{\lim} \\ u_{\lim}, & \text{if } u(t) > u_{\lim} \end{cases} \tag{9}$$

where $u_{\lim}$ is the control input limit. The matrices are

$$A(t) = \begin{bmatrix} \hat{A} & \hat{B}_2 C_a \\ \hat{A}_a & A_a(t) \end{bmatrix} \quad B_1 = \begin{bmatrix} \hat{B}_1 \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ B_a \end{bmatrix}.$$

It is noted that the system matrix $A(t)$ is a nonlinear and time-varying matrix due to the nonlinear time-varying behavior of the actuator dynamics and the variation of the sprung mass.

III. T–S Fuzzy Modeling

The full-car electrohydraulic suspension model (8) incorporates well-characterized and essential actuator nonlinearities, and the controller design is required to consider both the parameter uncertainty and the control input constraint, which leads to a challenging control problem. In order to design a controller for the model through the fuzzy approach, the T–S fuzzy modeling technique will be applied, and the idea of "sector nonlinearity" [18] is employed to construct an exact T–S fuzzy model for the nonlinear uncertain suspension system (8).

Suppose that the actuator force $F_i(t)$ (where i denotes fl, fr, rl, and rr, respectively) is bounded in practice by its minimum value $F_{i\min}$ and its maximum value $F_{i\max}$; the nonlinear function $f_i(t)$ is then bounded by its minimum value $f_{\min}$ and its maximum value $f_{\max}$. Thus, using the idea of "sector nonlinearity" [18], $f_i(t)$ can be represented by

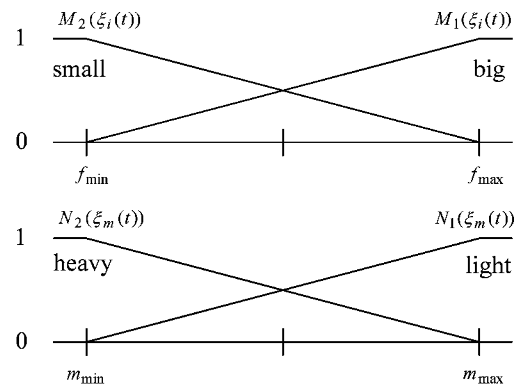

Fig. 2. Membership functions.

$$f_i(t) = M_{1i}(\xi_i(t))f_{\max} + M_{2i}(\xi_i(t))f_{\min} \tag{10}$$

where $\xi_i(t) = f_i(t)$ is a premise variable, $M_{1i}(\xi_i(t))$ and $M_{2i}(\xi_i(t))$ are membership functions, and

$$M_{1i}(\xi_i(t)) = \frac{f_i(t) - f_{\min}}{f_{\max} - f_{\min}} \qquad M_{2i}(\xi_i(t)) = \frac{f_{\max} - f_i(t)}{f_{\max} - f_{\min}}. \tag{11}$$

Similarly, the uncertain sprung mass $m_s(t)$ is bounded by its minimum value $m_{s\min}$ and its maximum value $m_{s\max}$, and can thus be represented by

$$\frac{1}{m_s(t)} = N_1(\xi_m(t))m_{\max} + N_2(\xi_m(t))m_{\min} \tag{12}$$

where $\xi_m(t) = 1/m_s(t)$ is also a premise variable, $m_{\max} = 1/m_{s\min}$, $m_{\min} = 1/m_{s\max}$, and $N_1(\xi_m(t))$ and $N_2(\xi_m(t))$ are membership functions that are defined as

$$N_1(\xi_m(t)) = \frac{1/m_s(t) - m_{\min}}{m_{\max} - m_{\min}}$$
$$N_2(\xi_m(t)) = \frac{m_{\max} - 1/m_s(t)}{m_{\max} - m_{\min}}. \tag{13}$$

For description brevity, we name the aforementioned membership functions $M_{1i}(\xi_i(t))$, $M_{2i}(\xi_i(t))$, $N_1(\xi_m(t))$, and $N_2(\xi_m(t))$, shown in Fig. 2, as 'big," "small," 'light," and 'heavy," respectively. The nonlinear uncertain suspension model (8) can then be represented by a T–S fuzzy model composed of 32 (2^5) fuzzy rules, as listed in Table I, where B, S, L, and H represent 'big," 'small," 'light," and 'heavy," respectively. To describe the T–S fuzzy model more clearly, several examples of the fuzzy IF-THEN rules corresponding to Table I are explained as follows.

Model Rule 1:

IF $\xi_i(t)$ (i denotes fl, fr, rl, and rr, respectively) are small and $\xi_m(t)$ is light,

THEN $\dot{x}(t) = A_1 x(t) + B_1 w(t) + B_2 \bar{u}(t)$

where matrix A_1 is obtained from matrix $A(t)$ in (8) by replacing f_i with $f_{\min}$ and $1/m_s$ with $m_{\max}$.

⋮

TABLE I
LIST OF FUZZY RULES

Rule No.	Premise variables					Rule No.	Premise variables				
	ξ_{fl}	ξ_{fr}	ξ_{rl}	ξ_{rr}	ξ_m		ξ_{fl}	ξ_{fr}	ξ_{rl}	ξ_{rr}	ξ_m
1	S	S	S	S	L	17	S	S	S	S	H
2	B	S	S	S	L	18	B	S	S	S	H
3	S	B	S	S	L	19	S	B	S	S	H
4	B	B	S	S	L	20	B	B	S	S	H
5	S	S	B	S	L	21	S	S	B	S	H
6	B	S	B	S	L	22	B	S	B	S	H
7	S	B	B	S	L	23	S	B	B	S	H
8	B	B	B	S	L	24	B	B	B	S	H
9	S	S	S	B	L	25	S	S	S	B	H
10	B	S	S	B	L	26	B	S	S	B	H
11	S	B	S	B	L	27	S	B	S	B	H
12	B	B	S	B	L	28	B	B	S	B	H
13	S	S	B	B	L	29	S	S	B	B	H
14	B	S	B	B	L	30	B	S	B	B	H
15	S	B	B	B	L	31	S	B	B	B	H
16	B	B	B	B	L	32	B	B	B	B	H

Model Rule 16:

IF $\xi_i(t)$ (i denotes $fl,\ fr,\ rl,$ and $rr,$ respectively) are big and $\xi_m(t)$ is light,

THEN $\dot{x}(t) = A_{16}x(t) + B_1 w(t) + B_2\bar{u}(t)$

where matrix A_{16} is obtained from matrix $A(t)$ in (8) by replacing f_i with $f_{\max}$ and $1/m_s$ with $m_{\max}$.

Model Rule 17:

IF $\xi_i(t)$ (i denotes $fl,\ fr,\ rl,$ and $rr,$ respectively) are small and $\xi_m(t)$ is heavy,

THEN $\dot{x}(t) = A_{17}x(t) + B_1 w(t) + B_2\bar{u}(t)$

where matrix A_{17} is obtained from matrix $A(t)$ in (8) by replacing f_i with $f_{\min}$ and $1/m_s$ with $m_{\min}$.

$$\vdots$$

Model Rule 32:

IF $\xi_i(t)$ (i denotes $fl,\ fr,\ rl,$ and $rr,$ respectively) are big and $\xi_m(t)$ is heavy,

THEN $\dot{x}(t) = A_{32}x(t) + B_1 w(t) + B_2\bar{u}(t)$

where matrix A_{32} is obtained from matrix $A(t)$ in (8) by replacing f_i with $f_{\max}$ and $1/m_s$ with $m_{\min}$.

Thus, the T–S fuzzy model that represents exactly the nonlinear uncertain suspension model (8) under the assumption of bounds on actuator forces $F_i(t) \in [F_{\min}, F_{\max}]$ and sprung mass $m_s(t) \in [m_{s\min}, m_{s\max}]$ is obtained as

$$\dot{x}(t) = \sum_{i=1}^{32} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2\bar{u}(t) \tag{14}$$

where

$$h_1(\xi(t)) = M_{2fl}(\xi_{fl}(t))M_{2fr}(\xi_{fr}(t))M_{2rl}(\xi_{rl}(t)) \times M_{2rr}(\xi_{rr}(t))N_1(\xi_m(t))$$

$$h_2(\xi(t)) = M_{1fl}(\xi_{fl}(t))M_{2fr}(\xi_{fr}(t))M_{2rl}(\xi_{rl}(t)) \times M_{2rr}(\xi_{rr}(t))N_1(\xi_m(t))$$

$$\vdots$$

$$h_{32}(\xi(t)) = M_{1fl}(\xi_{fl}(t))M_{1fr}(\xi_{fr}(t))M_{1rl}(\xi_{rl}(t)) \times M_{1rr}(\xi_{rr}(t))N_2(\xi_m(t)),$$

$$h_i(\xi(t)) \geq 0, \quad i = 1, 2, \ldots, 32, \text{ and } \sum_{i=1}^{32} h_i(\xi(t)) = 1.$$

In practice, the actuator force $F_i(t)$, the spool valve position $x_{vi}(t)$, and the sprung mass $m_s(t)$ can be measured; thus, the T–S fuzzy model (14) can be realized.

It is noted that the T–S fuzzy model (14) is obtained via the "sector nonlinearity" approach based on the analysis of the nonlinear function $f_i(t)$ and the variation of sprung mass m_s, the bounds of which can be estimated in a real operating situation. The construction of a T–S fuzzy model from a given nonlinear dynamic model can also utilize the idea of "local approximation" or a combination of "sector nonlinearity" and "local approximation" [18]. In general, these are analytic transformation techniques, which can be applied only to models described analytically. Since analytic techniques need problem-dependent human intuition and cannot be easily solved in some cases, recently, a higher order-singular-value-decomposition (HOSVD)-based tensor product (TP) model transformation approach was proposed to automatically and numerically transform a general dynamic system model into a TP model form, including polytopic and T–S model forms [22]. The TP model representation has shown various advantages for LMI-based controller design [23], [24], and relaxed LMI conditions can be further obtained for closed-loop fuzzy systems with TP structure [25]. There is also a MATLAB Toolbox for TP model transformation (available for download together with documentation and examples at *http://tptool.sztaki.hu/tpde)*.

For our problem, in fact, using the convex normalized (CNO) type of TP model transformation can obtain the same membership functions as those described before when $f_i(t)$ and $1/m_s(t)$ are used as time-varying parameter variables. However, if $m_s(t)$ is used as a time-varying parameter variable instead of $1/m_s(t)$, different membership functions can be generated with the TP model transformation, as shown in Fig. 3. It can be seen from Fig. 3 that the membership functions are nonlinear, and they are different from those shown in Fig. 2. Nevertheless, using this new type of membership function does not alter the LMI-based controller design process or the design results obtained with the membership functions defined in (13). Generally, $F_i(t)$ and $x_{vi}(t)$ can be directly used as the time-varying parameter variables to obtain the T–S model using the TP model transformation. However, the TP model transformation should consider the tradeoff between approximation accuracy and complexity. For the studied problem, when using the derived membership functions (11) and (13), only 32 fuzzy rules need to be applied. Therefore, in this paper, the derived membership functions (11) and (13) are used. And, despite the authors' effort, no other types of membership functions are found to yield better performance than the derived membership functions (11) and (13),

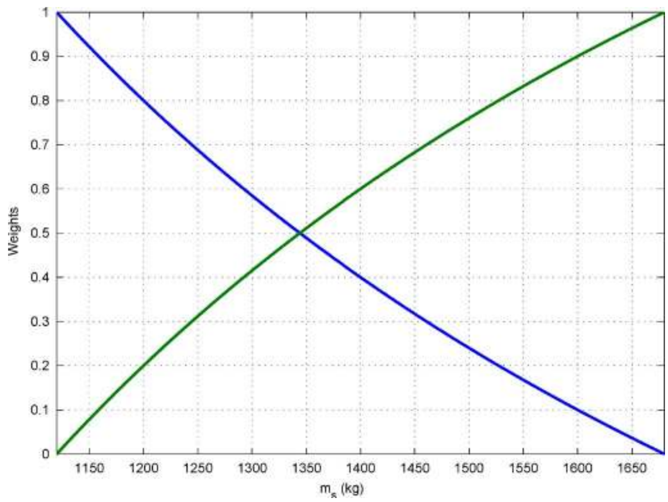

Fig. 3. Membership functions for sprung mass obtained from TP model transformation.

although there may exist methods to automatically generate affine decompositions.

IV. Fuzzy Controller Design

In order to avoid the problem associated with having a large number of inequalities involved in the controller design, the norm-bounded approach [20], [21] is used to handle the saturation nonlinearity defined in (9). Hence, (14) will be written as

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{32} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2 \bar{u}(t)\\ &= \sum_{i=1}^{32} h_i(\xi(t))A_i x(t) + B_1 w(t) + B_2 \frac{1+\varepsilon}{2}u(t)\\ &\quad + B_2\left(\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right)\\ &= A_h x(t) + B_1 w(t) + B_2\frac{1+\varepsilon}{2}u(t) + B_2 v(t) \quad (15)\end{aligned}$$

where $A_h = \sum_{i=1}^{32} h_i(\xi(t))A_i$ and $v(t) = \bar{u}(t) - \frac{1+\varepsilon}{2}u(t), 0 < \varepsilon < 1$. And for designing the controller, the following lemma will be used.

Lemma 1 [20]: For the saturation constraint defined by (9), as long as $|u(t)| \le \frac{u_{\lim}}{\varepsilon}$, we have

$$\left\|\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right\| \le \frac{1-\varepsilon}{2}\|u(t)\| \quad (16)$$

and hence

$$\left[\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right]^T\left[\bar{u}(t) - \frac{1+\varepsilon}{2}u(t)\right] \le \left(\frac{1-\varepsilon}{2}\right)^2 u^T(t)u(t) \quad (17)$$

where $0 < \varepsilon < 1$.

The fuzzy controller design for the T–S fuzzy model (15) is carried out based on the so-called PDC scheme [18]. For the T–S fuzzy model (15), we construct the fuzzy state feedback controller via the PDC as

$$u(t) = \sum_{i=1}^{32} h_i(\xi(t))K_i x(t) = K_h x(t) \quad (18)$$

where $K_h = \sum_{i=1}^{32} h_i(\xi(t))K_i$ and K_i is the state feedback gain matrix to be designed.

Since ride comfort is an important performance requirement for a vehicle suspension and it can usually be quantified by the sprung mass acceleration, the sprung mass acceleration is chosen as the control output, i.e.

$$z(t) = \ddot{z}_s(t) = \sum_{i=1}^{32} h_i(\xi(t))C_i x(t) = C_h x(t) \quad (19)$$

where $C_h = \sum_{i=1}^{32} h_i(\xi(t))C_i$ and C_i is extracted from the eighth to the tenth row of matrix A_i, $i = 1, 2, \ldots, 32$.

In order to design an active suspension to perform adequately in a wide range of shock and vibration environments, the L_2 gain of the system (15) with (19) is chosen as the performance measure, which is defined as

$$\|T_{zw}\|_\infty = \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} \quad (20)$$

where $\|z\|_2^2 = \int_0^\infty z^T(t)z(t)dt$ and $\|w\|_2^2 = \int_0^\infty w^T(t)w(t)dt$, and the supermum is taken over all nonzero trajectories of the system (15) with $x(0) = 0$. Our goal is to design a fuzzy controller (18) such that the fuzzy system (15) with controller (18) is quadratically stable and the L_2 gain (20) is minimized.

To design the controller, the following lemma will be used.

Lemma 2: For any matrices (or vectors) X and Y with appropriate dimensions, we have

$$X^T Y + Y^T X \le \epsilon X^T X + \epsilon^{-1} Y^T Y$$

where $\epsilon > 0$ is any scalar.

Theorem 3: For a given number $\gamma > 0$, $0 < \varepsilon < 1$, the T–S fuzzy system (15) with controller (18) is quadratically stable and the L_2 gain defined by (20) is less than γ if there exist matrices $Q > 0$, Y_i, $i = 1, 2, \ldots, 32$, and scalar $\epsilon > 0$, such that, (21) and (22), as shown at the bottom of the next page.

Moreover, the fuzzy state feedback gains can be obtained as $K_i = Y_i Q^{-1}$, $i = 1, 2, \ldots, 32$.

Proof: Let us define a Lyapunov function for the system (15) as

$$V(x(t)) = x^T(t)Px(t) \quad (23)$$

where P is a positive definite matrix. By differentiating (23), we obtain

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)\\ &= \left[A_h x(t) + B_1 w(t) + B_2\frac{1+\varepsilon}{2}u(t) + B_2 v(t)\right]^T Px(t)\\ &\quad + x^T(t)P\Big[A_h x(t) + B_1 w(t) + B_2\frac{1+\varepsilon}{2}u(t)\\ &\quad + B_2 v(t)\Big]. \quad (24)\end{aligned}$$

By Lemma 1, Lemma 2, and definition (18), we have

$$\begin{aligned}\dot V(x(t)) \le\ & x^T(t)\Big[A_h^T P + PA_h + \Big(B_2\frac{1+\varepsilon}{2}K_h\Big)^T P \\ & + PB_2\frac{1+\varepsilon}{2}K_h\Big]x(t) \\ & + w^T(t)B_1^T Px(t) + x^T(t)PB_1 w(t) + \epsilon v^T(t)v(t) \\ & + \epsilon^{-1}x^T(t)PB_2B_2^T Px(t) \\ \le\ & x^T(t)\Theta x(t) + w^T(t)B_1^T Px(t) + x^T(t)PB_1 w(t)\end{aligned} \tag{25}$$

where

$$\Theta = \Big[A_h^T P + PA_h + \Big(B_2\frac{1+\varepsilon}{2}K_h\Big)^T P + PB_2\frac{1+\varepsilon}{2}K_h + \epsilon\Big(\frac{1-\varepsilon}{2}\Big)^2 K_h^T K_h + \epsilon^{-1}PB_2B_2^T P\Big] \tag{26}$$

and ϵ is any positive scalar.

Adding $z^T(t)z(t) - \gamma^2 w^T(t)w(t)$ on both sides of (25) yields

$$\begin{aligned}\dot V(x(t)) &+ z^T(t)z(t) - \gamma^2 w^T(t)w(t) \\ &\le [\,x^T(t)\quad w^T(t)\,]\begin{bmatrix}\Theta + C_h^T C_h & PB_1\\ B_1^T P & -\gamma^2 I\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\end{bmatrix}.\end{aligned} \tag{27}$$

Let us consider

$$\Pi = \begin{bmatrix}\Theta + C_h^T C_h & PB_1\\ B_1^T P & -\gamma^2 I\end{bmatrix} < 0 \tag{28}$$

then, $\dot V(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \le 0$ and the L_2 gain defined in (20) is less than $\gamma > 0$ with the initial condition $x(0) = 0$ [26]. When the disturbance is zero, i.e., $w(t) = 0$, it can be inferred from (27) that if $\Pi < 0$, then $\dot V(x(t)) < 0$, and the fuzzy system (15) with the controller (18) is quadratically stable.

Pre- and postmultiplying (28) by diag(P^{-1} I) and its transpose, respectively, and defining $Q = P^{-1}$ and $Y_h = K_h Q$, the condition $\Pi < 0$ is equivalent to

$$\Sigma = \begin{bmatrix}QA_h^T + A_hQ + \frac{1+\varepsilon}{2}Y_h^T B_2^T + \frac{1+\varepsilon}{2}B_2Y_h + \epsilon\Big(\frac{1-\varepsilon}{2}\Big)^2 Y_h^T Y_h + \epsilon^{-1}B_2B_2^T + QC_h^T C_h Q & B_1\\ B_1^T & -\gamma^2 I\end{bmatrix} < 0. \tag{29}$$

By the Schur complement, $\Sigma < 0$ is equivalent to (Ψ), as shown at the bottom of this page.

By the definitions $A_h = \sum_{i=1}^{32} h_i(\xi(t))A_i$, $C_h = \sum_{i=1}^{32} h_i(\xi(t))C_i$, $Y_h = \sum_{i=1}^{32} h_i(\xi(t))Y_i$, and the fact that $h_i(\xi(t)) \ge 0$ and $\sum_{i=1}^{32} h_i(\xi(t)) = 1$, $\Psi < 0$ is equivalent to (21).

On the other hand, from (18), the constraint $|u(t)| \le \frac{u_{\lim}}{\varepsilon}$ can be expressed as

$$\left|\sum_{i=1}^{32} h_i(\xi(t))K_i x(t)\right| \le \frac{u_{\lim}}{\varepsilon}. \tag{31}$$

It is obvious that if $|K_i x(t)| \le \frac{u_{\lim}}{\varepsilon}$, then (31) holds. Let $\Omega(K) = \{x(t)|\, |x^T(t)K_i^T K_i x(t)| \le (\frac{u_{\lim}}{\varepsilon})^2\}$; then the equivalent condition for an ellipsoid $\Omega(P,\rho) = \{x(t)|\, x^T(t)Px(t) \le \rho\}$ being a subset of $\Omega(K)$, i.e., $\Omega(P,\rho) \subset \Omega(K)$, is [19]

$$K_i\Big(\frac{P}{\rho}\Big)^{-1}K_i^T \le \Big(\frac{u_{\lim}}{\varepsilon}\Big)^2. \tag{32}$$

$$\begin{bmatrix}QA_i^T + A_iQ + \frac{1+\varepsilon}{2}\left[Y_i^T B_2^T + B_2Y_i\right] + \epsilon^{-1}B_2B_2^T & Y_i^T & QC_i^T & B_1\\ * & -\epsilon^{-1}\Big(\frac{2}{1-\varepsilon}\Big)^2 I & 0 & 0\\ * & * & -I & 0\\ * & * & * & -\gamma^2 I\end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix}\Big(\frac{u_{\lim}}{\varepsilon}\Big)^2 I & Y_i\\ Y_i^T & \rho^{-1}Q\end{bmatrix} \ge 0. \tag{22}$$

$$\Psi = \begin{bmatrix}QA_h^T + A_hQ + \frac{1+\varepsilon}{2}\left[Y_h^T B_2^T + B_2Y_h\right] + \epsilon^{-1}B_2B_2^T & Y_h^T & QC_h^T & B_1\\ * & -\epsilon^{-1}\Big(\frac{2}{1-\varepsilon}\Big)^2 I & 0 & 0\\ * & * & -I & 0\\ * & * & * & -\gamma^2 I\end{bmatrix} < 0. \tag{30}$$

By the Schur complement, inequality (32) can be written as

$$\begin{bmatrix} \left(\frac{u_{\lim}}{\varepsilon}\right)^2 I & K_i \left(\frac{P}{\rho}\right)^{-1} \\ \left(\frac{P}{\rho}\right)^{-1} K_i^T & \left(\frac{P}{\rho}\right)^{-1} \end{bmatrix} \geq 0. \tag{33}$$

Using the definitions $Q = P^{-1}$ and $Y_i = K_i Q$, inequality (33) is equivalent to (22). This completes the proof. □

The minimization of γ can be realized as

$$\min \gamma \text{ subject to LMIs (21) and (22).} \tag{34}$$

This problem can be solved very efficiently by means of the MATLAB LMI Toolbox software.

Remark 1: It is noted that once the solution of problem (34) is feasible, then the value of γ can be obtained using the LMI Toolbox software and the designed controller can guarantee the L_2 gain (20) to be less than γ in terms of the LMI condition (21). When the designed controller is applied to the system (14), the real value of the L_2 gain (20) for the system (14) under a given disturbance can be evaluated by measuring the control output response (19) and calculating the value using equation (20).

Remark 2: In the paper, the common quadratic Lyapunov function approach, where the Lyapunov function candidate is defined as in (23), is utilized to derive the controller synthesis conditions. It has been noted that common quadratic Lyapunov functions tend to be conservative and, even worse, might not exist for some complex highly nonlinear systems [17] . This is one of the main limitations of this kind of approach. With regard to overcoming the drawback of common quadratic Lyapunov functions, piecewise quadratic Lyapunov functions and fuzzy Lyapunov functions have received increasing attention recently [27], [28]. However, controller synthesis conditions based on piecewise quadratic Lyapunov functions and fuzzy Lyapunov functions are generally given by bilinear matrix inequalities (BMIs), which have to be solved by way of, e.g., a cone complementarity linearisation approach [28] or a descriptor system approach [27]. Therefore, the computation cost and complexity of using piecewise quadratic Lyapunov function and fuzzy Lyapunov function approaches would be much higher in general. In terms of the possible requirement on real-time computation for a practical system and the feasible solutions checking on the given system, this paper keeps using the common quadratic Lyapunov function approach regardless of its conservatism.

V. Application Example

In this section, we will apply the proposed approach to design a fuzzy state feedback controller for a full-car electrohydraulic suspension model, as described in Section II. The full-car suspension model parameter values are listed in Table II, and the parameter values for each hydraulic actuator used in the simulation are given in Table III.

In this study, we suppose that the input voltage of each spool valve is limited to $u_{\lim} = 2.5$ V, and each actuator output force is limited to 2000 N. The bounds of the nonlinear function $f_i(t)$ are estimated as $f_{\min} = 2800$ and $f_{\max} = 4000$. The sprung mass is assumed to be varied between $m_s\text{min} = 1120$ kg and $m_s\text{max} = 1680$ kg, which is $\pm 20\%$ variation of the nominal sprung mass. Using the controller design approach presented in Section IV, and choosing $\varepsilon = 0.97$ by trial and error, we obtain the controller gain matrices K_i that consist of 32 matrices, each with dimensions of 4 × 22. It is noted that the controller gain matrices are constant matrices that do not need to be recalculated in a real-time implementation and can be easily stored in a microprocessor memory (RAM or ROM). The calculation of the control outputs, i.e., input voltages sent to the actuators, in terms of the measured state variables and the calculated membership functions is also quite straightforward. Therefore, the required computational power will not be very high, which enables the implementation of the controller on a DSP, e.g., the Texas Instruments TMS320C30.

TABLE II
Parameter Values of the Full-Car Suspension Model

Parameter	Value	Unit	Parameter	Value	Unit
m_s	1400	kg	c_{sf}	1000	Nm/s
I_θ	2100	kg·m^2	c_{sr}	1100	Nm/s
I_ϕ	460	kg·m^2	k_{sf}	23500	N/m
m_{uf}, m_{ur}	40	kg	k_{sr}	25500	N/m
l_f	0.96	m	k_{tf}, k_{tr}	190000	N/m
l_r	1.44	m	t_f, t_r	0.71	m

TABLE III
Parameter Values of the Hydraulic Actuator

Parameter	α	β	γ	p_s	A_s	τ	K_c
Unit	N/m^5	s^{-1}	N/m$^{5/2}$/kg$^{1/2}$	Pa	m^2	s	m/V
Value	4.515×10^{13}	1	1.545×10^9	10342500	3.35×10^{-4}	0.003	0.001

To compare the suspension performance, an optimal H_∞ controller is designed for the linear full-car suspension model (5) without considering the electrohydraulic actuator dynamics (7). By defining the control output as the sprung mass acceleration and using the bounded real lemma (BRL), this controller gain matrix is obtained as (35), as shown at the bottom of the next page.

Since the optimal H_∞ controller design theory can be found in many references, the controller design process is omitted here for brevity.

In the simulation, a test road disturbance, which is given as

$$z_r(t) = 0.0254 \sin 2\pi t + 0.005 \sin 10.5\pi t + 0.001 \sin 21.5\pi t \text{ (m)} \tag{36}$$

is used first. This road disturbance is close to the car body resonance frequency (1 Hz), with high-frequency disturbance added to simulate the rough road surface. To observe the roll motion, this road disturbance is assumed to pass the wheels on the left side of the car only. The simulation program is realized by MATLAB/Simulink.

For the nominal sprung mass under the specified road disturbance (36), the time-domain responses for three suspensions, i.e., passive suspension, active suspension with H_∞ controller (35), and active suspension with the electrohydraulic actuators

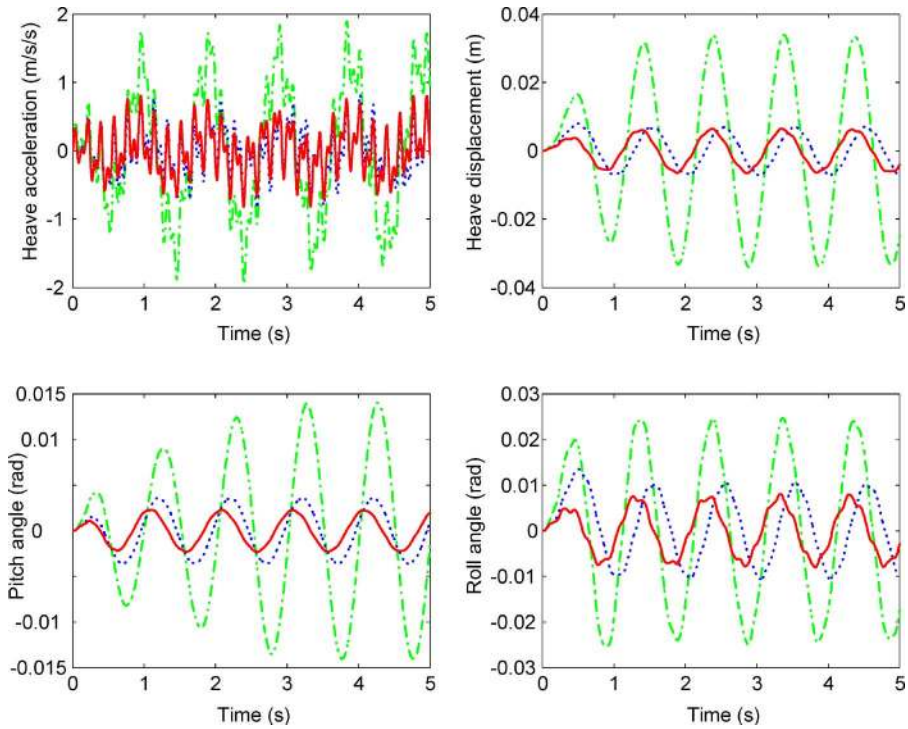

Fig. 4. Time-domain responses under a test road profile. Solid line is for active suspension with fuzzy controller. Dotted line is for active suspension with H_∞ controller. Dot-dashed line is for passive suspension.

and the designed fuzzy controller are compared. The responses, consisting of the sprung mass heave acceleration, heave displacement, pitch angle, and roll angle, are plotted in Fig. 4. It is observed from Fig. 4 that the proposed fuzzy control strategy reduces the sprung mass acceleration and displacement magnitudes significantly compared to the passive suspension under the same road disturbance. The active suspension with the designed fuzzy controller also achieves a very similar suspension performance to the active suspension with an optimal H_∞ controller. This confirms that the proposed fuzzy control strategy can realize good suspension performance with highly nonlinear electrohydraulic actuators.

Fig. 5 shows the actuator output forces for the active suspension with the designed fuzzy controller. It is observed that the four actuators provide different control forces in accordance with the fuzzy rules and the measurements of the state variables. The proposed fuzzy control strategy provides effective actuator forces that aim to optimize the sprung mass heave acceleration, pitch acceleration, and roll acceleration to improve ride comfort performance. Fig. 6 shows the control input voltages, and Fig. 7 shows the nonlinear function outputs. It can be seen from Figs. 6 and 7 that the control input voltages are within the defined input voltage range and the nonlinear function outputs are located within the estimated bounds.

Now, consider the case of an isolated bump in an otherwise smooth road surface. The corresponding ground displacement for the wheel is given by

$$z_r(t) = \begin{cases} \frac{a}{2}\left(1-\cos\left(\frac{2\pi v_0}{l}t\right)\right), & 0 \le t \le \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases} \tag{37}$$

where a and l are the height and the length of the bump. We choose $a = 0.1$ m, $l = 10$ m, and the vehicle forward velocity as $v_0 = 45$ km/h.

For the nominal sprung mass under the road disturbance (37), Fig. 9 shows the bump responses of the sprung mass heave

$$10^4 \times \begin{bmatrix} -1.0324 & -0.2478 & -1.2670 & 0.8961 & 0.2431 & 2.2499 & -2.0930 & 0.2879 & 0.3718 & 0.0665 & 0.0044 & -0.0023 & 0.0078 & -0.0072 \\ -1.0324 & -0.2478 & 1.2670 & 0.2431 & 0.8961 & -2.0930 & 2.2499 & 0.2879 & 0.3718 & -0.0665 & -0.0023 & 0.0044 & -0.0072 & 0.0078 \\ -1.3299 & 1.3735 & -1.5342 & 0.4602 & -0.4434 & -0.6042 & 1.6712 & 0.2097 & -0.3700 & 0.0504 & 0.0128 & -0.0124 & 0.0110 & -0.0081 \\ -1.3299 & 1.3735 & 1.5342 & -0.4434 & 0.4602 & 1.6712 & -0.6042 & 0.2097 & -0.3700 & -0.0504 & -0.0124 & 0.0128 & -0.0081 & 0.0110 \end{bmatrix}. \tag{35}$$

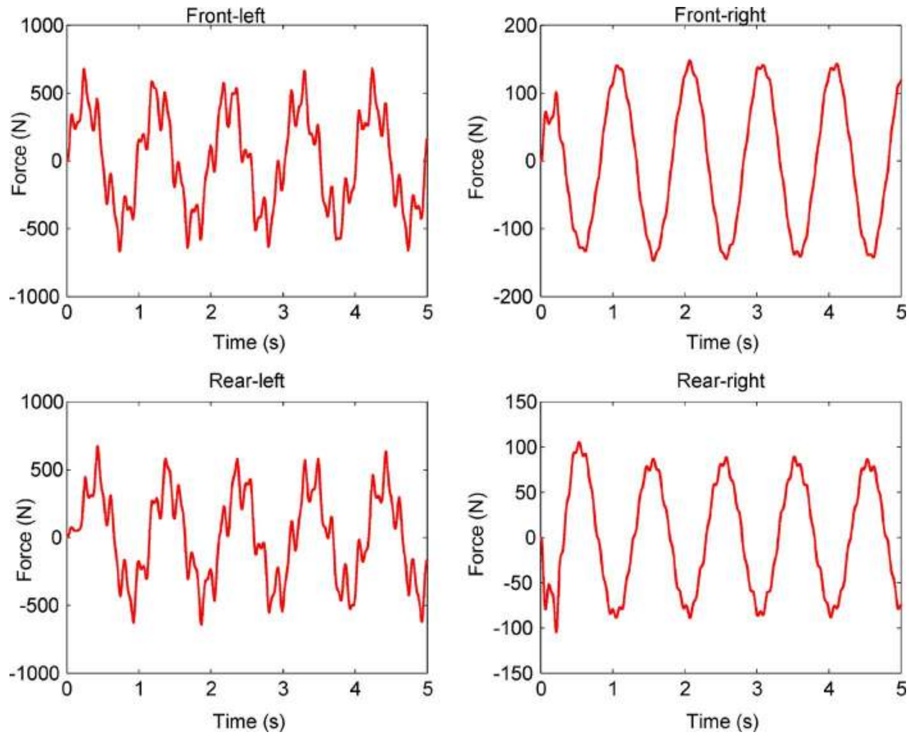

Fig. 5. Actuator output forces for active suspension with fuzzy controller.

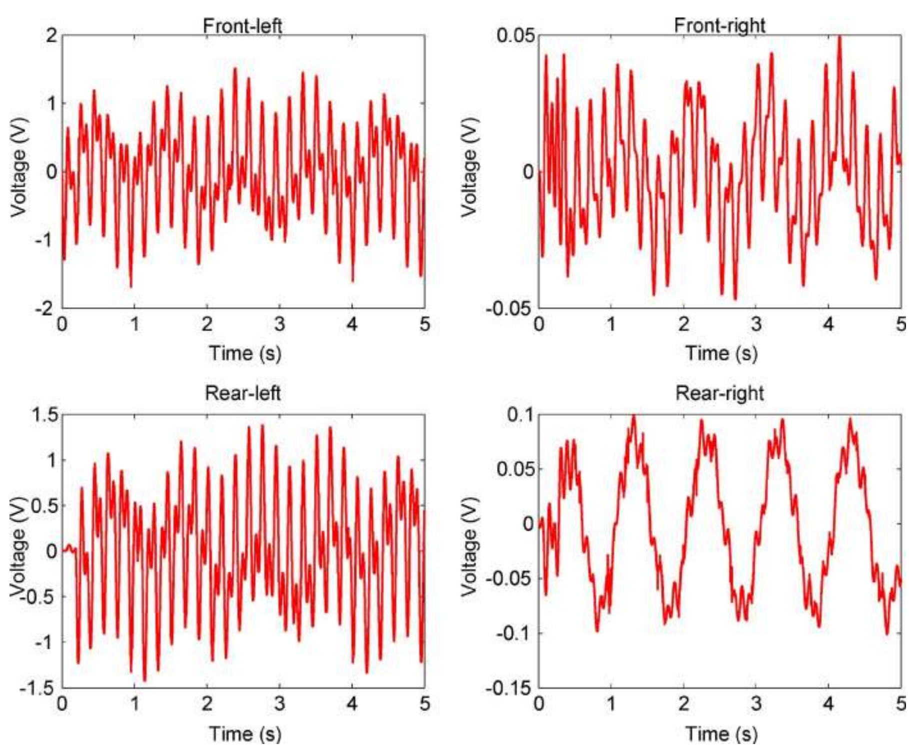

Fig. 6. Actuator control voltages for active suspension with fuzzy controller.

acceleration, pitch acceleration, and roll acceleration for the passive suspension, the active suspension with H_∞ controller (35), and the active suspension with the electrohydraulic actuators and the designed fuzzy controller. It is again observed that the proposed fuzzy control strategy achieves suspension performance very similar to the active suspension with an optimal H_∞ controller. The active suspension performance is significantly improved compared to the passive suspension.

To illustrate the effect of sprung mass variation, Fig. 9 shows the bump responses of the sprung mass heave acceleration for

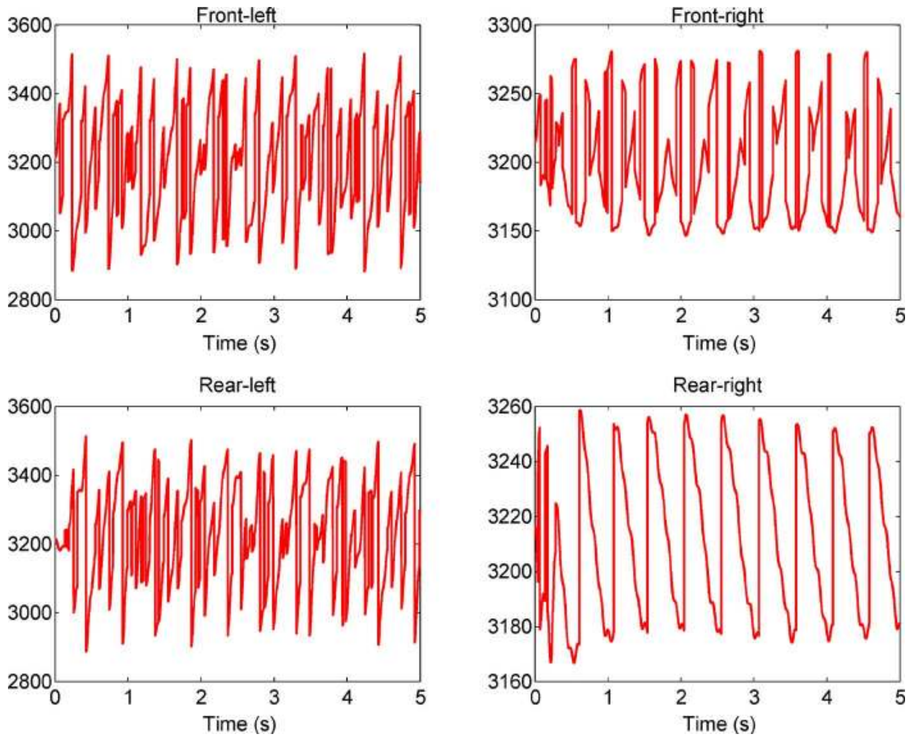

Fig. 7. Nonlinear function values for active suspension with fuzzy controller.

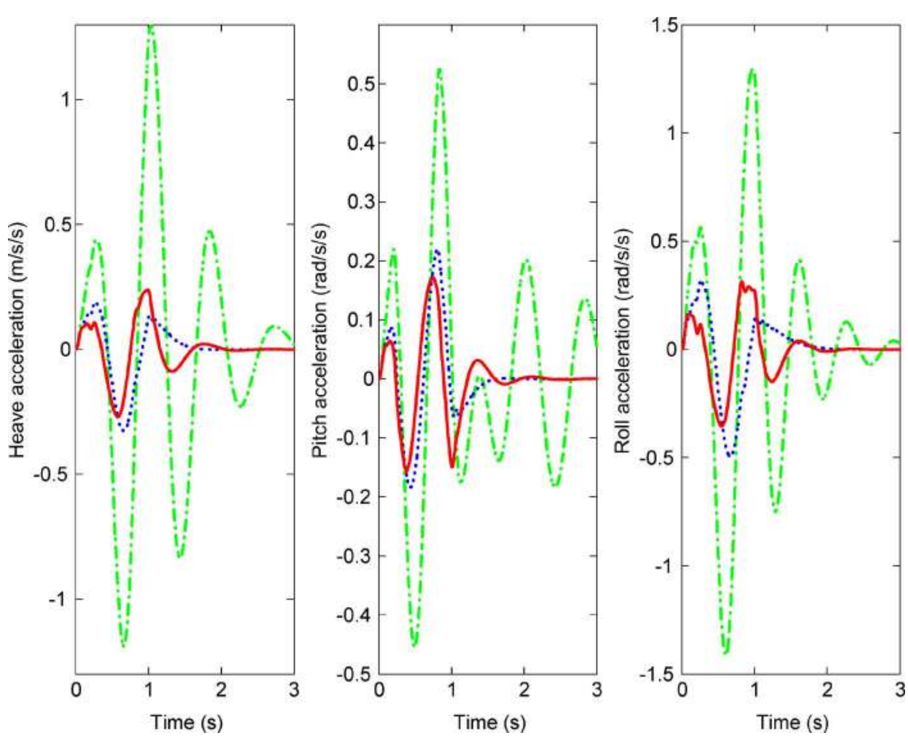

Fig. 8. Acceleration responses under a bump road profile. Solid line is for active suspension with fuzzy controller. Dotted line is for active suspension with H_∞ controller. Dot-dashed line is for passive suspension.

the passive suspension (passive) and the active suspension with electrohydraulic actuator and fuzzy controller (active) when the sprung mass is 1120 and 1680 kg. It is observed that, despite the change in sprung mass, the designed fuzzy controller achieves significantly better performance on heave acceleration, where a lower peak and shorter settling time are obtained. Figs. 10 and 11 show the bump responses of the sprung mass pitch acceleration and roll acceleration, respectively. It can be seen from Fig. 10 that the sprung mass affects the pitch acceleration significantly. However, the active suspension can keep the pitch acceleration

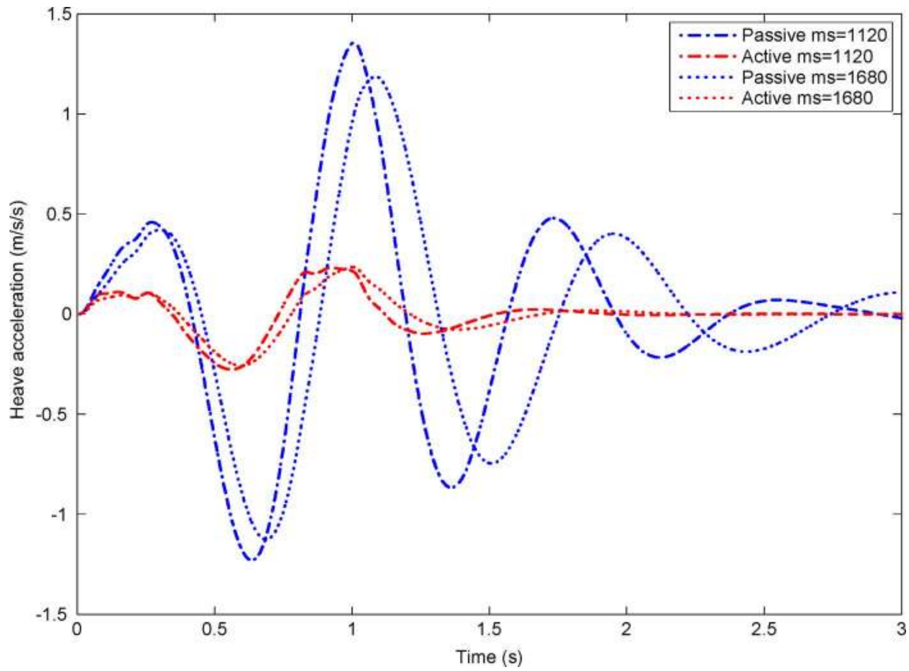

Fig. 9. Heave acceleration responses for different sprung mass.

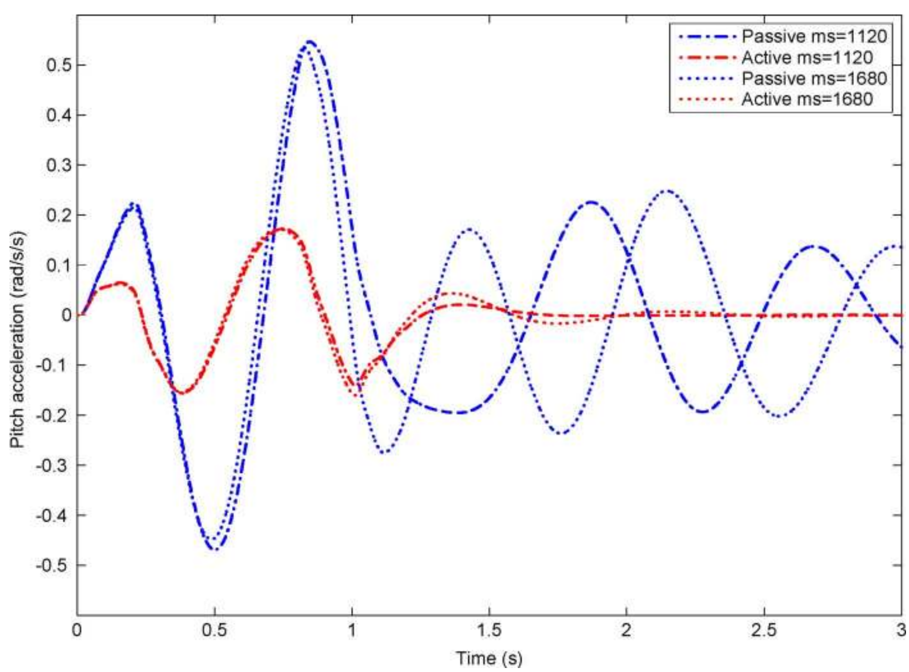

Fig. 10. Pitch acceleration responses for different sprung mass.

low, regardless of the sprung mass variation. In Fig. 11, the active suspension achieves lower roll acceleration compared to the passive suspension, although the sprung mass variation does not affect roll acceleration due to the symmetric distribution of the sprung mass about the vehicle's roll axis. Figs. 9–11 indicate that the improvement in ride comfort can be maintained by the designed active suspension for large changes in load conditions.

When the road disturbance is considered as random vibration, it is typically specified as a stationary random process that can be represented by

$$\dot{z}_r(t) = 2\pi q_0 \sqrt{G_0 V} \omega(t) \tag{38}$$

where G_0 stands for the road roughness coefficient, q_0 is the reference spatial frequency, V is the vehicle forward velocity, and $\omega(t)$ is zero-mean white noise with identity power spectral density. For a given road roughness $G_0 = 512 \times 10^{-6}$ m^3 and a given vehicle forward velocity $V = 20$ m/s, the rms values for sprung mass heave acceleration, pitch acceleration, and roll

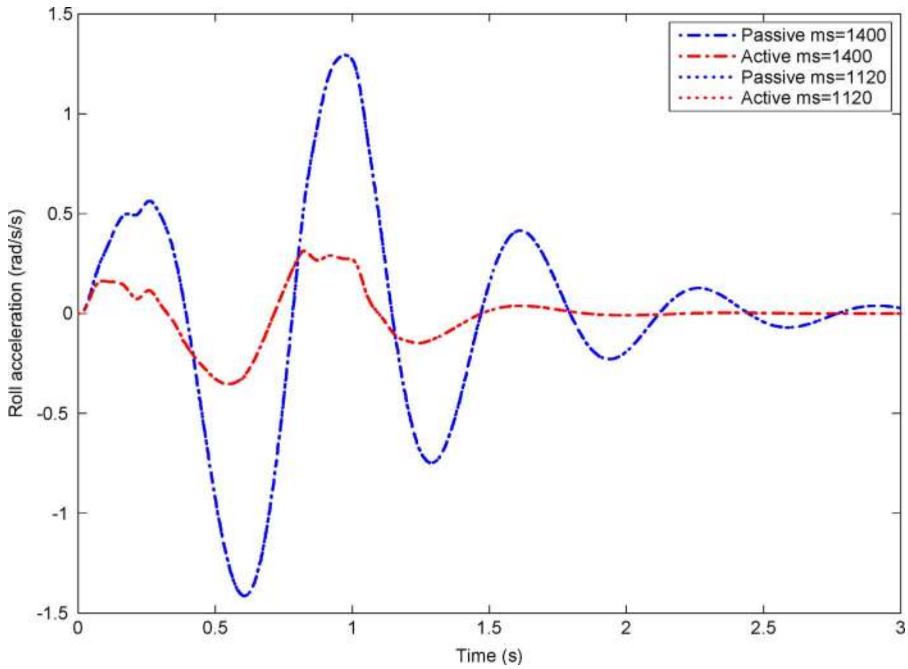

Fig. 11. Roll acceleration responses for different sprung mass.

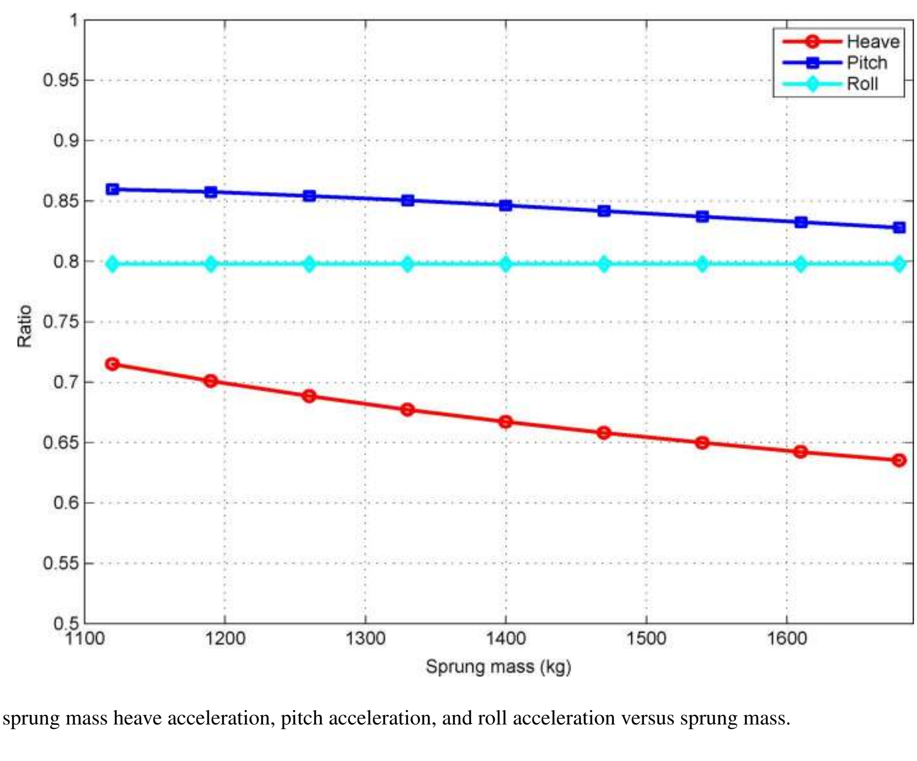

Fig. 12. RMS ratios for sprung mass heave acceleration, pitch acceleration, and roll acceleration versus sprung mass.

acceleration are calculated as the sprung mass changes from 1120 to 1680 kg. The rms ratios between the active suspension with fuzzy controller and the passive suspension are plotted against sprung mass in Fig. 12. It can be seen that the designed fuzzy controller maintains a ratio below 1, regardless of the large variations in the sprung mass. When the road roughness and vehicle forward velocity are given different values, very similar results are obtained. For brevity, these results are not shown. Fig. 12 further validates the claim that the proposed fuzzy control strategy can realize good ride comfort performance for electrohydraulic suspension even when the sprung mass is varied significantly.

VI. Conclusion

In this paper, we have presented a fuzzy state feedback control strategy for electrohydraulic active suspensions to deal with the nonlinear actuator dynamics, sprung mass variation, and control input constraint problems. First, using the idea of "sector nonlinearity," the nonlinear uncertain electrohydraulic actuator was represented by a T–S fuzzy model in defined regions. Thus, by means of the PDC scheme, a fuzzy state feedback controller was designed for the obtained T–S fuzzy model to optimize the H_∞ performance of ride comfort. At the same time, the actuator input voltage constraint was incorporated into the controller design process. The sufficient conditions for designing such a controller were expressed by LMIs. Simulations were used to validate the effectiveness of the designed controller.

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