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Fuzzy Ideal Extensions of Γ- Semigroups via its Operator Semigroups

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Abstract

The main purpose of this paper is to explore new relationships between a Γ-semigroup and its operator semigroups in terms of fuzzy ideals so as to apply them, together with the already obtained such relationships, in the study of fuzzy ideal extension of Γ-semigroups.

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1 Introduction

The notion of fuzzy set was introduced by L.A.Zadeh[11], and since then this concept has been applied to various algebraic structures. In [3], Kuroki characterized several classes of semigroups in terms of fuzzy left, right and two-sided ideals and bi-ideals. M. K. Sen and N. K. Saha[9] introduced the concept of Γ-semigroup as a generalization of semigroup and ternary semigroup. A natural example of a Γ-semigroup is the set of all mappings from any non-empty set A to a non-empty set B where Γ is the set of all mappings from B to A and ternary composition $a\alpha b$ or $\alpha a\beta$ $(a, b \in S, \alpha, \beta \in \Gamma)$ is the usual mapping composition. Uckun Mustafa, Ali Mehmet, Jun Young Bae[4] investigated some properties of fuzzy ideals in Γ-semigroups. T. K. Dutta and N. C. Adhikari[2] studied different properties of Γ-semigroup via its operator semigroups. In order to make the operator semigroups work in the context of fuzzy subsets of a Γ-semigroup as it did for conventional subsets we have investigated in [7], various relationships between the fuzzy ideals of a Γ-semigroup and that of its operator semigroups including an inclusion preserving bijection(Theorem 5.14.and 5.17[7]) between the set of all fuzzy ideals of a Γ-semigroup and that of its operator semigroups. Motivated by the work of Xie [10] on semigroups and the fact that Γ-semigroups encompass semigroups we extended the notion of fuzzy ideal extension to Γ-semigroups in [8]. The purpose of this paper is to bring operator semigroups into action to re-establish many results of [8]. To do this we first obtain some new relationships between a Γ-semigroup and its operator semigroups. These results together with those already obtained in [8] were used to work out as stated above. Among other results we have obtained characterization of a prime ideal of a Γ-semigroup in terms of fuzzy ideal extension.

2 Preliminaries

We recall the following definitions and results which will be used in the sequel.

Definition 2.1 [2] *Let S and Γ be two non-empty sets. S is called a Γ-semigroup if there exist mappings from $S \times \Gamma \times S$ to S, written as $(a, \alpha, b) \longrightarrow a\alpha b$, and from $\Gamma \times S \times \Gamma$ to Γ, written as $(\alpha, a, \beta) \longrightarrow \alpha a\beta$ satisfying the following associative laws $(a\alpha b)\beta c = a(\alpha b\beta)c = a\alpha(b\beta c)$ and $\alpha(a\beta b)\gamma = (\alpha a\beta)b\gamma = \alpha a(\beta b\gamma)$ for all $a, b, c \in S$ and for all $\alpha, \beta, \gamma \in \Gamma$.*

Example (a): Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers. Then S is a Γ-semigroup where $a\alpha b$ and $\alpha a\beta(a, b \in S, \alpha, \beta \in \Gamma)$ denote usual multiplication of integers.
Example (b): Let S=$\{5n + 4 : n$ is a positive integer$\}$ and $\Gamma = \{5n + 1 : n$ is a positive integer$\}$. Then S is a Γ-semigroup where $a\alpha b = a + \alpha + b$ and $\alpha a\beta = \alpha + a + \beta$ $(a, b \in S$, and $\alpha, \beta \in \Gamma$, $+$ is the usual addition of integers).

Definition 2.2 [11] *A fuzzy subset of a non-empty set X is a function $\mu : X \rightarrow [0, 1]$.*

Definition 2.3 [10] *The set of all fuzzy subsets of a set* X *with the relation* $f \subseteq g \iff f(x) \leq g(x)\ \forall x \in X$ *is a complete lattice where, for a non-empty family* $\{\mu_i : i \in I\}$ *of fuzzy subsets of* X*, the* $\inf\{\mu_i : i \in I\}$ *and the* $\sup\{\mu_i : i \in I\}$ *are the fuzzy subsets of* X *defined by:*

$\inf\{\mu_i : i \in I\} : X \longrightarrow [0,1],\ \ x \longrightarrow \inf\{\mu_i(x) : i \in I\}$

$\sup\{\mu_i : i \in I\} : X \longrightarrow [0,1],\ x \longrightarrow \sup\{\mu_i(x) : i \in I\}$

Definition 2.4 [4] *A non-empty fuzzy subset* μ *of a* Γ*-semigroup* S *is called a fuzzy left ideal(right ideal) of* S *if* $\mu(x\gamma y) \geq \mu(y)$*(resp.* $\mu(x\gamma y) \geq \mu(x)$*)* $\forall x, y \in S$, $\forall \gamma \in \Gamma$.

Definition 2.5 [4] *A non-empty fuzzy subset* μ *of a* Γ*-semigroup* S *is called a fuzzy ideal of* S *if it is both fuzzy left ideal and fuzzy right ideal of* S.

Definition 2.6 [5] *A fuzzy ideal* μ *of a* Γ*-semigroup* S *is called a fuzzy prime ideal if* $\inf\limits_{\gamma \in \Gamma} \mu(x\gamma y) = \max\{\mu(x), \mu(y)\}\ \forall x, y \in S$.

Example (c): Let S be the set of all 1×2 matrices over GF_2(the finite field with two elements) and Γ be the set of all 2×1 matrices over GF_2. Then S is a Γ-semigroup where $a\alpha b$ and $\alpha a\beta (a, b \in S$ and $\alpha, \beta \in \Gamma)$ denotes the usual matrix product. Let $\mu : S \rightarrow$[0,1] be defined by

$$\mu(x) = \begin{cases} 0.3 & \text{if } a=(0,0) \\ 0.2 & \text{otherwise} \end{cases}$$

Then μ is a fuzzy prime ideal of S.

Definition 2.7 [6] *A fuzzy ideal* μ *of a* Γ*-semigroup* S *is called fuzzy semiprime ideal if* $\mu(x) \geq \inf\limits_{\gamma \in \Gamma} \mu(x\gamma x)\ \forall x \in S$.

Proposition 2.8 [7] *Let* S *be a* Γ*-semigroup and* $\phi \neq I \subseteq S$*. Then* I *is a ideal of* S *iff* μ_I *is a fuzzy ideal of* S*, where* μ_I *is the characteristic function of* I.

Definition 2.9 [2] *Let* S *be a* Γ*-semigroup. Then an ideal* I *of* S *is said to be* (i) *prime if for ideals* A, B *of* S*,* $A\Gamma B \subseteq I$ *implies that* $A \subseteq I$ *or* $B \subseteq I$. (ii) *semiprime if for an ideal* A *of* S*,* $A\Gamma A \subseteq I$ *implies that* $A \subseteq I$.

Proposition 2.10 [5, 6] *Let* S *be a* Γ*-semigroup and* $\phi \neq I \subseteq S$*. Then* I *is a prime ideal(semiprime ideal) of* S *iff* μ_I *is a fuzzy prime ideal(resp. fuzzy semiprime ideal) of* S*, where* μ_I *is the characteristic function of* I.

Theorem 2.11 [5, 6] *Let* I *be an ideal of a* Γ*-semigroup* S*. Then the following are equivalent:*

(i) *I is prime(semiprime).*

(ii) *for* $x, y \in S, x\Gamma y \subseteq I \Rightarrow x \in I$ *or* $y \in I$ *(resp.* $x\Gamma x \subseteq I \Rightarrow x \in I$*).*

(ii) *for* $x, y \in S, x\Gamma S\Gamma y \subseteq I \Rightarrow x \in I$ *or* $y \in I$ *(resp.* $x\Gamma S\Gamma x \subseteq I \Rightarrow x \in I$*).*

Definition 2.12 [8] *Let S be a Γ-semigroup, μ be a fuzzy subset of S and $x \in S$, then the fuzzy subset $< x, \mu >: S \to [0,1]$ defined by $< x, \mu > (y) = \inf_{\gamma \in \Gamma} \mu(x\gamma y)$ is called the extension of μ by x.*

3 Corresponding Ideal Extensions

Let S be a Γ-semigroup and ρ a relation on $S \times \Gamma$ defined as : $(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α). Let $L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. This semigroup L is called the left operator semigroup[2] of the Γ-semigroup S.

Dually the right operator semigroup R of Γ-semigroup S is defined, where the multiplication is defined by $[\alpha, a][\beta, b] = [\alpha a\beta, b]$.

In [7], for fuzzy subsets of a Γ-semigroup S and of its left, right operator semigroups L, R respectively, we have defined four functions namely, $()^*, ()^+, ()^{*'}, ()^{+'}$, which are defined as follows:

For a fuzzy subset μ of R (σ of L) a fuzzy subset μ^*(respectively σ^+) of S by $\mu^*(a) = \inf_{\gamma \in \Gamma} \mu([\gamma, a])$(respectively $\sigma^+(a) = \inf_{\gamma \in \Gamma} \sigma([a, \gamma])$) where $a \in S$ and for a fuzzy subset η of S we define a fuzzy subset $\eta^{+'}$ ($\eta^{*'}$)of L(respectively R) by $\eta^{+'}([a, \alpha]) = \inf_{s \in S} \eta(a\alpha s)$ (respectively $\eta^{*'}([\alpha, a]) = \inf_{s \in S} \eta(s\alpha a)$).
It has subsequently been proved that $()^*, ()^+, ()^{*'}, ()^{+'}$ preserve fuzzy ideals. In fact we have shown that $()^{*'}$ and $()^{+'}$ are inclusion preserving bijections (cf. Theorem 5.14,5.13[7]) with $()^*$ and $()^+$ are respectively their inverses. In [10] Xie defined for a semigroup S and a fuzzy subset μ of S, a fuzzy subset $< x, \mu >$, called the extension of μ by x, by $< x, \mu > (y) := \mu(xy)\ \forall y \in S$. It is also proved that if μ is a fuzzy ideal of S then so is $< x, \mu >$.
Now we can deduce the following propositions easily:

Proposition 3.1 *Let S be a commutative Γ-semigroup and $L(R)$ the left (respectively the right) operator semigroups of S. Let μ be a fuzzy (left, right, two sided)ideal of S then $< x, \mu^{+'} >$ (respectively $< x, \mu^{*'} >$) is a fuzzy (left, right, two sided) ideal of L(respectively R) for all $x \in L(R)$.*

Proposition 3.2 (*With the same notation as in the above proposition*) *If σ is a fuzzy (left, right, two sided) ideal of $L(R)$ then $< x, \sigma^+ >$ (respectively $< x, \sigma^* >$) is a fuzzy (left, right, two sided) ideal of S for all $x \in S$.*

Now we obtain the following lemmas on the relationships between a Γ-semigroup and its operator semigroups which play crucial role to the development of this paper.

Lemma 3.3 *Let* μ *be a non-empty fuzzy subset of a commutative* Γ*-semigroup* S*. Then for all* $x \in S$

$(i) < x, \mu >^{*'} \subseteq < [\alpha, x], \mu^{*'} > \forall \alpha \in \Gamma.$

$(ii) < x, \mu >^{*'} = \inf_{\alpha\in\Gamma} < [\alpha, x], \mu^{*'} > .$

Proof. (i) Let $[\beta, y] \in R$. Then

$$\begin{aligned} < \ & x, \mu >^{*'} ([\beta, y]) = \inf_{s\in S} < x, \mu > (s\beta y) \\ = \ & \inf_{s\in S}\inf_{\gamma\in\Gamma}\mu(x\gamma s\beta y) = \inf_{\gamma\in\Gamma}\inf_{s\in S}\mu(x\gamma s\beta y) \end{aligned}$$

Again

$$\begin{aligned} < \ & [\alpha, x], \mu^{*'} > ([\beta, y]) = \mu^{*'}([\alpha, x][\beta, y]) \\ = \ & \mu^{*'}([\alpha, x\beta y]) = \inf_{s\in S}\mu(s\alpha x\beta y) \\ = \ & \inf_{s\in S}\mu(x\alpha s\beta y)(\text{using commutativity of } S) \end{aligned}$$

Since, $\inf_{\gamma\in\Gamma}\inf_{s\in S}\mu(x\gamma s\beta y) \le \inf_{s\in S}\mu(x\alpha s\beta y)$, we have

$$< x, \mu >^{*'} ([\beta, y]) \le < [\alpha, x], \mu^{*'} > ([\beta, y])$$

whence $< x, \mu >^{*'} \subseteq < [\alpha, x], \mu^{*'} >$.

(ii) Let $[\beta, y] \in R$. Then

$$\begin{aligned} \inf_{\alpha\in\Gamma} \ < \ & [\alpha, x], \mu^{*'} > ([\beta, y]) = \inf_{\alpha\in\Gamma}\mu^{*'}([\alpha, x][\beta, y]) \\ = \ & \inf_{\alpha\in\Gamma}\mu^{*'}([\alpha, x\beta y]) = \inf_{\alpha\in\Gamma}\inf_{s\in S} \mu(s\alpha x\beta y) \\ = \ & \inf_{s\in S} < x, \mu > (s\beta y) = < x, \mu >^{*'} ([\beta, y]) \end{aligned}$$

Thus $< x, \mu >^{*'} = \inf_{\alpha\in\Gamma} < [\alpha, x], \mu^{*'} >$. ■

Lemma 3.4 *Let* σ *be a non-empty fuzzy subset of the right operator semigroup* R *of a* Γ*-semigroup* S *then for all* $x \in S$, $< [\beta, x], \sigma >^{*} \supseteq < x, \sigma^{*} > \forall \beta \in \Gamma$.

Proof. Let $p \in S$. Then

$$\begin{aligned} < \ & [\beta, x], \sigma >^{*} (p) = \inf_{\gamma\in\Gamma} < [\beta, x], \sigma > ([\gamma, p]) \\ = \ & \inf_{\gamma\in\Gamma}\sigma([\beta, x][\gamma, p]) = \inf_{\gamma\in\Gamma}\sigma([\beta, x\gamma p]) \end{aligned}$$

Again

$$< \ x, \sigma^* > (p) = \inf_{\gamma \in \Gamma} \sigma^*(x\gamma p) = \inf_{\gamma \in \Gamma} \inf_{\beta \in \Gamma} \sigma([\beta, x\gamma p])$$
$$= \ \inf_{\beta \in \Gamma} \inf_{\gamma \in \Gamma} \sigma([\beta, x\gamma p])$$

Since

$$\inf_{\gamma \in \Gamma} \sigma([\beta, x\gamma p]) \geq \inf_{\beta \in \Gamma} \inf_{\gamma \in \Gamma} \sigma([\beta, x\gamma p])$$

Thus we have $< [\beta, y], \sigma >^* (p) \geq \ < x, \sigma^* > (p)$. Consequently, $< [\beta, y], \sigma >^* \supseteq < x, \sigma^* >$. ■

For the sake of convenience, from [1] we recall here that there are four functions namely $()^+$, $()^{+'}$, $()^*$, $()^{*'}$ for a Γ-semigroup S and its left and right operator semigroups L, R respectively. They are defined as follows: For $I \subseteq R, I^* = \{s \in S, [\alpha, s] \in I \forall \alpha \in \Gamma\}$; for $P \subseteq S, P^{*'} = \{[\alpha, x] \in R : s\alpha x \in P \forall s \in S\}$; for $J \subseteq L, J^+ = \{s \in S, [s, \alpha] \in J \forall \alpha \in \Gamma\}$; for $Q \subseteq S, Q^{+'} = \{[x, \alpha] \in L : x\alpha s \in Q \forall s \in S\}$.

Lemma 3.5 *Let I be an ideal, μ a fuzzy ideal of a Γ-semigroup S and R the right operator semigroup of S. Then $(\mu_I)^{*'} = \mu_{I^{*'}}$ where μ_I is the characteristic function of I.*

Proof. Let $[\beta, y] \in R$. Then $(\mu_I)^{*'}([\beta, y]) = \inf_{s \in S} \mu(s\beta y)$. Suppose $[\beta, y] \in I^{*'}$. Then $s\beta y \in I$ for all $s \in S$. Hence $\mu_I(s\beta y) = 1$ for all $s \in S$. This gives $\inf_{s \in S} \mu(s\beta y) = 1$ whence $(\mu_I)^{*'}([\beta, y]) = 1$. Also $\mu_{I^{*'}}([\beta, y]) = 1$. Hence $(\mu_I)^{*'}([\beta, y]) = \mu_{I^{*'}}([\beta, y])$. Suppose $[\beta, y] \notin I^{*'}$. Then for some $t \in S, t\beta y \notin I$. So $\mu_I(t\beta y) = 0$. This gives $\inf_{s \in S} \mu_I(s\beta y) = 0$ i.e., $(\mu_I)^{*'}([\beta, y]) = 0$. Again $\mu_{I^{*'}}([\beta, y]) = 0$. Thus in this case also $(\mu_I)^{*'}([\beta, y]) = \mu_{I^{*'}}([\beta, y])$. Hence $(\mu_I)^{*'} = \mu_{I^{*'}}$. ■

Lemma 3.6 *Let $\{A_\alpha\}_{\alpha \in \Lambda}$ be a family of ideals of a Γ-semigroup S. Then $(\bigcap_{\alpha \in \Lambda} A_\alpha)^{*'} = \bigcap_{\alpha \in \Lambda} A_\alpha^{*'}$.*

Proof. Let $[\alpha, x] \in (\bigcap_{\alpha \in \Lambda} A_\alpha)^{*'}$. Then $s\alpha x \in \bigcap_{\alpha \in \Lambda} A_\alpha \ \forall s \in S$. Hence $\forall s \in S$, $s\alpha x \in A_\alpha \ \forall \alpha \in \Lambda$. This gives $[\alpha, x] \in \bigcap_{\alpha \in \Lambda} A_\alpha^{*'}$. Hence $(\bigcap_{\alpha \in \Lambda} A_\alpha)^{*'} \subseteq \bigcap_{\alpha \in \Lambda} A_\alpha^{*'}$. Reversing the above argument we deduce that $\bigcap_{\alpha \in \Lambda} A_\alpha^{*'} \subseteq (\bigcap_{\alpha \in \Lambda} A_\alpha)^{*'}$. Hence $(\bigcap_{\alpha \in \Lambda} A_\alpha)^{*'} = \bigcap_{\alpha \in \Lambda} A_\alpha^{*'}$. ■

Lemma 3.7 *Let S be a Γ-semigroup, R its right operator semigroup and $\mu := inf\{\mu_i : i \in I\}$ a non-empty family of fuzzy subsets of S. Then $\mu^{*'} = inf\{\mu_{i^{*'}} : i \in I\}$.*

Proof. Let $[\alpha, x] \in R$. Then

$$\mu^{*'}[\alpha, x] = (\inf\{\mu_i : i \in I\})^{*'}[\alpha, x]$$

$$= \inf_{s\in S}(\inf\{\mu_i : i \in I\}(s\alpha x))$$
$$= \inf_{s\in S}\inf_{i\in I}\mu_i(s\alpha x)$$

Now

$$\inf\{\mu_i^{*'} : i \in I\}([\alpha, x]) = \inf_{i\in I}(\mu_i^{*'}[\alpha, x])$$
$$= \inf_{i\in I}\inf_{s\in S}\mu_i(s\alpha x) = \inf_{s\in S}\inf_{i\in I}\mu_i(s\alpha x)$$

Hence the lemma. ■

Now by using the above lemmas together with results obtained in [5], [6] on correspondence between the fuzzy prime(semiprime) ideals of S and that of the operator semigroups and theorem 5.14, 5.13[7] we deduce the following results on fuzzy ideal extension of Γ-semigroups

Proposition 3.8 *Let S be a commutative* Γ*-semigroup with unities and μ a fuzzy ideal of S. Then $< x, \mu >$ is a fuzzy ideal of S for all $x \in S$.*

Proof. Let R be the right operator semigroup of S. Since S is commutative, its right operator semigroup R is also commutative. Now $\mu^{*'}$ is a fuzzy ideal of R(cf. Proposition 5.10[7]). Let $x \in S$. Then for any $\alpha \in \Gamma$, $< [\alpha, x], \mu^{*'} >$ is a fuzzy ideal of R(cf. Proposition 3.2[10])and hence $\inf_{\alpha\in\Gamma} < [\alpha, x], \mu^{*'} >$ is a fuzzy ideal of R. This together with Lemma 3.3 implies that $< x, \mu >^{*'}$ is a fuzzy ideal of R. Consequently, $(< x, \mu >^{*'})^*$ is a fuzzy ideal of S (cf. Proposition 5.9[7]),i.e., $< x, \mu >$ is a fuzzy ideal of S(cf. Theorem 5.13, 5.14[7]). ■

Now by applying Proposition 5.3[3], Corollary 3.9[8] and Lemma 3.3 we deduce the following proposition.

Proposition 3.9 *Let S be a commutative* Γ*-semigroup. If μ is a fuzzy semiprime ideal of S, then $< x, \mu >$ is a fuzzy semiprime ideal of S for every $x \in S$.*

Theorem 3.10 *Let S be a commutative* Γ*-semigroup, $\{\mu_i\}_{i\in I}$ be a non-empty family of fuzzy semiprime ideals of S and let $\mu = \inf\{\mu_i : i \in I\}$. Then for any $x \in S$, $< x, \mu >$ is a fuzzy semiprime ideal of S.*

Proof. Let R be the right operator semigroup of S. Since S is commutative, its right operator semigroup R is commutative. Now $\{\mu_i^{*'}\}_{i\in I}$ is a non-empty family of fuzzy semiprime ideals of R[6]. Hence $\inf\{\mu_i^{*'}\}_{i\in I}$ is a fuzzy semiprime ideal of R. Again by Lemma 3.7, $\inf\{\mu_i^{*'}\}_{i\in I} = \mu^{*'}$. Thus we see that $\mu^{*'}$ is a fuzzy semiprime ideal of R. This means that for any $[\alpha, x] \in R$, $< [\alpha, x], \mu^{*'} >$ and hence $\inf\limits_{\alpha\in\Gamma} < [\alpha, x], \mu^{*'} >$ is a fuzzy semiprime ideal of R. This together with the Lemma 3.3 implies that $< x, \mu >^{*'}$ is a fuzzy semiprime ideal of R. Hence $(< x, \mu >^{*'})^*$[6] i.e., $< x, \mu >$ ($cf.$ Theorem 5.14[7]) is a fuzzy semiprime ideal of S. ■

Theorem 3.11 *Let S be a commutative Γ-semigroup, $\{S_i\}_{i\in I}$ a non-empty family of semiprime ideals of S and $A := \cap_{i\in I} S_i \neq \phi$. Then $< x, \mu_A >$ is a fuzzy semiprime ideal of S for all $x \in S$ where μ_A is the characteristic function of A.*

Proof. Since $\forall i \in I$, S_i is a semiprime ideal of S, $S_i^{*'}$ is a semiprime ideal of the right operator semigroup R of $S, \forall i \in I$[6]. Now $A := \cap_{i\in I} S_i$. $A^{*'} = (\cap_{i\in I} S_i)^{*'} = \cap_{i\in I} S_i^{*'}$ ($cf.$ Lemma 3.6) $\neq \phi$. So by Corollary 3.11[10], $< [\alpha, x], \mu_{A^{*'}} >$ is a fuzzy semiprime ideal of $R, \forall \alpha \in \Gamma$. Hence $\inf\limits_{\alpha\in\Gamma} < [\alpha, x], \mu_{A^{*'}} >$ is a fuzzy semiprime ideal of R. This together with Lemma 3.5 implies that $\inf\limits_{\alpha\in\Gamma}$ $< [\alpha, x], (\mu_A)^{*'} >$, i.e.,$< x, \mu_A >^{*'}$ ($cf.$ Lemma 3.3(ii)) is a fuzzy semiprime ideal of R. Hence $(< x, \mu_A >^{*'})^*$ is a fuzzy semiprime ideal of S[6]. Consequently, by Theorem 5.14[7], $< x, \mu_A >$ is a fuzzy semiprime ideal of S. ■

Theorem 3.12 *Let S be a Γ-semigroup and I be an ideal of S. Then I is prime if and only if for $x \in S$ with $x \notin I, < x, \mu_I >= \mu_I$, where μ_I is the characteristic function of I.*

Proof. Let I be a prime ideal of S and $x \notin I$. Then by dual of Lemma 3.12[2], $I^{*'}$ is a prime ideal of the right operator semigroup R of S[1]. Also $[\alpha, x] \notin I^{*'}$ for some $\alpha \in \Gamma$. Then $< [\alpha, x], \mu_{I^{*'}} >= \mu_{I^{*'}}$($cf.$ Corollary 3.6[10]). Now $(\mu_I)^{*'} = \mu_{I^{*'}}$ ($cf.$ Lemma 3.5). It follows that $< [\alpha, x], \mu_{I^{*'}} >= (\mu_I)^{*'}$. Hence $< [\alpha, x], \mu_{I^{*'}} >^* = ((\mu_I)^{*'})^* = \mu_I$($cf.$ Theorem 5.13[7]). Now by Lemma 3.4, $< x, (\mu_{I^{*'}})^* > \subseteq < [\alpha, x], \mu_{I^{*'}} >^*$. Hence $< x, (\mu_{I^{*'}})^* > \subseteq \mu_I$. This together with Lemma 3.5 implies that $< x, ((\mu_I)^{*'})^* > \subseteq \mu_I$. Hence by applying Theorem 5.14[7] we deduce that $< x, \mu_I > \subseteq \mu_I$. Also $\mu_I \subseteq < x, \mu_I >$ ($cf.$ Proposition 3.7[5]). Hence $< x, \mu_I >= \mu_I$.

Conversely, suppose $< z, \mu_I >= \mu_I$ for all z in S with $z \notin I$. Let $x\Gamma y \subseteq I$ where $x, y \in S$. Then $\mu_I(x\gamma y) = 1\ \forall \gamma \in \Gamma$. Let $x \notin I$. Then by hypothesis $< x, \mu_I >= \mu_I$. This gives $< x, \mu_I > (y) = \mu_I(y)$,i.e.,$\inf\limits_{\gamma\in\Gamma} \mu_I(x\gamma y) = \mu_I(y)$. Hence $\mu_I(y) = 1$ whence $y \in I$. Consequently I is a prime ideal of S($cf.$ Theorem 3.12). ■

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