International Mathematical Forum, 4, 2009, no. 42, 2093 - 2100

Fuzzy Ideal Extensions of Γ-Semigroups

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Abstract

In this paper the concept of the extensions of fuzzy ideals in a semigroup has been extended to a Γ-semigroup. Among other results characterization of prime ideals in a Γ-semigroup in terms of fuzzy ideal extension has been obtained.

Mathematics Subject Classification: 20M12, 03F55, 08A72

Keywords: Γ-Semigroup, Fuzzy ideal extension, Fuzzy left(right) ideal, Fuzzy semiprime ideal

1 Introduction

Γ-semigroup was introduced by Sen and Saha[9] as a generalization of semigroup and ternary semigroup. Many results of semigroups could be extended to Γ-semigroups directly and via operator semigroups[2] of a Γ-semigroup. Many results of semigroups have been studied in terms of fuzzy sets[11]. Kuroki[3,4] is the main contributor to this study. Motivated by Kuroki [3,4], Xie[10], Mustafa et all[5] we have initiated the study of Γ-semigroups in terms of fuzzy sets. This paper is a continuation of [6],[7],[8]. In this paper, the concept of the extensions of fuzzy ideals in a semigroup, introduced by Xie, has been extended to the general situation of Γ-semigroup. We have investigated some of its properties in terms of fuzzy prime and fuzzy semiprime ideals of Γ-semigroup. Among other results we have obtained characterization of prime ideals in a Γ-semigroup in terms of fuzzy ideal extension.

2 Preliminaries

We recall the following definitions and results which will be used in the sequel.

Definition 2.1 [2] *Let S and Γ be two non-empty sets. S is called a Γ-semigroup if there exist mappings from $S\times\Gamma\times S$ to S, written as $(a,\alpha,b)\longrightarrow a\alpha b$, and from $\Gamma\times S\times\Gamma$ to Γ, written as $(\alpha,a,\beta)\longrightarrow\alpha a\beta$ satisfying the following associative laws $(a\alpha b)\beta c=a(\alpha b\beta)c=a\alpha(b\beta c)$ and $\alpha(a\beta b)\gamma=(\alpha a\beta)b\gamma=\alpha a(\beta b\gamma)$ for all $a,b,c\in S$ and for all $\alpha,\beta,\gamma\in\Gamma$.*

Definition 2.2 [11] *A fuzzy subset of a nonempty set X is a function $\mu: X\to[0,1]$.*

Definition 2.3 [10] *The set of all fuzzy subsets of a set X with the relation $f\subseteq g\iff f(x)\leq g(x)\ \forall x\in X$ is a complete lattice where, for a non-empty family $\{\mu_i : i\in I\}$ of fuzzy subsets of X, the $\inf\{\mu_i : i\in I\}$ and the $\sup\{\mu_i : i\in I\}$ are the fuzzy subsets of X defined by:*
$\inf\{\mu_i : i\in I\}: X\longrightarrow[0,1],\ \ x\longrightarrow\inf\{\mu_i(x): i\in I\}$
$\sup\{\mu_i : i\in I\}: X\longrightarrow[0,1],\ x\longrightarrow\sup\{\mu_i(x): i\in I\}$

Definition 2.4 [8] *A non-empty fuzzy subset μ of a Γ-semigroup S is called a fuzzy left ideal(right ideal) of S if $\mu(x\gamma y)\geq\mu(y)$(resp. $\mu(x\gamma y)\geq\mu(x)$) $\forall x,y\in S$, $\forall\gamma\in\Gamma$.*

Definition 2.5 [8] *A non-empty fuzzy subset μ of a Γ-semigroup S is called a fuzzy ideal of S if it is both fuzzy left ideal and fuzzy right ideal of S.*

Definition 2.6 [6] *A fuzzy ideal μ of a Γ-semigroup S is called fuzzy prime ideal if $\inf\limits_{\gamma\in\Gamma}\mu(x\gamma y)=\max\{\mu(x),\mu(y)\}\ \forall x,y\in S$.*

Definition 2.7 [7] *A fuzzy ideal μ of a Γ-semigroup S is called fuzzy semiprime ideal if $\mu(x)\geq\inf\limits_{\gamma\in\Gamma}\mu(x\gamma x)\ \forall x\in S$.*

Definition 2.8 [2] *Let S be a Γ-semigroup. Then an ideal I of S is said to be (i) prime if for ideals A,B of S, $A\Gamma B\subseteq I$ implies that $A\subseteq I$ or $B\subseteq I$. (ii) semiprime if for an ideal A of S, $A\Gamma A\subseteq I$ implies that $A\subseteq I$.*

Proposition 2.9 [6, 7] *Let S be a Γ-semigroup and $\phi \neq I \subseteq S$. Then I is an ideal(prime ideal, semiprime ideal) of S iff μ_I is a fuzzy ideal(resp. fuzzy prime ideal, fuzzy semiprime ideal) of S, where μ_I is the characteristic function of I.*

Theorem 2.10 [6, 7] *Let I be an ideal of a Γ-semigroup S. Then the following are equivalent:*

(*i*) *I is prime(semiprime).*

(*ii*) *for $x, y \in S, x\Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$(resp. $x\Gamma x \subseteq I \Rightarrow x \in I$).*

(*ii*) *for $x, y \in S, x\Gamma S\Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$(resp. $x\Gamma S\Gamma x \subseteq I \Rightarrow x \in I$).*

3 Fuzzy Ideal Extensions

Definition 3.1 *Let S be a Γ-semigroup, μ be a fuzzy subset of S and $x \in S$, then the fuzzy subset $< x, \mu >: S \to [0, 1]$ defined by $< x, \mu > (y) = \inf_{\gamma \in \Gamma} \mu(x\gamma y)$ is called the extension of μ by x.*

Example (a): Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers. Then S is a Γ-semigroup where $a\gamma b$ and$\alpha a \beta$ denote the usual multiplication of integers a, γ, b and α, a, β respectively with $a, b \in S$ and $\alpha, \beta, \gamma \in \Gamma$. Let μ be a fuzzy subset of S, defined as follows $\mu(x) = \begin{cases} 1, \text{if } x = 0 \\ 0.1, \text{if } x = -1, -2 \\ 0.2, \text{if } x < -2 \end{cases}$. Then the fuzzy subset μ of S is a fuzzy ideal of S.

For $x = 0 \in S$,$< x, \mu > (y) = 1 \forall y \in S$. For all other $x \in S$, $< x, \mu > (y) = 0.2 \forall y \in S$.
Thus $< x, \mu >$ is a fuzzy ideal extension of μ by x.

Proposition 3.2 *Let μ be a fuzzy ideal of a commutative Γ-semigroup S and $x \in S$. Then $< x, \mu >$ is a fuzzy ideal of S.*

Proof. Let μ be a fuzzy ideal of a commutative Γ-semigroup S and $p, q \in S, \beta \in \Gamma$. Then $< x, \mu > (p\beta q) = \inf_{\gamma \in \Gamma} \mu(x\gamma p\beta q) \geq \inf_{\gamma \in \Gamma} \mu(x\gamma p) =< x, \mu > (p)$. Thus $< x, \mu >$ is a fuzzy right ideal of S. Hence S being commutative $< x, \mu >$ is a fuzzy ideal of S. ■

Remark 3.3 *Commutativity of Γ-semigroup S is not required to prove that $< x, \mu >$ is a fuzzy right ideal of S when μ is a fuzzy right ideal of S.*

Proposition 3.4 *Let S be a commutative Γ-semigroup and μ be a fuzzy prime ideal of S. Then $< x, \mu >$ is fuzzy prime ideal of S for all $x \in S$.*

Proof. Let μ be a fuzzy prime ideal of S. Then by Proposition 3.2, $< x, \mu >$ is a fuzzy ideal of S. Let $y, z \in S$. Then $\inf_{\beta\in\Gamma} < x, \mu > (y\beta z) = \inf_{\beta\in\Gamma} \inf_{\gamma\in\Gamma} \mu(x\gamma y\beta z)(cf.$ Definition 3.1) $= \inf_{\beta\in\Gamma} \max\{\mu(x), \mu(y\beta z)\}(cf.$ Definition 2.6) $= \max\{\mu(x), \inf_{\beta\in\Gamma} \mu(y\beta z)\} = \max[\mu(x), \max\{\mu(y), \mu(z)\}] = \max[\max\{\mu(x), \mu(y)\}, \max\{\mu(x), \mu(z)\}] = \max\{\inf_{\delta\in\Gamma} \mu(x\delta y), \inf_{\varepsilon\in\Gamma} \mu(x\varepsilon z)\} = \max\{< x, \mu > (y), < x, \mu > (z)\}$. Hence by Definition 2.6, $< x, \mu >$ is a fuzzy prime ideal of S. ■

Definition 3.5 *Suppose S is a Γ-semigroup and μ is a fuzzy subset of S. Then we define supp $\mu = \{x \in S : \mu(x) > 0\}$.*

Proposition 3.6 *Let S be a Γ-semigroup, μ be a fuzzy ideal of S and $x \in S$. Then we have the following:*

(1) $\mu \subseteq < x, \mu >$.
(2) $< (x\alpha)^n x, \mu > \subseteq < (x\alpha)^{n+1} x, \mu > \forall\alpha \in \Gamma, \forall n \in N$.
(3) *If $\mu(x) > 0$ then supp $< x, \mu >= S$.*

Proof. (1) Let $y \in S$. Then $< x, \mu > (y) = \inf_{\gamma\in\Gamma} \mu(x\gamma y) \geq \mu(y)$(since μ is a fuzzy ideal of S). Hence $\mu \subseteq < x, \mu >$.

(2) $< (x\alpha)^{n+1} x, \mu > (y) = \inf_{\gamma\in\Gamma} \mu((x\alpha)^{n+1} x\gamma y) = \inf_{\gamma\in\Gamma} \mu(x\alpha(x\alpha)^n x\gamma y) \geq \inf_{\gamma\in\Gamma} \mu((x\alpha)^n x\gamma y)$(since μ is a fuzzy ideal of S) $=< (x\alpha)^n x, \mu > (y)$. Hence $< (x\alpha)^n x, \mu > \subseteq < (x\alpha)^{n+1} x, \mu >$.

(3) Since $< x, \mu >$ is a fuzzy subset of S, by definition, supp $< x, \mu > \subseteq S$. Let $y \in S$. Since μ is a fuzzy ideal of S, we have, $< x, \mu > (y) = \inf_{\gamma\in\Gamma} \mu(x\gamma y) \geq \mu(x) > 0$. Then $< x, \mu > (y) > 0$ and so $y \in$supp $< x, \mu >$. ■

Remark 3.7 *If we consider $(x\alpha)^0 x = x$ then (2) is also true for $n = 0$.*

Definition 3.8 *Suppose S is a Γ-semigroup, $A \subseteq S$ and $x \in S$. We define $< x, A >= \{y \in S \mid x\Gamma y \subseteq A\}$, where $x\Gamma y := \{x\alpha y : \alpha \in \Gamma\}$.*

Proposition 3.9 *Let S be a Γ-semigroup and $\phi \neq A \subseteq S$. Then $< x, \mu_A >= \mu_{<x,A>}$ for every $x \in S$, where μ_A denotes the characteristic function of A.*

Proof. Let $x, y \in S$. Then two cases may arise viz. *Case (i)* $y \in< x, A >$. *Case (ii)* $y \notin< x, A >$.

Case (i) $y \in< x, A >$. Then $x\Gamma y \subseteq A$. Hence $x\gamma y \in A \ \forall\gamma \in \Gamma$. This means that $\mu_A(x\gamma y) = 1 \ \forall\gamma \in \Gamma$. Hence $\inf_{\gamma\in\Gamma} \mu_A(x\gamma y) = 1$ whence $< x, \mu_A > (y) = 1$. Also $\mu_{<x,A>}(y) = 1$.

Case (ii) $y \notin< x, A >$. Then there exists $\gamma \in \Gamma$ such that $x\gamma y \notin \dot{A}$. So $\mu_A(x\gamma y) = 0$. Hence $\inf_{\gamma\in\Gamma} \mu_A(x\gamma y) = 0$. Thus $< x, \mu_A > (y) = 0$. Again $\mu_{<x,A>}(y) = 0$. Thus we conclude $< x, \mu_A >= \mu_{<x,A>}$. ■

Proposition 3.10 *Let S be a Γ-semigroup and μ be a nonempty fuzzy subset of S. Then for any $t \in Im(\mu)$, $< x, \mu_t >=< x, \mu >_t$ for all $x \in S$.*

Proof. Let $y \in < x, \mu >_t$. Then $< x, \mu > (y) \geq t$. Hence $\inf\limits_{\gamma \in \Gamma} \mu(x\gamma y) \geq t$. This gives $\mu(x\gamma y) \geq t$ for all $\gamma \in \Gamma$ and hence $x\gamma y \in \mu_t$ for all $\gamma \in \Gamma$. Consequently, $y \in < x, \mu_t >$. It follows that $< x, \mu >_t \subseteq < x, \mu_t >$. Reversing the above argument we can deduce that $< x, \mu_t > \subseteq < x, \mu >_t$. Hence $< x, \mu >_t =< x, \mu_t >$. ■

Proposition 3.11 *Let S be a commutative Γ-semigroup i.e., $a\alpha b = b\alpha a\ \forall a, b \in S, \forall \alpha \in \Gamma$ and μ be a fuzzy subset of S such that $< x, \mu >= \mu$ for every $x \in S$. Then μ is a constant function.*

Proof. Let $x, y \in S$. Then by hypothesis we have $\mu(x) =< y, \mu > (x) = \inf\limits_{\gamma \in \Gamma} \mu(y\gamma x) = \inf\limits_{\gamma \in \Gamma} \mu(x\gamma y) =< x, \mu > (y) = \mu(y)$. Hence μ is a constant function. ■

Corollary 3.12 *Let S be a commutative Γ-semigroup, μ be a fuzzy prime ideal of S. If μ is not constant, then μ is not a maximal fuzzy prime ideal of S.*

Proof. Let μ be a fuzzy prime ideal of S. Then, by Proposition 3.4 for each $x \in S$, $< x, \mu >$ is a fuzzy prime ideal of S. Now by Proposition 3.6(1), $\mu \subseteq < x, \mu >$ for all $x \in S$. If $\mu =< x, \mu >$ for all $x \in S$ then by Proposition 3.11, μ is constant which is not the case by hypothesis. Hence there exists $x \in S$ such that $\mu \subsetneq < x, \mu >$. This completes the proof. ■

Proposition 3.13 *Let S be a commutative Γ-semigroup. If μ is a fuzzy semiprime ideal of S, then $< x, \mu >$ is a fuzzy semiprime ideal of S for every $x \in S$.*

Proof. Let μ be a fuzzy semiprime ideal of S and $x, y \in S$. Then $\inf\limits_{\gamma \in \Gamma} < x, \mu > (y\gamma y) = \inf\limits_{\gamma \in \Gamma} \inf\limits_{\delta \in \Gamma} \mu(x\delta y\gamma y) \leq \inf\limits_{\gamma \in \Gamma} \inf\limits_{\delta \in \Gamma} \mu(x\delta y\gamma y\delta x)$(since μ is a fuzzy ideal of S) $= \inf\limits_{\gamma \in \Gamma} \inf\limits_{\delta \in \Gamma} \mu(x\delta y\gamma x\delta y)$(using commutativity of S and Definition 2.7)) $=< x, \mu > (y)$. Again by Proposition 3.2, $< x, \mu >$ is a fuzzy ideal of S. Consequently, $< x, \mu >$ is a fuzzy semiprime ideal of S for all $x \in S$. ■

Corollary 3.14 *Let S be a commutative Γ-semigroup, $\{\mu_i\}_{i \in I}$ be a non-empty family of fuzzy semiprime ideals of S and let $\mu = \inf\{\mu_i : i \in I\}$. Then for any $x \in S$, $< x, \mu >$ is a fuzzy semiprime ideal of S.*

Proof. Since each $\mu_i(i \in I)$ is a fuzzy ideal, $\mu_i(0) \neq 0\ \forall i \in I$(Each μ_i is non-empty, so there exists $x_i \in S$ such that $\mu_i(x_i) \neq 0\ \forall i \in I$. Also $\mu_i(0) = \mu_i(0\gamma x_i) \geq \mu_i(x_i)\ \forall i \in I$. Hence $\forall i \in I$, $\mu_i(0) \neq 0$). Consequently, $\mu(0) \neq 0$. Thus μ is non-empty. Now let $x, y \in S$. Then $\mu(x\gamma y) = \inf\{\mu_i : i \in I\}(x\gamma y) = \inf\{\mu_i(x\gamma y) : i \in I\} \geq \inf\{\mu_i(x) : i \in I\} = \mu(x)$. Hence S being a commutative Γ-semigroup, μ is a fuzzy ideal of S.

Now if $a \in S$ then $\mu(a) = \inf\{\mu_i : i \in I\}(a) = \inf\{\mu_i(a) : i \in I\} \geq \inf\{\inf\limits_{\gamma\in\Gamma} \mu_i(a\gamma a) : i \in I\}$(since each μ_i is a fuzzy semiprime ideal(cf. Definition 2.7)) $= \inf\limits_{\gamma\in\Gamma}[\inf\{\mu_i : i \in I\}(a\gamma a)] = \inf\limits_{\gamma\in\Gamma} \mu(a\gamma a)$. This means, μ is a fuzzy semiprime ideal of S. Hence by Proposition 3.13, for any $x \in S$, $< x, \mu >$ is a fuzzy semiprime ideal of S. ■

Remark 3.15 *The proof of the above Corollary shows that in a Γ-semigroup intersection of arbitrary family of fuzzy semiprime ideals is a fuzzy semiprime ideal.*

Corollary 3.16 *Let S be a commutative Γ-semigroup, $\{S_i\}_{i\in I}$ a non-empty family of semiprime ideals of S and $A := \cap_{i\in I} S_i \neq \phi$. Then $< x, \mu_A >$ is a fuzzy semiprime ideal of S for all $x \in S$ where μ_A is the characteristic function of A.*

Proof. By supposition $A \neq \phi$. Then for any ideal P of S, $P\Gamma P \subseteq A$ implies that $P\Gamma P \subseteq S_i\ \forall i \in I$. Since each S_i is a semiprime ideal of S, $P \subseteq S_i\ \forall i \in I$($cf$. Definition 2.8). So $P \subseteq \cap_{i\in I} S_i = A$. Hence A is a semiprime ideal of S(cf. Definition 2.8). So the characteristic function μ_A of A is a fuzzy semiprime ideal of S(cf. Proposition 2.9). Hence by Proposition 3.13,$\forall x \in S, < x, \mu_A >$ is a fuzzy semiprime ideal of S. ■

Alternative Proof: $A = \cap_{i\in I} S_i \neq \phi$(by the given condition). Hence $\mu_A \neq \phi$. Let $x \in S$. Then $x \in A$ or $x \notin A$. If $x \in A$ then $\mu_A(x) = 1$ and $x \in S_i\ \forall i \in I$. Hence $\inf\{\mu_{S_i} : i \in I\}(x) = \inf\limits_{i\in I} \{\mu_{S_i}(x)\} = 1 = \mu_A(x)$. If $x \notin A$ then $\mu_A(x) = 0$ and for some $i \in I$, $x \notin S_i$. It follows that $\mu_{S_i}(x) = 0$. Hence $\inf\{\mu_{S_i} : i \in I\}(x) = \inf\limits_{i\in I} \{\mu_{S_i}(x)\} = 0 = \mu_A(x)$. Thus we see that $\mu_A = \inf\{\mu_{S_i} : i \in I\}$. Again μ_{S_i} is a fuzzy semiprime ideal of S for all $i \in I$(cf. Proposition 2.9). Consequently by Corollary 3.14, for all $x \in S, < x, \mu_A >$ is a fuzzy semiprime ideal of S.

Theorem 3.17 *Let S be a Γ-semigroup. If μ is a fuzzy prime ideal of S and $x \in S$ such that $\mu(x) = \inf\limits_{y\in S} \mu(y)$, then $< x, \mu >= \mu$. Conversely, if μ is a fuzzy ideal of S such that $< y, \mu >= \mu\ \forall y \in S$ with $\mu(y)$ not maximal in $\mu(S)$ then μ is prime.*

Proof. Let μ be a fuzzy prime ideal of S and $x \in S$ be such that $\mu(x) = \inf_{y\in S} \mu(y)$(it can be noted here that since each $\mu(y) \in [0,1]$, a closed and bounded subset of R, $\inf_{y\in S} \mu(y)$ exists). Let $z \in S$. Then $\mu(x) \leq \mu(z)$. Hence $\max\{\mu(x), \mu(z)\} = \mu(z)$.........$(*)$. Now $< x, \mu > (z) = \inf_{\gamma\in\Gamma} \mu(x\gamma z)$. Since μ is a fuzzy prime ideal of S, $\inf_{\gamma\in\Gamma} \mu(x\gamma z) = \max\{\mu(x), \mu(z)\} = \mu(z)$(using $(*)$). Hence $< x, \mu > (z) = \mu(z)$. Consequently, $< x, \mu >= \mu$.

Conversely, let μ be a fuzzy ideal of S such that $< y, \mu >= \mu\ \forall y \in S$ with $\mu(y)$ is not maximal in $\mu(S)$ and let $x_1, x_2 \in S$. Then μ being a fuzzy ideal of S, $\mu(x_1\gamma x_2) \geq \mu(x_1)$ and $\mu(x_1\gamma x_2) \geq \mu(x_2)\ \forall \gamma \in \Gamma$. This leads to $\inf_{\gamma\in\Gamma} \mu(x_1\gamma x_2) \geq \mu(x_1)$.......$(**)$ and $\inf_{\gamma\in\Gamma} \mu(x_1\gamma x_2) \geq \mu(x_2)$......$(***)$. Now two cases may arise viz. *Case* (i) Either $\mu(x_1)$ or $\mu(x_2)$ is maximal in $\mu(S)$. *Case* (ii) Neither $\mu(x_1)$ nor $\mu(x_2)$ is maximal in $\mu(S)$. *Case* (i) Without loss of generality, let $\mu(x_1)$ be maximal in $\mu(S)$. Then $\inf_{\gamma\in\Gamma} \mu(x_1\gamma x_2) \leq \mu(x_1)$. Consequently $\inf_{\gamma\in\Gamma} \mu(x_1\gamma x_2) = \mu(x_1) = \max\{\mu(x_1), \mu(x_2)\}$. *Case* (ii) By the hypothesis $< x_1, \mu >= \mu$ and $< x_2, \mu >= \mu$. Hence $< x_1, \mu > (x_2) = \mu(x_2) \Rightarrow \inf_{\gamma\in\Gamma} \mu(x_1\gamma x_2) = \mu(x_2) = \max\{\mu(x_1), \mu(x_2)\}$(using $(**)$). Thus we conclude that μ is a fuzzy prime ideal of S. ■

To end this paper we get the following characterization theorem of a prime ideal of a Γ-semigroup which follows as a corollary to the above theorem.

Corollary 3.18 *Let S be a Γ-semigroup and I be an ideal of S. Then I is prime iff for $x \in S$ with $x \notin I$, $< x, \mu_I >= \mu_I$, where μ_I is the characteristic function of I.*

Proof. Let I be a prime ideal of S. Then, by Proposition 2.9, μ_I is a fuzzy prime ideal of S. For $x \in S$ such that $x \notin I$, we have $\mu_I(x) = 0 = \inf_{y\in S} \mu_I(y)$. Then by Theorem 3.17, $< x, \mu_I >= \mu_I$.

Conversely, let $< x, \mu_I >= \mu_I$ for all x in S with $x \notin I$. Let $y \in S$ be such that $\mu_I(y)$ is not maximal in $\mu_I(S)$. Then $\mu_I(y) = 0$ and so $y \notin I$. So $< y, \mu_I >= \mu_I$. So by the Theorem 3.17, μ_I is a fuzzy prime ideal of S. So I is a prime ideal of S(cf. Proposition 2.9). ■

Acknowledgement: We are thankful to Prof. Xiang-Yun Xie of the Department of Mathematics and Physics, Wuyi University, P.R. China; for his co-operation and encouragement to write this paper.

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Received: March, 2009