

Fuzzy logic applications

Gordon Hayward and Valerie Davidson

School of Engineering, University of Guelph, Guelph, Ontario, CANADA N1G 2W1

Fuzzy logic is a modeling method well suited for the control of complex and non-linear systems. This paper illustrates some of the power of fuzzy logic through a simple control example. For the analytical chemist, fuzzy logic incorporates imprecision from measurement noise as well as from linguistic process descriptions to produce operational control systems.

Introduction

The analytical chemist is often asked to develop measurement methods to provide information required for the control of complex systems ranging from large scale chemical processes to medical diagnoses. Although the behavior of complex or non-linear systems is difficult or impossible to describe using numerical models, quantitative observations are often required to make quantitative control decisions. These decisions could be the determination of a flow rate for a chemical process or a drug dosage in medical practice. In the development of appropriate methodology, the analytical chemist must address the issue of precision. This requires the understanding of the form of the control model as well as measurement noise.

In any system, measurement noise is a significant source of imprecision. Since noise has a mean of zero, increasing the number of readings and averaging them decreases the influence of the random component. With sufficient replicates the result approaches a 'good' measurement. Statistical methods can be used to determine the number of replicates required to achieve an acceptable level of precision.

The form of the control model also determines the appropriate level of precision in the result obtained. Numerical models provide high precision, but the complexity or non-linearity of a process may make a numerical model unfeasible. In these cases, linguistic models provide an alternative. Here the process is described in common language. In human activities such as economics, this has served well but the automation of a process based on linguistic models is difficult. This is, in part, due to the vagueness associated with the words used. Each user of a descriptive word has a somewhat different definition of the value carried by that word. To use a common example, "hot", when used to describe the temperature of a room, conveys very

different thermometer readings to someone from a nordic climate than to someone from the tropics. This lack of precision is not random so vagueness can not be treated by statistical means.

Fuzzy logic was first presented by Lotfi Zadeh in 1965,¹ to formalize a mathematical approach to deal with these complex or ill defined systems² and is, therefore, a relatively new mathematical paradigm. Its basis is a consideration of overlapping sets and the definition of operators to manipulate these sets. The linguistic model is built from a set of IF-THEN rules which describe the control model. Although Zadeh was attempting to model human activities, Mamdani³ showed that fuzzy logic could be used to develop operational automatic control systems.

The mechanics of fuzzy logic

The mechanics of fuzzy mathematics involve the manipulation of fuzzy variables through a set of linguistic equations which can take the form of IF-THEN rules. Much of the fuzzy literature uses set theory notation which obscures the ease of the formulation of a fuzzy controller. Although the controllers are simple to construct, the proof of stability and other validations remain important topics. The outline of fuzzy

operations will be shown here through the design of a familiar room thermostat.⁴

A fuzzy variable is one of the parameters of a fuzzy model which can take one or more fuzzy values, each represented by a fuzzy set and a word descriptor. The room temperature is the variable shown in Fig. 1. Three fuzzy sets: 'hot', 'cold' and 'comfortable' have been defined by membership distributions over a range of actual temperatures.

The power of a fuzzy model is the overlap between the fuzzy values. A single temperature value at an instant in time can be a member of both of the overlapping sets. In conventional set theory, an object (in this case a temperature value) is either a member of a set or it is not a member. This implies a crisp boundary between the sets. In fuzzy logic, the boundaries between sets are blurred. In the overlap region, an object can be a partial member of each of the overlapping sets. The blurred set boundaries give fuzzy logic its name. By admitting multiple possibilities in the model, the linguistic imprecision is taken into account.

The membership functions defining the three fuzzy sets shown in Fig. 1 are triangular. There are no constraints on the specification of the form of the membership distribution. The Gaussian form from statistics has been used,⁵ but

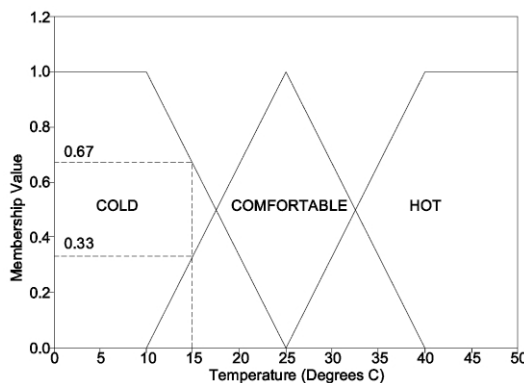


Fig. 1 Room temperature

the triangular form is commonly chosen as its computation is simple. The number of values and the range of actual values covered by each one is also arbitrary. Finer resolution is possible with additional sets, but the computation cost increases.

Guidance for these choices is provided by Zadeh's "Principle of Incompatibility":⁶

"As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics."

The operation of a fuzzy controller proceeds in three steps. The first is fuzzification, where measurements are converted into memberships in the fuzzy sets. The second step is the application of the linguistic model, usually in the form of IF-THEN rules. Finally the resulting fuzzy output is converted back into physical values through a defuzzification process.

Fuzzification. For a single measured value, the fuzzification process is simple, as shown in Fig. 1. The membership functions are used to calculate the memberships in all of the fuzzy sets. Thus, a temperature of 15 °C becomes three fuzzy values, 0.66 'cold', 0.33 'comfortable' and 0.00 'hot'.

Foulloy and Galichet⁷ present the use of fuzzy inputs rather than single measured values. Here the input was a distribution which incorporates the measured value and its confidence interval. A different approach would be to collect a series of measurements as a histogram and use this as the fuzzy input as shown in Fig. 2. In either case, the fuzzy inference is extended to include the uncertainty due to measurement error as well as the vagueness in the linguistic descriptions.

In Fig. 2 the measurement data histogram as been normalized so that its peak is a membership value of 1.0 and it can be used as a fuzzy set. The membership of the histogram in 'cold' is

given by:¹

$$\text{Max}\{\text{Min}[\mu_{\text{COLD}}(T), \mu_{\text{HISTOGRAM}}(T)]\}$$

where the maximum and minimum operations are taken using the membership values at each point T over the temperature range of the two distributions. The minimum operation yields the overlap region of the two sets and the maximum operation gives the highest membership in the overlap. The membership of the histogram in 'cold', indicated by the arrow in Fig. 2, is 0.73. By similar operations, the membership of the histogram in 'comfortable' and 'hot' are 0.40 and 0.00. It is interesting to note that there is no requirement that the sum of all memberships be 1.00.

Rule application. The linguistic model of a process is commonly made of a series of IF-THEN rules. These use the measured state of the process, the rule antecedents, to estimate the extent of control action, the rule consequents. Although each rule is simple, there must be a rule to cover every possible combination of fuzzy input values. Thus, the simplicity of the rules trades off against the number of rules. For complex systems the number of rules required may be very large.

The rules needed to describe a process are often obtained through consultation with workers who have expert knowledge of the process operation. These experts include the process designers, but more importantly, the process operators. The rules can include both the normal operation of the process as well as the experience obtained through upsets and other abnormal conditions. Exception handling is a particular strength of fuzzy control systems.

For very complex systems, the experts may not be able to identify their thought processes in sufficient detail for rule creation. Rules may also be generated from operating data by searching for clusters in the input data space. Chen and Liu⁸ used a principal component analysis to develop IF-THEN rules for the fault

analysis of a chemical reactor system. The result identified faults from simulated data with very few errors.

A simple temperature control model can be constructed from the example of Fig. 1;⁴

Rule 1: IF (Temperature is Cold) THEN (Heater is On)

Rule 2: IF (Temperature is Comfortable) THEN (Heater is Off)

Rule 3: IF (Temperature is Hot) THEN (Heater is Off)

In Rule 1, (Temperature is Cold) is the membership value of the actual temperature in the 'cold' set. Rule 1 transfers the 0.66 membership in 'cold' to become 0.66 membership in the heater setting 'on'. Similar values from rules 2 and 3 are 0.33 and 0.00 in the 'off' setting for the heater. When several rules give membership values for the same output set, Mamdani³ used the maximum of the membership values. The result for the three rules is then 0.66 membership in 'on' and 0.33 membership in 'off'.

The rules presented in the above example are simple yet effective. To extend these to more complex control models, compound rules may be formulated. For example, if humidity was to be included in the room temperature control example, rules of the form: IF (Temperature is Cold) AND (Humidity is High) THEN (Heater is ON) might be used. Zadeh¹ defined the logical operators as AND = $\text{Min}(\mu_A, \mu_B)$ and OR = $\text{Max}(\mu_A, \mu_B)$, where μ_A and μ_B are membership values in sets A and B respectively. In the above rule, the membership in 'on' will be the minimum of the two antecedent membership values. Zadeh also defined the NOT operator by assuming that complete membership in the set A is given by $\mu_A = 1$. The membership in NOT(A) is then given by $\mu_{\text{NOT}(A)} = 1 - \mu_A$. This gives the interesting result that A AND NOT(A) does not vanish, but gives a distribution corresponding to the overlap between A and its adjacent sets.

Defuzzification. The results of rule application are membership values in each of the consequent or output sets. These can be used directly where the membership values are viewed as the strength of the recommendations provided by the rules. It is possible that several outputs are recommended and some may be contradictory (e.g. heater on and heater off). In automatic control, one physical value of a controller output must be chosen from multiple recommendations. In decision support systems, there must be a consistent method to resolve conflict and define an appropriate compromise. Defuzzification is the process for

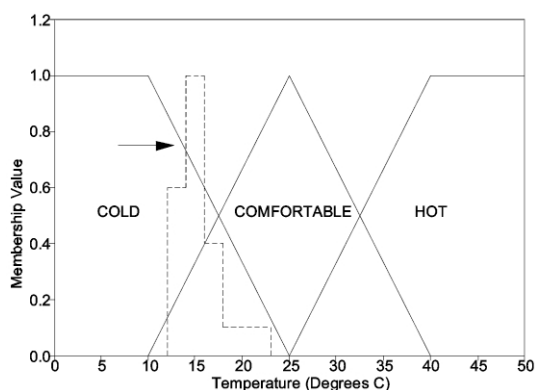


Fig. 2 Fuzzification with measurement noise.

converting fuzzy output values to a single value or final decision. Two methods are commonly used.

The first is the maximum membership method. All of the output membership functions are combined using the OR operator and the position of the highest membership value in the range of the output variable is used as the controller output. This method fails when there are two or more equal maximum membership values for different recommendations. Here the method becomes indecisive and does not produce a satisfactory result.

The second method uses the center of gravity of the combined output distribution to resolve this potential conflict and to consider all recommendations based on the strengths of their membership values. The center of gravity is given by: $X_F = \frac{\int x\mu(x)dx}{\int \mu(x)dx}$ where x is a point in the output range and X_F is the final control value. These integrals are taken over the entire range of the output. By taking the center of gravity, conflicting rules essentially cancel and a fair weighting is obtained.

The output values used in the thermostat example are singletons.⁴ Singletons are fuzzy values with a membership of 1.00 at a single value rather than a membership function between 0 and 1 defined over an interval of values. In the example there were two, 'off' at 0% power and 'on' at 100% power. With singletons, the center of gravity equation integrals become a simple weighted average. Applying the rules gave $\mu_{ON} = 0.67$ and $\mu_{OFF} = 0.33$. Defuzzifying these gives a control output of 67% power. Although only two singleton output functions were used, with center of gravity defuzzification, the heater power decreases smoothly between fully on and fully off as the temperature increases between 10 °C and 25 °C.

In the histogram input case, applying the same rules gave $\mu_{ON} = 0.73$ and $\mu_{OFF} = 0.40$. Center of gravity defuzzification gave, in this case, a heater power of 65%. The sum of the membership functions was normalized by the denominator of the center of gravity calculation.

Applications of fuzzy logic

In the following section, three examples of fuzzy control are considered. The first is the control of a complex system. The second is a non-linear control system and the third is a medical diagnosis and treatment support system.

Almardy⁹ used a fuzzy control system to apply current to a series of anodes to protect a long buried pipeline. The goal was to maintain protection, but at the same time to minimize the power used to protect the pipeline. The disturbances were

localized rain events which increased the soil conductivity in small regions along the pipeline. Fuzzy control was used as modeling the widely variable soil conditions along a pipeline would be too complex for a practical control system.

The fuzzy model to control an experimental pipeline with three anodes consisted of 126 rules. The results from simulation trials and experimental data agreed well and the controller gave adequate performance in maintaining protection. The controller gain, which determines the control system stability, was tuned by adjusting the output membership functions.

The pH control of flowing waste water is a non-linear system. In addition to the non-linearity of the titration curve, other components of the waste change the buffering capacity of the waste water. The controller described by Adroer *et al.*¹⁰ used the fuzzy error, the difference between the desired and actual pH, and the fuzzy change in the error to calculate the change in neutralization flow required to control the pH. As the pH approaches the neutralization point the controller gain must be changed. They used a separate tuning element which calculated a multiplier applied to the fuzzy output. An alternate scheme would be to use variable width membership functions to increase the control action as the error increases. The tuning element scheme implemented by Adroer *et al.*¹⁰ was found to provide acceptable pH control with a small mixer with a short residence time.

Warren *et al.*¹¹ present a decision support system for automating the application of clinical practice guidelines based on fuzzy methods. These guidelines are not algorithms but rather composite scenarios as the case might present to a physician. This presents several difficulties in addition to the inherent imprecision of linguistic description. The results of many medical tests lack both selectivity and sensitivity. The tests, therefore, yield likelihood estimates rather than a confirmation of the presence or absence of disease. The recommendations for action suggested in the guidelines are particularly vague and sometimes conflicting.

These difficulties are handled well by fuzzy methods which implicitly incorporate the vagueness of the guidelines. Likelihood estimates can be handled as membership values and used as such in the fuzzy inference model. The conflicting recommendations are also handled well as the output represents a weighted average which lies between the extremes. If the result is not continuous but a binary yes or no type decision, each will have a membership value which may

be reported to indicate the strength of the decision. The automated decision will then carry with it information on the strength of the decisions to further guide the physician.

Conclusions

Fuzzy logic is a formal mathematical approach for modeling complex systems that has been used successfully in many control systems. Linguistic descriptions in the form of membership functions and rules make up the model. The rules are generated *a priori* from expert knowledge or from data through system identification methods. Input membership functions are based on estimates of the vagueness of the descriptors used. Output membership functions can be initially set, but can be revised for controller tuning.

Once these are defined, the operating procedures for the calculations are well set out. Measurement data are converted to memberships through fuzzification procedures. The rules are applied using formalized operations to yield memberships in output sets. Finally, these are combined through defuzzification to give a final control output.

Fuzzy values account for the imprecision from the use of linguistic terms. In analytical chemistry applications, the use of fuzzy logic includes the imprecision of a measurement method in the calculation of a final output. This ability can make the use of a noisy measurement system feasible.

References

- 1 L. A. Zadeh, *Inf. Control*, 1965, **8**, 338.
- 2 L. A. Zadeh, NASA CR-1432, 1969.
- 3 E. H. Mamdani, *Proc. IEE*, 1974, **121**, 1585.
- 4 W. Banks and G. Hayward, *Fuzzy Logic in Embedded Microcomputers and Control Systems*, Byte Craft Limited, Waterloo, Ontario, 1993.
- 5 M. Sugeno, *Inf. Sci.*, 1985, **36**, 59.
- 6 L. A. Zadeh, *IEEE Transactions on Systems, Man and Cybernetics*, 1973, **3**, 28.
- 7 L. Foulloy and S. Galichet, *IEEE Transactions on Fuzzy Systems*, 2003, **11**, 437.
- 8 J. Chen and J. Liu, *Ind. Eng. Chem. Res.*, 2000, **39**, 235.
- 9 M. S. Almardy, *Design of Fuzzy Logic Controller for the Cathodic Protection of Underground Pipelines*, MSc Thesis, University of Guelph, 1999.
- 10 M. Adroer, A. Alsina, J. Aumatell and M. Poch, *Ind. Eng. Chem. Res.*, 1999, **38**, 2709.
- 11 J. Warren, G. Beliakov and B. van der Zwaag, Proc. 33rd Hawaii International Conference on System Sciences, IEEE, 2000, p. 1578.