

# Fuzzy Matrix Game: A Fast Approach using Artificial Hybrid Neural-Net Logic-Gate Switching Circuit

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## Research Article

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# Fuzzy matrix game: A fast approach using artificial hybrid neural-net logic-gate switching circuit

Ankan Bhaumik<sup>†</sup> · Sankar Kumar Roy<sup>\*</sup>

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**Abstract** Introducing neuro-fuzzy concept in decision making problems, makes a new way in artificial intelligence and expert systems. Sometimes, neural networks are used to optimize certain performances. In general, knowledge acquisition becomes difficult when problem's variables, constraints, environment, decision maker's attitude and complex behaviour are encountered with. A sense of fuzziness prevails in these situations; sometimes numerically and sometimes linguistically. Neural networks (or neural nets) help to overcome this problem. Neural networks are explicitly and implicitly hyped to draw out fuzzy rules from numerical information and linguistic information. Logic-gate and switching circuit mobilize the fuzzy data in crisp environment and can be used in artificial neural network, also. Game theory has a tremendous scope in decision making; and consequently decision makers' hesitant characters play an important role in it. In this paper, a game situation is clarified under artificial neural network through logic-gate switching circuit in hesitant fuzzy environment with a suitable example; and this concept can be applied in future for real-life situations.

**Keywords** Artificial neural networks · Logic gates · Fuzzy matrix games · Takagi-Sugeno model.

## 1 Introduction

Basic decision making issues related with game hypotheses have large-scale implementations in science, engineering, management science and sociology. Fuzzy set (FS) (Zadeh (1965)) gives assistance to a brilliant and wonderful role

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to investigate the inner situations of issues identified with regular day to day existence's. Although it has a few constraints to deal with imprecise information, hazy and murky data when various kinds of fuzziness, murkiness and unpredictability yield up at the same time. Analysts have nurtured the fuzzy sets and stretched out the FS to intuitionistic fuzzy set (IFS) (Atanassov (1986)), hesitant fuzzy set (HFS) (Torra (2010)), etc. HFS effectively actualized the problems' environments where FS, IFS flunk to depict issues about fuzziness, uncertainty. HFS portrays a group of values from  $[0, 1]$  instead of single one to every member of corresponding set. Artificial neural frameworks or artificial neural nets are physical cell frameworks generally able to acquire, store and use exploratory information towards knowledge. The learning is acquired in stable mappings inserted in network framework. Neurons or nodes are the basic unit or element of net. In brain-neuron system, i.e., in neural net systems, activity starts at networks' polarization, then the firing rate of neuron is considered through a set of input connections using synapses on cells and the corresponding dendrite; then neurons are given internal resting space and consequently neuron's axonal projections are done. In artificial neural nets, every processing element is marked by an activity level, an output cost or value, a group of input links, a bias cost, i.e., an artificial resting stage of corresponding neuron, and a bunch of output links. Each of these characteristics of the unit is expressed mathematically by means of real numbers. Thus, every connection possesses weight, may be positive or negative, i.e., synaptic influence which decides the impact of the approaching contribution on the enactment level of the unit. Weights determine excitatory or inhibitory initiation. Artificial neural network may be classified as the generalization of brain-style computational methods in mathematical sciences, mainly in applied sciences. McCulloch and Pitts (1943), Hebb (1949) originated the idea of brain-style computation. Minsky and Papert (1969) proposed artificial

intelligence as symbol processing and it became a dominant theme in artificial intelligence in 1970s.

Contemporarily, von Neumann and Morgenstern (1944) pioneered *Game Theory*. A game constitutes with players, their turns and returns. These turns are called strategies and returns, quantitatively are termed as payoffs. Players assume pure strategies randomly, and when with some probabilities, these strategies are defined to be mixed one. Matrix game and related duality hypothesis in linear programming problem (LPP) have a linkage in crisp form of intricacies of issues yet reality wants various dubious natures. Because of the vulnerability, imprecision attributes of frameworks in question, and the vagueness, ambiguity of adjudications of decision players, we understand inclusion of aversion, hesitation environments in game problems. Campos (1989) first illuminated fuzzy matrix game. Li (2013, 2014) solved matrix games with fuzzy payoffs. Bhaumik et al. (2017), Bhaumik and Roy (2021), Bhaumik et al. (2021), Jana and Roy (2019), Roy and Mula (2016), Roy and Jana (2021) understood matrix games and effectively applied them in real-life circumstances. Several articles (cf. Azam and Yao (2015), Collins and Hu (2008), Tang and Li (2020), Xia (2019)) have been published on fuzzy game theory.

Hirota and Pedrycz (1993) discussed on logic-based neural networks. Neural nets have been applied in fuzzy logic-system, soft-computing (cf. Buckley and Hayashi (1992), Buckley and Hayashi (1995), Jang (1993), Lin and Lee (1991)), function approximation (Wu and Er (2000)), fuzzy modeling (cf. Sugeno and Yasukawa (1993), Takagi and Sugeno (1985)), etc., but hybrid-neural net has not been applied in matrix game using logic-gate switching circuits. Also, matrix game under hesitant triangular intuitionistic fuzzy environment has not been discussed using neural net in literatures. These can be considered as the research gaps from the others in literatures. The main aims of this study are as follows:

- (i) To construct a game model using artificial neural nets.
- (ii) To apply switching circuit gates in neural nets.
- (iii) To compute a quick geometric way for defuzzification of a set of hesitant fuzzy elements.

The rest of the paper is set as: Preliminaries related to triangular intuitionistic fuzzy set, triangular norm and conorm, hesitant fuzzy set are talked about in short in Section 2. Section 3 describes classical matrix game shortly. In Section 4, we discuss on neural network model with two subsections; one for biological neural network and another for artificial neural network, briefly. In Section 5, logic-gate switching circuits are defined with the elementary gate operators; and in Section 6, we develop fuzzy logic-gate switching circuit oriented artificial neural network model. Matrix games with artificial neural network and logic-gate-switch are described in Section 7. A numerical problem is simulated and exercised in Section 8. Section 9 is confined under the consid-

eration of results and the corresponding discussion to the problem [having some comparative analyses with others](#) and Section 10 finishes up this work with extent of future research scopes.

## 2 Preliminaries

We introduce here the basic definitions and some properties on triangular intuitionistic fuzzy set, triangular norm and conorm, and hesitant fuzzy set.

Let  $X$  denotes a universe of discourse. A fuzzy set  $\check{A}$  in  $X$  is distinguished by a membership function  $\mu_{\check{A}} : X \rightarrow [0, 1]$ . A fuzzy set  $\check{A}$  in  $X$  can be demonstrated as follows:

$$\check{A} = \{(x, \mu_{\check{A}}(x)) : \mu_{\check{A}}(x) \in [0, 1], x \in X\}.$$

Membership degrees  $\mu_{\check{A}}(x)$  of  $\check{A}$  are crisp numbers.

**Definition 21 Triangular IFN (Li (2013)):** An IF number  $\hat{A} = \langle (\underline{\varphi}, \varphi, \overline{\varphi}); \varepsilon_{\hat{A}}, \rho_{\hat{A}} \rangle$  having membership & non membership degrees of an element  $x = \varphi$  in  $\hat{A}$ , being  $\varepsilon_{\hat{A}}$  and  $\rho_{\hat{A}}$ , is said to be triangular IFN if its membership and non-membership functions respectively are defined as pursues:

$$\phi_{\hat{A}}(x) = \begin{cases} \varepsilon_{\hat{A}} \left( \frac{x-\underline{\varphi}}{\varphi-\underline{\varphi}} \right), & \text{if } \underline{\varphi} \leq x < \varphi, \\ \varepsilon_{\hat{A}}, & \text{if } x = \varphi, \\ \varepsilon_{\hat{A}} \left( \frac{\overline{\varphi}-x}{\overline{\varphi}-\varphi} \right), & \text{if } \varphi < x \leq \overline{\varphi}, \\ 0, & \text{if } x < \underline{\varphi} \text{ or } x > \overline{\varphi}, \end{cases} \quad (1)$$

and

$$\Phi_{\hat{A}}(x) = \begin{cases} \frac{\varphi-x+\rho_{\hat{A}}(x-\underline{\varphi})}{\varphi-\underline{\varphi}}, & \text{if } \underline{\varphi} \leq x < \varphi, \\ \rho_{\hat{A}}, & \text{if } x = \varphi, \\ \frac{x-\varphi+\rho_{\hat{A}}(\overline{\varphi}-x)}{\overline{\varphi}-\varphi}, & \text{if } \varphi < x \leq \overline{\varphi}, \\ 1, & \text{if } x < \underline{\varphi} \text{ or } x > \overline{\varphi}. \end{cases} \quad (2)$$

Here,  $\varepsilon_{\hat{A}}$  and  $\rho_{\hat{A}}$ , in Eqs. (1) and (2), satisfy the conditions:  $0 \leq \varepsilon_{\hat{A}} \leq 1$ ,  $0 \leq \rho_{\hat{A}} \leq 1$ ,  $0 \leq \varepsilon_{\hat{A}} + \rho_{\hat{A}} \leq 1$ . Also,  $\pi_{\hat{A}}(x) = 1 - \phi_{\hat{A}}(x) - \Phi_{\hat{A}}(x)$  is defined as intuitionistic fuzzy index of an element  $x \in \hat{A}$ .

### Arithmetic Operations on Triangular IFNs:

Let  $\hat{\varphi} = \langle (\underline{\varphi}, \varphi, \overline{\varphi}); \varepsilon_{\hat{\varphi}}, \rho_{\hat{\varphi}} \rangle$  and  $\hat{\vartheta} = \langle (\underline{\vartheta}, \vartheta, \overline{\vartheta}); \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\vartheta}} \rangle$  appear for two triangular IFNs, then the addition, subtraction, multiplication, division and scalar multiplication of the numbers are conveyed as below:

#### Addition:

$$\hat{\varphi} \oplus \hat{\vartheta} = \langle (\underline{\varphi} + \underline{\vartheta}, \varphi + \vartheta, \overline{\varphi} + \overline{\vartheta}); \varepsilon_{\hat{\varphi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}} \rangle. \quad (3)$$

#### Subtraction:

$$\hat{\varphi} \ominus \hat{\vartheta} = \langle (\underline{\varphi} - \underline{\vartheta}, \varphi - \vartheta, \overline{\varphi} - \overline{\vartheta}); \varepsilon_{\hat{\varphi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}} \rangle. \quad (4)$$

#### Multiplication:

$$\hat{\varphi} \otimes \hat{\vartheta} = \begin{cases} \langle (\underline{\varphi}\underline{\vartheta}, \varphi\vartheta, \overline{\varphi}\overline{\vartheta}); \varepsilon_{\hat{\varphi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\varphi} > 0, \hat{\vartheta} > 0, \\ \langle (\underline{\varphi}\underline{\vartheta}, \varphi\vartheta, \overline{\varphi}\underline{\vartheta}); \varepsilon_{\hat{\varphi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\varphi} < 0, \hat{\vartheta} > 0, \\ \langle (\overline{\varphi}\overline{\vartheta}, \varphi\vartheta, \underline{\varphi}\underline{\vartheta}); \varepsilon_{\hat{\varphi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\varphi} < 0, \hat{\vartheta} < 0. \end{cases} \quad (5)$$

**Division:**

$$\hat{\phi} \oslash \hat{\vartheta} = \begin{cases} \langle (\underline{\varphi}/\underline{\vartheta}, \varphi/\vartheta, \overline{\varphi}/\overline{\vartheta}); \varepsilon_{\hat{\phi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\phi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\phi} > 0, \hat{\vartheta} > 0, \\ \langle (\overline{\varphi}/\overline{\vartheta}, \varphi/\vartheta, \underline{\varphi}/\underline{\vartheta}); \varepsilon_{\hat{\phi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\phi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\phi} < 0, \hat{\vartheta} > 0, \\ \langle (\overline{\varphi}/\overline{\vartheta}, \varphi/\vartheta, \underline{\varphi}/\underline{\vartheta}); \varepsilon_{\hat{\phi}} \wedge \varepsilon_{\hat{\vartheta}}, \rho_{\hat{\phi}} \vee \rho_{\hat{\vartheta}} \rangle, & \text{if } \hat{\phi} < 0, \hat{\vartheta} < 0. \end{cases}$$

where “ $\wedge$ ” and “ $\vee$ ” individually denote min and max operators in fuzzy sense.

**Scalar Multiplication:** For any real number  $k$ ,

$$k\hat{\phi} = \begin{cases} \langle (k\underline{\varphi}, k\varphi, k\overline{\varphi}); \varepsilon_{\hat{\phi}}, \rho_{\hat{\phi}} \rangle, & \text{if } k \geq 0, \\ \langle (k\overline{\varphi}, k\varphi, k\underline{\varphi}); \varepsilon_{\hat{\phi}}, \rho_{\hat{\phi}} \rangle, & \text{if } k < 0. \end{cases} \quad (7)$$

Where the assumption of  $\hat{\phi} > 0$  or  $< 0$  are decided by the extension principle of fuzzy set (Li (2013)).

t-norm and t-conorm are two sorts of operations in fuzzy sets. They are otherwise called as triangular norm and triangular conorm, respectively.

**Definition 22 Triangular norm (Fuller (2000)):** A mapping  $T$  is a triangular norm such that,  $T : [0, 1] \times [0, 1] \rightarrow [0, 1], \forall x, y, x_1, y_1, z \in [0, 1]$ , with the following conditions as axioms:

2.2.1:  $T(x, 0) = 0, T(x, 1) = x$ ; *Boundary condition.*

2.2.2:  $T(x, y) = T(y, x)$ ; *Condition for symmetricity.*

2.2.3:  $T(x, T(y, z)) = T(T(x, y), z)$ ; *Condition for associativity.*

2.2.4:  $T(x, y) \leq T(x_1, y_1)$  if  $x \leq x_1, y \leq y_1$ ; *Condition for monotonicity.*

**Definition 23 Triangular co-norm (Fuller (2000)):** A mapping  $T$  is a triangular conorm such that,  $T : [0, 1] \times [0, 1] \rightarrow [0, 1], \forall x, y, x_1, y_1, z \in [0, 1]$ , with the following conditions as axioms:

2.3.1:  $T(x, 0) = x, T(x, 1) = 1$ ; *Boundary condition.*

2.3.2:  $T(x, y) = T(y, x)$ ; *Condition for symmetricity.*

2.3.3:  $T(x, T(y, z)) = T(T(x, y), z)$ ; *Condition for associativity.*

2.3.4:  $T(x, y) \leq T(x_1, y_1)$  if  $x \leq x_1, y \leq y_1$ ; *Condition for monotonicity.*

**Definition 24 Complement of fuzzy set:** Considering a fuzzy sentence  $p$ ; we describe its complement as some sentence fulfilling the uniformity:  $M^c(p) = W - M(p)$ , where  $M^c(p)$  means  $M(p)$ 's complementary set;  $W$  is the entire set of sentences;  $M$  is a membership function that partners  $p$  with the members of  $M(p)$ .

**Definition 25 Hesitant Fuzzy Set (Torra (2010)):** Based on reference set  $X$ ,  $A_{HF}$  is defined to be a hesitant fuzzy set described by the function  $h_A(x)$ . Here,  $h_A(x)$  is applied to  $X$  and gives a subset of  $[0, 1]$ , i.e.,  $A_{HF} = \{ \langle x, h_A(x) \rangle : x \in X \}$  where  $h_A(x)$  is named as hesitant fuzzy element (HFE), an essential unit of HFS, is a set fitted with various merits in  $[0, 1]$  represent the conceivable membership degrees to component  $x \in X$ .

**Example 21**  $A_{HF} = \{ \langle x_1, 0.1, 0.4, 0.3 \rangle, \langle x_2, 0.3, 0.35 \rangle, \langle x_3, 0.2, 0.4, 0.6, 0.69, 0.8 \rangle \}$  is a HFS;  $\{x_1, x_2, x_3\} \in X$ , a reference set and  $h_A(x_1) = \{0.1, 0.4, 0.3\}$ ,  $h_A(x_2) = \{0.3, 0.35\}$ ,  $h_A(x_3) = \{0.2, 0.4, 0.6, 0.69, 0.8\}$  are hesitant fuzzy elements.

**Property 2.1:** Considering  $h, h_1$  and  $h_2$  as three HFEs, a few tasks are characterized by Torra (2010) as pursues:

2.1.1:  $h^c = \{1 - \gamma : \gamma \in h\}$ , complement of  $h$ ;

2.1.2:  $h_1 \cup h_2 = \{\gamma_1 \vee \gamma_2 : \gamma_1 \in h_1, \gamma_2 \in h_2\}$ ;

2.1.3:  $h_1 \cap h_2 = \{\gamma_1 \wedge \gamma_2 : \gamma_1 \in h_1, \gamma_2 \in h_2\}$ ;

Furthermore, in order to aggregate hesitant fuzzy information, Xu and Xia (2011) defined some new operations on  $h, h_1$  and  $h_2$  with  $\lambda > 0$  as below:

2.1.4:  $h_1 \oplus h_2 = \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2 : \gamma_1 \in h_1, \gamma_2 \in h_2\}$ ;

2.1.5:  $h_1 \otimes h_2 = \{\gamma_1 \gamma_2 : \gamma_1 \in h_1, \gamma_2 \in h_2\}$ ;

2.1.6:  $h^\lambda = \{\gamma^\lambda : \gamma \in h\}$ ;

2.1.7:  $\lambda h = \{1 - (1 - \gamma)^\lambda : \gamma \in h\}$ .

**3 Classical matrix game**

In this part, we describe some basics on classical game theory. A matrix game is communicated as  $A = (a_{ij})$  where  $i = 1, 2, \dots, p$ ,  $j = 1, 2, \dots, q$  with components as real numbers and the corresponding matrix is termed as payoff matrix. We think about two players. Players I and II play row  $i$  and column  $j$ , individually and the results to players I and II are  $a_{ij}$  and  $-a_{ij}$ , respectively in case of zero-sum concept. Strategies that advantage player's individual adjustments are picked by players. Expecting the game with arrangement of unadulterated techniques  $S_1$  and  $S_2$  and that of blended or mixed strategies  $X$  and  $Y$  for players I and II individually, we characterize,  $S_1 = \{\alpha_1, \alpha_2, \dots, \alpha_p\}$ ,  $S_2 = \{\beta_1, \beta_2, \dots, \beta_q\}$ ,  $X = \{(X_1, X_2, \dots, X_p)^T : \sum_{i=1}^p X_i = 1, X_i \geq 0, i = 1, 2, \dots, p\}$ ,  $Y = \{(Y_1, Y_2, \dots, Y_q)^T : \sum_{j=1}^q Y_j = 1, Y_j \geq 0, j = 1, 2, \dots, q\}$ .

Here  $X_i$  ( $i = 1, 2, \dots, p$ ) and  $Y_j$  ( $j = 1, 2, \dots, q$ ) are probabilities in which the players I and II sort-out their pure strategies  $\alpha_i \in S_1$  ( $i = 1, 2, \dots, p$ ) and  $\beta_j \in S_2$  ( $j = 1, 2, \dots, q$ ) individually and game is enunciated as  $G \equiv (X, Y, A)$ .

Basically, we wish to get the most favourable strategy(ies) for players' and the value of considered game. The estimation of the game is characterized to be the maximum ensured gain to maximizing player I or the minimum conceivable loss to minimizing player II; and here the best strategic procedures are utilized by both players. If a player records the most exceedingly awful potential results of all things considering his or her prospective strategies, the individual in question will pick that strategy, the most reasonable for the person in question. This idea is observed as maximin and minimax principle. When maximin for player I and minimax for player II be equal then the existence of a saddle point in corresponding game is certified (von Neumann and Morgenstern (1944) noted the term *saddle point*).

1 Expect that player I uses any mixed strategy from  $X$ . Clearly,  
 2 player I's normal increase floor is  $\min(X^T A Y)$  and if shortly  
 3 be denoted by  $v$ , we need to maximize  $v$ , state  $v^*$  for certain  
 4  $X^* \in X$ , as  $v(X^*) = \max(\min\{\sum_{i=1}^p a_{ij} X_i : j = 1, 2, \dots, q\})$ .  
 5 Such  $X^*$  and  $v^*$ , respectively called player I's maximin strat-  
 6 egy and game-value, are obtained from the accompanying  
 7 LPP in Model 1 as:

#### 9 Model 1

$$11 \text{ maximize } v \quad (8)$$

$$12 \text{ subject to } \sum_{i=1}^p a_{ij} X_i \geq v \quad (j = 1, 2, \dots, q), \quad (9)$$

$$15 \sum_{i=1}^p X_i = 1, \quad (10)$$

$$18 X_i \geq 0 \quad (i = 1, 2, \dots, p). \quad (11)$$

20 What's more, with same contention, player II's optimal or  
 21 minimax strategy, say  $Y^* \in Y$ , and game value, state  $w^*$  are  
 22 depicted from Model 2 as:

#### 23 Model 2

$$25 \text{ minimize } w \quad (12)$$

$$27 \text{ subject to } \sum_{j=1}^q a_{ij} Y_j \leq w \quad (i = 1, 2, \dots, p), \quad (13)$$

$$30 \sum_{j=1}^q Y_j = 1, \quad (14)$$

$$32 Y_j \geq 0 \quad (j = 1, 2, \dots, q). \quad (15)$$

## 35 4 Neural network model

### 37 4.1 Biological network system

39 A typical neuron or nerve cell belongs to the vertebrate ner-  
 40 vous system which contains the nucleus (genetic informer)  
 41 and offers to two sorts of cell processes, *axon* and *dendron*.  
 42 Axon acts as transmitting element or output element whereas  
 43 dendron as input element. Branches of the axon of one neu-  
 44 ron communicate with signals to other neuron at a site is  
 45 called the synapse. Synapses are the elementary signal pro-  
 46 cessing devices.

48 Though the brain with its nervous system makes up for the  
 49 slow rate of operation with a few factors, the brain is an  
 50 exceptionally perplexing, non-linear, parallel data handling  
 51 framework. From early stage of childhood to adult stage,  
 52 synapses are modified gradually through the learning pro-  
 53 cess. And these motivate the scientists to use neural net-  
 54 works and the related sciences in artificial intelligence like  
 55 pattern recognition, perception, and motor controlling in fuzzy  
 56 sets and systems. Thus neural networks motivate to gener-  
 57 ate fuzzy rules from examples (cf. Sugeno and Yasukawa  
 58 (1993), Wang and Mendel (1992)).

### 4.2 Artificial neural network

In mathematics, biological structures of neural systems in-  
 fluence mathematical modeling to construct network func-  
 tions as forward and backward calculations. And this leads  
 to artificial neural network (ANN). ANN was found its roots  
 almost 75 years ago in the works of McCulloch and Pitts  
 (1943) and later by others (cf. Buckley and Hayashi (1992),  
 Buckley and Hayashi (1995), Kwan and Cai (1994)).

**Definition 41 Artificial neural network(ANN):** ANNs are  
 physical cell frameworks which can acquire, store and use  
 experimental information, knowledge and complex utilitar-  
 ian relations by summing up from a restricted amount of  
 preparing data.

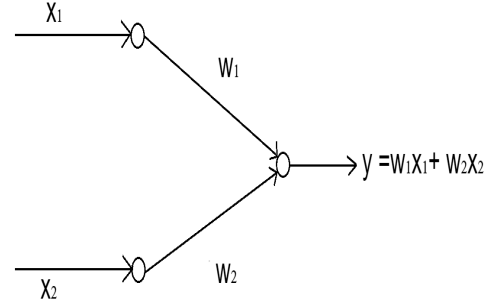


Fig. 1. A simple neural network model.

**Definition 42 Hybrid neural-net (Fuller (2000)):** In a sim-  
 ple net, as picturesquely in Fig. 1, all input criteria like  
 signals as well as weights are reals. Signals interact with  
 weights and pass through one to another layer using sig-  
 moidal function. This straightforward neural net with in-  
 crease, expansion, and sigmoidal function is called regu-  
 lar (or standard) neural net. If triangular-norm, triangular-  
 conorm or their combination are employed and used in next  
 layer we call it hybrid neural net.

**Definition 43 Fuzzy hybrid neural-net:** When weights are  
 crisp and signals are fuzzy then hybrid neural net is termed  
 as fuzzy hybrid neural net. A fuzzy hybrid neural net may not  
 use multiplication, addition and sigmoidal function.

In ANN, signal flows or transfers on the basis of the net's  
 activity, sometimes, termed as an activation or transfer func-  
 tion. The output of the flow persists if the value of activa-  
 tion function remains greater than some parameters, say, the  
 threshold-level.



**Definition 44 Max fuzzy neuron (Kwan and Cai (1994)):**  
 The signal  $X_i$  interfaced with the weight  $W_i$  produces  $p_i = W_i X_i, i = 1, 2$ . The input value  $p_i$  is aggregated utilizing the most extreme conorm  $z = \max\{p_1, p_2\} = \max\{W_1 X_1, W_2 X_2\}$  and the  $j$ -th yield of the neuron is registered by  $y_j = g_j(f(z - \theta)) = g_j(f(\max\{W_1 X_1, W_2 X_2\} - \theta))$ , where  $f$  is an initiation capacity and  $\theta$  is known as the threshold-level.

**Definition 45 Min fuzzy neuron (Kwan and Cai (1994)):**  
 The signal  $X_i$  communicated with the weight  $W_i$  produces  $p_i = W_i X_i, i = 1, 2$ . The input value  $p_i$  is amassed utilizing the minimum norm  $z = \min\{p_1, p_2\} = \min\{W_1 X_1, W_2 X_2\}$  and the  $j$ -th output of neuron is processed by  $y_j = g_j(f(z - \theta)) = g_j(f(\min\{W_1 X_1, W_2 X_2\} - \theta))$ , where  $f$  and  $\theta$  are the same as in Definition 44.

In this paper, we consider Max fuzzy neuron and Min fuzzy neuron with t-norm and t-conorm in processing of the problem's solution.

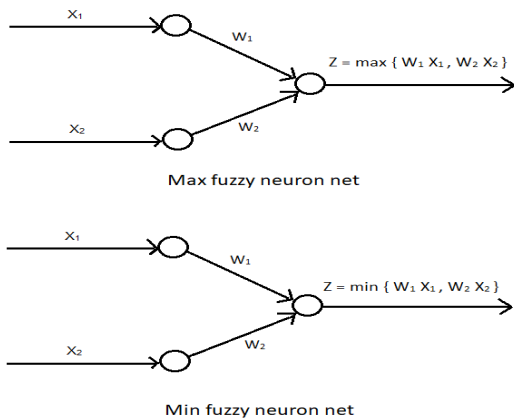


Fig. 2. Max fuzzy and Min fuzzy neuron nets.

### 5 Logic-gate switching circuit

In algebra of switching circuits, electrical or electronic switching circuits are depicted mathematically or planned to get an outline for circuit having some criteria. In switching circuits, we can consider conductor-nonconductor, charged-uncharged, decidedly and contrarily polarized components. These days, semi-conductor components in switching circuits have more importance. In specific situation, switches are meant as so-called *gates*, or combination of gates. This can be treated as emblematic portrayal. In this way a gate (or combination of gates) is a polynomial  $p$  which has the elements  $x_i$  for each  $i$ . We depict the gate as an acknowledgement of a switching function. In the event that, as for worth,  $p = 1$  (or 0), we have

current (or no current) through switching circuit  $p$ . Examples of switching gates with properties as output, are given in Fig. 3 ( $X_i$ s are treated here as input variables). Since, hu-

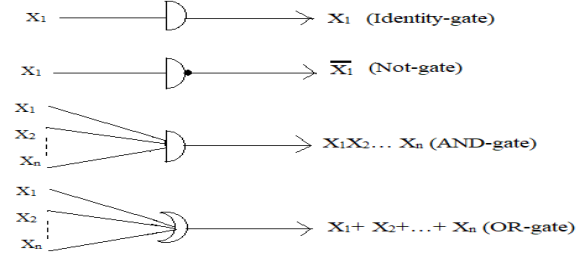


Fig. 3. Examples of some special gates.

man thinking, nowadays are not confined within 1-0 logical concept, a consequent fuzzy approach tends the switching circuit output towards linguistic variables like fuzzy sets. So the output in switching circuit also collaborates the crisp and fuzzy concept.

### 6 Fuzzy logic-gate switching circuit oriented artificial neural network (FGSC-ANN) model

Here, we discuss the steps algorithmically to collaborate the fuzzy data through the artificial neural net. From the collected data, the required optimal value is obtained applying the following Algorithm 1.

**Algorithm 1:** Construction of FGSC-ANN using fuzzy numbers.

- Step 1.1:** Collection of input data.
- Step 1.2:** Weight assign with input-data according to problem, if required.
- Step 1.3:** Weighted-data summation.
- Step 1.4:** Summed data are divided with two switches namely, original and corresponding NOT gate.
- Step 1.5:** All combinations are get together.
- Step 1.6:** All combined data set forms a geometrical figure, may be any polygon.
- Step 1.7:** Each vertex of the polygon is ranked according to their distances from centroid of the polygon.
- Step 1.8:** Signal flows through the minimum distance.
- Step 1.9:** Optimum vertex is obtained.

### 7 ANN based fuzzy matrix game

In ANNs, weighted interconnections are established by mathematical formulation, termed as rules. Rules are basically

governed by two approaches, crisp approach and fuzzy approach. A mathematical model when uses fuzzy set in a way is termed as a fuzzy approach of model rather than the crisp set oriented approach. When ambiguous, uncertain, imprecise conditions are applied in *if-then* relationship between the variables of the rules in ANNs, fuzzy neural networks (FNNs) are originated. FNNs are classified by using two models namely, Mamdani model (Mamdani and Assilian (1975)) and Takagi-Sugeno model (cf. Sugeno and Yasukawa (1993), Takagi and Sugeno (1985)), depend on the structures of *if-then* rules. If the antecedent (*if* part) and the consequent (*then* part) are fuzzy propositions like:

$R_i$  : If  $x$  is  $A_i$  then  $y$  is  $B_i, i = 1(1)k$ , where  $A_i$  and  $B_i$  are from linguistic fuzzy sets and  $k$  is the number of rules in the model, then the Mamdani model is useful to apply. When knowledge is acquired in quantitative or data-based information, then knowledge can be accessed through the Takagi-Sugeno-Kang (TSK) model.

Here we consider zero-order TSK fuzzy model with four rules and say it as *Modified TSK Model*.

#### Modified TSK Model:

**Assumption:** If  $x$  and  $y$  are inputs then output is  $z$ .

#### Rules:

$R_1$  : If  $x$  is  $X_1$  and  $y$  is  $Y_1$  then  $z$  is  $a_{11}$ , i.e., the output is  $((X_1, Y_1), a_{11})$

$R_2$  : If  $x$  is  $X_2$  and  $y$  is  $Y_1$  then  $z$  is  $a_{21}$ , i.e., the output is  $((X_2, Y_1), a_{21})$

$R_3$  : If  $x$  is  $X_1$  and  $y$  is  $Y_2$  then  $z$  is  $a_{12}$ , i.e., the output is  $((X_1, Y_2), a_{12})$

$R_4$  : If  $x$  is  $X_2$  and  $y$  is  $Y_2$  then  $z$  is  $a_{22}$ , i.e., the output is  $((X_2, Y_2), a_{22})$

If all of  $a_{11}, a_{12}, a_{21}$  and  $a_{22}$  are assumed as fuzzy numbers, the fuzzy game, in matrix form can be written as the following with  $X_1, X_2$  as player I's strategies and  $Y_1, Y_2$  are that for player II,

$$G = \begin{matrix} & Y_1 & Y_2 \\ \begin{matrix} X_1 \\ X_2 \end{matrix} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{matrix}$$

Here, for example, the payoff  $a_{11}$  emerged as the outcome when player I plays his/her strategy  $X_1$  and player II plays  $Y_1$ . Consider player I's strategies have weights  $w_1$  and  $w_2$  and player II's strategies have weights  $w_3$  and  $w_4$  respectively. Therefore, according to the concept of game theory, discussed in Section III, we must have

$$\begin{cases} f_1 = w_1 a_{11} X_1 + w_2 a_{21} X_2 \geq v_I; \\ f_2 = w_1 a_{12} X_1 + w_2 a_{22} X_2 \geq v_I; \\ g_1 = w_3 a_{11} Y_1 + w_4 a_{12} Y_2 \leq v_{II}; \\ g_2 = w_3 a_{21} Y_1 + w_4 a_{22} Y_2 \leq v_{II}. \end{cases}$$

Assuming that each rectangular game has a solution and assuming  $v_I$  and  $v_{II}$  to be the game values for players I and

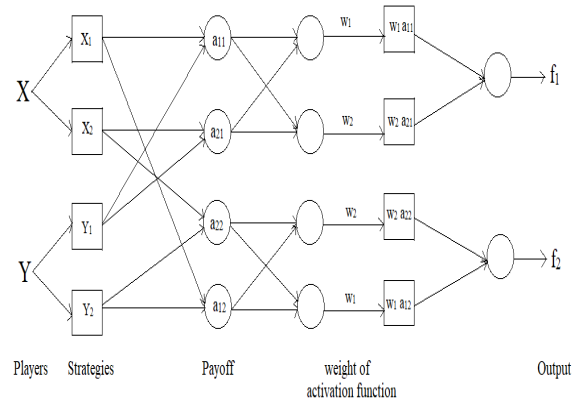


Fig. 4. Diagrammatic form of Player I's problem.

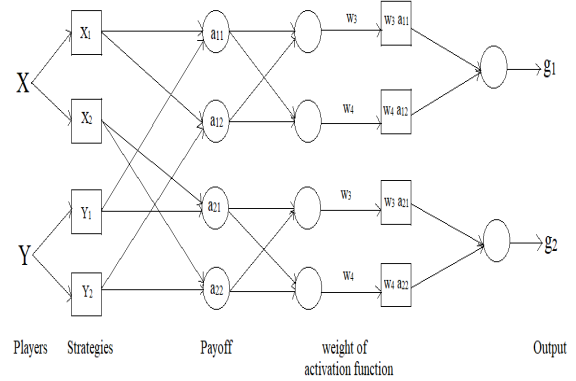


Fig. 5. Pictorial form of Player II's problem.

II, respectively which are to be optimized. So, in Figs. 4 and 5,  $f_1, f_2$  and  $g_1, g_2$  are the combined form in Max fuzzy neuron and Min fuzzy neuron respectively according to the existence of the saddle point(s) or can be summed according to the non-existence of saddle point to derive the optimum results through Algorithm 2.

#### Algorithm 2: Construction of matrix game solution.

**Step 2.1:** Construction of rules of the matrix game according to the strategies of the players.

**Step 2.2:** Combination of rules to form the payoffs of the matrix game.

**Step 2.3:** Application of the concept of the saddle point or mixed strategy to the matrix game.

**Step 2.4:** Achieving optimum strategies and obtaining the game value.

#### Algorithm 3: Construction of ANN-logic gate-switching circuit oriented matrix game solution.

**Step 3.1:** Follow Algorithm 1, stepwise.

**Step 3.2:** Follow Algorithm 2, stepwise.

**8 Numerical simulation**

Consider the existence of two business houses  $H_1$  and  $H_2$ . By selling their products, both these houses are aiming to increase their profits in terms of market-shares. One wishes to maximize his gain and the other aims to cut his lost. The two houses consider various strategies. House  $H_1$  considers  $X_1$ : Advertisement,  $X_2$ : Reducing the printed price. And house  $H_2$  chooses  $Y_1$ : Attracting packaging features,  $Y_2$ : Giving promotion-pack to customers free of cost. Again we consider that these two houses have their own managing bodies which call meetings regularly (say, every three months or every six months) to put some weights on their decisions. The decisions after each meeting may vary from the previous meeting's decisions or not. So, hesitant environment arises. Both the houses have efforts to increase their market-shares considering the fact that when one house profits, the other loses. So the outcome, after applying strategies, are the profit-percentage of the houses in terms of market shares. If we consider this problem as a game issue with two players I and II representing houses  $H_1$  and  $H_2$ , respectively, then the payoff matrix is as follows:

$$\check{G} = \begin{matrix} & Y_1 & Y_2 \\ X_1 & \check{C}_{11} & \check{C}_{12} \\ X_2 & \check{C}_{21} & \check{C}_{22} \end{matrix}$$

Here, player I has strategies  $X_1$  and  $X_2$ ; player II has  $Y_1$  and  $Y_2$ . And the payoff elements are hesitant triangular intuitionistic fuzzy elements  $\check{C}_{ij}$ ,  $i, j = 1, 2$  with their corresponding weights, separated by second semicolon, are given below:

$$\begin{aligned} \check{C}_{11} &= \{ \langle (5.7, 7.7, 9.3); 0.7, 0.2 \rangle; 0.4, \langle (5, 7, 9); 0.6, 0.3 \rangle; 0.3, \langle (5.7, 7.7, 9); 0.8, 0.1 \rangle; 0.3 \}; \\ \check{C}_{12} &= \{ \langle (8, 9, 10); 0.6, 0.3 \rangle; 0.5, \langle (8.3, 9.7, 10); 0.7, 0.2 \rangle; 0.3, \langle (7, 9, 10); 0.6, 0.2 \rangle; 0.2 \}; \\ \check{C}_{21} &= \{ \langle (8.33, 9.67, 10); 0.6, 0.4 \rangle; 0.4, \langle (3, 5, 7); 0.6, 0.3 \rangle; 0.4, \langle (6.5, 8.6, 10); 0.4, 0.5 \rangle; 0.2 \}; \\ \check{C}_{22} &= \{ \langle (6.5, 8.2, 9.3); 0.8, 0.1 \rangle; 0.3, \langle (7, 9, 10); 0.7, 0.2 \rangle; 0.4, \langle (6.3, 8.3, 9.7); 0.7, 0.2 \rangle; 0.3 \}. \end{aligned}$$

Here,  $\check{C}_{12} = \{ \langle (8, 9, 10); 0.6, 0.3 \rangle; 0.5, \langle (8.3, 9.7, 10); 0.7, 0.2 \rangle; 0.3, \langle (7, 9, 10); 0.6, 0.2 \rangle; 0.2 \}$  indicates that if player I assumes  $X_1$  and player II considers  $Y_2$ , then the profit will be 90% with minimum 80% to maximum 100% having 6% positive chance and 3% pessimistic chance if the managing body gives 5% weight to their decisions. In the following meeting the decisions remain same and no problem arises but if weight are given 3% then the profit percentage is 97, lying between 83 and 100 having 7% positive chance. The remaining member of the set can be depicted likewise.

Now, using the regular neural net structure, we combine the data in hesitant fuzzy set and using the mean averaging operator, we get from  $\check{C}_{11}$ ,  $x_1 = \langle (5.49, 7.49, 9.12); 0.6, 0.3 \rangle$ .

Similarly the others are obtained as:

$$\begin{cases} x_2 = \langle (7.89, 9.21, 10.00); 0.6, 0.3 \rangle \\ x_3 = \langle (5.83, 7.58, 8.80); 0.4, 0.5 \rangle \\ x_4 = \langle (6.64, 8.55, 9.70); 0.7, 0.2 \rangle \end{cases}$$

Since every switching circuit has two inputs as 'on' and 'off', we consider the  $x_i$  as 'on' and the  $\bar{x}_i$  as 'off' to maintain the circuit rational. This consideration is important on the basis of neural net since in the course of passing signal from one neuron to another, only the predefined/prefixed neuron is in on mode, others are in off mode.

$$\begin{cases} \bar{x}_1 = \langle (0.88, 2.51, 4.51); 0.3, 0.6 \rangle \\ \bar{x}_2 = \langle (0.00, 0.79, 2.11); 0.3, 0.6 \rangle \\ \bar{x}_3 = \langle (1.20, 2.42, 4.17); 0.5, 0.4 \rangle \\ \bar{x}_4 = \langle (0.30, 1.45, 3.36); 0.2, 0.7 \rangle \end{cases}$$

Now, using the multiplication operations on triangular intuitionistic fuzzy numbers using Eq. (5), we compute the values of the following sixteen combinations:

$$x_1x_2x_3x_4, \bar{x}_1x_2x_3x_4, x_1\bar{x}_2x_3x_4, x_1x_2\bar{x}_3x_4, x_1x_2x_3\bar{x}_4, \bar{x}_1\bar{x}_2x_3x_4, \bar{x}_1x_2\bar{x}_3x_4, \bar{x}_1x_2x_3\bar{x}_4, x_1\bar{x}_2\bar{x}_3x_4, x_1\bar{x}_2x_3\bar{x}_4, x_1x_2\bar{x}_3\bar{x}_4, \bar{x}_1\bar{x}_2\bar{x}_3x_4, \bar{x}_1\bar{x}_2x_3\bar{x}_4, x_1\bar{x}_2\bar{x}_3\bar{x}_4, \bar{x}_1x_2\bar{x}_3\bar{x}_4 \text{ and } \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4.$$

For example,  $\bar{x}_1x_2x_3\bar{x}_4 = \langle (12.14, 254.08, 1333.51); 0.2, 0.7 \rangle$  and the others.

These set of values of sixteen fuzzy numbers can be assigned as sixteen vertices of a solid figure as in Fig. 6.

Now, inspired from the articles (cf. Coupland and John

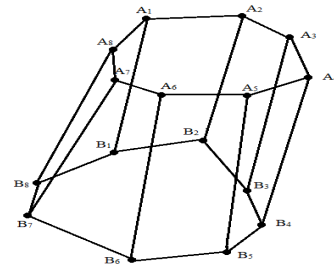


Fig. 6. A solid three-dimensional figure with sixteen vertices.

(2008), Wu et al. (2012)), the centroid of the figure is computed using the formulae:  $\frac{\sum V_i}{i}, i = 1(1)n$ ; Here,  $n = 16$  and  $V$  denote the vertices (In Figs. 6 and 7, denoted by  $A_s$  and  $B_t$ ;  $s, t = 1, \dots, 8$ ).

The centroid of the figure is the triangular fuzzy number  $\langle (153.25, 624.99, 1747.43); 0.2, 0.7 \rangle$ . Now, computing the Euclidean distances of all vertices from the centroid, the shortest distance arises for the vertex  $\bar{x}_1x_2\bar{x}_3x_4$  and the farthest for the vertex  $x_1x_2x_3x_4$ . Now considering the vertex  $\bar{x}_1x_2\bar{x}_3x_4$ , we form the payoff matrix, given in TABLE I.



TABLE I: Payoff matrix of the game problem.

Payoff matrix		
Players' strategies	$Y_1$	$Y_2$
$X_1$	$\langle(0.88, 2.51, 4.51); 0.3, 0.6\rangle$	$\langle(7.89, 9.21, 10); 0.6, 0.3\rangle$
$X_2$	$\langle(1.20, 2.42, 4.17); 0.5, 0.4\rangle$	$\langle(6.64, 8.55, 9.70); 0.7, 0.2\rangle$

Considering player I's strategies have weights  $w_1 = 0.5, w_2 = 0.5$  and player II's strategies have weights  $w_3 = 0.5, w_4 = 0.5$  respectively and using Definition 43, 44 and 45, Max fuzzy neuron {Min fuzzy neuron} =  $\max\{\min\{w_3a_{11}, w_4a_{12}\}, \min\{w_3a_{21}, w_4a_{22}\}\} = (0.5)(2.63)$ , and Min fuzzy neuron {Max fuzzy neuron} =  $\min\{\max\{w_1a_{11}, w_2a_{21}\}, \max\{w_1a_{12}, w_2a_{22}\}\} = (0.5)(2.63)$ . Thus, we get, Max fuzzy neuron {Min fuzzy neuron} = Min fuzzy neuron {Max fuzzy neuron}.

The existence of the saddle point gives the strategy sets for players I and II, respectively  $X_1$  and  $Y_1$  and the value of the game in triangular intuitionistic fuzzy form is  $\langle(0.88, 2.51, 4.51); 0.3, 0.6\rangle$ .

But, considering player I's strategies have weights  $w_1 = 0.6, w_2 = 0.4$  and player II's strategies have weights  $w_3 = 0.5, w_4 = 0.5$  respectively, we obtain,

Max fuzzy neuron {Min fuzzy neuron} =  $\max\{\min\{w_3a_{11}, w_4a_{12}\}, \min\{w_3a_{21}, w_4a_{22}\}\} = \max\{\min\{1.315, 4.515\}, \min\{1.295, 4.145\}\} = 1.315$ , and Min fuzzy neuron {Max fuzzy neuron} =  $\min\{\max\{w_1a_{11}, w_2a_{21}\}, \max\{w_1a_{12}, w_2a_{22}\}\} = \min\{\max\{1.578, 1.036\}, \max\{5.418, 3.316\}\} = 1.578$ . And again get, Max fuzzy neuron {Min fuzzy neuron}  $\neq$  Min fuzzy neuron {Max fuzzy neuron}; but we infer that the defuzzified crisp value  $V$  of the game satisfies  $1.315 \leq V \leq 1.578$ . If we consider the weights  $w_1 = 0.9, w_2 = 0.1, w_3 = 0.25, w_4 = 0.75$ , then using Definition 43, 44 and 45, we derive, Max fuzzy neuron {Min fuzzy neuron} =  $\max\{\min\{w_3a_{11}, w_4a_{12}\}, \min\{w_3a_{21}, w_4a_{22}\}\} = 0.658$ , and Min fuzzy neuron {Max fuzzy neuron} =  $\min\{\max\{w_1a_{11}, w_2a_{21}\}, \max\{w_1a_{12}, w_2a_{22}\}\} = 2.371$ .

Payoff matrices for player I and player II are diverse because of different weights, earmarked for the strategies of the players, and consequently we get different values of the game. But in each cases, optimal strategies for player I from player I's payoff matrix and optimal strategies for player II from corresponding payoff matrix are, respectively:  $(X_1^*, X_2^*) = (1, 0)$ ,  $(Y_1^*, Y_2^*) = (1, 0)$ . If we consider different weights to different strategies, we get the different resolutions. The whole procedure is picturesquely represented in Fig. 7 (Here,  $\check{a}_{ij}$  is hesitant triangular IF numbered-set and  $^k a_{ij}$  are its members), where the game model is performed through Algorithm 2.

## 9 Results and discussion

In this work, we contemplate fuzzy matrix game with respect of ANN and fuzzy logic gate switching circuit. De-

fuzzification technique using the centroid concept is applied and achieved a fine result to the matrix game problems.

Here we notice that the weights assigned to the strategies of the players or decision makers, when changed, give an interesting resolution. As the weights are changed, the crisp value of the game are changed, simultaneously. When we consider player I's strategies with weights  $w_1 = 0.5, w_2 = 0.5$  and player II's strategies with weights  $w_3 = 0.5, w_4 = 0.5$  respectively, we see the crisp value of the game as 1.315 and the profits in terms of market-shares is 25.1% with minimum 8.8% and maximum 45.1% in addition with 3% optimistic and 6% pessimistic chance. But if we apply the weights as  $w_1 = 0.6, w_2 = 0.4, w_3 = 0.5, w_4 = 0.5$ , we observe that the crisp game value lies between 1.315 and 1.578 with fuzzy value of the game within  $\langle(0.88, 2.51, 4.51); 0.3, 0.6\rangle$  and  $\langle(1.20, 2.42, 4.17); 0.5, 0.4\rangle$  with corresponding weights. This significantly suggests that the value of a decision, here game, depends upon the decision makers', here players', choices of weights of the alternatives, here strategies, of the game.

### 9.1 Comparative Analysis

Our proposed study, in comparison with others, has some significant advantages. Our study is based on hesitant fuzzy sets and elements by considering artificial neural network and logic gate switching circuits. This kind of idea has not been thought of before. For this reason, the result-relative comparison is beyond the scope. The superiority of the proposed study can be obtained from the following discussions.

- (i) Esfahlani et al. (2019) considered the intuitive neuro-rehabilitation video game employing the fusion of artificial neural network (ANN), inverse kinematics and fuzzy logic algorithms. Particularly, this paper manifests an approach to rectify incorrect positioning through real-time visual feedback on the screen of video game. But in our proposed study, we have considered the extended version of fuzzy sets as hesitant fuzzy sets. We have developed game model using artificial neural nets applying switching circuit gates in neural nets by defuzzifying of hesitant fuzzy elements.
- (ii) Abu-Khalaf et al. (2008) in their paper presented an application of neural networks to find closed-form representation of feedback strategies for a zero-sum game that appears in the  $H_\infty$  control. Uncertain environments are not considered in this paper whereas, in our proposed study, hesitant fuzzy environment is assumed in real-life problematic situations.
- (iii) In the paper by Mansoori et al. (2019), the fuzzy constrained matrix game problems were assumed in triangular fuzzy numbers using the concepts of recurrent neural network (RNN). In our proposed work, we have considered artificial hybrid neural-net logic-gate switching

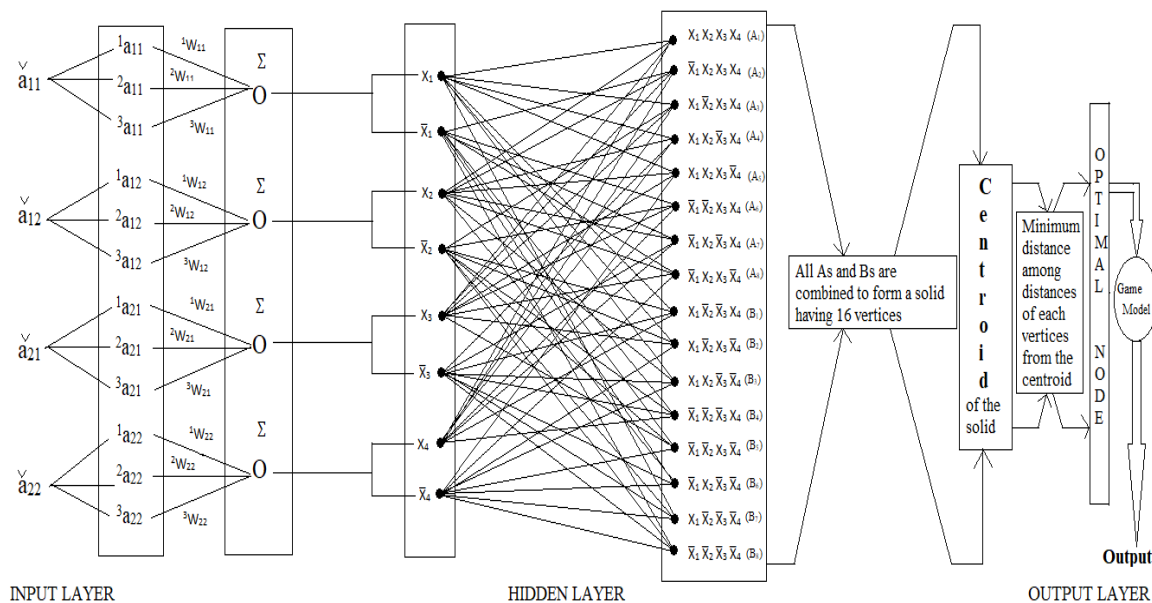


Fig. 7. Combined model for logic-gate-fuzzy-ANN system.

circuit in fuzzy matrix game under hesitant triangular intuitionistic fuzzy environment.

### 10 Conclusion

A few articles have been publicized using game theory in different fields, (cf. Bhaumik et al. (2020), Jana and Roy (2018), Roy and Bhaumik (2018), Roy (2010), Sadeghi and Zandieh (2011)) successfully. Hesitant fuzzy concept is also an important tool to represent the decision makers' hesitance characteristics and successfully have been applied in different aspects (cf. Chen and Xu (2014), Chen et al. (2013), Rodriguez et al. (2012), Torra and Narukawa (2009), Yu et al. (2013)). The major objectives of this work, to explore the potentiality of the neuro-fuzzy systems in modeling game phenomenon and to access its behavioural structures through artificial neural network and logic-gate switching circuits, are fulfilled. From the model description, our suggested methodology is unique in the following manners:

- (i) This is (probably) the main endeavour to explain fuzzy matrix game using max fuzzy neuron and min fuzzy neuron in hybrid fuzzy neural network.
- (ii) This is the fast approach to combine the hesitant fuzzy elements using neural network.
- (iii) The applied defuzzification method is unique in the sense that it can be applied easily.
- (iv) In future research works, this model can be applied in marketing, finance, medical sciences, engineering, etc.

The analysis of the results indicates that the rendition of FGSC-ANN model in game theory would be significantly improved if the input data are transformed into the normal or real domain prior to model formulation. The results of the proposed study highly encourage the researchers with a suggestion that ANN is viable for modeling daily life problems in the light of game theory.

**Abbreviations:** IFS: Intuitionistic Fuzzy Set, IFN: Intuitionistic Fuzzy Number, HFS: Hesitant Fuzzy Set, LPP: Linear Programming Problem, ANN: Artificial Neural Network, TSK: Takagi-Sugeno-Kang, FGSC-ANN: Fuzzy logic Gate Switching Circuit oriented Artificial Neural Network

### Compliance with Ethical Standards

**Conflict of Interest:** The authors would like to announce that there is no conflict of interest.

**Ethical Approval:** This article does not contain any examinations with human members or creatures performed by any of the others.

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