Applied Mathematical Sciences, Vol. 5, 2011, no. 67, 3335 - 3346

Fuzzy Parameterized Interval-Valued Fuzzy Soft Set

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Abstract

In this work, we introduce the concept of fuzzy parameterized interval-valued fuzzy soft set theory (*fpivfss*) and study their operations. We then define *fpivfss*-aggregation operator to form *fpivfss*-decision making method that allows constructing more efficient decision processes. Finally, some numerical examples are employed to substantiate the conceptual arguments.

Keywords: *fpivfs*-aggregation operator, *fpivfs*-decision making; *fpivfs*-sets, fuzzy parameterized soft sets, fuzzy soft set; interval-valued fuzzy set; interval-valued fuzzy soft set; soft set.

1 Introduction

There are some mathematical tools for dealing with uncertainties; two of them are fuzzy set theory, developed by Zadeh [11], and soft set theory, introduced by Molodtsov [9]. In [12] Zadeh introduced and used interval-valued fuzzy set. After that many authors study the mathematical tools and their applications.

For soft set theory, Maji et al. [7] defined operations of soft sets to make a detailed theoretical study on the soft sets. Also Maji et al. [6] defined a fuzzy soft set and they gave the application of fuzzy soft set in decision making problem in [8]. By using these definitions, the applications of soft set theory have been studied increasingly. Cagman and Enginoglu [1] studied the soft decision making and Cagman et al [2] also gave an application of soft set theory in decision making. Chen et al. [3], discussed the parameterization reduction of soft sets and its applications. An adjustable approach to fuzzy soft set based on decision making is given by Feng et al. [4]. Cagman et al. [2] defined the concept of fuzzy parameterized fuzzy soft set (*fpfs*-set). The purpose of this paper is to combine the interval-valued fuzzy soft sets and *fpfs*-set, from which we can obtain a new soft set model: fuzzy parameterized interval-valued fuzzy soft set theory. In this paper, we define *fpivfs*-sets in which the approximate functions are defined from fuzzy parameters set to the interval-valued fuzzy subsets of the universal set. We also define their operations and soft aggregation operator to form *fpivfs*-decision making method that allows constructing more efficient decision processes. We finally present examples which show that the methods can be successfully applied to many problems that contain uncertainties.

2 Preliminary

Molodtsov [9] defined soft set in the following way. Let U be a universe set and E a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 [9]. *A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.*

Definition 2.2 [6]. *Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F, E) is called a fuzzy soft set over U where F is a mapping given by*

$$F : A \to I^U.$$

Definition 2.3 [12]. *An interval-valued fuzzy set $\tilde{\mathrm{X}}$ on a universe U is a mapping such that*

$$\tilde{\mathrm{X}} : U \to Int\left([0, 1]\right),$$

where $Int([0, 1])$ stands for the set of all closed subintervals of $[0, 1]$, the set of all interval-valued fuzzy sets on U is denoted by $\tilde{P}(U)$.

Suppose that $\tilde{X} \in \tilde{P}(U), \forall x \in U, \mu_x(x) = [\mu_x^-(x), \mu_x^+(x)]$ is called the degree of membership of an element x to $\tilde{X}$ where $\mu_x^-(x)$ and $\mu_x^+(x)$ are the lower and upper degrees of membership of x to $\tilde{X}$ respectively such that

$$0 \leq \mu_x^-(x) \leq \mu_x^+(x) \leq 1.$$

The complement, intersection and union of the interval-valued fuzzy sets are defined in [5] as follows: Let $\tilde{X}, \tilde{Y} \in \tilde{P}(U)$ then

1. the complement of $\tilde{X}$ is denoted by $\tilde{X}^c$ where

$$\mu_{\tilde{X}^c}(x) = 1 - \mu_{\tilde{X}}(x) = \left[1 - \mu_{\tilde{X}}^+(x), 1 - \mu_{\tilde{X}}^-(x)\right];$$

2. the intersection of $\tilde{X}$ and $\tilde{Y}$ is denoted by $\tilde{X} \cap \tilde{Y}$ where

$$\mu_{\tilde{X} \cap \tilde{Y}}(x) = \inf\left[\mu_{\tilde{X}}(x), \mu_{\tilde{Y}}(x)\right]$$

$$= \left[\inf\left(\mu_{\tilde{X}}^-(x), \mu_{\tilde{Y}}^-(x)\right), \inf\left(\mu_{\tilde{X}}^+(x), \mu_{\tilde{Y}}^+(x)\right)\right];$$

3. the union of $\tilde{X}$ and $\tilde{Y}$ is denoted by $\tilde{X} \cup \tilde{Y}$ where

$$\mu_{\tilde{X} \cup \tilde{Y}}(x) = \sup\left[\mu_{\tilde{X}}(x), \mu_{\tilde{Y}}(x)\right]$$

$$= \left[\sup\left(\mu_{\tilde{X}}^-(x), \mu_{\tilde{Y}}^-(x)\right), \sup\left(\mu_{\tilde{X}}^+(x), \mu_{\tilde{Y}}^+(x)\right)\right].$$

Definition 2.4 [2]. *Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function*

$$\mu_X : E \to [0, 1],$$

and let γ_X be a fuzzy set over U for all $x \in E$. Then an fpfs-set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping $\gamma_X : E \to F(U)$ such that

$$\gamma_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, γ_X is called a fuzzy approximate function of the fpfs-set Γ_X, and the value $\gamma_X(x)$ is a set called x-element of the fpfs-set for all $x \in E$. Thus, an fpfs-set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E,\ \gamma_X(x) \in F(U), \mu_X(x) \in [0, 1]\}.$$

It must be noted that the set of all *fpfs*-sets over U will be denoted by $FPFS(U)$.

Definition 2.5 [8]. *Let $\Gamma_X \in FPFS(U)$. Then fpfs-aggregation operator, denoted by $FPFS_{agg}$, is defined by*

$$FPFS_{agg} : F(E) \times FPFS(U) \to F(U),$$

$$FPFS_{agg}(X, \Gamma_X) = \Gamma_X^*$$

where

$$\Gamma_X^* = \left\{ \mu_{\Gamma_X^*}(u)/u : u \in U \right\}$$

which is a fuzzy set over U. The value Γ_X^* is called an aggregate fuzzy set of Γ_X. Here, the membership degree $\mu_{\Gamma_X^*}(u)$ of u is defined as follows:

$$\mu_{\Gamma_X^*}(u) = \frac{1}{|E|} \sum_{x \in E} \mu_X(x) \mu_{\gamma X(x)}(u),$$

where $|E|$ is the cardinality of E.

Definition 2.6 [12]. *Let U be an initial universe and E be a set of parameters. $\tilde{P}(U)$ denotes the set of all interval-valued fuzzy sets of U. Let $A \subseteq E$. A pair $\left(\tilde{F}, A\right)$ is an interval-valued fuzzy soft set over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \to \tilde{P}(U)$.*

3 Fuzzy parameterized interval-valued fuzzy soft set

In this section, we shall define fuzzy parameterized interval-valued fuzzy soft sets (*fpivfs*-sets) and their operations with examples.

Definition 3.1 *Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function: $\mu_X : E \to [0, 1]$ and let η_X be an interval-valued fuzzy set over U for all $x \in E$. Then a fuzzy parameterized interval-valued fuzzy soft set (fpivfs-set) Ψ_X over U is a set defined by a function η_X representing a mapping*

$$\eta_X : E \to Int(U)$$

such that $\eta_X(x) = \emptyset$ if $\mu_X(x) = 0$.

Here, $\eta_X = \left[\eta_X^-, \eta_X^+\right]$ is called an interval-valued fuzzy approximate function of the *fpivfs*-set Ψ_X, and the value $\eta_X(x)$ is a set called an x-element of the *fpivfs*-set for all $x \in E$. Thus a *fpivfs*-set Ψ_X over U can be represented by the set of ordered pairs

$$\Psi_X = \{(x/\mu_X(x), \eta_X(x)) : x \in E, \eta_X(x) \in Int(U), \mu_X(x) \in [0,1]\}.$$

It must be noted that the set of all *fpivfs*-sets over U will be denoted by $FPIVFSS(U)$.

Example 3.2 *Let* $U = \{u_1, u_2, u_3, u_4\}$ *be a set of universe,* $E = \{x_1, x_2, x_3, x_4\}$ *be a set of qualities and let* $\mu : E \to [0,1]$. *Suppose* $X = \left\{\frac{x_1}{0.3}, \frac{x_2}{0.1}, \frac{x_3}{0.4}, \frac{x_4}{1}\right\}$ *and* $\eta_X(x)$ *is defined as follows:*

$$\eta_X(x_1) = \left\{\frac{u_1}{[0.3,\ 0.6]}, \frac{u_2}{[0.7,\ 0.8]}, \frac{u_3}{[0.5,\ 0.8]}, \frac{u_4}{[0.2,\ 0.5]}\right\},$$

$$\eta_X(x_2) = \left\{\frac{u_1}{[0,\ 0.7]}, \frac{u_2}{[0.2,\ 0.3]}, \frac{u_3}{[0.3,\ 0.5]}, \frac{u_4}{[0.1,\ 0.3]}\right\},$$

$$\eta_X(x_3) = \left\{\frac{u_1}{[0.1,\ 0.4]}, \frac{u_2}{[0.4,\ 0.6]}, \frac{u_3}{[0.3,\ 0.8]}, \frac{u_4}{[0.4,\ 0.5]}\right\},$$

$$\eta_X(x_4) = \left\{\frac{u_1}{[0.5,\ 0.7]}, \frac{u_2}{[0.1,\ 0.4]}, \frac{u_3}{[0,\ 0.4]}, \frac{u_4}{[0.8,\ 0.9]}\right\}.$$

Then the fpivfs-set Ψ_X *is given by*

$$\begin{aligned}\Psi_X = \Big\{&\left(\frac{x_1}{0.3}, \left\{\frac{u_1}{[0.3,\ 0.6]}, \frac{u_2}{[0.7,\ 0.8]}, \frac{u_3}{[0.5,\ 0.8]}, \frac{u_4}{[0.2,\ 0.5]}\right\}\right),\\ &\left(\frac{x_2}{0.1}, \left\{\frac{u_1}{[0,\ 0.7]}, \frac{u_2}{[0.2,\ 0.3]}, \frac{u_3}{[0.3,\ 0.5]}, \frac{u_4}{[0.1,\ 0.3]}\right\}\right),\\ &\left(\frac{x_3}{0.4}, \left\{\frac{u_1}{[0.1,\ 0.4]}, \frac{u_2}{[0.4,\ 0.6]}, \frac{u_3}{[0.3,\ 0.8]}, \frac{u_4}{[0.4,\ 0.5]}\right\}\right),\\ &\left(\frac{x_4}{1}, \left\{\frac{u_1}{[0.5,\ 0.7]}, \frac{u_2}{[0.1,\ 0.4]}, \frac{u_3}{[0,\ 0.4]}, \frac{u_4}{[0.8,\ 0.9]}\right\}\right)\Big\}.\end{aligned}$$

Definition 3.3 *Let* Ψ_X *and* Ψ_Y *be two FPIVFSS(U). Then* Ψ_X *is said to be a fuzzy parameterized interval-valued fuzzy soft subset (fpivfs-subset) of* Ψ_Y *and we write* $\Psi_X \subseteq \Psi_Y$ *if*

1. $\mu_X(x) \leq \mu_Y(x), \forall x \in E$;
2. $\eta_X(x) \subseteq \eta_Y(x), \forall x \in E$.

Definition 3.4 *Two FPIVFSS(U)* Ψ_X *and* Ψ_Y *are said to be equal and we write* $\Psi_X = \Psi_Y$ *if* Ψ_X *is a fpivfs-subset of* Ψ_Y *and* Ψ_Y *is a fpivfs-subset of* Ψ_X. *In other words,* $\Psi_X = \Psi_Y$ *if the following conditions are satisfied:*

1. $\mu_X(x) = \mu_Y(x), \forall x \in E$,
2. $\eta_X(x) = \eta_Y(x), \forall x \in E$.

Definition 3.5 *Let* $\Psi_X \in FPIVFSS(U)$. *If* $\eta_X(x) = \emptyset, \ \forall x \in E$, *then* Ψ_X *is called an* X*-empty fpivfs-set, denoted by* $\Psi_{\emptyset_X}$. *If* $X = \emptyset$, *then the* X*-empty fpivfs-set* $(\Psi_{\emptyset_X})$ *is called an empty fpivfs-set,denoted by* $\Psi_\emptyset$.

Definition 3.6 *Let* $\Psi_X \in FPIVFSS(U)$. *If* $\eta_X(x) = U, \ \forall x \in E$, *then* Ψ_X *is called an* X*-universal fpivfs-set, denoted by* $\Psi_{\tilde{X}}$. *If* $X = E$, *then the* X*-universal fpivfs-set* $\Psi_{\tilde{X}}$ *is called a universal fpivfs-set, denoted by* $\Psi_{\tilde{E}}$.

Proposition 3.7 *Let* Ψ_X, Ψ_Y *and* Ψ_Z *be any three FPIVFSS*(U). *Then the following results hold:*

1. $\Psi_X \subseteq \Psi_{\tilde{E}}$,
2. $\Psi_{\emptyset_X} \subseteq \Psi_X$,
3. $\Psi_\emptyset \subseteq \Psi_X$,
4. $\Psi_X \subseteq \Psi_X$,
5. $\Psi_X \subseteq \Psi_Y$ *and* $\Psi_Y \subseteq \Psi_Z \Rightarrow \Psi_X \subseteq \Psi_Z$,
6. $(\Psi_X = \Psi_Y$ *and* $\Psi_Y = \Psi_Z) \Leftrightarrow \Psi_X = \Psi_Z$,
7. $(\Psi_X \subseteq \Psi_Y$ *and* $\Psi_Y \subseteq \Psi_X) \Leftrightarrow \Psi_X = \Psi_Y$.

Proof: The proof is straightforward.

Definition 3.8 *Let* $\Psi_X \in FPIVFSS(U)$. *Then the complement of* Ψ_X, *denoted by* Ψ_X^c, *is defined by* $c(\mu_X(x))$ *and* $\tilde{c}(\eta_X(x))$, $\forall x \in E$, *where* c *is a fuzzy complement and* $\tilde{c}$ *is an interval-valued fuzzy complement.*

Example 3.9 *Consider Example 3.2. By using the basic fuzzy complement for* $\mu_X(x)$ *and interval-valued fuzzy complement for* $\eta_X(x)$ *we have*

$$\Psi_X^c = \left\{ \left(\frac{x_1}{0.7}, \left\{ \frac{u_1}{[0.4,\,0.7]}, \frac{u_2}{[0.2,\,0.3]}, \frac{u_3}{[0.2,\,0.5]}, \frac{u_4}{[0.5,\,0.8]} \right\} \right), \right.$$
$$\left(\frac{x_2}{0.9}, \left\{ \frac{u_1}{[0.3,\,1]}, \frac{u_2}{[0.7,\,0.8]}, \frac{u_3}{[0.5,\,0.7]}, \frac{u_4}{[0.7,\,0.9]} \right\} \right),$$
$$\left(\frac{x_3}{0.6}, \left\{ \frac{u_1}{[0.6,\,0.9]}, \frac{u_2}{[0.4,\,0.6]}, \frac{u_3}{[0.2,0.7]}, \frac{u_4}{[0.5,\,0.6]} \right\} \right),$$
$$\left. \left(\frac{x_4}{0}, \left\{ \frac{u_1}{[0.3,\,0.5]}, \frac{u_2}{[0.6,\,0.9]}, \frac{u_3}{[0.6,\,1]}, \frac{u_4}{[0.1,\,0.2]} \right\} \right) \right\}.$$

Proposition 3.10 *Let* $\Psi_X \in FPIVFSS(U)$. *Then the following results hold:*

1. $(\Psi_X^c)^c = \Psi_X$,

2. $\Psi^{c}_{\emptyset} = \Psi_{\tilde{E}}$.

Proof: The proof is straightforward.

Definition 3.11 *The union of two FPIVFSS(U)* Ψ_X *and* Ψ_Y*, denoted by* $\Psi_X \cup \Psi_Y$*, is defined by*

$$\mu_{X \cup Y}(x) = s(\mu_X(x), \mu_Y(x))$$

and

$$\eta_{X \tilde{\cup} Y}(x) = \eta_X(x) \tilde{\cup} \eta_Y(x)$$

where s is an s-norm and $\tilde{\cup}$ *is an interval-valued fuzzy union.*

Example 3.12 *Consider* Ψ_X *as in Example 3.2. and let* Ψ_Y *be another FPIVFSS(U) defined as follows:*

$$\begin{aligned}\Psi_Y = \Big\{ & \Big(\tfrac{x_1}{0.4}, \Big\{\tfrac{u_1}{[0.3,\ 0.6]}, \tfrac{u_2}{[0.5,\ 0.7]}, \tfrac{u_3}{[0.4,\ 0.5]}, \tfrac{u_4}{[0.1,\ 0.3]}\Big\}\Big), \\ & \Big(\tfrac{x_2}{0.7}, \Big\{\tfrac{u_1}{[0.3,\ 0.5]}, \tfrac{u_2}{[0,\ 0.2]}, \tfrac{u_3}{[0.5,\ 0.6]}, \tfrac{u_4}{[0.4,\ 0.6]}\Big\}\Big), \\ & \Big(\tfrac{x_3}{0.5}, \Big\{\tfrac{u_1}{[0.4,\ 0.6]}, \tfrac{u_2}{[0.2,0.4]}, \tfrac{u_3}{[0.1,\ 0.3]}, \tfrac{u_4}{[0.5,\ 0.6]}\Big\}\Big), \\ & \Big(\tfrac{x_4}{0.2}, \Big\{\tfrac{u_1}{[0.5,\ 0.6]}, \tfrac{u_2}{[0.3,\ 0.5]}, \tfrac{u_3}{[0.3,\ 0.4]}, \tfrac{u_4}{[0.2,\ 0.3]}\Big\}\Big)\Big\}.\end{aligned}$$

By using the basic fuzzy union (maximum) and the interval-valued fuzzy union we have

$$\begin{aligned}\Psi_X \cup \Psi_Y = \Big\{ & \Big(\tfrac{x_1}{0.4}, \Big\{\tfrac{u_1}{[0.3,\ 0.6]}, \tfrac{u_2}{[0.7,\ 0.8]}, \tfrac{u_3}{[0.5,\ 0.8]}, \tfrac{u_4}{[0.2,\ 0.5]}\Big\}\Big), \\ & \Big(\tfrac{x_2}{0.7}, \Big\{\tfrac{u_1}{[0.3,\ 0.7]}, \tfrac{u_2}{[0.2,\ 0.3]}, \tfrac{u_3}{[0.5,\ 0.6]}, \tfrac{u_4}{[0.4,\ 0.6]}\Big\}\Big), \\ & \Big(\tfrac{x_3}{0.5}, \Big\{\tfrac{u_1}{[0.4,\ 0.6]}, \tfrac{u_2}{[0.4,\ 0.6]}, \tfrac{u_3}{[0.3,\ 0.8]}, \tfrac{u_4}{[0.5,\ 0.6]}\Big\}\Big), \\ & \Big(\tfrac{x_4}{1}, \Big\{\tfrac{u_1}{[0.5,\ 0.7]}, \tfrac{u_2}{[0.3,\ 0.5]}, \tfrac{u_3}{[0.5,\ 0.6]}, \tfrac{u_4}{[0.8,\ 0.9]}\Big\}\Big)\Big\}.\end{aligned}$$

Proposition 3.13 *Let* Ψ_X, Ψ_Y *and* Ψ_Z *be any three FPIVFSS(U). Then the following results hold:*

1. $\Psi_X \cup \Psi_X = \Psi_X$,

2. $\Psi_{\emptyset_X} \cup \Psi_X = \Psi_X$,

3. $\Psi_{\emptyset} \cup \Psi_X = \Psi_X$,

4. $\Psi_X \cup \Psi_{\tilde{E}} = \Psi_{\tilde{E}}$,

5. $\Psi_X \cup \Psi_Y = \Psi_Y \cup \Psi_X$.

Proof: The proof is straightforward.

Definition 3.14 *The intersection of two FPIVFSS(U) Ψ_X and Ψ_Y, denoted by $\Psi_X \cap \Psi_Y$, is defined by*

$$\mu_{X\cap Y}(x) = t\left(\mu_X(x), \mu_Y(x)\right)$$

and

$$\eta_{X\tilde{\cap}Y}(x) = \eta_X(x)\,\tilde{\cap}\eta_{\ Y}(x)$$

where t is a t-norm and $\tilde{\cap}$ is an interval-valued fuzzy intersection.

Example 3.15 *Consider example 3.12 again. By using the basic fuzzy intersection (minimum) and the interval-valued fuzzy intersection we have*

$$\Psi_X \cap \Psi_Y = \left\{\left(\frac{x_1}{0.3}, \left\{\frac{u_1}{[0.3,\ 0.6]}, \frac{u_2}{[0.5,\ 0.7]}, \frac{u_3}{[0.4,\ 0.5]}, \frac{u_4}{[0.1,\ 0.3]}\right\}\right),\right.$$

$$\left(\frac{x_2}{0.1}, \left\{\frac{u_1}{[0,\ 0.6]}, \frac{u_2}{[0,\ 0.2]}, \frac{u_3}{[0.3,\ 0.5]}, \frac{u_4}{[0.1,\ 0.3]}\right\}\right),$$

$$\left(\frac{x_3}{0.4}, \left\{\frac{u_1}{[0.1,\ 0.4]}, \frac{u_2}{[0.2,\ 0.4]}, \frac{u_3}{[0.1,\ 0.3]}, \frac{u_4}{[0.4,\ 0.5]}\right\}\right),$$

$$\left.\left(\frac{x_4}{0.2}, \left\{\frac{u_1}{[0.5,\ 0.6]}, \frac{u_2}{[0.1,\ 0.4]}, \frac{u_3}{[0,\ 0.4]}, \frac{u_4}{[0.2,\ 0.3]}\right\}\right)\right\}.$$

Proposition 3.16 *Let Ψ_X, Ψ_Y and Ψ_Z be any three FPIVFSS(U). Then the following results hold:*

1. $\Psi_X \cap \Psi_X = \Psi_X$,

2. $\Psi_{\emptyset_X} \cap, \Psi_X = \Psi_X$,

3. $\Psi_{\emptyset} \cap \Psi_X = \Psi_X$,

4. $\Psi_X \cap \Psi_{\tilde{E}} = \Psi_{\tilde{E}}$,

5. $\Psi_X \cap \Psi_Y = \Psi_Y \cap \Psi_X$.

Proof: The proof is straightforward.

Proposition 3.17 *Let Ψ_X, Ψ_Y be any two FPIVFSS(U). Then De Morgan's law is valid:*

1. $(\Psi_X \cup \Psi_Y)^c = \Psi_X^c \cap \Psi_Y^c$,

2. $(\Psi_X \cap \Psi_Y)^c = \Psi_X^c \cup \Psi_Y^c$.

Proof: 1. For all $x \in E$,

$$\begin{aligned}\mu_{(X\cup\, Y)^c}(x) &= c\left(\mu_{(X\cup Y)}(x)\right)\\ &= c\left(s\left(\mu_X(x), \mu_{\,Y}(x)\right)\right)\\ &= t\left(c\left(\mu_X(x)\right), c\left(\mu_{\,Y}(x)\right)\right)\\ &= t\left(\mu_{X^c}(x), \mu_{\,Y^c}(x)\right)\\ &= \mu_{X^c\cap\, Y^c}(x)\end{aligned}$$

and

$$\begin{aligned}\eta_{(X\tilde{\cup}\, Y)^{\tilde{c}}}(x) &= \tilde{c}\left(\eta_{(X\tilde{\cup} Y)}(x)\right)\\ &= \tilde{c}\left(\eta_X(x)\,\tilde{\cup}\eta_{\,Y}(x)\right)\\ &= \tilde{c}\left(\eta_X(x)\right)\tilde{\cap}\tilde{c}\left(\eta_Y(x)\right)\\ &= 1-\left(\eta_X(x)\right)\tilde{\cap}1-\left(\eta_Y(x)\right)\\ &= \eta_{X^{\tilde{c}}}(x)\,\tilde{\cap}\eta_{Y^{\tilde{c}}}(x)\\ &= \eta_{X^{\tilde{c}}\tilde{\cap}\, Y^{\tilde{c}}}(x).\end{aligned}$$

Likewise, the proof of 2 can be made similarly.

Proposition 3.18 *Let Ψ_X, Ψ_Y and Ψ_Z be any three FPIVFSS(U). Then the following results hold:*

1. $\Psi_X \cup (\Psi_Y \cap \Psi_Z) = (\Psi_X \cup \Psi_Y) \cap (\Psi_X \cup \Psi_Z)$,
2. $\Psi_X \cap (\Psi_Y \cup \Psi_Z) = (\Psi_X \cap \Psi_Y) \cup (\Psi_X \cap \Psi_Z)$.

Proof: 1. For all $x \in E$,

$$\begin{aligned}\mu_{X\cup(Y\cap Z)}(x) &= s\left(\mu_X(x), t\left(\mu_{\,Y}(x), \mu_Z(x)\right)\right)\\ &= t\left(s\left(\mu_X(x), \mu_{\,Y}(x)\right), s\left(\mu_X(x), \mu_Z(x)\right)\right)\\ &= \mu_{(X\cup\, Y)\cap\,(X\cup Z)}(x)\end{aligned}$$

and

$$\begin{aligned}\eta_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= \eta_X(x)\,\tilde{\cup}\left(\eta_{\,Y}(x)\,\tilde{\cap}\eta_Z(x)\right)\\ &= \left(\eta_X(x)\,\tilde{\cup}\eta_{\,Y}(x)\right)\tilde{\cap}\left(\eta_X(x)\,\tilde{\cup}\eta_Z(x)\right)\\ &= \eta_{(X\tilde{\cup}\, Y)\tilde{\cap}\,(X\tilde{\cup}Z)}(x)\end{aligned}$$

Likewise, the proof of 2 can be made similarly.

4 *fpivfs*-Aggregation operator

In this section, we define an aggregate interval-valued fuzzy set of an *fpivfs*-set. We also define *fpivfs*-aggregation operator that produces an aggregate interval-valued fuzzy set from an *fpivfs*-set and its fuzzy parameter set. Also we give an application of this operator in decision making problem.

Definition 4.1 *Let $\Psi_X \in FPIVFSS\,(U)$. Then a fpivfs-aggregation operator, denoted by $FPIVFS_{agg}$, is defined by*

$$FPIVFS_{agg} : F\,(E) \times FPIVFSS\,(U) \to Int\,(U)\,,$$

$$FPIVFS_{agg}\,(X, \Psi_X) = \Psi_X^*$$

where

$$\Psi_X^* = \left\{ u/\mu_{\Psi_X^*}\,(u) : u \in U \right\}$$

which is an interval-valued fuzzy set over U. The value Ψ_X^* is called an aggregate interval-valued fuzzy set of Ψ_X. Here, the membership degree $\mu_{\Psi_X^*}\,(u)$ of u is defined as follows:

$$\mu_{\Psi_X^*}\,(u) = \left[c^- = \frac{1}{|E|} \sum_{x \in E} \mu_X\,(x) \mu_{\eta^- X(x)}\,(u)\,, c^+ = \frac{1}{|E|} \sum_{x \in E} \mu_X\,(x) \mu_{\eta^+ X(x)}\,(u) \right],$$

where $|E|$ is the cardinality of E.

In the following example, we present an application for the *fpivfs*-decision making method.

Example 4.2 *A company wants to fill a position. There are four candidates who form the set of universe, $U = \{u_1, u_2, u_3, u_4\}$. The hiring committee considers a set of parameters, $E = \{x_1, x_2, x_3, x_4\}$. The parameters $x_i\,(i = 1, 2, 3, 4)$ stand for "experience", "computer knowledge", "young age", and "good speaking", respectively. After a serious discussion each candidate is evaluated from point of view of the goals and the constraint according to a chosen fuzzy subset $X = \left\{ \frac{x_1}{0.3}, \frac{x_2}{0.1}, \frac{x_3}{0.4}, \frac{x_4}{1} \right\}$ of E. Finally, the committee constructs the following fpivfs-set over U.*

Step 1 *Let the constructed fpivfs-set, Ψ_X, be given as follows:*

$$\Psi_X = \left\{ \left(\tfrac{x_1}{0.3}, \left\{ \tfrac{u_1}{[0.3,\ 0.6]}, \tfrac{u_2}{[0.7,\ 0.8]}, \tfrac{u_3}{[0.5,\ 0.8]}, \tfrac{u_4}{[0.2,\ 0.5]} \right\} \right), \right.$$

$$\left(\tfrac{x_2}{0.1}, \left\{ \tfrac{u_1}{[0,\ 0.7]}, \tfrac{u_2}{[0.2,\ 0.3]}, \tfrac{u_3}{[0.3,\ 0.5]}, \tfrac{u_4}{[0.1,\ 0.3]} \right\} \right),$$

$$\left(\tfrac{x_3}{0.4}, \left\{ \tfrac{u_1}{[0.1,\ 0.4]}, \tfrac{u_2}{[0.4,\ 0.6]}, \tfrac{u_3}{[0.3,\ 0.8]}, \tfrac{u_4}{[0.4,\ 0.5]} \right\} \right),$$

$$\left(\frac{x_4}{1}, \left\{\frac{u_1}{[0.5,\ 0.7]}, \frac{u_2}{[0.1,\ 0.4]}, \frac{u_3}{[0,\ 0.4]}, \frac{u_4}{[0.8,\ 0.9]}\right\}\right)\right\}.$$

Step 2 *The aggregate interval-valued fuzzy set can be found as*

$$\Psi_X^* = \left\{\frac{u_1}{[0.1575,\ 0.2775]}, \frac{u_2}{[0.1225,\ 0.2275]}, \frac{u_3}{[0.075,\ 0.2525]}, \frac{u_4}{[0.2575,\ 0.32]}\right\}$$

Step 3 $\forall u_i \in U$, *compute the score* r_i *of* u_i *such that*

$$r_i = \sum_{u_j \in U} \left(\left(c_i^- - c_j^-\right) + \left(c_i^+ - c_j^+\right)\right)$$

Thus, we have

$$r_1 = 0.047,\ r_2 = -0.29,\ r_3 = -0.38,\ r_4 = 0.62.$$

Step 4 *The decision is any one of the elements in* S *where* $S = \max_{u_i \in U}\{r_i\}$. *In our example, candidate* u_4 *is the best choice because* $\max_{u_i \in U}\{r_i\} = r_4$. *Hence candidate* u_4 *is selected for the job.*

ACKNOWLEDGEMENT

The authors would like to acknowledge the financial support received from Universiti Kebangsaan Malaysia under the research grant UKM-ST-06-FRGS0104-2009.

References

[1] N. Cagman and S. Enginoglu, Soft matrices and its decision makings, *Computers and Mathematics with Applications,* **59** (2010), 3308 - 3314.

[2] N. Cagman F. Citak, and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy Systems,* **1** (2010), 21 - 35.

[3] D. Chen, E.C.C. Tsang, D.S. Yeung and X. Wang, The parameterization reduction of soft sets and its applications, *Computers and Mathematics with Applications,* **49** (2005), 757 - 763.

[4] F. Feng, Y.B. Jun, X. Liu and L. Li, An adjustable approach to fuzzy soft set based decision making, *Journal of Computational and Applied Mathematics,* **234** (2010), 10 - 20.

[5] M.B. Gorzalczany, A method of inference in approximate reasoning based on interval valued fuzzy sets, *Fuzzy Sets and Systems,* **21** (1987), 1 - 17.

[6] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, *Journal of Fuzzy Mathematics,* **9** (2001), 589 - 602.

[7] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, *Computers and Mathematics with Applications,* **45** (2003), 555 - 562.

[8] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, *Computers and Mathematics with Applications,* **44** (2002), 1077 - 1083.

[9] D.A. Molodtsov, Soft set theory - first results, *Computers and Mathematics with Applications,* **37** (1999), 19 - 31.

[10] X. Yang, T.Y. Lin, J. Yang, Y. Li and D. Yu, Combination of interval-valued fuzzy set and soft set, *Computers and Mathematics with Applications,* **58** (2009), 521 - 527.

[11] L.A. Zadeh, Fuzzy sets, *Information and Control,* **8** (1965), 338 - 353.

[12] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning - I, *Information Sciences,* **8** (1975), 199 - 249.

Received: February, 2011