Fuzzy-modal control for the ball and beam system

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Abstract. The article is devoted to the development of a fuzzy-modal controller for the ball-beam position control system. This nonlinear dynamic plant is often considered when developing various control strategies. One option here is to use modal controls based on the linearized system model. However, the peculiarity of the linear controller is that the duration of the transient process does not depend on the initial state of the system. The proposed nonlinear control algorithm is based on the use of a set of modal control laws synthesized for different eigenvalues of a closed loop system. The signals of the modal controllers are matched by a fuzzy inference circuit using an a priori linguistic description of the states of the system. The simulation results show that fuzzy-modal control provides a transient time proportional to the initial deviation of the system.

1 Introduction

For the modern stage of the development of the theory of automatic control (TAU), the interaction of methods of intellectual control and classical sections of the TAU is characteristic. One of the variants of such interaction is the fuzzy-modal control considered in this paper.

The simulation of the "ball and beam" system is of great practical importance, because the transient processes here are similar to the dynamics of the aircraft during takeoff and landing, as well as when moving in the turbulent zone [1, 2]. To solve this problem, various approaches can be used, including modal control and fuzzy logic.

The space-state method is an effective design tool that ensures high accuracy of control systems [3, 4]. However, the modal control law is linear, and the reaction time of the system does not depend on its initial state. For essentially nonlinear objects such an idealization is unsatisfactory, which stimulates the use of fuzzy [5, 6] and neural network [7] methods for solving this problem.

Fuzzy logic allows you to manage complex objects using nonlinear algorithms [8, 9]. But the task of constructing fuzzy rules is not strictly formalized, and the accuracy of control may not be high enough.

In this paper, we investigate the option of organizing a control system for a nonlinear object, based on the use of a set of modal regulators whose signals are matched by a fuzzy logic inference scheme. This variant resembles a fuzzy logic controller (FLC) of the Takagi-Sugeno type [10, 11], in which the set of rules corresponds to a set of linear models, each of which describes the local area of the object's phase space. The construction of Takagi-Sugeno type FLCs for nonlinear objects was considered in [12, 13], where PID regulators were used as local ones. The advantage of modal control lies in the possibility of an analytic calculation of parameters that allows the poles of a closed system to be placed in a preselected position, thus ensuring the necessary characteristics of transient processes.

2 Mathematical model of control plant

Consider a mathematical model of the control object. We introduce the following notation: θ is the tilt angle of the trough, m is the ball mass, r is the radius of the ball, p is the coordinate along the beam axis, l is the length of the beam, J_1 and J_2 are the moments of inertia of the beam and ball, and τ is the control moment. The scheme of the forces acting in the system is shown in Figure 1.
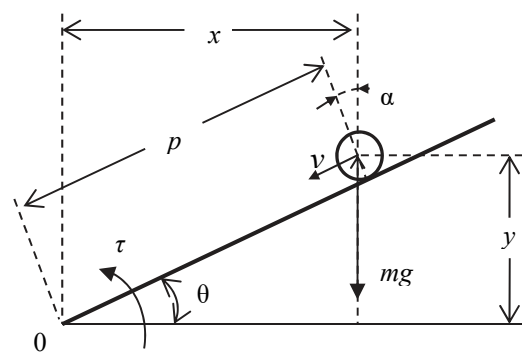

Fig. 1. Schematic representation of the ball-beam system.

A nonlinear model of the system dynamics was obtained, for example, in [1] on the basis of the Lagrange equations, it has the form:

$$\begin{cases} \left(mp^2 + J_1\right)\ddot{\theta} + 2mp\dot{p}\dot{\theta} + mgp\cos\theta = \tau, \\ \left(\dfrac{J_2}{r^2} + m\right)\ddot{p} + mg\sin\theta - mp\dot{\theta}^2. \end{cases} \quad (1)$$

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We introduce the state variables:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} p \\ \dot{p} \\ \theta \\ \dot{\theta} \end{bmatrix}.$$

Then it follows from (1) that

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{m\left(x_1 x_4^2 - g \sin x_3\right)}{\dfrac{J_2}{r^2} + m} \\ x_4 \\ \dfrac{-2mx_1 x_2 x_4 - mgx_1 \cos x_3 + \tau}{mx_1^2 + J_1} \end{bmatrix}. \qquad (2)$$

Let us consider linearization (2):

$$\dot{X} = F(X,\tau), \qquad A = \frac{\partial F(X,\tau)}{\partial X}; \qquad B = \frac{\partial F(X,\tau)}{\partial \tau}.$$

Let the working point have the coordinates of the state $X_0 = [p_0\ 0\ 0\ 0]^T$, then from (2) it follows that

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-mg}{\dfrac{J_2}{r^2} + m} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{-mg}{mp_0^2 + J_1} & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{mp_0^2 + J_1} \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T. \qquad (3)$$

Thus, a linearized description of the dynamics of the system is obtained, which depends on the operating point along the length of the beam.

3 Modal control of the ball-beam system

Consider the system with parameters: $m = 0.1$ kg, $r = 0.015\ m$, $J_1 = 0.00001\ kg \cdot m^2$, $J_2 = 0.025\ kg \cdot m^2$ and operating point $X_0 = [0\ 0\ 0\ 0]^T$, then substituting in (3) we have

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -6.8 & 0 \\ 0 & 0 & 0 & 1 \\ -39.2 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 40 \end{bmatrix},$$

We select the position of the poles of the closed loop system: $P = [-3\ -3\ -3\ -3]$, then by the Ackerman

formula we can obtain the vector of feedback coefficients $K = [-1.28, -0.4, 1.35, 0.3]$.

In Figure 2 is a block diagram of the control system simulation, in Figure 3 and 4 - transient processes with different initial deviations of the ball.
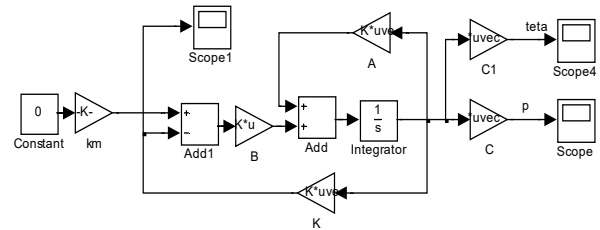

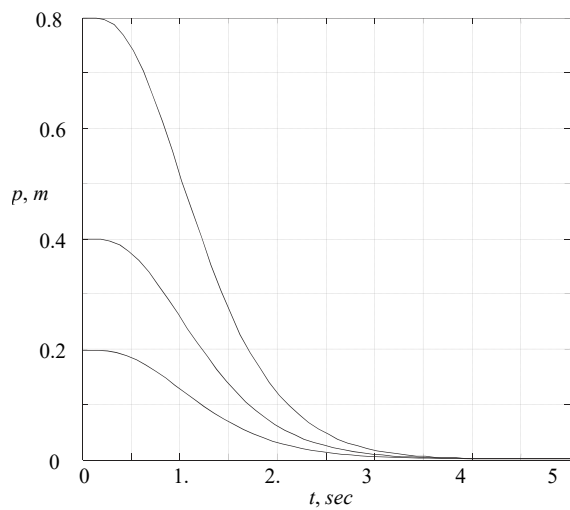**Fig. 2.** The scheme of modal control in Simulink MatLab.

Fig. 3. Transient processes with respect to the position of the ball for various initial deviations.
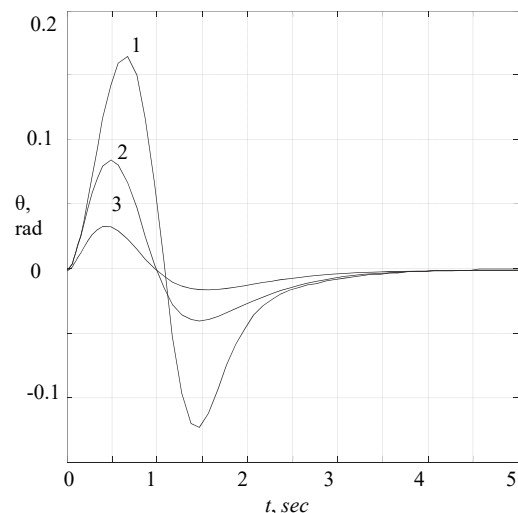

Fig. 4. Transient processes on the slope of the bar for various initial deviations of the ball: 1 - 0.8 m, 2 - 0.4 m, 3 - 0.2 m.

As Figure 3 shows, the time of the transient process is the same regardless of the initial position of the ball.

Consider the use of standard Newtonian polynomials for the choice of the position of the poles of a closed

system [3]. Each pole of λ must be negative, and the magnitude of its modulus λ_0 is determined by the speed requirements - the larger λ_0, the shorter the transient time.

Table 1 presents 5 variants of the arrangement of the poles of the closed loop system and the corresponding feedback coefficients of the modal controller. Controller no. 5 is the most slow, and controller no. 1, respectively, the fastest.

Table 1. Parameters of local modal regulators.

No	Vector of poles	Vector of feedbacks gain
1	[–18 –18 –18 –18]	[–387, –85.87, 48.6, 1.8]
2	[–12 –12 –12 –12]	[–77.3, –25.44, 21.6, 1.2]
3	[–9 –9 –9 –9]	[–25.13, –10.73, 12.15, 0.9]
4	[–6 –6 –6 –6]	[–5.75, –3.18, 5.4, 0.6]
5	[–3 –3 –3 –3]	[–1.28, –0.4, 1.35, 0.3]

Examples of transient processes under the control of various regulators are shown in Figure 5. When modeling, the nonlinear model of the plant specified by the system (1) is used.
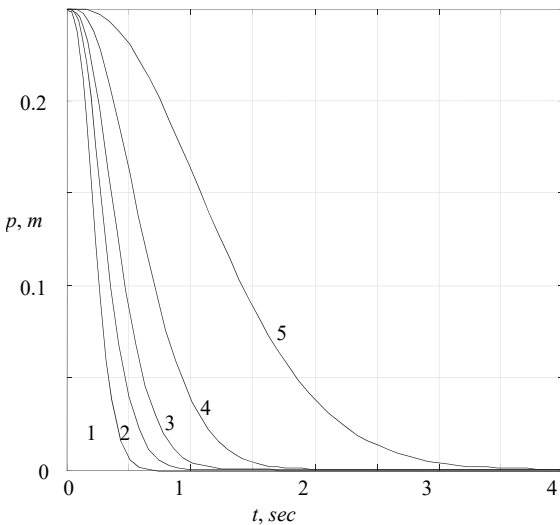

Fig. 5. Illustration of the work of controllers 1 – 5.

Let us formulate the control problem as follows: the duration of the transient process must be directly proportional to the initial deviation of the ball while maintaining the shape of the transient process. In other words, the average velocity of the ball should be approximately constant, regardless of the magnitude of the initial deviation. To solve this problem, it is proposed to simultaneously use a variety of modal controllers whose signals are matched by a fuzzy inference system.

4 Structure of fuzzy modal controller

The use of Takagi-Sugeno's FLC suggests that the fuzzy control law is described by a set of rules in which the conclusion is some analytic dependence on the input variables:

$$if \ \ Y(t) = C_i(X), \ \ then \ u_i = f_i(X),$$

where Y is the output vector of the plant, C is a fuzzy set, f is a function, usually linear, and X is a state vector.

In the problem under consideration

$$u_i = K_i X(t) = k_1^i p(t) + k_2^i \dot{p}(t) + k_3^i \theta(t) + k_4^i \dot{\theta}(t), \ i = \overline{1,5}.$$

A fuzzy set C describes a certain region of the state space of a plant, so it must be defined in a 4-dimensional space:

$$C_i = C_i(p, \dot{p}, \theta, \dot{\theta}).$$

Consider a simplified version when $C_i = C_i(p(t))$. In this case, the fuzzy partitioning option shown in Figure 6 (where T_i are fuzzy sets corresponding to the range of the individual regulators, μ is the degree of membership, Δp is the distance between the current point and the set point).
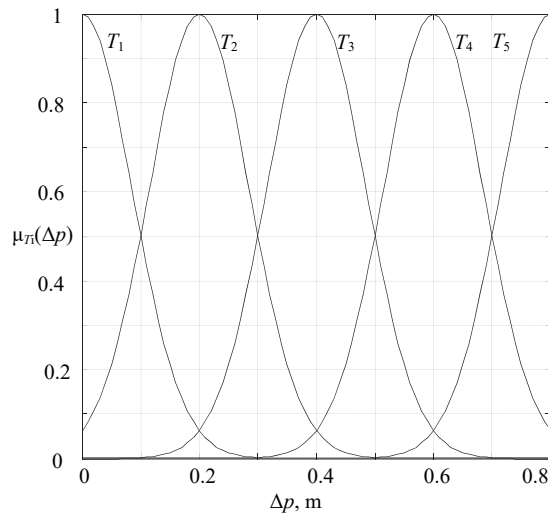

Fig. 6. Variant of fuzzy description of controllers activity areas.

The output of the fuzzy system at each time is calculated by the formula:

$$u(t) = \frac{\sum_{i=1}^{5} \mu_{Ti}(p(t)) u_i}{\sum_{i=1}^{5} \mu_{Ti}(p(t))} = w_1 u_1 + w_2 u_2 + \dots + w_5 u_5, \quad (3)$$

where $\mu_{Ti}(p(t))$ is the degree of membership of the input value to the i-th fuzzy region; w_i is the weighting factor of the i-th fuzzy controller.

As shown in (3), the output of the FLC here is the weighted sum of the outputs of the linear controllers. The structure of the FLC is shown in Figure 7, where MC is the modal controller, p^* is the setpoint.
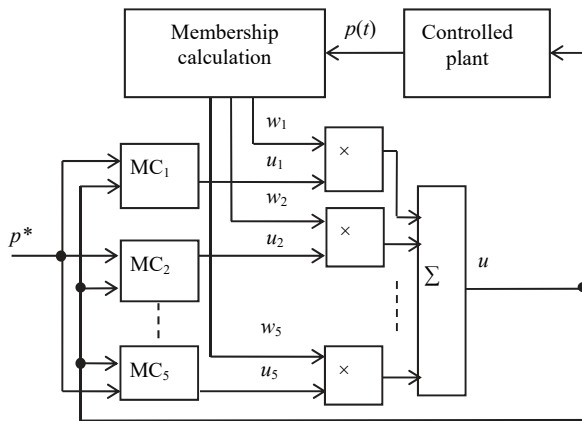
Fig. 7. Structure of fuzzy modal controller.

Thus, each local controller is responsible for its range of deviation of the ball from the target position. Regions overlap, since fuzzy sets are used to describe them.

Figures 8 and 9 show an example of modeling the operation of a fuzzy modal controller under various initial conditions.
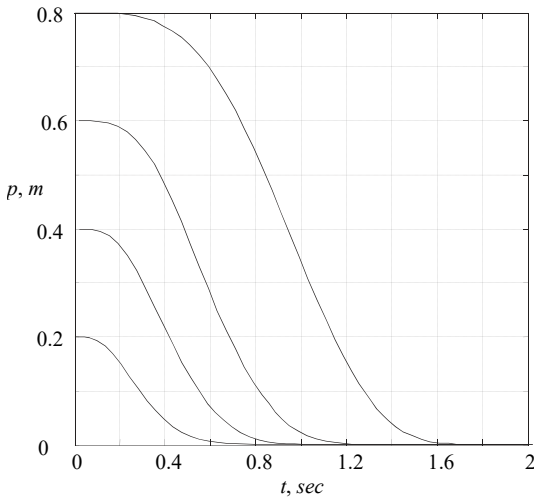

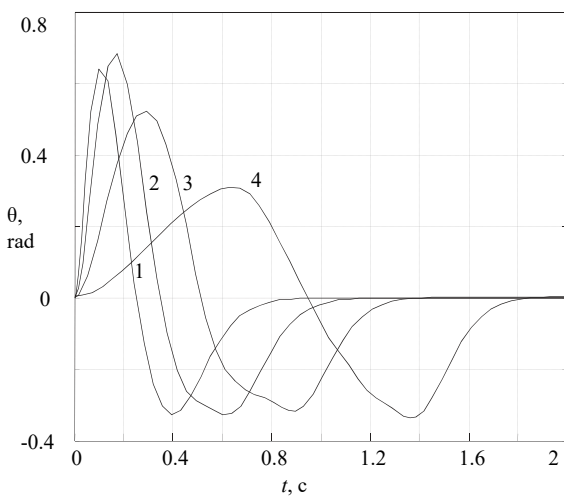**Fig. 8.** Dynamics of a ball under a fuzzy-modal control.

Fig. 9. Transient processes on the slope of the bar for various initial deviations of the ball: 1 - 0.2 *m*, 2 - 0.4 *m*, 3 - 0.6 *m*, 4 - 0.8 *m*.

As Figure 9, the angle of inclination of the strip during the transient process does not exceed the permissible value.

In contrast to the linear modal regulator (Figure 3), here the time of the transient process depends on the initial deviation of the ball.

Figure 10 shows an example of changing the value of the membership of the current position of the ball to the controller activity areas when moving on the beam from 0.8 m to 0 m, and Figure 11 - the control signal generated at the same time.
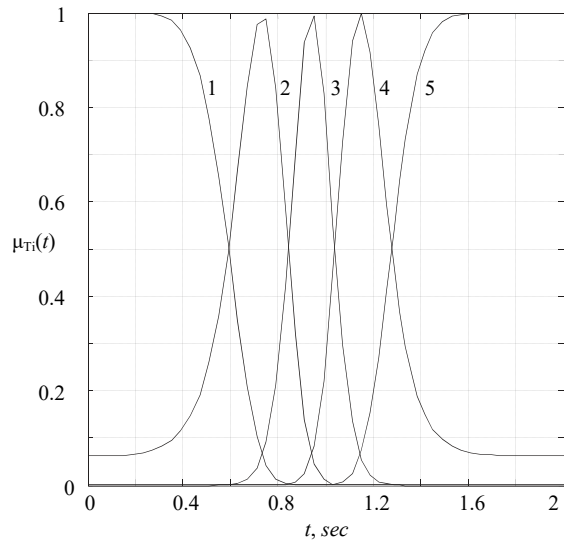

Fig. 10. Dynamics of the degree of belonging of the coordinate of the ball when moving from the point 0.8 m to zero.
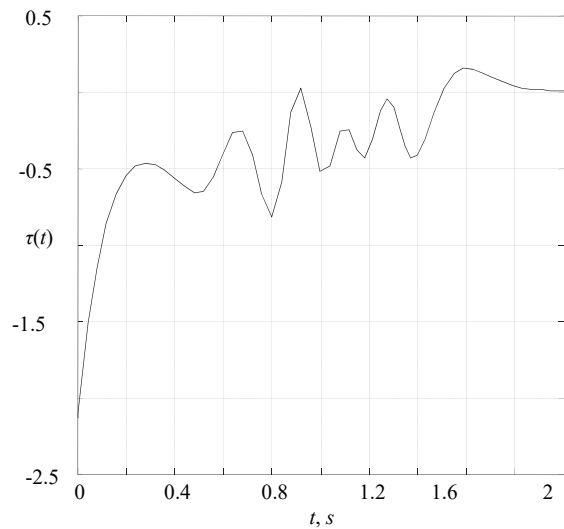

Fig. 11. Control moment of fuzzy modal controller.

Figure 12 compares the velocities of ball motion under various control laws. The fuzzy modal regulator provides an insignificant increase in the time of the transient process in comparison with the fast modal regulator (Fig. 12). However, this results in a smooth change in speed, which does not cause the risk of loss of stability.
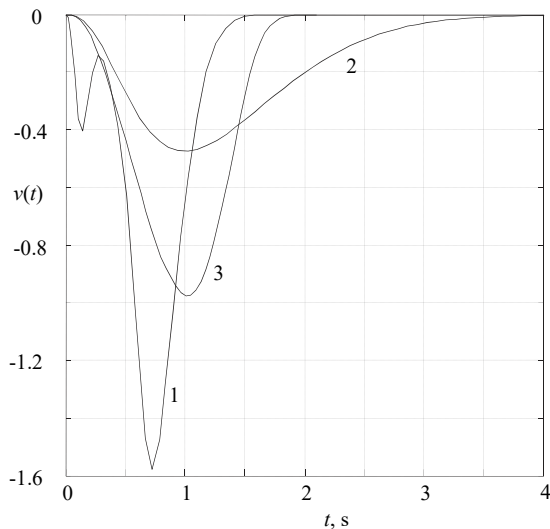
Fig. 12. Velocity of the ball: 1 - fast modal regulator (no. 2), 2 - slow modal regulator (no. 5), 3 - fuzzy modal regulator.

5 Conclusion

The structure of the fuzzy modal controller for controlling a nonlinear dynamic object, a ball on the bar, is proposed in the article.

The proposed scheme allows you to get the following options:

− Perform analytical analysis of local linear regulators for specified quality indicators.

− To provide different speed of local controllers at different points in the state space of the object.

− Use a limited number of local controllers by matching their signals using a fuzzy output circuit.

− Eliminate the drawback of linear control law, in which small deviations of the object are compensated for at the same time as large deviations.

The last property of the fuzzy modal controller is not rigidly established, since it is possible to describe the desired speed for an arbitrary point in the state space. This is the main advantage of the proposed approach, which makes it possible to recommend its use for controlling nonlinear plants of various physical nature.

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