Fuzzy LP with a Non-Linear MF for Product-Mix solution: A Case-Based Re-modelling and Solution

Sani Susanto[1] Pandian Vasant[2] Arijit Bhattacharya[3] Fransiscus Rian Pratikto[4]

[1]Senior Lecturer, Department of Industrial Engineering, Faculty of Industrial Technology, Parahyangan Catholic University, Jln. Ciumbuleuit 94, Bandung – 40141, Indonesia. E-mail: sjrh@bdg.centrin.net.id
[2]Research Lecturer, Electrical & Electronic Engineering Program, Universiti Teknologi Petronas, 31750 Tronoh, BSI, Perak DR, Malaysia. E-mail: pvasant@gmail.com
[3]Examiner of Patents & Designs, The Patent Office, Bouddhik Sampada Bhawan, CP – 2, Sector V, Salt Lake, Kolkata – 700 091, West Bengal, India. E-mail: arijit.bhattacharya2005@gmail.com
[4]Lecturer, Department of Industrial Engineering, Faculty of Industrial Technology, Parahyangan Catholic University, Jln. Ciumbuleuit 94, Bandung – 40141, Indonesia. E-mail: frianp@yahoo.com

Abstract

This paper deals with re-modelling of a fuzzy linear programming (FLP) for an optimal product-mix decision problem and its solution. Database of a chocolate exporting company has been used here to show the practicability of using the proposed model. The proposed model includes a non-linear membership function (MF), a logistic function, which resemblances the realistic behaviour of the solution. A software platform LINGO® has been utilized to find the optimal solution.

Keywords: Product-mix, Optimization, Fuzzy sets, S-curve MF, Fuzzy LP.

1. Introduction

The theory of fuzzy linear programming (FLP) was developed to tackle imprecise or vague problems using the fundamental concept of artificial intelligence. Solutions to such fuzzy decision-making problems include research works of Bellman and Zadeh [1], Tanaka *et al.* [14, 15], Negoita and Sularia [9], Negoita and Ralescu [10], Negoita [11], Freeling [6], Ross [12], Klir and Yuan [8], Yager *et al.* [20], Zimmermann [24], Chen and Chou [4] and Dubois and Prade [5].

Buckley *et al.* [3] solved multi-objective fully fuzzified LP problems. Triangular fuzzy numbers were used in their solution.

An attempt was made by Vasant [17], Vasant and Barsoum [19] to deal with the product-mix problem of the firm Chocoman Inc [13]. The said work was simulated in MATLAB® platform using *S*-curve MF. The present work is different from the prior works in a sense that it incorporates the non-linear logistic MF in the constraints of the LP model. In this work the LP model has been re-modelled in a fashion so as to get a synergistic effect in the optimized solution.

2. The case study

In this section we set out a non-linear fuzzy optimization problem as a case study that describes a possible situation in a chocolate exporting company.

The data for this problem have been adopted from the databank of Chocoman Inc, USA [13]. Chocoman produces varieties of chocolate bars, candy and wafer using a number of raw materials and processes. There are ‘n’ number of products to be manufactured by mixing ‘m’ number of raw materials having different proportion and by using ‘k’ number of different kind of processing techniques. Limitations in resources of raw materials exist. There are also some constraints imposed by marketing department such as product-mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. The objective is to formulate the linear programming model using a fuzzy *S*-curve MF in order to obtain optimal unit of products.

The firm Chocoman, Inc. manufactures 8 different kinds of chocolate products. There are 8 raw materials to be mixed in different proportions and 9 processes (facilities) to be utilized. The product demand, discount, profit, revenue/sales and objective coefficients are illustrated in Table 1. Table 2 depicts required materials & facility usage, and availability of the raw materials for manufacturing each of the products.

The following constraints were established by the sales department of Chocoman, Inc.:

(i) **Product-mix requirements**: Large-sized products (250g) of each type should not exceed 60% (non fuzzy value) of the small-sized product (100 g), such that:

$x_1 \le 0.6\, x_2$ … (1)
$x_3 \le 0.6\, x_4$ … (2)
$x_5 \le 0.6\, x_6$ … (3)

(ii) **Main product line requirement**: The total sales from candy and wafer products should not exceed 15% (non-fuzzy value) of the total revenues of the chocolate bar products, such that:

$400x_7 + 150x_8 \le 0.15\,(375x_1 + 150x_2 + 400x_3 + 160x_4 + 420x_5 + 175x_6)$ … (4)

2.1. Re-modelling the Problem

The problem of Tabucanon [13] has been re-modelled in this paper. The linear programming formulation adopts fuzzification using a non-linear membership function (MF). This MF is the logistic function described by Goguen [7] and Zadeh [21,22,23].

$$f(x) = \frac{B}{1 + Ce^{\gamma x}}$$

B and *C* are scalar constants and γ, $0 < \gamma < \propto$ is a fuzzy parameter for measuring degree of imprecision.

The logistic MF is modified and redesigned in the following fashion so as to fit into the LP model. This MF behaves like a *S*-curve.

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\gamma x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases}$$

For further details on this modified MF as well as the logistics MF readers are referred to Vasant *et al.* [16,18], Vasant and Barsoum [19] and Bhattacharya and Vasant [2].

The following FLP is constructed using the modified *S*-curve MF:

Maximize $\sum_{i=1}^{8} (c_i x_i - d_i x_i^2)$

Subject to:

$$\sum_{i=1}^{8} \left[a_{ij}^{l} + \left(\frac{a_{ij}^{h} - a_{ij}^{l}}{\gamma}\right) ln \frac{1}{C}\left(\frac{B}{\mu} - 1\right) \right] x_i - b_j \le 0,\ j = 1, \ldots .17$$

$$\sum_{k=7}^{8} r_k x_k - 0.15 \sum_{k=1}^{6} r_k x_k \le 0$$

$$x_1 - 0.6x_2 \le 0$$

$$x_3 - 0.6x_4 \le 0$$

$$x_5 - 0.6x_6 \le 0$$

$$0 \le x_i \le u_i, \quad i = 1, \ \ldots\ldots ,8$$

2.2. Results

Table 3: Products at disparate μ values and impact of FLP computation on cost

μ	Variable	Value	Reduced Cost
0.001	x_1	239.7161	0.000000
	x_2	399.5268	0.000000
	x_3	198.9859	0.6470742E-07
	x_4	331.6432	0.000000
	x_5	141.1270	0.000000
	x_6	235.2116	0.000000
	x_7	139.2046	0.000000
	x_8	11.70292	0.000000
0.1	x_1	279.4339	0.000000
	x_2	465.7232	0.000000
	x_3	234.7411	0.000000
	x_4	391.2352	0.000000
	x_5	155.3713	0.1630958E-05
	x_6	258.9522	0.000000
	x_7	159.3681	0.000000
	x_8	16.73137	0.000000
0.5	x_1	302.5503	0.000000
	x_2	504.2506	0.000000
	x_3	255.6712	0.000000
	x_4	426.1187	0.000000
	x_5	163.7803	0.000000
	x_6	272.9672	0.000000
	x_7	171.2133	0.000000
	x_8	19.52965	0.2514186E-04
0.9	x_1	329.6295	0.000000
	x_2	549.3824	0.000000
	x_3	280.2814	0.000000
	x_4	467.1356	0.000000
	x_5	173.7323	0.000000
	x_6	289.5538	0.000000
	x_7	185.1788	0.000000
	x_8	22.70207	0.000000
0.999	x_1	414.3502	0.000000
	x_2	690.5837	0.000000
	x_3	354.0144	0.000000
	x_4	590.0239	0.000000
	x_5	200.0325	0.000000
	x_6	333.3874	0.000000
	x_7	200.0000	0.000000
	x_8	54.48504	0.000000

The values of γ, B, C are found after a rigorous algebraic computation. It was observed that $\gamma = 13.8135$, B = 1 and C = 0.001001001. The values of MF, i.e., μ, is referred as the degree of possibility and $0<\mu<1$. For step-wise calculation we adopt $\mu = 0.001$, 0.1 to 0.9 in a step of 0.1, and 0.999. Limitations of page restrict the authors to illustrate all the values of all variables at all μ. Variable values at $\mu = 0.001$, 0.1, 0.5, 0.9 and 0.999 are only shown in Table 3. The results illustrated on Tables 3 and 4 are found by LINGO® software platform. Table 4 depicts product-mix and f-values at disparate μ values.

3. Discussion and Conclusion

It is understood from Tables 3 and 4 that a decision-maker has many choices open in his/her hand. Both the Tables 3 and 4 illustrate sensitivity of the judgement of a decision-maker while making a product-mix decision of the chocolate manufacturing firm. Therefore, trading off the fuzziness values (γ) as well as the degree of possibility (μ) of the choices will make the DM to apply an ample judgement under this unstructured environment.

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Table 1: Profit (c_k), Discount (d_k), Demand (u_k) and Revenues/Sales (r_k) in US $ per 10^3 units

Product (x_k)	Synonym	Profit (c_k)	Discount (d_k)	Demand (u_k)	Revenues/Sales (r_k)
x_1 = Milk chocolate, 250g	MC 250	$c_1 = 180$	$d_1 = 0.18$	u1 = 500	r1 = 375
x_2 = Milk chocolate, 100g	MC 100	$c_2 = 83$	$d_2 = 0.05$	u2 = 800	r2 = 150
x_3 = Crunchy chocolate, 250g	CC 250	$c_3 = 153$	$d_3 = 0.15$	u3 = 400	r3 = 400
x_4 = Crunchy chocolate, 100g	CC 100	$c_4 = 72$	$d_4 = 0.06$	u4 = 600	r4 = 160
x_5 = Chocolate with nuts, 250g	CN 250	$c_5 = 130$	$d_5 = 0.13$	u5 = 300	r5 = 420
x_6 = Chocolate with nuts, 100g	CN 100	$c_6 = 70$	$d_6 = 0.14$	u6 = 500	r6 = 175
x_7 = Chocolate candy	CANDY	$c_7 = 208$	$d_7 = 0.21$	u7 = 200	r7 = 400
x_8 = Chocolate wafer	WAFER	$c_8 = 83$	$d_8 = 0.1$	u8 = 400	r8 = 150

Table 2: Raw material and Facility usage required (per 10^3 units) ($\tilde{a}_{ij} = [a^l_{ij}, a^h_{ij}]$) and Availability ($b_j$)

Material or Facility	MC 250	MC 100	CC 250	CC 100	CN 250	CN 100	Candy	Wafer	Availability
Cocoa (kg)	[66, 109]	[26, 44]	[56, 94]	[22, 37]	[37, 62]	[15, 25]	[45, 75]	[9, 21]	100000
Milk (kg)	[47, 78]	[19, 31]	[37, 62]	[15, 25]	[37, 62]	[15, 25]	[22, 37]	[9, 21]	120000
Nuts (kg)	[0, 0]	[0, 0]	[28, 47]	[11, 19]	[56, 94]	[22, 37]	[0, 0]	[0, 0]	60000
Cons. sugar (kg)	[75, 125]	[30, 50]	[66, 109]	[26, 44]	[56, 94]	[22, 37]	[157,262]	[18, 30]	200000
Flour (kg)	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[54, 90]	20000
Alum. foil (ft^2)	[375, 625]	[0, 0]	[375, 625]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[187, 312]	500000
Paper (ft^2)	[337, 562]	[0, 0]	[337, 563]	[0, 0]	[337, 562]	[0, 0]	[0, 0]	[0, 0]	500000
Plastic (ft^2)	[45, 75]	[95, 150]	[45, 75]	[90, 150]	[45, 75]	[90, 150]	[1200, 2000]	[187, 312]	500000
Cooking (ton-hours)	[0.4, 0.6]	[0.1, 0.2]	[0.3, 0.5]	[0.1, 0.2]	[0.3, 0.4]	[0.1, 0.2]	[0.4, 0.7]	[0.1,0.12]	1000
Mixing (ton-hours)	[0, 0]	[0, 0]	[0.1, 0.2]	[0.04, 0.07]	[0.2, 0.3]	[0.07, 0.12]	[0, 0]	[0, 0]	200
Forming (ton-hours)	[0.6, 0.9]	[0.2, 0.4]	[0.6, 0.9]	[0.2, 0.4]	[0.6, 0.9]	[0.2, 0.4]	[0.7, 1.1]	[0.3, 0.4]	1500
Grinding (ton-hours)	[0, 0]	[0, 0]	[0.2, 0.3]	[0.07, 0.12]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	200
Wafer making (ton-hours)	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.4]	100
Cutting (hours)	[0.07, 0.12]	[0.07, 0.12]	[0.07, 0.12]	[0.07, 0.12]	[0.07, 0.12]	[0.07, 0.12]	[0.15, 0.25]	[0, 0]	400
Packaging 1 (hours)	[0.2, 0.3]	[0, 0]	[0.2, 0.3]	[0, 0]	[0.2, 0.3]	[0, 0]	[0, 0]	[0, 0]	400
Packaging 2 (hours)	[0.04, 0.06]	[0.2, 0.4]	[0.04, 0.06]	[0.2, 0.4]	[0.04, 0.06]	[0.2, 0.4]	[1.9, 3.1]	[0.1, 0.2]	1200
Labour (hours)	[0.2, 0.4]	[0.2, 0.4]	[0.2, 0.4]	[0.2, 0.4]	[0.2, 0.4]	[0.2, 0.4]	[1.9, 3.1]	[1.9, 3.1]	1000

Table 4: Product-mix and f-values at disparate μ

μ	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	f
0.001	239.7161	399.5268	198.9859	331.6432	141.1270	235.2116	139.2046	11.70292	150089.2
0.1	279.4339	465.7232	234.7411	391.2352	155.3713	258.9522	159.3681	16.73137	165662.6
0.2	287.5612	479.2687	242.0906	403.4844	158.3183	263.8638	163.5240	17.72523	168585.9
0.3	293.2170	488.6950	247.2111	247.2111	160.3752	267.2920	166.4218	18.41035	170566.9
0.4	298.0141	496.6902	251.5579	419.2632	162.1237	270.2061	168.8830	18.98738	172212.8
0.5	302.5503	504.2506	255.6712	426.1187	163.7803	272.9672	171.2133	19.52965	173740.0
0.6	307.2207	512.0344	259.9091	433.1819	165.4891	275.8152	173.6153	20.08459	175282.7
0.7	312.4697	520.7829	264.6756	441.1260	167.4136	279.0226	176.3184	20.70434	176980.6
0.8	319.1108	531.8513	270.7111	451.1852	169.8541	283.0901	179.7432	21.48254	179074.1
0.9	329.6295	549.3824	280.2814	467.1356	173.7323	289.5538	185.1788	22.70207	182264.4
0.999	414.3502	690.5837	354.0144	590.0239	200.0325	333.3874	200.0000	54.48504	200116.4