# Fuzzy Controller Design by Means of Genetic Optimization and NFN-Based Estimation Technique

**Sung-Kwun Oh, Seok-Beom Rho, and Hyun-Ki Kim**

**Abstract:** In this study, we introduce a noble neurogenetic approach to the design of the fuzzy controller. The design procedure dwells on the use of Computational Intelligence (CI), namely genetic algorithms and neurofuzzy networks (NFN). The crux of the design methodology is based on the selection and determination of optimal values of the scaling factors of the fuzzy controllers, which are essential to the entire optimization process. First, tuning of the scaling factors of the fuzzy controller is carried out, and then the development of a nonlinear mapping for the scaling factors is realized by using GA based NFN. The developed approach is applied to an inverted pendulum nonlinear system where we show the results of comprehensive numerical studies and carry out a detailed comparative analysis.

**Keywords:** Computational Intelligence (CI), estimation algorithm, fuzzy controller, genetic algorithms, neurofuzzy networks (NFN), optimization process, scaling factors.

## 1. INTRODUCTON

The ongoing challenges we face when designing advanced control systems has resulted in a diversity of underlying methodologies, development platforms and detailed algorithms. In parallel to PID controllers that are regarded nowadays as the standard control constructs of numeric control [1-4], fuzzy controllers have positioned themselves in a similar dominant role at the knowledge-rich end of the entire spectrum of control algorithms. The design goals of PID control and fuzzy control are similar yet the same problem is approached from two different angles. At the final stage of the design phase, one realizes that two different threads are being served. PID controllers are superb when it comes to linear systems or nonlinear systems with an operation mode confined to a small neighborhood around a given set point. The advantages of the fuzzy controllers are situated at opposite ends of the scale as we envision their full strength in the setting of nonlinear systems (as these controllers are nonlinear mappings in the first place) and when dealing with high deviations from the set point. These advantages of fuzzy controllers stem directly from the nonlinear type of characteristics of the linguistic rules and the associated membership functions used in the description of linguistic terms.

The intent of this study is to develop, optimize and experiment with the fuzzy controllers (fuzzy PD controller or fuzzy PID controller) when developing a general design scheme of Computational Intelligence. One of the difficulties in the construction of the fuzzy controller is to derive a set of optimal control parameters of the controller such as linguistic control rules, scaling factors, and membership functions of the fuzzy controller. In the application of the conventional design method, a control expert proposes some linguistic rules and decides upon the type and parameters of the associated membership functions. With an attempt to enhance the quality of the control knowledge conveyed by the expert (and this usually applies to the matter of calibration of such initial domain knowledge), genetic algorithms (GAs) have already started playing a pivotal role. More specifically, considering a vast number of parameters of the fuzzy controller, they are instrumental in carrying out a global search in the overall parameter space. One should stress however that evolutionary computing (such as GAs) is computationally intensive and this may be a point of concern when dealing with the amount of time available for such search. For instance, when controlling a nonlinear plant such as an inverted pendulum of which initial states vary in each case, the search time required by GAs could be prohibitively high when dealing with dynamic systems. As a consequence, the parameters of the fuzzy controller cannot be easily adapted to the

Manuscript received October 20, 2003; revised June 25, 2004; accepted July 20, 2004. Recommended by Editorial Board member Jong Tae Lim under the direction of Editor Jin Bae Park. This work has been supported by KESRI (R-2003-0-285), which is funded by MOCIE (Ministry of Commerce, Industry and Energy).

Sung-Kwun Oh and Seok-Beom Rho are with the Dept. of Electrical Electronic & Information Engineering, Wonkwang University, 344-2 Shinyong-dong, Iksan, Chonbuk 570-749, Korea (e-mails: {ohsk, nado}@wonkwang.ac.kr).

Hyun-Ki Kim is with the Dept. of Electrical Engineering, University of Suwon, San 2-2, Wau-ri, Bongdam-eup, Hwanseong-si, Gyeonggi-do 445-743, Korea (e-mail: hkkim @suwon.ac.kr).

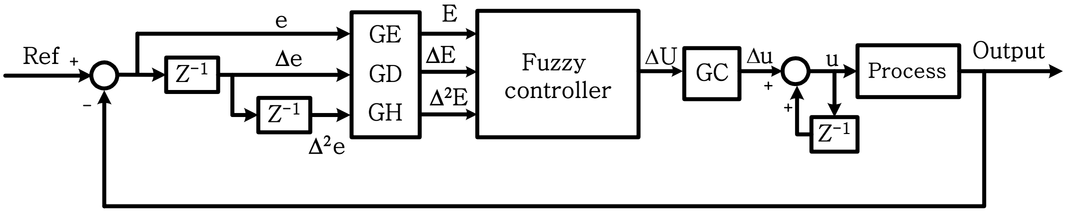



Fig. 1. An overall architecture of the fuzzy PID controller.

changing initial states of this system such as an angular position and an angular velocity of the pendulum. To alleviate this shortcoming, we introduce a nonlinear mapping from the initial states of the system and the corresponding optimal values of the parameters. With anticipation of the nonlinearity residing within such transformation, in its realization we consider GA-based NFN. Bearing this in mind, the development process consists of two main phases. First, using genetic optimization we determine optimal parameters of the fuzzy controller for various initial states (conditions) of the dynamic system. Second, we build up a nonlinear model that captures a relationship between the initial states of the system and the corresponding genetically optimized control parameters. The paper includes the experimental study dealing with the inverted pendulum having the initial states changed. We carry out experimentation with several categories of the controllers such as PID controller, fuzzy PD controller, and fuzzy PID controller. The performance of systems under control is evaluated and compared from the viewpoint of ITAE (Integral of the Time multiplied by the Absolute value of Error), overshoot and rising time [1].

## 2. THE FUZZY CONTROLLER

The block diagram of a fuzzy PID controller is shown in Fig. 1. We confine ourselves to the following notation: e denotes the error between reference and response (output of the system under control), Δe is the first-order difference of error signal while $\Delta^2$e is the second-order difference of the error. Note that the input variables to the fuzzy controller are transformed by the scaling factors (GE, GD, GH, and GC) whose role is to allow the fuzzy controller to properly "perceive" the external world to be controlled.

The above fuzzy PID controller consists of rules of the following form, cf. [5,6] $R_j$: if E is $A_{1j}$ and ΔE is $A_{2j}$ and $\Delta^2$E is $A_{3j}$ then $\Delta U_j$ is $D_j$ The capital letters existing in the rule ($R_j$) denote fuzzy variables (linguistic terms) whereas D is a numeric value (singleton) of the control action. In each control rule, a level of its activation is computed in a standard fashion given by (1). Subsequently, the inferred value of consequence part is converted into numeric values with the aid of (2a) [7].

$$\omega_i = \min\{\mu_{A_i}(E), \mu_{B_i}(\Delta E), \mu_{C_i}(\Delta^2 E)\}\,, \tag{1}$$

$$\Delta U^* = \frac{\sum_{i=1}^{n} \omega_i D_i}{\sum_{i=1}^{n} \omega_i}\,, \tag{2a}$$

$$\Delta u(k) = \Delta U^*(k) \cdot GC\,. \tag{2b}$$

An overall operation of a fuzzy PID controller can be described in the format so that the resulting control is formed incrementally based on the previous control

$$u(k) = u(k-1) + \Delta u(k). \tag{3}$$

Here the input variables are denoted by $E$ and $\Delta E$ while their membership functions are as follows.
NB: Negative Big, NM: Negative Medium, NS: Negative Small, ZO: Zero, PS: Positive Small, PM: Positive Medium, and PB: Positive Big. When dealing with the three input variables of the fuzzy controller, namely E, ΔE, and $\Delta^2$E, the membership functions are denoted as follows N: Negative, Z: Zero, and P: Positive.

The membership functions of the output variable of the controller, that is, the changes of control are NB(-m3), NM(-m2), NS(-m1), ZO(0), PS(m1), PM(m2) and PB(m3). The initial parameters of these membership functions are equal to m1, m2, and m3, respectively. The collection of the rules is shown in Table 1.

We use triangular membership functions defined in the input and output spaces; see Figs. 2 and 3. Here these spaces are normalized to the [-1, 1] interval.

## 3. AUTO-TUNING OF THE FUZZY CONTROLLER BY GAs

Genetic algorithms (GAs) are the search algorithms inspired by nature in the sense that we exploit a fundamental concept of a survival of the fittest as being encountered in selection mechanisms among species. In GAs, the search variables are encoded in bit strings called chromosomes. They deal with a

Table 1. Fuzzy control rules.

(a) 2 input variables.

| | | ΔE | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | NB | NM | NS | ZO | PS | PM | PB |
| E | NB | $-m_3$ | $-m_3$ | $-m_3$ | $-m_3$ | $-m_2$ | $-m_1$ | 0 |
| | NM | $-m_3$ | $-m_3$ | $-m_3$ | $-m_2$ | $-m_1$ | 0 | $m_1$ |
| | NS | $-m_3$ | $-m_3$ | $-m_2$ | $-m_1$ | 0 | $m_1$ | $m_2$ |
| | ZO | $-m_3$ | $-m_2$ | $-m_1$ | 0 | $m_1$ | $m_2$ | $m_3$ |
| | PS | $-m_2$ | $-m_1$ | 0 | $M_1$ | $m_2$ | $m_3$ | $m_3$ |
| | PM | $-m_1$ | 0 | $m_1$ | $M_2$ | $m_3$ | $m_3$ | $m_3$ |
| | PB | 0 | $m_1$ | $m_2$ | $M_3$ | $m_3$ | $m_3$ | $m_3$ |

(b) 3 input variables.

$\Delta^2E$=N

| | | ΔE | | |
|---|---|---|---|---|
| | | N | Z | P |
| E | N | $-m_3$ | $-m_3$ | $-m_2$ |
| | Z | $-m_2$ | $-m_1$ | 0 |
| | P | 0 | $m_1$ | $m^3$ |

$\Delta^2E$ = Z

| | | ΔE | | |
|---|---|---|---|---|
| | | N | Z | P |
| E | N | $-m_3$ | $-m_3$ | $-m_2$ |
| | Z | $-m_2$ | $-m_1$ | 0 |
| | P | 0 | $m_1$ | $m_3$ |

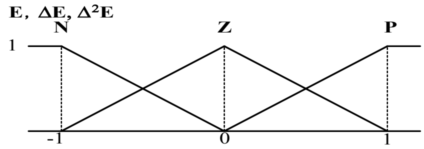
$\Delta^2E$ =P

| | | ΔE | | |
|---|---|---|---|---|
| | | N | Z | P |
| E | N | $-m_3$ | $-m_3$ | $-m_2$ |
| | Z | $-m_2$ | $-m_1$ | 0 |
| | P | 0 | $m_1$ | $m^3$ |

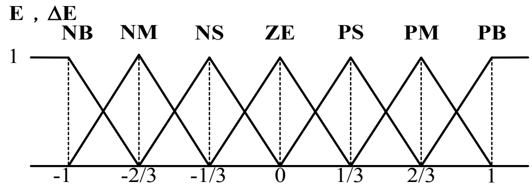
population of chromosomes with each representing a possible solution for a given problem. Each chromosome has a fitness value that indicates how good a solution represented by it is. In control applications, the chromosome represents the controller's adjustable parameters and the fitness value is a quantitative measure of the performance of the controller.

In general, the population size, number of bits used for binary coding, crossover rate, and mutation rate are essential parameters whose values are specified in advance. The genetic search is guided by reproduction, mutation, and crossover. Each of these phases comes with a set of specific numeric parameters characterizing the phase. In this study, the number of generations is set at 100, crossover rate is equal to 0.6, while the mutation rate is taken as 0.1. The number of bits used in the coding is equal to 10.

Fig. 4 portrays an overall auto-tuning scheme. Let us recall that this involves tuning of the scaling factors




(a) In case of E, ΔE and $\Delta^2$E.




(b) In case of E and ΔE.

Fig. 2. Membership functions of the premise input variables.

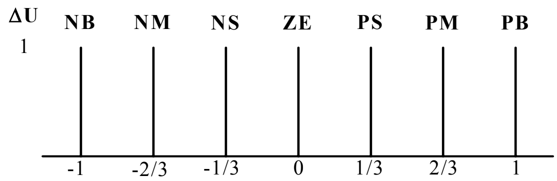



Fig. 3. Membership functions (singletons) defined in the consequence variable, ΔU.

and a construction of the control rules. These are genetically optimized. We set the initial individuals of GAs using three types of parameter estimation modes such as a basic mode, contraction mode and expansion mode. In the case of the basic mode (BM), we use scaling parameters that normalize error between reference and output, one level error difference and two level error difference by [-1, 1] for the initial individuals in the GA. In the contraction mode (CM), we use scaling parameters reduced by 25% in relation to the basic mode. In the expansion mode (EM), we use scaling parameters enlarged by 25% from a basic mode. The standard ITAE expressed for the reference and the output of the system under control is treated as a fitness function [2].

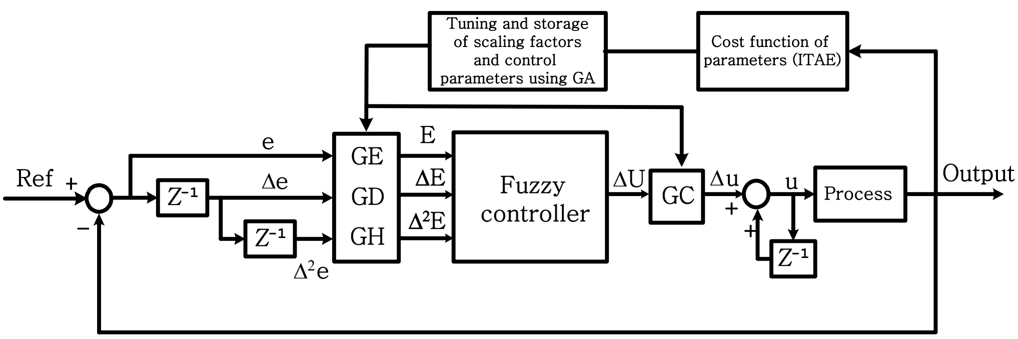



Fig. 4. The scheme of auto-tuning of the fuzzy PID controller involving estimation of the scaling factors.

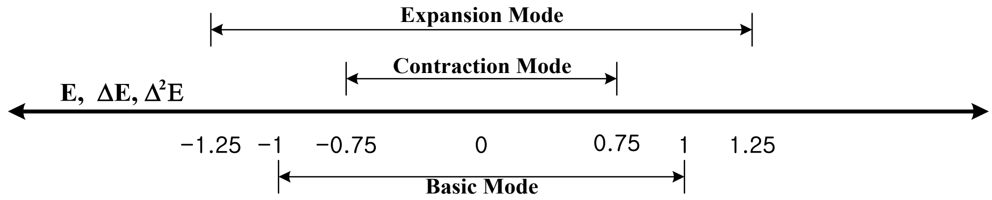



Fig. 5. Three types of estimation modes for the scaling factors: basic, expansion, and contraction.

The overall design procedure of the fuzzy PID controller realized by means of GAs is illustrated in Fig. 4. It consists of the following steps.

[Step 1] Select the general structure of the fuzzy controller according to the purpose of control and dynamics of the process. In particular, we consider architectural options. (PID, FPD (Fuzzy PD), and FPID (Fuzzy PID) controller)

[Step 2] Define the number of fuzzy sets for each variable and set up initial control rules. Refer to Figs. 2 and 3.

[Step 3] Form a collection of initial individuals of GAs. We set the initial individuals of GAs for the scaling factors of the fuzzy controller. The scaling factors can be described as normalized coefficients. Each scaling factor is expressed by (4).

Fig. 5 illustrates three types of estimation modes of the scaling factors being used in setting the initial individuals of GAs describing the fuzzy controller.

$$E(kT) = e \times GE, \tag{4a}$$

$$\Delta E(kT) = [e(kT) - e((k-1)T)] \times GD, \tag{4b}$$

$$\Delta^2 E(kT) = [e(kT) - 2e((k-1)T) + e((k-2)T)] \times GH, \tag{4c}$$

$$U(kT) = U((k-1)T) + \Delta U(kT) \times GC. \tag{4d}$$

[Step 4] Here, all the control parameters such as the scaling factors GE, GD, GH and GC are tuned simultaneously.

## 4. THE ESTIMATION ALGORITHM BY MEANS OF GA-BASED NEUROFUZZY NETWORKS (NFN)

Let us consider an extension of the network with the fuzzy partition realized by fuzzy relations. Fig. 6 visualizes the architecture of two-input and one-output NFNs, where each input assumes three membership functions. The circles denote processing units of the NFN. The node indicated $\prod$ denotes a Cartesian product, whose output is the product of all the incoming signals. N denotes the normalization of the membership grades.

In the language of the rule-based systems, the structure is equivalent to the following collection of rules

$$\begin{aligned}
&R^1 : If\ \ x_1\ \ is\ \ A_{11}\ \ and\ \cdots\ x_k\ \ is\ \ A_{1k}\ \ then\ \ y_1 = w_1 \\
&\ \vdots\quad \vdots \\
&R^j : If\ \ x_1\ \ is\ \ A_{j1}\ \ and\ \cdots\ x_k\ \ is\ \ A_{jk}\ \ then\ \ y_j = w_j \\
&\ \vdots\quad \vdots \\
&R^n : If\ \ x_1\ \ is\ \ A_{n1}\ \ and\ \cdots\ x_k\ \ is\ \ A_{nk}\ \ then\ \ y_n = w_n.
\end{aligned} \tag{5}$$

The fuzzy rules in equation (5) constitute the overall networks of modified NFNs such as are shown in Fig. 6. The output $f_i$ of each node generates a final output $\hat{y}$ of the form

$$\hat{y} = \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} \overline{\mu}_i \cdot w_i = \sum_{i=1}^{n} \frac{\mu_i \cdot w_i}{\sum_{i=1}^{n} \mu_i}. \tag{6}$$

The learning of the NFN is realized by adjusting connections of the neurons and as such it follows a standard Back-Propagation (BP) algorithm. In this study, we use the Euclidean error distances

$$E_p = (y_p - \hat{y}_p)^2, \tag{7}$$

$$E = \sum_{p=1}^{N} (y_p - \hat{y}_p)^2, \tag{8}$$

where $E_p$ is an error measure for the *p-th* data, $y_p$ is the *p-th* target output data, $\hat{y}_p$ stands for the *p-th* actual output of the model for this specific data point, $N$ is total input-output data pairs, and $E$ is a sum of the errors. As far as learning is concerned, the

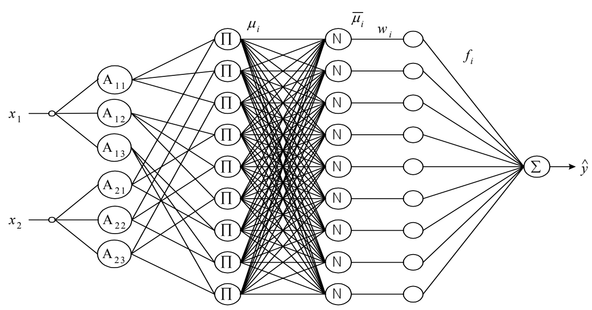



Fig. 6. NFN structure by means of the fuzzy space partition realized by fuzzy relations.

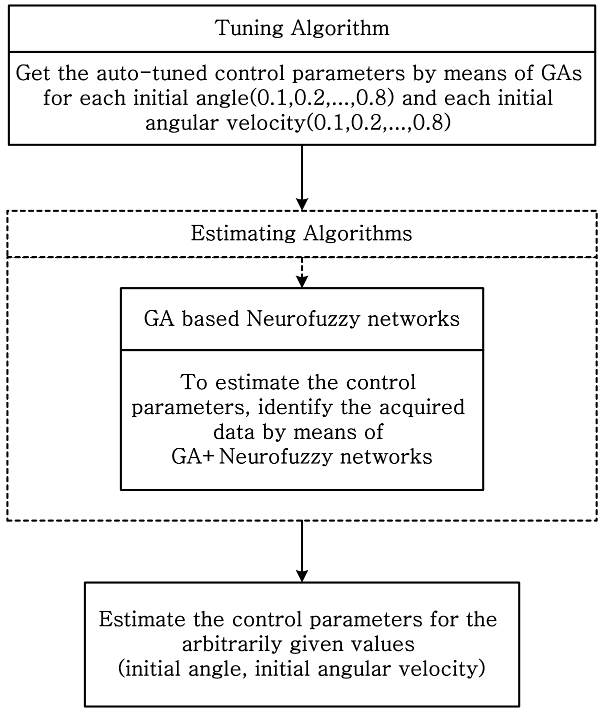



Fig. 7. Overall organization of the optimization process.

connections change as follows

$$w(new) = w(old) + \Delta w\,, \tag{9}$$

where the update formula follows the gradient descent method

$$\begin{aligned}\Delta w_{ij} &= \eta \cdot \left( -\frac{\partial E_p}{\partial w_i} \right) = -\eta \cdot \frac{\partial E_p}{\partial \hat{y}_p} \cdot \frac{\partial \hat{y}_p}{\partial f_i} \cdot \frac{\partial f_i}{\partial w_i} \\ &= 2 \cdot \eta \cdot (y_p - \hat{y}_p) \cdot \overline{\mu}_i\end{aligned} \tag{10}$$

with $\eta$ being a positive learning rate.

Quite commonly to accelerate convergence, a momentum term is being added to the learning expression. Combining (10) and a momentum term, the complete update formula merging the already discussed components is

$$\Delta w_{ij} = 2 \cdot \eta \cdot (y_p - \hat{y}_p) \cdot \mu_i + \alpha (w_{ij}(t) - w_{ij}(t-1)). \tag{11}$$

(Here the momentum coefficient,$\alpha$, is constrained to the unit interval).

In this algorithm, to optimize the learning rate, we use the genetic algorithm for the momentum term and fuzzy membership function of the above NFN. We use 100 generations, 60 populations, 10 bits per string, crossover rate equal to 0.6, and mutation probability equal to 0.1. Fig. 7. depicts the detailed flowchart of the overall optimization process.

## 5. EXPERIMENTAL STUDIES

The proposed control scheme can be applied to a variety of control problems. In this section, we demonstrate the effectiveness of the fuzzy PD/PID controller by applying it to the inverted pendulum system. The inverted pendulum system is composed of a rigid pole and a cart on which the pole is hinged [4,8]. The cart moves on the rail tracks to its right or left, depending on the force exerted on the cart. The pole is hinged to the car through a frictionless free joint such that it has only one degree of freedom. The control goal is to balance the pole starting from nonzero conditions by supplying appropriate force to the cart. In this study, the dynamics of the inverted pendulum system are characterized by two state variables: $\theta$ (angle of the pole with respect to the vertical axis), $\dot{\theta}$ (angular velocity of the pole). The behavior of these two state variables is governed by the following second-order equation.

The dynamic equation of the inverted pendulum comes in the form

$$\ddot{\theta} = \frac{g \sin\theta + \cos\theta \left( \dfrac{-F - ml\dot{\theta}^2 \sin\theta}{m_c + m} \right)}{l \left( \dfrac{4}{3} - \dfrac{m \cos^2\theta}{m_c + m} \right)}, \tag{12}$$

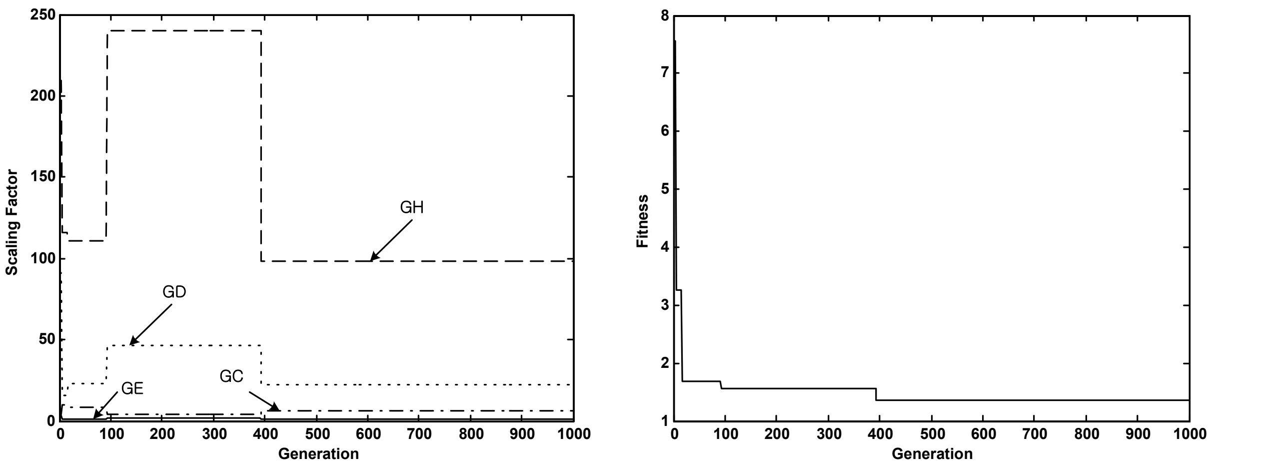
where $g$ (acceleration due to gravity) is $9.8m/s^2$, $m_c$ (mass of cart) is $1.0kg$, $m$ (mass of pole) is $0.5kg$, and F is the applied force expressed in Newtons.

Our control goal here is to balance the pole without regard to the cart's position and velocity, and we compare the fuzzy PID controller and the fuzzy PD controller with the conventional PID controller under identical conditions to validate the fuzzy PID controller and the fuzzy PD controller.

**Tuning of control parameters and estimation**

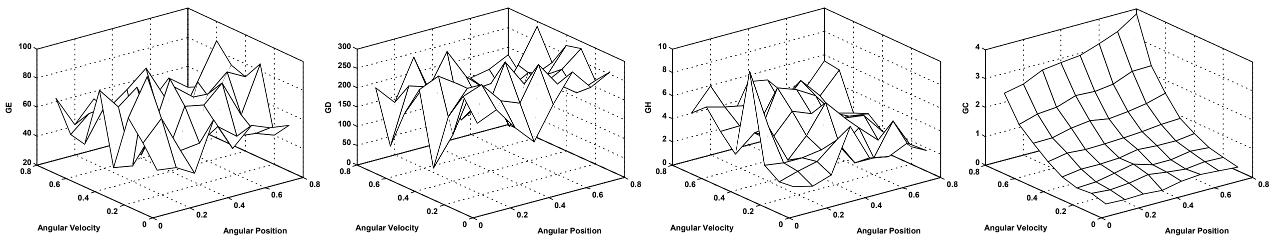
We genetically optimize control parameters (namely GE, GD, GH, and GC) with a clear intent of achieving the best performance of the controller [9]. GAs are powerful nonlinear optimization techniques.

However, their high performance is obtained at the expense of computing time. This essentially rules out the use of GAs in an on-line mode. Rather than that we select a collection of "representative" control scenarios (viz. initial conditions of the pendulum) to off-line genetically optimize the controller and then use these results as a training set to form a nonlinear mapping between the initial conditions of the system and the corresponding scaling factors of the fuzzy controller. The form of the mapping can be experimented




(a) (b)

Fig. 8. (a) fitness function, (b) tuning procedure of scaling factors in successive generations ($\theta$ = 0.6 rad and $\dot{\theta}$ =0.4 rad/sec).




(a)GE (b) GD (c)GH (d) GC

Fig. 9. Auto-tuned scaling factors according to the change of initial angles and angular velocity in the fuzzy PID controller (a) GE, (b) GD, (c) GH and (d) GC.

with the help of a GA-based NFN. In the sequel, first we select several initial angular positions and angular velocities and then we obtain the auto-tuned control parameters by means of GAs according to the change of each selected initial angular positions and angular velocities. Next we build a table. Secondly, we use a GA-based NFN to estimate the control parameters, which are explained in the next section [10,11]. Proceeding with the genetic optimization, we consider the ITAE (Integral of the Time multiplied by the Absolute value of Error), overshoot and rising time as the three underlying criteria of the PI (Performance Index) of the controller. We decided to select 0.1 rad., 0.2 rad.,…,0.7 rad., and 0.8 rad. as a collection of initial angular positions and 0.1 rad/sec, 0.2 rad/sec ,…, 0.7 rad/sec, and 0.8 rad/sec as the corresponding family of values of the initial angular velocity. We also tuned (adjust) the control parameters of each controller (fuzzy PID controller, fuzzy PD controller and PID controller).

Table 2 presents the scaling factors of the fuzzy controller tuned by using GAs, ITAE, overshoot and rising time in case that the initial angular position of the inverted pendulum is 0.1 rad, 0.2 rad,…, 0.7 rad, and 0.8 rad and the initial angular velocity is 0.1 rad/sec, 0.2 rad/sec,…, 0.7 rad/sec, and 0.8 rad/sec, respectively. Using these 64 data, the auto-tuned values of scaling factors are obtained by using GAs for estimating control parameters.

Fig. 8 shows (a) the performance of a fitness function in case of $\theta$ = 0.6 (rad) and $\dot{\theta}$ =0.4 (rad/sec) and (b) the tuning procedure of scaling factors such as GE, GD, GH and GC according to successive generation with the aid of GAs. Refer to Table 2.

Fig. 9 visualizes the value of the scaling factors treated as a function of initial angular position and angular velocity of the inverted pendulum in the fuzzy PID controller. Evidently there are nonlinear characteristics.

Table 3 summarizes the scaling factors of the fuzzy PD controller that are tuned by using GAs under the same initial condition as those of the fuzzy PID controller, ITAE, overshoot and rising time.

Table 4 shows the control parameters of the PID controller that are tuned by using GAs under the same initial condition as those of the fuzzy PID controller, ITAE, overshoot and rising time.

Table 2. The control parameters, ITAE, overshoot and rising time of the fuzzy PID controller after genetic optimization in the case that the initial angular position of the inverted pendulum is 0.1 rad, 0.2 rad,…, 0.7 rad, and 0.8 rad and the initial angular velocity 0.1 rad/sec, 0.2 rad/sec ,…, 0.7 rad/sec, and 0.8 rad/sec.

| $\theta$ | $\dot{\theta}$ | GE | GD | GH | GC | ITAE | Overshoot (%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|---|
| 0.100000 | 0.100000 | 1.650049 | 45.161293 | 215.542511 | 1.735093 | 0.097273 | 0.000255 | 0.19108 |
| 0.100000 | 0.200000 | 1.668622 | 43.108505 | 236.656891 | 1.319648 | 0.074026 | 1.057175 | 0.14779 |
| 0.100000 | 0.300000 | 1.557185 | 36.070381 | 177.126099 | 1.832844 | 0.074897 | 0.520979 | 0.14178 |
| 0.100000 | 0.400000 | 7.351906 | 69.208214 | 127.272728 | 5.571847 | 0.031921 | 0.284381 | 0.076249 |
| 0.100000 | 0.500000 | 1.575758 | 45.747803 | 211.730194 | 2.468231 | 0.177262 | 0.001417 | 0.22292 |
| 0.100000 | 0.600000 | 1.854350 | 58.651028 | 281.524933 | 2.126100 | 0.220245 | 0.000361 | 0.24818 |
| 0.100000 | 0.700000 | 1.947214 | 53.372433 | 229.912018 | 2.834800 | 0.181537 | 0.003848 | 0.2131 |
| 0.100000 | 0.800000 | 2.040078 | 51.319649 | 256.598236 | 1.735093 | 0.138513 | 0.597917 | 0.16876 |
| 0.200000 | 0.100000 | 2.244379 | 59.237537 | 268.328430 | 1.588465 | 0.220017 | 0.002452 | 0.19383 |
| 0.200000 | 0.200000 | 3.321603 | 72.434013 | 283.870972 | 2.003910 | 0.176821 | 0.084697 | 0.16606 |
| 0.200000 | 0.300000 | 1.891496 | 47.214077 | 229.618759 | 2.443793 | 0.233764 | 0.572674 | 0.18522 |
| 0.200000 | 0.400000 | 1.668622 | 48.680351 | 210.263931 | 3.787879 | 0.358009 | 0.000340 | 0.23948 |
| 0.200000 | 0.500000 | 2.114369 | 56.598240 | 297.360718 | 1.441838 | 0.253218 | 0.523023 | 0.18571 |
| 0.200000 | 0.600000 | 1.928641 | 46.334312 | 232.844574 | 1.441838 | 0.194606 | 0.575528 | 0.1466 |
| 0.200000 | 0.700000 | 1.928641 | 44.868038 | 197.360718 | 4.178886 | 0.262681 | 0.529929 | 0.17919 |
| 0.200000 | 0.800000 | 1.538612 | 39.882702 | 216.129044 | 1.515152 | 0.243090 | 0.813163 | 0.15522 |
| 0.300000 | 0.100000 | 2.987292 | 72.140762 | 198.240479 | 2.443793 | 0.194699 | 0.168146 | 0.11374 |
| 0.300000 | 0.200000 | 3.080156 | 96.774193 | 257.771271 | 2.199414 | 0.228670 | 0.541496 | 0.12085 |
| 0.300000 | 0.300000 | 1.538612 | 37.536655 | 184.457474 | 2.174976 | 0.335907 | 0.476415 | 0.16969 |
| 0.300000 | 0.400000 | 2.448680 | 69.501465 | 299.413483 | 2.419355 | 0.491584 | 0.001374 | 0.21956 |
| 0.300000 | 0.500000 | 2.820137 | 66.862167 | 229.912018 | 2.199414 | 0.278169 | 0.016132 | 0.12592 |
| 0.300000 | 0.600000 | 3.154448 | 78.299118 | 214.662750 | 3.470186 | 0.276092 | 0.682007 | 0.11368 |
| 0.300000 | 0.700000 | 2.281525 | 53.079178 | 241.348969 | 1.930596 | 0.392080 | 0.401829 | 0.16297 |
| 0.300000 | 0.800000 | 2.857283 | 84.750732 | 281.231659 | 1.710655 | 0.343879 | 0.931851 | 0.12301 |
| 0.400000 | 0.100000 | 2.337243 | 33.137829 | 47.507332 | 9.946237 | 0.344132 | 0.983603 | 0.12211 |
| 0.400000 | 0.200000 | 1.650049 | 30.791788 | 119.354843 | 4.667644 | 0.445385 | 0.557061 | 0.15545 |
| 0.400000 | 0.300000 | 1.928641 | 48.973606 | 223.460419 | 1.417400 | 0.439613 | 0.764274 | 0.14312 |
| 0.400000 | 0.400000 | 2.430108 | 83.577713 | 226.392960 | 3.054741 | 0.457985 | 0.822324 | 0.13338 |
| 0.400000 | 0.500000 | 1.130010 | 25.219942 | 102.639297 | 5.498534 | 0.596329 | 0.123811 | 0.1798 |
| 0.400000 | 0.600000 | 2.374389 | 66.275658 | 229.325516 | 3.714565 | 0.533014 | 0.669856 | 0.1411 |
| 0.400000 | 0.700000 | 2.411535 | 78.005859 | 210.557175 | 3.494624 | 0.522671 | 0.438267 | 0.13076 |
| 0.400000 | 0.800000 | 2.114369 | 70.967743 | 272.434021 | 1.686217 | 0.612176 | 0.021387 | 0.1484 |
| 0.500000 | 0.100000 | 1.891496 | 81.818184 | 234.017593 | 2.223851 | 0.636086 | 0.223738 | 0.15523 |
| 0.500000 | 0.200000 | 1.928641 | 66.862167 | 255.718475 | 3.616813 | 0.747243 | 0.334514 | 0.1702 |
| 0.500000 | 0.300000 | 1.947214 | 76.832840 | 222.873901 | 2.834800 | 0.703863 | 0.279617 | 0.15312 |
| 0.500000 | 0.400000 | 2.040078 | 49.266865 | 154.252197 | 4.618768 | 0.723054 | 0.941279 | 0.14564 |
| 0.500000 | 0.500000 | 2.021505 | 70.087975 | 158.651016 | 3.983382 | 0.753718 | 0.752549 | 0.14271 |
| 0.500000 | 0.600000 | 1.891496 | 59.530792 | 179.765396 | 2.834800 | 0.795945 | 0.282009 | 0.14634 |
| 0.500000 | 0.700000 | 1.854350 | 52.785923 | 189.149551 | 2.859238 | 0.870931 | 0.278803 | 0.15069 |
| 0.500000 | 0.800000 | 1.891496 | 74.486801 | 210.557175 | 2.663734 | 0.911025 | 0.314776 | 0.14768 |
| 0.600000 | 0.100000 | 1.575758 | 39.882702 | 153.665680 | 3.616813 | 1.044728 | 0.004459 | 0.18124 |
| 0.600000 | 0.200000 | 1.687194 | 61.583576 | 211.730194 | 3.641251 | 1.066975 | 0.153832 | 0.1748 |
| 0.600000 | 0.300000 | 1.390029 | 35.483871 | 175.659821 | 2.419355 | 1.219189 | 0.529370 | 0.19305 |
| 0.600000 | 0.400000 | 1.018573 | 22.287392 | 98.533730 | 6.036168 | 1.291297 | 0.595427 | 0.19592 |
| 0.600000 | 0.500000 | 1.761486 | 43.695015 | 127.565987 | 5.474096 | 1.181221 | 0.279334 | 0.16266 |
| 0.600000 | 0.600000 | 1.705767 | 39.589439 | 129.618774 | 4.472141 | 1.260368 | 0.796205 | 0.16061 |
| 0.600000 | 0.700000 | 1.594330 | 62.170086 | 200.586517 | 2.565982 | 1.358645 | 0.001934 | 0.16766 |
| 0.600000 | 0.800000 | 1.557185 | 86.803520 | 287.683289 | 2.101662 | 1.556496 | 0.127577 | 0.18196 |
| 0.700000 | 0.100000 | 1.427175 | 42.228741 | 53.372433 | 5.131965 | 1.465122 | 1.354618 | 0.17854 |
| 0.700000 | 0.200000 | 1.538612 | 53.079178 | 195.894424 | 4.692082 | 1.647075 | 0.410465 | 0.19333 |
| 0.700000 | 0.300000 | 1.575758 | 52.785923 | 172.434021 | 4.398827 | 1.707701 | 0.906766 | 0.18489 |
| 0.700000 | 0.400000 | 1.427175 | 62.463341 | 258.357788 | 3.079179 | 1.947195 | 0.362886 | 0.20453 |
| 0.700000 | 0.500000 | 1.538612 | 73.900291 | 179.472153 | 3.567937 | 1.900622 | 0.604560 | 0.18265 |
| 0.700000 | 0.600000 | 1.594330 | 50.733139 | 136.363647 | 5.229716 | 1.982359 | 0.690369 | 0.17907 |
| 0.700000 | 0.700000 | 1.018573 | 23.460411 | 112.023460 | 3.494624 | 2.255980 | 0.444391 | 0.2019 |
| 0.700000 | 0.800000 | 1.594330 | 50.733139 | 136.363647 | 5.229716 | 2.274349 | 1.094785 | 0.17625 |
| 0.800000 | 0.100000 | 1.427175 | 62.756596 | 185.923752 | 4.032258 | 2.319965 | 0.348073 | 0.20954 |
| 0.800000 | 0.200000 | 1.445748 | 40.762466 | 136.070389 | 5.962854 | 2.451787 | 0.062809 | 0.21003 |
| 0.800000 | 0.300000 | 1.260020 | 55.131966 | 239.296188 | 2.321603 | 2.768469 | 0.366498 | 0.2222 |
| 0.800000 | 0.400000 | 1.371457 | 34.310852 | 125.219940 | 5.449658 | 2.848641 | 0.013807 | 0.21421 |
| 0.800000 | 0.500000 | 1.408602 | 30.205278 | 88.563049 | 5.156403 | 2.930422 | 1.509672 | 0.19847 |
| 0.800000 | 0.600000 | 1.334311 | 42.228741 | 145.747803 | 3.372434 | 3.164248 | 0.050040 | 0.20457 |
| 0.800000 | 0.700000 | 1.371457 | 55.425220 | 154.545456 | 3.519062 | 3.402342 | 0.285634 | 0.20153 |
| 0.800000 | 0.800000 | 1.445748 | 45.454548 | 180.645172 | 5.474096 | 3.833133 | 1.429595 | 0.20405 |

Tables 2-4 show the control parameters such as the scaling factors and PID parameter of each controllers obtained from GAs, and the performance indexes (ITAE, overshoot, and rising time) for each controller. In general, fuzzy PD and fuzzy PID controllers are preferred architectures. However, PID controller is also satisfactory in comparison to fuzzy PID controller within a linear range of $\theta < 0.4$, while in case of a nonlinear range of $\theta > 0.6$ fuzzy PID controller architecture is superior to both fuzzy PD and PID controller.

Fig. 10(a) and (b) illustrate the dynamics of output of the system controlled by each controller after genetic optimization in the case of $\theta$ = 0.3 (rad), $\dot{\theta}$ =0.7 (rad/sec) and (b) $\theta$ = 0.7 (rad), $\dot{\theta}$ =0.5 (rad/sec), respectively.

In Fig. 10, we know that the fuzzy PID controller and fuzzy PD controller are superior to the conventional PID controller from the viewpoint of ITAE, overshoot and rising time.

Now, we consider the case in which the initial angular positions and angular velocities of the inverted pendulum are not included in Tables 2, 3 and 4 (in other words, selected arbitrarily within the given range). Here we show that the control parameters under the arbitrarily selected initial condition are not tuned by the GAs and the control parameters of each

Table 3. The control parameter and performance index (ITAE, overshoot and rising time) of the fuzzy PD controller after genetic optimization in the case of $\theta = 0.1,\ldots,0.8$ (rad) and $\dot{\theta} = 0.1,\ldots,0.8$ (rad/sec).

| $\theta$ | $\dot{\theta}$ | GE | GD | GC | ITAE | Overshoot(%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|
| 0.100000 | 0.100000 | 14.93646 | 0.899866 | 1.185454 | 0.034373 | 0.000092 | 0.09920 |
| 0.100000 | 0.200000 | 10.26392 | 0.802163 | 1.052545 | 0.045998 | 0.100769 | 0.11809 |
| 0.100000 | 0.300000 | 6.549364 | 0.622390 | 1.287091 | 0.073581 | 0.000000 | 0.15279 |
| 0.100000 | 0.400000 | 17.12610 | 1.046420 | 0.978273 | 0.049569 | 0.026306 | 0.10701 |
| 0.100000 | 0.500000 | 7.820137 | 0.765036 | 0.927455 | 0.087333 | 0.076760 | 0.15233 |
| 0.100000 | 0.600000 | 3.734115 | 0.247211 | 3.831909 | 0.047277 | 0.134572 | 0.09530 |
| 0.100000 | 0.700000 | 9.716520 | 0.675150 | 1.377000 | 0.054430 | 0.073070 | 0.09997 |
| 0.100000 | 0.800000 | 13.43108 | 0.911590 | 1.033000 | 0.065572 | 0.073025 | 0.10502 |
| 0.200000 | 0.100000 | 4.672532 | 0.305833 | 3.151727 | 0.095997 | 0.077961 | 0.10776 |
| 0.200000 | 0.200000 | 9.638318 | 0.626298 | 1.549000 | 0.127201 | 0.051684 | 0.12571 |
| 0.200000 | 0.300000 | 5.571847 | 0.454341 | 1.810909 | 0.135217 | 0.074449 | 0.13046 |
| 0.200000 | 0.400000 | 5.337244 | 0.426984 | 1.955546 | 0.139306 | 0.074474 | 0.12701 |
| 0.200000 | 0.500000 | 10.45943 | 0.661471 | 1.517727 | 0.158252 | 0.018422 | 0.12827 |
| 0.200000 | 0.600000 | 5.298142 | 0.374225 | 2.448091 | 0.139665 | 0.072501 | 0.11224 |
| 0.200000 | 0.700000 | 6.392962 | 0.462157 | 1.932091 | 0.153668 | 0.101304 | 0.11511 |
| 0.200000 | 0.800000 | 6.295210 | 0.626298 | 1.111182 | 0.248850 | 0.081654 | 0.16335 |
| 0.300000 | 0.100000 | 3.069404 | 0.256981 | 3.128273 | 0.242936 | 0.076407 | 0.14274 |
| 0.300000 | 0.200000 | 6.920821 | 0.460203 | 2.065000 | 0.252706 | 0.060785 | 0.13752 |
| 0.300000 | 0.300000 | 6.001955 | 0.395719 | 2.428545 | 0.241516 | 0.055312 | 0.12708 |
| 0.300000 | 0.400000 | 6.510263 | 0.442617 | 2.139273 | 0.276194 | 0.037955 | 0.13542 |
| 0.300000 | 0.500000 | 3.558162 | 0.256981 | 3.511364 | 0.267076 | 0.064160 | 0.12602 |
| 0.300000 | 0.600000 | 4.437928 | 0.380087 | 2.096273 | 0.325881 | 0.054341 | 0.14534 |
| 0.300000 | 0.700000 | 6.451612 | 0.512963 | 1.631091 | 0.392499 | 0.056445 | 0.15815 |
| 0.300000 | 0.800000 | 5.591398 | 0.342960 | 2.956273 | 0.299164 | 0.044844 | 0.11366 |
| 0.400000 | 0.100000 | 7.526882 | 0.499284 | 1.912545 | 0.538816 | 0.036244 | 0.1821 |
| 0.400000 | 0.200000 | 3.753666 | 0.335144 | 2.287818 | 0.469680 | 0.059967 | 0.16405 |
| 0.400000 | 0.300000 | 3.343109 | 0.303879 | 2.487182 | 0.486880 | 0.062577 | 0.16267 |
| 0.400000 | 0.400000 | 4.320626 | 0.305833 | 2.964091 | 0.445922 | 0.080275 | 0.13948 |
| 0.400000 | 0.500000 | 3.460411 | 0.282384 | 2.925000 | 0.498048 | 0.055342 | 0.14933 |
| 0.400000 | 0.600000 | 4.633431 | 0.399628 | 1.963364 | 0.616099 | 0.060080 | 0.17118 |
| 0.400000 | 0.700000 | 3.734115 | 0.294108 | 2.854636 | 0.545544 | 0.080466 | 0.1447 |
| 0.400000 | 0.800000 | 4.985337 | 0.305833 | 3.296364 | 0.533104 | 0.070599 | 0.12827 |
| 0.500000 | 0.100000 | 5.767351 | 0.423076 | 2.104091 | 0.842767 | 0.036113 | 0.1983 |
| 0.500000 | 0.200000 | 4.242424 | 0.337098 | 2.487182 | 0.749735 | 0.049547 | 0.17579 |
| 0.500000 | 0.300000 | 4.281525 | 0.288246 | 3.296364 | 0.690006 | 0.043878 | 0.15353 |
| 0.500000 | 0.400000 | 3.323558 | 0.253073 | 3.390182 | 0.733171 | 0.084664 | 0.15592 |
| 0.500000 | 0.500000 | 1.779081 | 0.180773 | 3.573909 | 0.899276 | 0.270279 | 0.18419 |
| 0.500000 | 0.600000 | 4.907136 | 0.319511 | 3.038364 | 0.843915 | 0.038558 | 0.15806 |
| 0.500000 | 0.700000 | 3.910069 | 0.286292 | 3.104818 | 0.877988 | 0.060032 | 0.15613 |
| 0.500000 | 0.800000 | 3.304008 | 0.243303 | 3.652091 | 0.902420 | 0.058428 | 0.15031 |
| 0.600000 | 0.100000 | 4.496579 | 0.307787 | 3.030545 | 1.048746 | 0.044971 | 0.18273 |
| 0.600000 | 0.200000 | 2.697947 | 0.217900 | 3.769364 | 1.045987 | 0.078194 | 0.17535 |
| 0.600000 | 0.300000 | 4.261975 | 0.272614 | 3.401909 | 1.075930 | 0.422543 | 0.16773 |
| 0.600000 | 0.400000 | 3.479961 | 0.262844 | 3.312000 | 1.171544 | 0.052938 | 0.17489 |
| 0.600000 | 0.500000 | 2.795699 | 0.229625 | 3.577818 | 1.240273 | 0.049336 | 0.17577 |
| 0.600000 | 0.600000 | 2.287390 | 0.221808 | 3.237727 | 1.407169 | 0.052847 | 0.19181 |
| 0.600000 | 0.700000 | 2.717498 | 0.239395 | 3.214273 | 1.425942 | 0.071779 | 0.18084 |
| 0.600000 | 0.800000 | 3.538612 | 0.258935 | 3.468364 | 1.456570 | 0.037791 | 0.16951 |
| 0.700000 | 0.100000 | 3.753666 | 0.268706 | 3.358909 | 1.552194 | 0.048066 | 0.19662 |
| 0.700000 | 0.200000 | 4.320626 | 0.282384 | 3.409727 | 1.645655 | 0.055660 | 0.19666 |
| 0.700000 | 0.300000 | 4.750733 | 0.290200 | 3.628637 | 1.741531 | 0.010769 | 0.19666 |
| 0.700000 | 0.400000 | 2.521994 | 0.227671 | 3.308091 | 1.860775 | 0.070872 | 0.20154 |
| 0.700000 | 0.500000 | 4.535679 | 0.288246 | 3.421455 | 1.966259 | 0.052065 | 0.1963 |
| 0.700000 | 0.600000 | 4.125122 | 0.290200 | 3.186909 | 2.134502 | 0.026069 | 0.20084 |
| 0.700000 | 0.700000 | 2.033236 | 0.200314 | 3.476182 | 2.282319 | 0.086637 | 0.20661 |
| 0.700000 | 0.800000 | 2.248289 | 0.221808 | 3.151727 | 2.432745 | 0.075062 | 0.20607 |
| 0.800000 | 0.100000 | 3.577713 | 0.253073 | 3.636455 | 2.346170 | 0.042356 | 0.21543 |
| 0.800000 | 0.200000 | 2.209189 | 0.200314 | 3.773273 | 2.528666 | 0.053902 | 0.22109 |
| 0.800000 | 0.300000 | 2.971652 | 0.223762 | 3.914000 | 2.592028 | 0.079453 | 0.20856 |
| 0.800000 | 0.400000 | 2.404692 | 0.225717 | 3.296364 | 2.927830 | 0.033023 | 0.22568 |
| 0.800000 | 0.500000 | 2.306940 | 0.204222 | 3.753727 | 3.045240 | 0.064020 | 0.21653 |
| 0.800000 | 0.600000 | 3.792766 | 0.264798 | 3.476182 | 3.301775 | 0.045469 | 0.21634 |
| 0.800000 | 0.700000 | 3.010753 | 0.235487 | 3.605182 | 3.490056 | 0.045494 | 0.21136 |
| 0.800000 | 0.800000 | 2.404692 | 0.204222 | 3.914000 | 3.748619 | 0.046755 | 0.20982 |

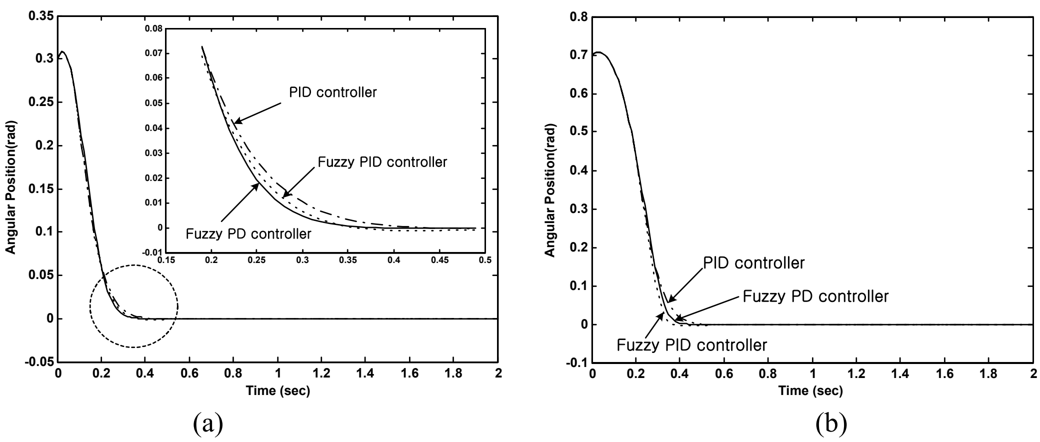



Fig. 10. The dynamics of output of the system controlled by each optimized controller in the case of (a) $\theta = 0.3$ (rad), $\dot{\theta} = 0.7$ (rad/sec) and (b) $\theta = 0.7$ (rad), $\dot{\theta} = 0.5$ (rad/sec).

Table 4. The control parameter and performance index (ITAE, overshoot and rising time) of the PID controller after genetic optimization in the case of $\theta$ = 0.1,…,0.8 (rad) and $\dot{\theta}$ = 0.1,…,0.8 (rad/sec).

| $\theta$ | $\dot{\theta}$ | K | Ti | Td | ITAE | Overshoot (%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|
| 0.100000 | 0.100000 | 166.177917 | 168.670975 | 0.105460 | 0.094016 | 0.076108 | 0.17165 |
| 0.100000 | 0.200000 | 168.670578 | 161.195160 | 0.104877 | 0.098847 | 0.080049 | 0.17077 |
| 0.100000 | 0.300000 | 167.174973 | 168.670975 | 0.105655 | 0.105030 | 0.078796 | 0.17169 |
| 0.100000 | 0.400000 | 168.172043 | 167.341934 | 0.106239 | 0.111790 | 0.080387 | 0.17288 |
| 0.100000 | 0.500000 | 167.174973 | 165.348389 | 0.105849 | 0.116218 | 0.085414 | 0.17029 |
| 0.100000 | 0.600000 | 168.670578 | 165.348389 | 0.104099 | 0.117358 | 0.096094 | 0.16486 |
| 0.100000 | 0.700000 | 170.000000 | 164.850006 | 0.105849 | 0.128590 | 0.090310 | 0.1688 |
| 0.100000 | 0.800000 | 168.338226 | 165.182266 | 0.106822 | 0.137882 | 0.094369 | 0.16985 |
| 0.200000 | 0.100000 | 167.839691 | 165.680649 | 0.104488 | 0.207465 | 0.086079 | 0.1716 |
| 0.200000 | 0.200000 | 168.504395 | 159.035477 | 0.104682 | 0.218555 | 0.088015 | 0.17226 |
| 0.200000 | 0.300000 | 169.335297 | 167.508072 | 0.104488 | 0.225252 | 0.085342 | 0.17157 |
| 0.200000 | 0.400000 | 169.833817 | 168.006454 | 0.103515 | 0.230809 | 0.095161 | 0.16843 |
| 0.200000 | 0.500000 | 169.501465 | 170.000000 | 0.105655 | 0.251252 | 0.085611 | 0.17366 |
| 0.200000 | 0.600000 | 167.507339 | 169.501617 | 0.105655 | 0.261690 | 0.090230 | 0.17212 |
| 0.200000 | 0.700000 | 167.839691 | 169.335480 | 0.105460 | 0.273272 | 0.093410 | 0.17106 |
| 0.200000 | 0.800000 | 165.180847 | 169.335480 | 0.105266 | 0.284905 | 0.104905 | 0.16889 |
| 0.300000 | 0.100000 | 169.002930 | 167.840332 | 0.104099 | 0.362849 | 0.092720 | 0.17604 |
| 0.300000 | 0.200000 | 169.501465 | 161.029037 | 0.104099 | 0.379521 | 0.096959 | 0.17575 |
| 0.300000 | 0.300000 | 168.172043 | 169.501617 | 0.103904 | 0.391446 | 0.102306 | 0.17478 |
| 0.300000 | 0.400000 | 169.169113 | 159.201614 | 0.104293 | 0.414550 | 0.102718 | 0.17535 |
| 0.300000 | 0.500000 | 167.673508 | 168.670975 | 0.104488 | 0.430043 | 0.103205 | 0.17497 |
| 0.300000 | 0.600000 | 169.501465 | 161.195160 | 0.104099 | 0.451165 | 0.108161 | 0.17388 |
| 0.300000 | 0.700000 | 169.002930 | 168.006454 | 0.103126 | 0.461775 | 0.122959 | 0.17047 |
| 0.300000 | 0.800000 | 168.836761 | 157.041931 | 0.104488 | 0.498353 | 0.116110 | 0.17307 |
| 0.400000 | 0.100000 | 169.833817 | 164.850006 | 0.103126 | 0.571272 | 0.112191 | 0.18146 |
| 0.400000 | 0.200000 | 169.667648 | 167.674194 | 0.102932 | 0.592150 | 0.117532 | 0.18075 |
| 0.400000 | 0.300000 | 170.000000 | 168.837097 | 0.103126 | 0.618764 | 0.114223 | 0.18098 |
| 0.400000 | 0.400000 | 167.008804 | 168.172577 | 0.102932 | 0.642883 | 0.144168 | 0.17913 |
| 0.400000 | 0.500000 | 170.000000 | 169.501617 | 0.103904 | 0.681876 | 0.110402 | 0.18173 |
| 0.400000 | 0.600000 | 167.839691 | 165.182266 | 0.104099 | 0.715386 | 0.122198 | 0.18081 |
| 0.400000 | 0.700000 | 167.174973 | 169.003220 | 0.102932 | 0.736046 | 0.151878 | 0.17733 |
| 0.400000 | 0.800000 | 169.501465 | 169.501617 | 0.101764 | 0.763473 | 0.167841 | 0.17431 |
| 0.500000 | 0.100000 | 169.002930 | 169.169357 | 0.104099 | 0.868561 | 0.111528 | 0.1929 |
| 0.500000 | 0.200000 | 167.507339 | 166.511292 | 0.103515 | 0.901292 | 0.131474 | 0.19141 |
| 0.500000 | 0.300000 | 169.335297 | 165.348389 | 0.103710 | 0.945491 | 0.122725 | 0.19163 |
| 0.500000 | 0.400000 | 168.005859 | 168.837097 | 0.103126 | 0.979462 | 0.140650 | 0.18977 |
| 0.500000 | 0.500000 | 169.667648 | 166.677429 | 0.101375 | 1.009529 | 0.181248 | 0.18561 |
| 0.500000 | 0.600000 | 169.833817 | 162.358063 | 0.102932 | 1.078726 | 0.143894 | 0.18844 |
| 0.500000 | 0.700000 | 167.341156 | 163.354828 | 0.104099 | 1.140496 | 0.141993 | 0.18926 |
| 0.500000 | 0.800000 | 169.335297 | 164.351608 | 0.102348 | 1.179671 | 0.166694 | 0.18525 |
| 0.600000 | 0.100000 | 168.670578 | 167.175812 | 0.103904 | 1.273187 | 0.128207 | 0.20418 |
| 0.600000 | 0.200000 | 167.507339 | 168.837097 | 0.102932 | 1.318895 | 0.154512 | 0.20204 |
| 0.600000 | 0.300000 | 167.507339 | 168.670975 | 0.104099 | 1.394525 | 0.136399 | 0.20372 |
| 0.600000 | 0.400000 | 168.670578 | 169.335480 | 0.102153 | 1.440542 | 0.172079 | 0.19977 |
| 0.600000 | 0.500000 | 169.501465 | 166.012909 | 0.101181 | 1.505114 | 0.202769 | 0.19719 |
| 0.600000 | 0.600000 | 168.172043 | 169.501617 | 0.102153 | 1.589511 | 0.185255 | 0.19793 |
| 0.600000 | 0.700000 | 168.504395 | 167.840332 | 0.101375 | 1.666328 | 0.215378 | 0.19575 |
| 0.600000 | 0.800000 | 167.673508 | 168.504837 | 0.103321 | 1.778220 | 0.169083 | 0.19828 |
| 0.700000 | 0.100000 | 169.002930 | 168.006454 | 0.104293 | 1.861877 | 0.138084 | 0.21974 |
| 0.700000 | 0.200000 | 163.519058 | 170.000000 | 0.104293 | 1.942323 | 0.168098 | 0.21785 |
| 0.700000 | 0.300000 | 169.169113 | 163.188705 | 0.102543 | 2.035579 | 0.174166 | 0.21553 |
| 0.700000 | 0.400000 | 164.017593 | 169.335480 | 0.103515 | 2.144568 | 0.193269 | 0.21567 |
| 0.700000 | 0.500000 | 167.673508 | 167.009674 | 0.101570 | 2.243018 | 0.225348 | 0.21206 |
| 0.700000 | 0.600000 | 166.842621 | 167.674194 | 0.104099 | 2.406587 | 0.170850 | 0.21563 |
| 0.700000 | 0.700000 | 169.002930 | 163.188705 | 0.102348 | 2.529273 | 0.200558 | 0.21162 |
| 0.700000 | 0.800000 | 166.842621 | 169.003220 | 0.101959 | 2.665989 | 0.235506 | 0.20932 |
| 0.800000 | 0.100000 | 165.845551 | 165.016129 | 0.102543 | 2.707035 | 0.212035 | 0.23376 |
| 0.800000 | 0.200000 | 170.000000 | 168.172577 | 0.100792 | 2.839842 | 0.227844 | 0.23121 |
| 0.800000 | 0.300000 | 167.341156 | 160.862900 | 0.102153 | 3.038635 | 0.223635 | 0.23243 |
| 0.800000 | 0.400000 | 168.836761 | 164.850006 | 0.100792 | 3.202980 | 0.257897 | 0.22947 |
| 0.800000 | 0.500000 | 169.335297 | 167.840332 | 0.102153 | 3.427777 | 0.210568 | 0.23097 |
| 0.800000 | 0.600000 | 166.344086 | 169.833878 | 0.103126 | 3.665044 | 0.216031 | 0.23096 |
| 0.800000 | 0.700000 | 168.338226 | 162.856445 | 0.101375 | 3.906416 | 0.264730 | 0.22702 |
| 0.800000 | 0.800000 | 169.833817 | 168.837097 | 0.100986 | 4.178266 | 0.262563 | 0.22532 |

controller are estimated by using the estimation algorithm of the GA-based NFN. We implement the optimal neurofuzzy networks for parameter estimation using GAs. In this algorithm, we adjust the learning rates, momentum coefficient, and apexes of the membership function of neurofuzzy networks by using GAs.

Table 5 shows the estimated scaling factors of the fuzzy PID controller and describes the performance index (ITAE, overshoot and rising time) of the fuzzy PID controller with the estimated scaling factors in the case of $\theta$ = 0.22, 0.45, 0.78 (rad) and $\dot{\theta}$ = 0.22, 0.45, 0.78 (rad/sec), respectively.

In the case of the fuzzy PD controller, the estimated scaling factors and performance index are shown in Table 6 when the initial angular position is 0.22 (rad), 0.45 (rad), or 0.78 (rad) and the initial angular velocity is 0.22 (rad/sec), 0.45 (rad/sec), or 0.78 (rad/sec), respectively.

In the case of the PID controller, the estimated scaling factors by means of the GA-based NFN are presented in Table 7 when the initial angular position is 0.22 (rad), 0.45 (rad), or 0.78 (rad) and the initial angular velocity is 0.22 (rad/sec), 0.45 (rad/sec), or 0.78 (rad/sec), respectively.

Fig. 11 demonstrates (a) pole angle (b) pole angular velocity for initial angle $\theta$ = 0.22 (rad) and initial angular velocity $\dot{\theta}$ =0.22 (rad/sec) (Case 1).

Table 5. The estimated parameters by means of the GA-based NFN and performance index (ITAE, overshoot and rising time) of the fuzzy PID controller in the case of $\theta$ = 0.22, 0.45, 0.78 (rad) and $\dot{\theta}$ = 0.22, 0.45, 0.78 (rad/sec).

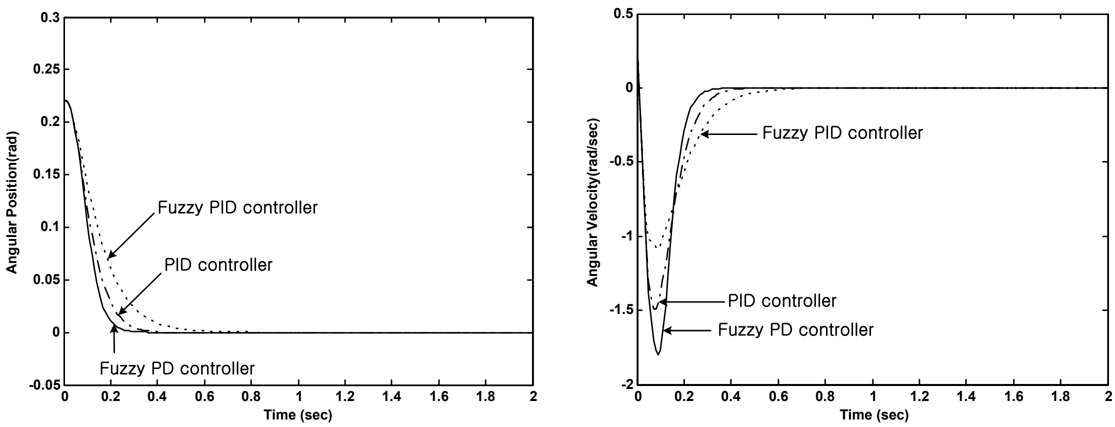
| Case | Initial angle (rad) | Initial angular velocity | GE | GD | GH | GC | ITAE | Overshoot (%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.22 | 0.22 | 2.032828 | 61.545998 | 237.387817 | 3.706272 | 0.419737 | 0.000000 | 0.26105 |
| 2 | 0.22 | 0.45 | 2.117639 | 61.290897 | 242.742706 | 3.047558 | 0.398423 | 0.000000 | 0.24208 |
| 3 | 0.22 | 0.78 | 2.185512 | 60.583847 | 250.425781 | 1.442666 | 0.363320 | 0.000000 | 0.20613 |
| 4 | 0.45 | 0.22 | 1.818810 | 57.950821 | 222.710297 | 4.021980 | 0.923726 | 0.000000 | 0.23562 |
| 5 | 0.45 | 0.45 | 1.868329 | 58.781029 | 228.355225 | 3.553186 | 0.898830 | 0.000000 | 0.21048 |
| 6 | 0.45 | 0.78 | 1.907958 | 61.082088 | 236.454498 | 2.411014 | 0.855770 | 0.000000 | 0.16716 |
| 7 | 0.78 | 0.22 | 1.372258 | 47.819809 | 174.743240 | 4.474953 | 2.432061 | 0.000000 | 0.21519 |
| 8 | 0.78 | 0.45 | 1.348141 | 51.708344 | 181.336151 | 4.278651 | 2.911324 | 0.000000 | 0.21672 |
| 9 | 0.78 | 0.78 | 1.328839 | 62.486092 | 190.795532 | 3.800384 | 3.726972 | 0.000000 | 0.21093 |

Table 6. The estimated parameters by means of the GA-based NFN and performance index (ITAE, overshoot and rising time) of the fuzzy PD controller in the case of $\theta$ = 0.22, 0.45, 0.78 (rad) and $\dot{\theta}$ = 0.22, 0.45, 0.78 (rad/sec).

| Case | Initial angle (rad) | Initial angular velocity | GE | GD | GC | ITAE | Overshoot (%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.22 | 0.22 | 7.437843 | 0.529057 | 1.854648 | 0.149772 | 0.000000 | 0.12902 |
| 2 | 0.22 | 0.45 | 7.003085 | 0.508195 | 2.066092 | 0.173872 | 0.000000 | 0.13051 |
| 3 | 0.22 | 0.78 | 6.693032 | 0.492697 | 2.270946 | 0.216222 | 0.000000 | 0.13200 |
| 4 | 0.45 | 0.22 | 4.914640 | 0.333972 | 2.715822 | 0.546678 | 0.155587 | 0.15296 |
| 5 | 0.45 | 0.45 | 4.470177 | 0.317409 | 2.791710 | 0.609271 | 0.133525 | 0.15100 |
| 6 | 0.45 | 0.78 | 4.153202 | 0.305104 | 2.865232 | 0.728531 | 0.102153 | 0.14932 |
| 7 | 0.78 | 0.22 | 3.597757 | 0.247494 | 3.913758 | 2.270186 | 0.026860 | 0.20643 |
| 8 | 0.78 | 0.45 | 3.148229 | 0.232836 | 3.801080 | 2.617699 | 0.076634 | 0.20401 |
| 9 | 0.78 | 0.78 | 2.827642 | 0.221947 | 3.691914 | 3.304682 | 0.140848 | 0.20195 |

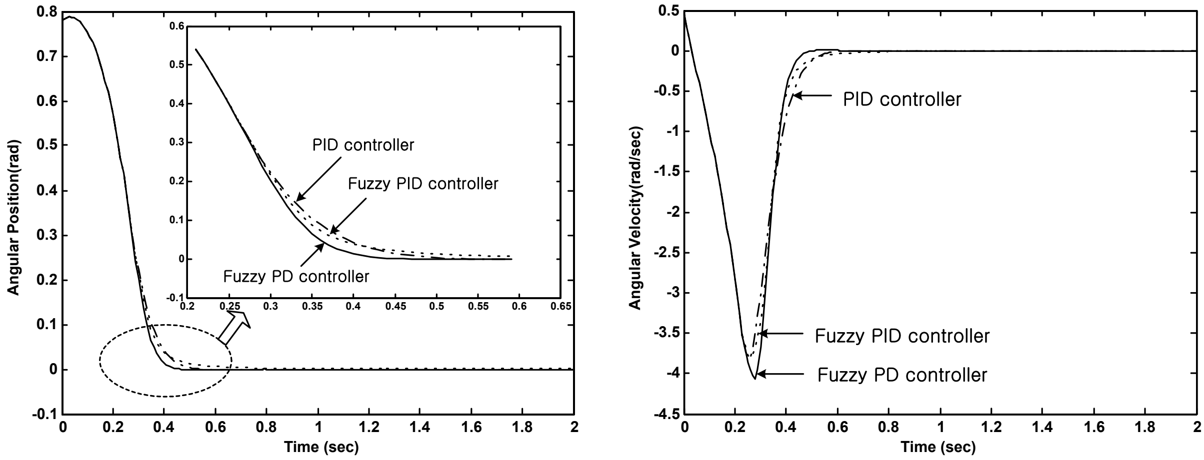
Table 7. The estimated parameters, ITAE, overshoot and rising time of the PID controller in the case of $\theta$ = 0.22, 0.45, 0.78 (rad) and $\dot{\theta}$ = 0.22, 0.45, 0.78 (rad/sec).

| Case | Initial angle (rad) | Initial angular velocity | K | Ti | Td | ITAE | Overshoot (%) | Rising time (sec) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.22 | 0.22 | 168.686066 | 164.717194 | 0.104505 | 0.247020 | 0.087829 | 0.17247 |
| 2 | 0.22 | 0.45 | 168.680069 | 164.879379 | 0.104704 | 0.273436 | 0.092216 | 0.17193 |
| 3 | 0.22 | 0.78 | 168.661194 | 164.987473 | 0.104988 | 0.319242 | 0.100283 | 0.16993 |
| 4 | 0.45 | 0.22 | 168.392883 | 166.310638 | 0.103440 | 0.742116 | 0.122528 | 0.18613 |
| 5 | 0.45 | 0.45 | 168.420395 | 166.041107 | 0.103302 | 0.819175 | 0.131957 | 0.18468 |
| 6 | 0.45 | 0.78 | 168.506943 | 165.861481 | 0.103104 | 0.953855 | 0.147016 | 0.18185 |
| 7 | 0.78 | 0.22 | 167.593369 | 167.404816 | 0.102826 | 2.674119 | 0.187619 | 0.23005 |
| 8 | 0.78 | 0.45 | 167.712250 | 166.838821 | 0.102493 | 3.051612 | 0.208242 | 0.22797 |
| 9 | 0.78 | 0.78 | 168.086258 | 166.461639 | 0.102016 | 3.762012 | 0.241529 | 0.22332 |




(a) (b)

Fig. 11. (a) pole angle, (b) pole angular velocity for initial angle $\theta$ = 0.22 (rad) and initial angular velocity $\dot{\theta}$ =0.22 (rad/sec) (Case 1).




(a) (b)

Fig. 12. (a) pole angle, (b) pole angular velocity for initial angle $\theta$ = 0.78 (rad) and initial angular velocity $\dot{\theta}$ =0.45 (rad/sec) (Case 8).

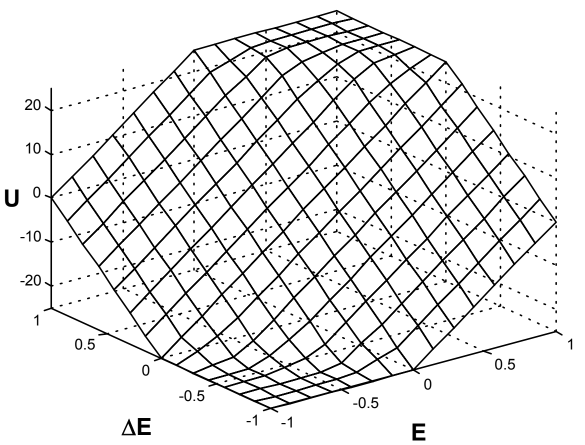



Fig. 13. The input-output relation of the fuzzy PD controller (GE, GD, GC = 1).

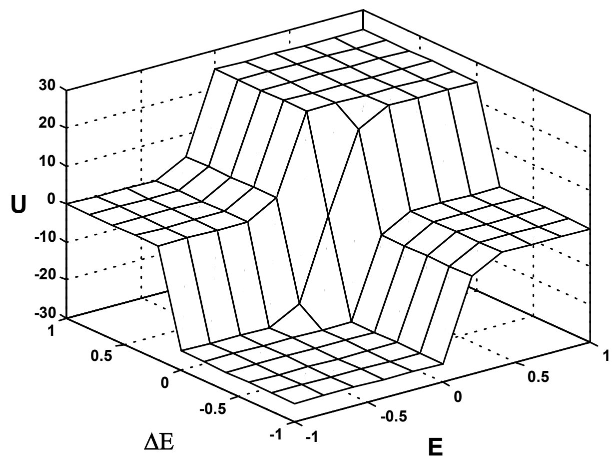



Fig. 14. The input-output relation of fuzzy PD controller (Case 6).

Fig. 12 demonstrates (a) pole angle (b) pole angular velocity for initial angle $\theta$ = 0.78 (rad) and initial angular velocity $\dot{\theta}$ =0.45 (rad/sec) (Case 8).

From the above Figs. 11 and 12, we know that the fuzzy PD and fuzzy PID effectively control the inverted pendulum system. The proposed estimation algorithm such as GA-based NFN generates the preferred model architectures. The output performance of the fuzzy controllers such as the fuzzy PD and the fuzzy PID controller including nonlinear characteristics are superior to that of the PID controller, especially in a nonlinear range of $\theta$>0.45 when using the nonlinear dynamic equation of the inverted pendulum. While in case of a linear range $\theta$<0.45, the PID controller is also satisfactory in comparison to the fuzzy PID controller. In particular the fuzzy PD controller is the most preferred one among the controllers when using NFN-based estimation techniques. Fig. 13 depicts the nonlinear characteristic of the fuzzy PD controller in cases where GE, GD, and GC are equal to 1.

Fig. 14 visualizes the input-output relation of the fuzzy PD controller when using Case 6. Note that the fuzzy PD comes with a significant nonlinear mapping between the inputs and the output.

## 6. CONCLUSIONS

In this paper, we have proposed a two-phase optimization scheme of the fuzzy PID and PD controllers. The parameters under optimization concern scaling factors of the input and output variables of the controller that are known to exhibit an immense impact on its quality. The first phase of the design of the controller employs genetic computing that aims at the global optimization of its scaling factors where they are optimized with regard to a finite collection of initial conditions of the system under control. In the second phase, we construct a nonlinear mapping between the initial conditions of the system and the corresponding values of the scaling factors. Based on the simulation studies, using genetic optimization by scaling factor estimation modes and the estimation algorithm of the GA-based neurofuzzy networks model, we demonstrated that the fuzzy PD/PID controller effectively controls the inverted pendulum system, particularly in a nonlinear range of $\theta$. While the study showed the development of the controller in the experimental framework of control of a specific dynamic system (inverted pendulum), this methodology is general and can be directly utilized to any other system. Similarly, one can envision a number of modifications that are worth investigating. For instance, a design of systems exhibiting a significant level of variability could benefit from the approach pursued in this study.

## REFERENCES


[1] S.-K. Oh, *Fuzzy Model & Control System by C-Programming*, Naeha Press, 2002.

[2] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, 1989.

[3] B.-J. Park, W. Pedrycz, and S.-K. Oh, "Identification of fuzzy models with the aid of evolutionary data granulation," *IEE Proceedings-CTA*, vol. 148, no. 5, pp. 406-418, 2001.

[4] J. R. Jang, "Self-learning fuzzy controllers based on temporal back propagation," *IEEE Trans. on Neural Networks*, vol. 3, no. 5, pp. 714-723, September 1992.

[5] T. J. Procyk and E. H. Mamdani, "A linguistic synthesis of fuzzy controller," *Automatica,* vol. 15, pp. 15-30, 1979.

[6] H. X. Li, "A comparative design and tuning for conventional fuzzy control," *IEEE Trans. on Syst., Man, Cybern. B*, vol. 27, pp. 884-889, October 1997.

[7] B.-G. Hu, G. K. I. Mann, and R. G. Gosine, "A systematic study of fuzzy PID controllers-function-based evaluation approach," *IEEE Trans. on Fuzzy Systems*, vol. 9, pp. 699-712, October 2001.

[8] L. Wang, "Stable adaptive fuzzy controllers with application to inverted pendulum tracking," *IEEE Trans. on Systems, Man and Cybernetics-Part B: Cybernetics*, vol. 26, no. 5, pp. 677-691, October 1996.

[9] S.-K. Oh, W. Pedrycz, and D.-K. Lee, "The genetic design of hybrid fuzzy controllers," *Cybernetics & Systems*, vol. 35, no. 4, pp. 333-361, June 2004.

[10] S.-K. Oh and W. Pedrycz, "The design of hybrid fuzzy controllers based on genetic algorithms and estimation techniques," *Kybernetes*, vol. 31, no. 6, pp. 909-917, 2002.

[11] S.-K. Oh, W. Pedrycz, and S.-B. Rho, "Parameter estimation of fuzzy controller and its application to inverted pendulum," *Engineering Applications of Artificial Intelligence*, vol. 17, no. 1, pp. 37-60, February 2004.


**Sung-Kwun Oh** received his B.Sc., M.Sc., and Ph.D. degrees in Electrical Engineering from Yonsei University, Seoul, Korea, in 1981, 1983 and 1993, respectively. During 1983-1989, he worked as the Senior Researcher in the R&D Lab. of Lucky-Goldstar Industrial Systems Co., Ltd. He was a Postdoctoral Fellow in the Department of Electrical and Computer Engineering at the University of Manitoba, Canada, from 1996 to 1997. He is currently an Associate Professor in the School of Electrical, Electronic and Information Engineering, Wonkwang University, Korea. His research interests include fuzzy systems, fuzzy-neural networks, automation systems, advanced computational intelligence, and intelligent control. He is a member of IEEE. He currently serves as an Associate Editor for the Korean Institute of Electrical Engineers and the Institute of Control, Automation & Systems Engineers, Korea.

**Seok-Beom Rho** received his M.S. in Computer Engineering from Wonkwang University, Korea in 1996. He is currently a Ph.D. student studying in the Department of Control and Instrumentation Engineering at Wonkwang University. His research interests include the theory and technology of intelligent systems, control, and soft computing.

**Hyun-Ki Kim** received his B.Sc., M.Sc., and Ph.D. degrees in Electrical Engineering from Yonsei University, Seoul, Korea, in 1977, 1985 and 1991, respectively. During 1999-2003, he worked as Chairman at the Korea Association of Small Business Innovation Research. He is currently a Professor in the Dept. of Electrical Engineering, Suwon University, Korea. His research interests include system automation and intelligent control. He currently serves as a Chief Editor for the Journal of Korea Association of Small Business Innovation Research.