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# Fuzzy Rule Interpolation Matlab Toolbox – FRI Toolbox

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*Abstract* — **In most fuzzy systems, the completeness of the fuzzy rule base is required to generate meaningful output when classical fuzzy reasoning methods are applied. This means, in other words, that the fuzzy rule base has to cover all possible inputs. Regardless of the way of rule base construction, be it created by human experts or by an automated manner, often incomplete rule bases are generated. One simple solution to handle sparse fuzzy rule bases and to make infer reasonable output is the application of fuzzy rule interpolation (FRI) methods. In this paper, we present a Fuzzy Rule Interpolation Matlab Toolbox, which is freely available. With the introduction of this Matlab Toolbox, different FRI methods can be used for different real time applications, which have sparse or incomplete fuzzy rule base.**

## I. INTRODUCTION

FUZZY systems use fuzzy rule base to make inference. A fuzzy rule base is fully covered (at level $\alpha$), if all input universes are covered by rules at level $\alpha$. Such fuzzy rule bases are also called dense or complete rule bases. In practice, it means that for all the possible observations there exists at least one (at least partially) matching rule, whose antecedent part overlaps the input data at level $\alpha$. If this condition is violated, the rule base is considered to be sparse, i.e. it contains gaps. The classical fuzzy reasoning techniques like Zadeh's, Mamdani's, Larsen's or even Sugeno's cannot generate an acceptable output for such cases. Fuzzy rule based interpolation (FRI) techniques were introduced to generate inference for sparse fuzzy rule base, thus extend the usage of fuzzy inference mechanisms for sparse fuzzy rule base systems. Basically, FRI techniques perform interpolative approximate reasoning by taking into consideration the existing fuzzy rules for cases where there is no matching fuzzy rule.

There are several FRI techniques that satisfy the general applicability conditions introduced in [13]. These techniques can be divided into two groups depending on whether they generate the estimated conclusion directly or in two steps: first creating an intermediate rule by interpolation, and then specifying the conclusion.

Relevant members of the first group are the KH method [1] proposed by Kóczy and Hirota, MACI [2] (Tikk and Baranyi), FIVE [3] (Kovács and Kóczy), IMUL [4], [19] (Wong, Gedeon, and Tikk), the method based on the conservation of the relative fuzziness [5] (Kóczy, Hirota, and Gedeon), the interpolative reasoning based on graduality [6] (Bouchon-Meunier, Marsala, and Rifqi), and VKK method [7] (Vass, Kalmár and Kóczy). The methods belonging to the second group are described best by the generalized methodology (GM) defined by Baranyi et al. in [8]. Other typical members of this group are the ST method [9] (Yan, Mizumoto, and Qiao), the IGRV [10] developed by Huang and Shen, and the technique proposed by Jenei [11]. More details on most of these methods will be described in Section III.

The rest of this paper is organized as follows. Section II presents the background of FRI and introduces numerous comparison conditions for such methods. Section III gives a short overview of FRI methods with special emphasis on those that will be included in our FRI Matlab toolbox. Section IV introduces the toolbox itself. Finally, Section V gives the conclusions.

## II. BACKGROUND OF FUZZY RULE INTERPOLATION

### A. Notation

We use the conventional notations for fuzzy sets. $A$ and $B$ denote fuzzy sets of input and output universes, respectively. An $n$-dimensional MIMO (multi input, multi output) fuzzy rule, $R_i$, is formulated as:

$$R_i : A_{i1}, A_{i2}, \dots, A_{in} \rightarrow B_{i1}, B_{i2}, \dots, B_{im} \quad (1)$$

where the first lower index refers to the rule, and the second index to the dimension. The observation and the conclusion are denoted by a star superscript: $A^*$, $B^*$. We refer to an $\alpha$-cut of a fuzzy set as $A_\alpha$, where $A$ denotes the set itself. The subscript indicating the cut precedes all other subscripts.

### B. Justification of FRI methods

The main purpose to introduce FRI was to break down the computational complexity required in most classical fuzzy reasoning methods [12]. Rule interpolation is efficient if the shape of the fuzzy set is simple, mostly piecewise linear, for example triangular or trapezoidal. In such cases, fuzzy sets can be described by only a few characteristic points. It should be noted that an α-cut based FRI method should determine the conclusion based on a sufficient number of α-cuts, i.e. based on the characteristic points of the involved fuzzy sets. Otherwise the calculation could become too

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"expensive". Although it could be expected that the conclusion preserves the linearity of the premises, it is not always satisfied; i.e., the shape of the conclusion can be different from the shape of the other involved sets.

### C. General conditions on rule interpolation methods

In this section, we briefly review the general conditions related to the interpolative methods introduced in [13] for the evaluation and comparison of the different techniques based on the same fundamentals. The conditions reflect an application-oriented viewpoint.

1. *Avoidance of the abnormal conclusion*. The estimated fuzzy set should be a valid one. This requisite can be described by the constraints (2) and (3).

$$\inf\{B^*_\alpha\} \le \sup\{B^*_\alpha\} \qquad \forall \alpha \in [0,1] \qquad (2)$$

$$\inf\{B^*_{\alpha 1}\} \le \inf\{B^*_{\alpha 2}\} \le \sup\{B^*_{\alpha 2}\} \le \sup\{B^*_{\alpha 1}\} \quad \forall \alpha_1 < \alpha_2 \in [0,1] \qquad (3)$$

   where $\inf\{B^*_\alpha\}$ and $\sup\{B^*_\alpha\}$ are the lower and upper endpoints of the actual α-cut of the estimated fuzzy set.

2. *The continuity of the mapping between the antecedent and consequent fuzzy sets should be consistent*. This condition indicates that similar observations should lead to similar results.

3. *Preserving the "in between"*. If the antecedent sets of two neighboring rules surround an observation, the approximated conclusion should be surrounded by the consequent sets of those rules as well.

4. *Compatibility with the rule base*. This condition requires the validity of the *modus ponent*, i.e. if an observation coincides with the antecedent part of a rule, the conclusion produced by the method should correspond to the consequent part of that rule.

5. *The fuzziness of the approximated result*. There are two opposite approaches in the literature related to this topic. According to the first subcondition (5.a), the less uncertain the observation is, the less fuzziness should have the approximated consequent. In other words, in case of a singleton observation the method should produce a singleton consequence. The second approach (5.b) originates the fuzziness of the estimated consequent from the nature of the fuzzy rule base. Thus, crisp conclusion can be expected only if all the consequents of the rules taken into consideration in the interpolation are singleton, i.e. the knowledge base produces certain information from fuzzy input data.

6. *Approximation capability* (*stability*). The estimated rule should approximate with the highest possible degree the relationship between universes of the antecedent and consequent. If the number of the measurement points tends to infinity, the result should converge to the approximated function independently from the position of the measurement points.

7. *Preserving the piece-wise linearity*. If the fuzzy sets of the rules taken into consideration are piece-wise linear, the approximated sets should preserve this feature.

8. *Applicability in case of multidimensional antecedent universe*. This condition indicates that an FRI technique should present similar characteristics when being extended and applied to multidimensional input spaces.

9. *Applicability without any constraint regarding to the shape of the fuzzy sets*. This condition can be weakened practically to the case of piece-wise linear, and Gauss-bell shaped fuzzy sets, being the most frequently encountered in the applications.

## III. Overview of Fuzzy Rule Interpolation Techniques

The first FRI technique was published by Kóczy and Hirota [1]. It is referred to as *KH method*. It is applicable to convex and normal fuzzy (CNF) sets. It determines the conclusion by its *α*-cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with the ones between the observation and the antecedents for all important *α*-cuts. The formula is

$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2), \qquad (4)$$

that is called the *fundamental equation of rule interpolation* (FERI), which can be solved for $B^*$ for relevant *α*-cuts after decomposition. Here $A_1 \to B_1$ and $A_2 \to B_2$ form the pair of flanking rules for the observation $A^*$, and $d$: $F(X)\times F(X) \to R$ is a distance function of fuzzy sets. The solution of (4) for the simplest SISO (single input, single output) case is:

$$B^*_{\alpha C} = \frac{\sum_{i=1}^{2} B_{\alpha iC}\left(\frac{1}{d_C(A^*_{\alpha C}, A_{\alpha iC})}\right)}{\sum_{k=1}^{2} \frac{1}{d_C(A^*_{\alpha C}, A_{\alpha kC})}}, \qquad (5)$$

These are called the formulae of linear KH interpolator. Here $C \in \{L, U\}$ where $L$ and $U$ denote "lower" and "upper" extreme of the *α*-cut or fuzzy distance, respectively.

It is shown in, e.g., [13] that KH method violates condition 1, i.e. the conclusion is not directly interpretable as fuzzy sets (see also Figure 1). This drawback motivated many researchers in finding alternative solutions. An obvious modification was proposed by Vass, Kalmár and Kóczy [7] (termed *VKK method*), where the conclusion is computed based on the distance of the centre points and the widths of the α-cuts, instead of lower and upper distances. The VKK method decreases the applicability limit of KH method, but does not eliminate it completely. The technique cannot be applied if any of the antecedent sets is singleton (the width of the antecedent's support must be nonzero).

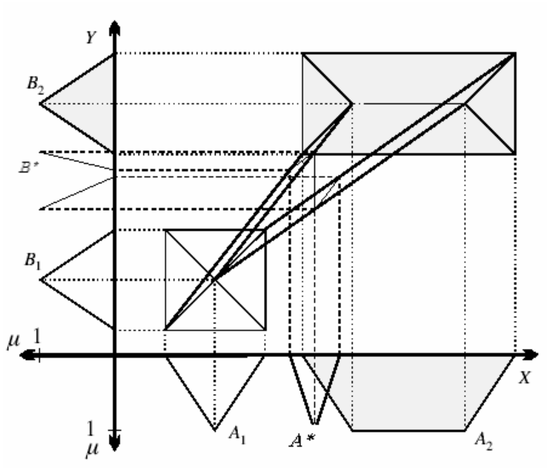


Fig. 1. Abnormal conclusion generated by the KH technique

Despite the above disadvantage, KH is popular because its simplicity that infers its advantageous complexity properties. It was generalized in several ways. Among them the *stabilized KH interpolator* (6) is emerged, as it is proved to hold the universal approximation property [15], [16].

$$B^*_{\alpha C} = \frac{\sum_{i=1}^{n} B_{\alpha iC} \left( \frac{1}{d_C^N (A^*_{\alpha C}, A_{\alpha iC})} \right)}{\sum_{k=1}^{n} \frac{1}{d_C^N (A^*_{\alpha C}, A_{\alpha kC})}} \quad (6)$$

This method takes into account all flanking rules of an observation in the calculation of the conclusion in extent to the inverse of the distance of antecedents and observation. The universal approximation property holds if the distance function is raised to the power of $N$ (input's dimension).

Another modification of KH is the modified alpha-cut based interpolation (*MACI*) method [2], which alleviates completely the abnormality problem. MACI's main idea is the following: it transforms fuzzy sets of the input and output universes to such a space where abnormality is excluded, then computes the conclusion there, which is finally transformed back to the original space. MACI uses vector representation of fuzzy sets and originally was applicable to CNF sets [17]. These latter conditions (convexity and normality of fuzzy sets) can be relaxed, but it increases the computational need of the method considerably [18] (cf. condition 9). MACI is one of the most applied FRI methods [19], since it preserves advantageous computational and approximate nature of KH, while it excludes its abnormality.

Another fuzzy interpolation technique was proposed by Kóczy et al. [5] that is related to condition 5a. It is called conservation of "relative fuzziness" (*CRF*) method, which notion means that the left (right) fuzziness of the approximated conclusion in proportion to the flanking fuzziness of the neighboring consequent should be the same as the (left) right fuzziness of the observation in proportion to the flanking fuzziness of the neighboring antecedent. The technique is applicable to CNF sets. The authors showed that this method has immediate connection with FERI.

A new improved fuzzy interpolation technique for multidimensional input spaces (*IMUL*) was proposed in [4], and described in details in [19]. IMUL applies a combination of CRF and MACI methods, and mixes advantages of both. The core of the conclusion is determined by MACI method, while its flanks by CRF. The main advantages of this method are its applicability for multi-dimensional problems and its relative simplicity. It is therefore ideal for real-word problems.

A rather different application oriented aspect of the fuzzy rule interpolation emerges in the concept of *FIVE*. The fuzzy reasoning method "FIVE" (Fuzzy Interpolation based on Vague Environment, originally introduced in [21], [22]) was developed to fit the speed requirements of direct fuzzy control, where the conclusions of the fuzzy controller are applied directly as control actions in a real-time system (see [3]).

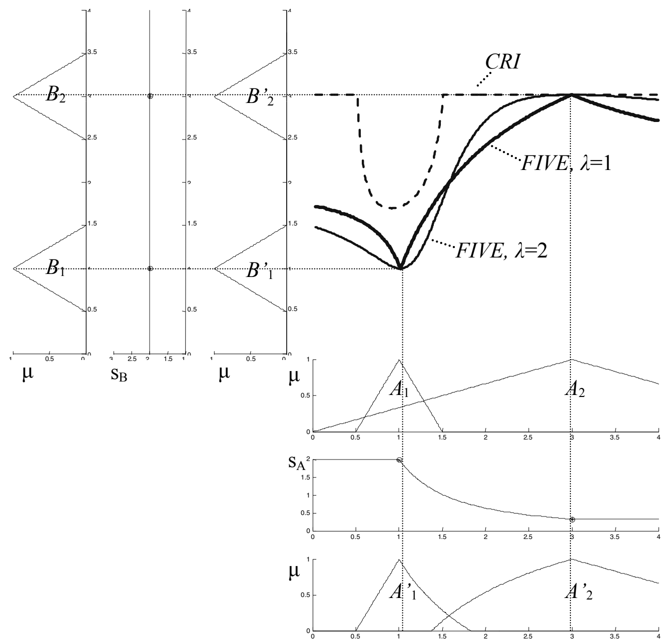


Fig. 2. Interpolation of two fuzzy rules ($R_i$: $A_i \rightarrow B_i$), by the Shepard operator based *FIVE*, and for comparison the min-max *CRI* with the centre of gravity defuzzification.

The main idea of the FIVE is based on the fact that most of the control applications serves crisp observations and requires crisp conclusions from the controller. Adopting the idea of the vague environment (VE) [20], FIVE can handle the antecedent and consequent fuzzy partitions of the fuzzy rule base by scaling functions [20] and therefore turn the fuzzy interpolation to crisp interpolation.

The idea of a VE is based on the similarity (in other words: indistinguishability) of the considered elements. In VE the fuzzy membership function $\mu_A(x)$ is indicating level of similarity of $x$ to a specific element $a$ that is a representative or prototypical element of the fuzzy set $\mu_A(x)$, or, equivalently, as the degree to which $x$ is indistinguishable from $a$ [20]. Therefore the α-cuts of the fuzzy set $\mu_A(x)$ are the sets which contain the elements that are $(1-\alpha)$-indistinguishable from $a$. Two values in a VE are ε-distinguishable if their distance is greater than ε. The distances in a VE are weighted distances. The weighting factor or function is called *scaling function (factor)* [20]. If VE of a fuzzy partition (the scaling function or at least the approximate scaling function [21], [22]) exists, the member sets of the fuzzy partition can be characterized by points in that VE (see e.g. scaling function $s$ on Figure 2). Therefore any crisp interpolation, extrapolation, or regression method can be adapted very simply for FRI [21], [22]. Because of its simple multidimensional applicability, in FIVE the *Shepard operator* based interpolation (first introduced in [23]) is adapted (see e.g. Figure 2).

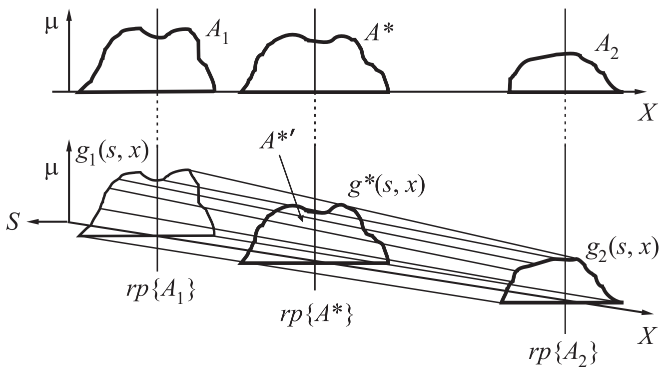


Fig. 3. Formation of solid in the input dimension and determination of $A^{*\prime}$

Conceptually different approaches were proposed by Baranyi et al [8] based on the relation and on the semantic and inter-relational features of the fuzzy sets. The family of these methods applies GM; this notation also reflects to the feature that these methods are able to process arbitrary shaped fuzzy sets. The basic concept is to calculate the reference point of the conclusion based on the ratio of the distances between the reference points of the observation and the antecedents. Due to the modular structure of the methodology several techniques can be applied in its two steps.

For example the solid cutting method (*SCM*) is used in the first step of the GM. Its key idea is that all involved sets are rotated by 90º around a vertical axis going through their reference point; then by connecting the corresponding points of antecedents and consequents two solids can be formed: one in the input and one in the output dimension. In figure 3, the solid formed in an input dimension is depicted. The solids are cut at the centres of the observation and at the location of the conclusion, respectively, which results in the set $A^{*\prime}$ in the input space and in the set $B^{*\prime}$ in the output space.

The fuzzy set approximation technique FEAT-p proposed by Johanyák and Kovács in [28] is also applicable in the first step of the GM. It comes from the assumption that a better set approximation can be attained by taking into consideration all the sets in the partition. First, all the sets are shifted horizontally in order to reach the coincidence of the abscissa of their reference points with the abscissa of the reference point of the observation. Next, the shape of the new set is determined from the collection of the overlapped sets by introducing the concept of the polar cut (see Figure 4) defined by the polar distance *rho* and the angle $\theta$ and assuming that a resolution and an extension principle can be defined for polar cuts, too. Its main advantages are that it can handle subnormal sets, and it is applicable for extrapolation, too.

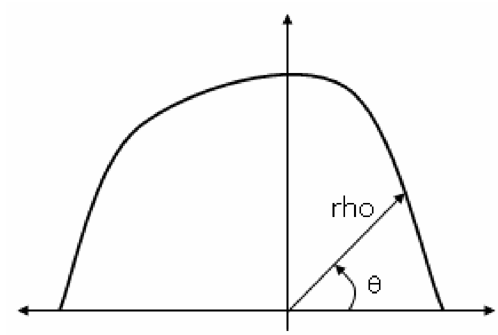


Fig. 4. Polar cut

In the second step of the GM a single rule reasoning method (e.g. revision function) is used to determine the final conclusion $B^*$ based on the similarity of the observation $A^*$ and the "interpolated" observation $A^{*\prime}$. The detailed description can be found for example in [8].

The methods following the GM have numerous advantages, such as:

- they always give an interpretable conclusion as a "real" fuzzy set, i.e., any abnormal shape of the conclusion is precluded;
- they can be applied to arbitrary shaped fuzzy sets, i.e., neither convexity nor normality is prescribed, only the centres of the sets have to be ordered. It means that some part of the observation can even exceed the support of antecedents;
- versions specialized for piecewise linear fuzzy sets produce piecewise linear fuzzy set as conclusions, hence methods are shape-invariants.

The only problematic point of some of these methods is that the calculation of the conclusion even for the special piecewise linear case requires considerable time, thus one of the most important reasons for inventing FRI techniques is violated or at least partly neglected.

## IV. FRI Matlab Toolbox

### A. General description

The Fuzzy Rule Interpolation Toolbox (FRI TB) is a collection of Matlab functions implementing interpolation based fuzzy inference techniques. The current version supports nine FRI methods (KH, the stabilized version of the KH, MACI, IMUL, CRF, FIVE, VKK, GM with SCM, FERI, and FPL, and GM with FEAT-p, FERI, and FPL), but the number of the included techniques is continuously growing. The whole toolbox is available for download under GNU General Public License from the web site [25]. The FRI TB was developed using Matlab 7 (R14) under Microsoft Windows XP.

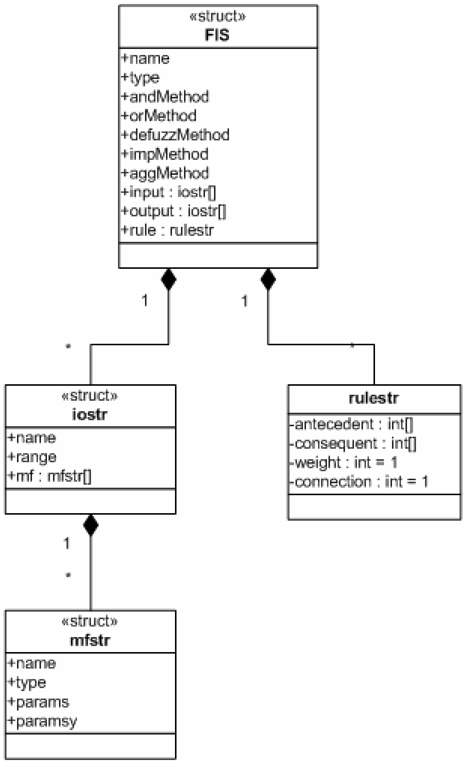


Fig. 5. FIS data structure

The FRI TB can be viewed as an extension of the standard Fuzzy Logic Toolbox (FL TB). It uses the FIS data structure extended with new parameters (see Figure 5) enabling to define subnormal membership functions (i.e. linguistic terms with height smaller than 1). Its data loader function (*ireadfis*) can equally use the new and the original FIS file format. This feature enables that a fuzzy system created by the FL TB can be reused, loaded, and evaluated by our program, which simplifies the comparison of results obtained by different inference methods. The new FIS data file format differs from the original only in the description of the membership functions. For example the line

MF1='mf1':'trimf',[0.4 0.6 0.8]![0 0.8 0]

describes a triangular shaped linguistic term, of which characteristic points are {0.4, 0.0},{0.6, 0.8}, and {0.8, 0.0}. This extension was necessary because the FL TB presume the normality of the sets and therefore the ordinate values of the characteristic points are not stored.

```
NumInputs=4
ObsName='Obs_4D_Trap_01'
[Observation]
OBS1='observation':'trapmf',[0.45 0.45 0.65 0.65]![0 1 1 0]
OBS2='observation':'trapmf',[0.55 0.65 0.70 0.75]![0 1 1 0]
OBS3='observation':'trapmf',[0.45 0.45 0.65 0.80]![0 1 1 0]
OBS4='observation':'trapmf',[0.45 0.60 0.65 0.70]![0 1 1 0]
```

Fig. 6. The observation data file

The FRI Toolbox contains a collection of sample FIS data files. Their naming convention can be explained in the easiest way through an example. The file

In_4D_Out_2D_N_01.fis

defines a FIS with 4 input and 2 output dimensions. Each set is normal. The digits at the end (*01*) denote that this one is the first from its group.

The data describing the observation are also read from a text file with a structure similar to the structure of the FIS file and having the extension *obs*. Figure 6 presents the data file of an observation with 4 dimensions. Each line in the section *[Observation]* describes a fuzzy set of the input in each dimension. The meaning of their elements is the same as in the case of the FIS data file. Let us review the naming convention of the sample observation files through an example. In

Obs_4D_Trap_01.obs

the meaning of the first part is obvious, *4D* denotes that it is 4 dimensional, *Trap* indicates that all the four sets are trapezoidal shaped, and the digits at the end (*01*) denote that this is the first from its group. In the memory the observation is stored as an array of structures (*obsstr*). The fields of this structure are presented in Figure 7.

The input and output universes can be multi-dimensional, the number of dimensions is not restricted. The system supports piece-wise linear membership functions (singleton, triangular, trapezoidal, and polygonal) for the most part of the methods. The method FIVE enables only singleton shaped observations.

The current version of the toolbox enables only the use of convex and normal fuzzy sets in the rules and in the observation, as well. The range of the linguistic variables has to be [0,1]. Extrapolation is not supported.

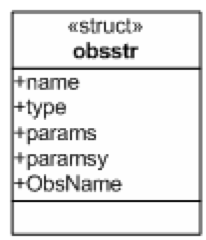


Fig. 7. The structure describing the observation

Each method requires the existence of at least two such rules, which surround the observation in each dimension. At interpolation, it is an important task is to find flanking rules. If the observation coincides with the antecedent part of a rule, the rule is viewed as right or left flanking depending on the existence of other left or right flanking rules. For example if there is no left flanking rule the actual rule is considered as left flanking one.

Most of the papers presenting the FRI methods specify an initial condition related to the ordering of the antecedent and consequent parts of the surrounding rules that can be expressed for the one dimensional case by (7) and (8).

$$A_1 \prec A^* \prec A_2 \quad (7)$$

$$B_1 \prec B_2 \quad (8)$$

where $A_i$ and $B_i$ denote the antecedent and consequent sets of the left respectively right flanking rules. In the real world applications, this condition cannot be always fulfilled.

Therefore most of our implementations do not require it.

### *B. Parameters of the method*

In the case of the α-cut based techniques the number of the α-levels for which the calculations are made can be set by the user. IMUL, MACI, and the techniques belonging to the GM family use a reference point for the characterization of the position of each fuzzy set. The type of this reference point is a parameter of their implementations. Most of the techniques calculate multidimensional distances in the Minkowski sense. The parameter $w$ of the formula can also be set by the user. Its default value is 2.

The method FIVE uses the Shepard interpolation. Its power factor can be given as parameter; by default it is equal to the antecedent dimensions of the rule base. The user can choose between linear and non-linear scaling factor approximations. The technique FEAT-p takes into consideration all fuzzy sets that belong to the partition with different weight values. The type of the weighting factor and its parameters also can be set by the user.

### *C. The usage of the software*

The functions can be used from command line or from a graphical interface. The current version of the GUI is simple and easy to use. It can be started by typing in the Command Window with the command *GraphTest*. First, the location of FIS and the observation data should be given (see Figure 8), which can be done through the standard file open dialog box.

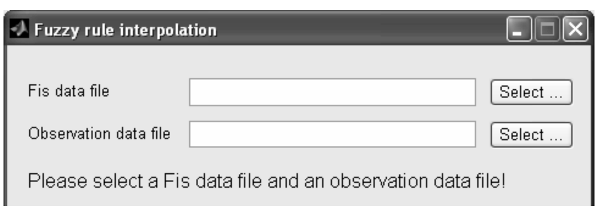


Fig. 8. Specifying FIS and observation

After the selection and load of the data, a new panel appears that enables to the user to choose an inference method (Figure 9).

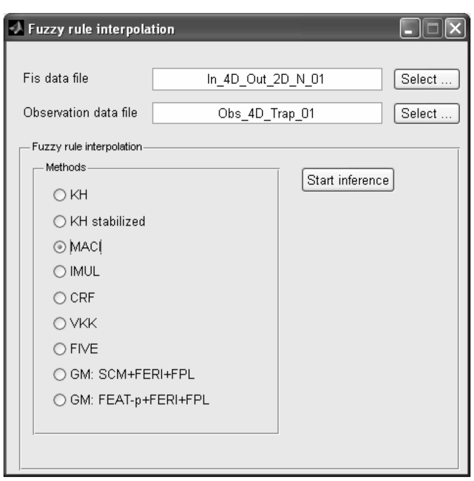


Fig. 9. Selection of inference method

The evaluation of the modeled fuzzy system, i.e. the inference process starts by pressing the *Start inference* button. The input and output universes are represented in two separate windows each containing the same number of diagrams as the dimension of the input and output respectively.

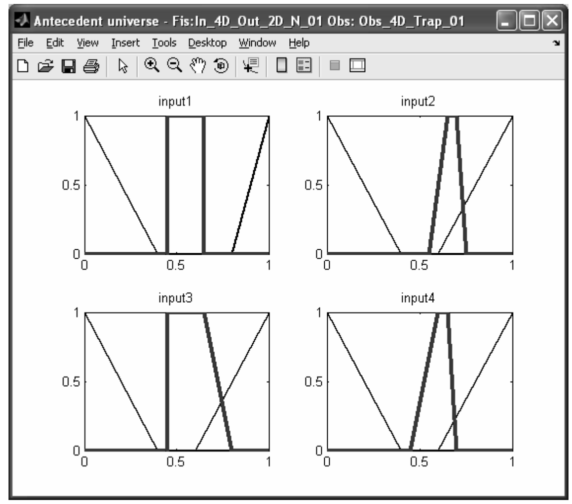


Fig. 10. Antecedent partitions and observation

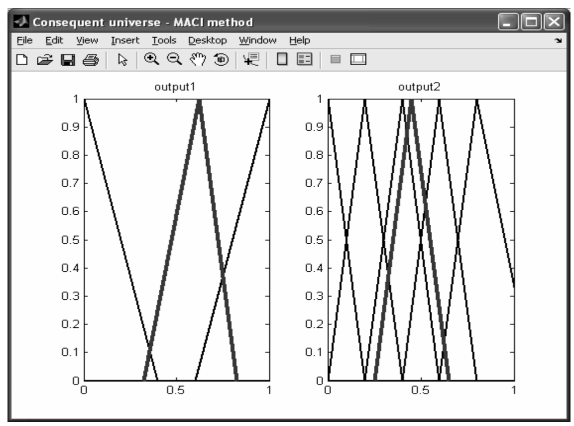


Fig. 11. Consequent partitions and conclusion

In the example presented on Figures 10 and 11 the fuzzy system has 4 input and 2 output dimensions, the knowledge base contains only two rules, and the observation is of trapezoid shape. The result was inferred by MACI method.

The sets representing the observation and the conclusion respectively; the sets belonging to the interpolated intermediate rule in the case of the techniques belonging to the group of two-steps methods described by the GM, are represented by thin and colored lines.

In the case of the method FIVE the scaling functions describing the input and output universes are visualized as, well

### *D. Further development plans*

The FRI Toolbox is under continuous development. We plan its development in three main directions.

- Extending the existent implementations in order to support all kinds of polygonal membership functions even the subnormal and non-convex ones in case of the methods whose definition cover these non-regular cases, too.

- Implementation of new methods and techniques. From the first group of FRI methods among others, we are going to include the interpolative reasoning based on graduality [6]. From the second group, we are going to implement the similarity transfer based method [9], the semantic revision based methods (SRM I-II) [26], and the α-cut based FEAT-α technique [27].
- The graphical user interface will also be extended in order to ensure the input of FIS and observation related data interactively.

## V. Conclusions

Fuzzy rule interpolation techniques extend the applicability of fuzzy rule based reasoning methods for the case when the rule base is sparse or incomplete. We gave a brief ovreview of the popular FRI methods and the comparison conditions. The paper introduced the FRI Matlab Toolbox, a freely available public tool that serves as the comparison test bed for FRI techniques and offers straightforward application possibility for real world problems.

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