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Fuzzy Separability and Axioms of Countability in Fuzzy Hyperspaces

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ABSTRACT

We study some relations between separability in fuzzy topological spaces and one in fuzzy hyperspaces. And we investigate some properties of axiom of countability in fuzzy hyperspaces.

Key words : fuzzy separability, axioms of countability, compactly C_{II}, fuzzy hyperspace.

1. Introduction and preliminaries

In 2000, K.Hur, J.R.Moon and J.H.Ryou[5] introduced the concept of a fuzzy hyperspace and studied some of its properties. In this paper, we study some relations between separability in fuzzy topological space and we in fuzzy hyperspaces. And we investigate some properties of axioms of countability in fuzzy hyperspaces.

We will list some concepts and properties needed in the later section.

Let $I=[0,1]$ and let $I_0=(0,1]$. For a set X, let I^X be the collection of all the mappings from X into I. Then each member of I^X, $A:X\to I$, is called a fuzzy set in X(cf. [2,10,13]). In particular, $\emptyset$ and X can be considered as fuzzy sets in X defined by $\emptyset(x)=0$ and $X(x)=1$ for each $x\in X$, respectively.

The concept of a fuzzy point and its properties refer to [8,10,12]. And we will denote the set of all fuzzy points in a set X as $F_p(X)$.

Definition 1.1[1]. Let X be a nonempty set. Then a fuzzy set A in X is called :

(1) *an upper fuzzy set* if $A(x)>\frac{1}{2}$ whenever $A(x)\neq 0$ for each $x\in X$.

(2) *a lower fuzzy set* if $A(x)<\frac{1}{2}$ whenever $A(x)\neq 1$ for each $x\in X$.

It is clear that the only fuzzy sets in X which are both upper and lower fuzzy sets are $\emptyset$ and X.

Throughout this paper, we use the fuzzy topological space defined by Chang[2]. For a fts X, we will denote the family of all F-open sets and F-closed sets in X as $FO(X)$ and $FC(X)$, respectively.

Definition 1.2[3]. A fts X is said to be *fuzzy* T_1(in short, FT_1) if for any two fuzzy points x_λ and y_μ in X ; $x\neq y$

(case 1) When , there exist $U, V\in FO(X)$ such that $x_\lambda\in U$, $y_\mu \bar{q} U$ and $y_\mu\in V$, $x_\lambda \bar{q} V$.

(case 2) When $x=y$ and $\lambda<\mu$(say), there exists a $U\in FO(X)$ such that $y_\mu q U$ and $x_\lambda \bar{q} U$.

Definition 1.3[4]. A fuzzy set A in a fts X is said to be *fuzzy compact*(in short, *F-compact*) in X if for each F-filter base β such that for any finite subcollection $\{B_i : i=1,\cdots,n\}$ of β,

$$(\bigcap_{i=1}^{n} B_i)qA,\ (\bigcap_{B\in\beta} \mathrm{cl}B)\cap A \neq \emptyset.$$

Definition 1.4. Let X be a fts.

(1) $\Omega\subset F_{p(X)}$ is said to be *dense*(resp. *Q-dense*) in X [11] if for each $\emptyset\neq U\in FO(X)$, there exists $x_\lambda\in\Omega$ such that $x_\lambda\in U$(resp. $x_\lambda q U$).

(2) $A\in I^X$ is said to be *fuzzy dense*(in short, *F-dense*) in X[9] if $\mathrm{cl}A=X$.

It is clear that the concept of being dense and that of being Q-dense do not imply each other.

Definition 1.5.

(1) *Separable*(*i*)(resp. Q-separable) [11] if there exists a sequence $\{x_{n,\lambda_n}\}_{n\in N}$ of fuzzy points in X such that $\{x_{n,\lambda_n}\}_{n\in N}$ is dense(resp. Q-dense) in X.

(2) *Separable*(*ii*)[12] if there exists a sequence $\{x_{n,\lambda_n}\}_{n\in N}$ of fuzzy points in X such that for each $\emptyset\neq U\in FO(X)$, there exists an x_{n,λ_n} such that $x_{n,\lambda_n}\in U$.

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It is clear that X is separable(i) if and only if it is separable(ii).

Although the concept of being dense and that of being Q-dense do not imply each other, but we have the following.

Result 1.A[11, Proposition 5.1]. A fts X is separable (i) if and only if it is Q-separable.

In the light of Result 1.A, from now on, we shall make no difference between separable(i)(*F-separable*) spaces and Q-separable spaces. For convenience, they are both called fuzzy separable(*F-separable*) spaces.

Definition 1.6. A fts X is said to:

(1) *satisfy the first axiom of countability* or be C_I [12] if every fuzzy point in X has a countable local base.

(2) *satisfy the Q-first axiom of countability* or be $Q-C_I$ [10] if every fuzzy point in has a countable Q-local base.

Result 1.B[10, Proposition 3.1]. If X is a C_I-space, then it is a $Q-C_I$-space.

Definition 1.7[12]. A fts (X, T) is said to *satisfy the second axiom of countability* or to be C_{II} if there exists a countable base B for T.

2. Separability and axioms of countability in fuzzy hyperspaces

Notations 2.1. For a fts X, let

$I_0^X = \{E: E$ is a nonempty F-closed set in $X\}$,

$I_0^A = \{E\in I_0^X: E\subset A\}$, where $A\in I^X$,

$K(X) = \{E\in I_0^X: E$ is F-compact in $X\}$,

$F_n(X) = \{E\in I_0^X: E$ has at most n elements $\}$,

$F(X) = \{E\in I_0^X: E$ is finite $\}$.

Definition 2.2[5]. Let X be a fts. Then the *fuzzy Vietoris*(or *finite*) *topology* T_v on I_0^X is the generated by the collection of the forms $\langle U_1, \cdots, U_n\rangle$ with $U_i\in FO(X)$ for each $i=1, \cdots, n$. The pair (I_0^X, T_v) is called *a fuzzy hyperspace with fuzzy Vietoris topology*(*in short, fuzzy hyperspace*).

It is clear that $K(X)$, $F_n(X)$ and $F(X)$ are subspaces of I_0^X.

Result 2.A[6, Theorem 3.7]. Let X be a FT_1-space and let $U_i, V_j\in I^X$ upper fuzzy sets in X for each $i=1, \cdots, n$ and each $j=1, \cdots, m$. Then $\langle U_1, \cdots, U_n\rangle \subset \langle V_1, \cdots, V_m\rangle$ if and only if $\bigcup_{i=1}^n U_i \subset \bigcup_{j=1}^m V_j$ and for each V_j there exists U_i such that $U_i\subset V_j$.

Result 2.B[5, Theorem 3.7]. Let X be a FT_1-space. $F(X)$ is dense in I_0^X.

Definition 2.3[7]. A fts X is said to be *finitely F-compact* if each finite fuzzy set in X is F-compact in X.

Theorem 2.4. If X is a finitely F-compact FT_1-space, then $K(X)$ is dense in I_0^X.

Proof. Let $E\in I_0^X$ and let $\mathrm{U} = \langle U_1, \cdots, U_n\rangle \cap K(X)\cap K(X)$ such that $E\in \mathrm{U}$, where $\langle U_1, \cdots, U_n\rangle$ is a base member for T. Then $E\subset \bigcup_{i=1}^n U_i$ and EqU_i for each $i=1, \cdots, n$. Let $x_{i,\lambda_i}qU_i$ for each $i=1, \cdots, n$. Let $F=\{x_{1,\lambda_1}, \cdots, x_{n,\lambda_n}\}$. Since X is FT_1, $F\in I_0^X$. Moreover, $F\subset \bigcup_{i=1}^n U_i$ and FqU_i for each $i=1, \cdots, n$. Then $F\in \langle U_1, \cdots, U_n\rangle$. Since F is finite, by the hypothesis, $F\in K(X)$. Then $F\in \langle U_1, \cdots, U_n\rangle \cap K(X) \neq \emptyset$. Thus $E\in \mathrm{cl}\, K(X)$, i.e., $I_0^X\subset \mathrm{cl}\, K(X)$. So $\mathrm{cl}\, K(X) = I_0^X$. Hence $K(X)$ is dense in I_0^X.

Definition 2.5[6]. A fts X is called a $(q,\in)$-*fuzzy topological space*(in short, $(q,\in)$-fts) if for each $U\in FO(X)$, U is a $(q,\in)$-fuzzy set in X, i.e., there exists $x_\lambda\in F_p(X)$ such that $x_\lambda qU$ if and only in $x_\lambda\in U$.

Theorem 2.6. Let X be a $(q,\in)-FT_1$-space. Then X is F-separable if and only if I_0^X is separable.

Proof. ($\Rightarrow$) : Suppose X is F-separable. Then, by Result 1.A, X is Q-separable. By Definition 1.5, there exists a sequence $D=\{x_{n,\lambda_n}\}_{n\in N}$ of fuzzy points in X such that D is Q-dense in X. Let β be the collection of finite subsets of D. Then β is countable. Let $\langle U_1, \cdots, U_m\rangle$ be a base member for T_v. Since D is Q-dense in X and $\emptyset \neq U_i\in FO(X)$ for each $i=1, , \cdots, m$ there exists $n_i\in N$ such that $x_{n_i,\lambda_i}\in D$ and $x_{n_i,\lambda_i}qU_i$ for each $i=1, \cdots, m$. Since X is a $(q,\in)$-fts, $x_{n_i,\lambda_i}\in U_i$ for each $i=1, \cdots, m$. Let $E=\{x_{n_1,\lambda_1}, \cdots, x_{n_m,\lambda_m}\}$ Since X is FT_1, $E\in I^X$. Moreover, EqU_i for each $i=1, \cdots, n$ and $E\subset \bigcup_{i=1}^n U_i$. Then $E\in \mathrm{B}\cap \langle U_1, \cdots, U_n\rangle$. Thus B is countable dense in I_0^X. Hence

I_0^X is separable.

($\Leftarrow$): Suppose I_0^X is separable. Let $\beta = \{A_n\}_{n \in N}$ be a countable dense subset of I_0^X. For each $n \in N$, choose a fuzzy point $a_{n, \lambda_n} \in A_n$. Let $D = \{a_{n, \lambda_n}\}_{n \in N}$. Now let $U \in FO(X)$. Then clearly $\langle U \rangle$ is open in I_0^X. Since β is dense in I_0^X, $\beta \cap \langle U \rangle \neq \emptyset$. Let $A_n \in \beta \cap \langle U \rangle$.

Then $A_n \in \langle U \rangle$. Thus $A_n q U$. Let $a_{n, \lambda_n} \in A_n$ such that $a_{n, \lambda_n} q U$. Then $a_{n, \lambda_n} \in D$ such that $a_{n, \lambda_n} q U$. Thus D is Q-dense in X. So D is countable Q-dense in X. Hence, by Result 1.D, X is F-separable.

Theorem 2.7. Let X be a $(q, \in) - FT_1$-space. Then X is F-separable if and only if $F(X)$ is separable.

Proof. ($\Rightarrow$): Suppose X is F-separable. Then, by Result 1.A, X is Q-separable. Let $D = \{x_{n, \lambda_n}\}_{n \in N}$ be a sequence of fuzzy points in X such that D is Q-dense in X. Let β be the collection of finite subset of D. Then clearly β is countable. Let $\mathrm{U} = \langle U_1, \cdots, U_m \rangle \cap F(X)$, where $\langle U_1, \cdots, U_m \rangle$ is a base member for T_v. Since D is Q-dense in X and $\emptyset \neq U_i \in FO(X)$ for each $i = 1, \cdots, m$, there exists $n_i \in N$ such that $x_{n_i, \lambda_n} \in D$ and $x_{n_i, \lambda_n} q U_i$ for each $i = 1, \cdots, m$. Let $E = \{x_{n_1, \lambda_{n_1}}, \cdots, x_{n_m, \lambda_{n_m}}\}$. Since X is FT_1, $E \in I_0^X$. Since X is a $(q, \in)$-fts, $x_{n_i, \lambda_n} \in U_i$ for each $i = 1, \cdots, m$. Then $E \subset \bigcup_{i=1}^{m} U_i$ and $E q U_i$ for each $i = 1, \cdots, m$. Thus $E \in \langle U_1, \cdots, U_m \rangle$. So $E \in \beta \cap \mathrm{U} \neq \emptyset$. Hence $F(X)$ is separable.

($\Leftarrow$): Suppose $F(X)$ is separable. Let $\beta = \{A_n\}_{n \in N}$ be a countable dense subset of $F(X)$. For each $n \in N$, choose $a_{n, \lambda_n} \in A_n$. Let $D = \{a_{n, \lambda_n}\}_{n \in N}$. Now let $U \in FO(X)$. Then $\langle U \rangle \cap F(X)$ is open in $F(X)$. Since β is dense in $F(X)$, B $\cap (\langle U \rangle \cap F(X)) \neq \emptyset$. Let $A_n \in \beta \cap (\langle U \rangle \cap F(X))$. Then $A_n \in \langle U \rangle$. Thus $A_n q U$. Let $a_{n, \lambda_n} \in A_n$ such that $a_{n, \lambda_n} q U$. Then $a_{n, \lambda_n} \in D$ such that $a_{n, \lambda_n} q U$. Thus D is Q-dense in X. So D is countable Q-dense in X. Hence, by Result 1.A, X is F-separable.

Theorem 2.8. Let X be a finitely F-compact $(q, \in) - FT_1$-space. Then X is F-separable if and only if $K(X)$ is separable.

Proof. ($\Rightarrow$) : Suppose X is F-separable. Then, by Result 1.A, X is Q-separable. Let $D = \{x_{n, \lambda_n}\}_{n \in N}$ be a sequence of fuzzy points in X such that D is Q-dense in X. Let β be the collection of finite subsets of D. Let $\mathrm{U} = \langle U_1, \cdots, U_m \rangle \cap K(X)$, where $\langle U_1, \cdots, U_m \rangle$ is a base member for T_v. Since D is Q-dense in X and $\emptyset \neq U_i \in FO(X)$ for each $i = 1, \cdots, m$, there exists $n_i \in N$ such that $x_{n_i, \lambda_n} \in D$ and $x_{n_i, \lambda_n} q U_i$ for each $i = 1, \cdots, m$. Let $E = \{x_{n_1, \lambda_{n_1}}, \cdots, x_{n_m, \lambda_{n_m}}\}$. Since X is T_{1w}, $E \in I_0^X$. Since X is a $(q, \in)$-fts, $x_{n_i, \lambda_n} \in U_i$ for each $i = 1, \cdots, m$. Then $E \subset \bigcup_{i=1}^{m} U_i$ and $E q U_i$ for each $i = 1, \cdots, m$. Thus $E \in \langle U_1, \cdots, U_m \rangle$. Since each finite fuzzy set in X is F-compact in X, $E \in K(X)$. Then $E \in \beta \cap (\langle U_1, \cdots, U_m \rangle \cap K(X)) = \beta \cap \mathrm{U} \neq \emptyset$. Hence $K(X)$ is separable.

($\Leftarrow$): Suppose $K(X)$ is separable. Let $\beta = \{A_n\}_{n \in N}$ be a countable dense subset of $K(X)$. For each $n \in N$, choose $a_{n, \lambda_n} \in A_n$. Let $D = \{a_{n, \lambda_n}\}_{n \in N}$. Let $\emptyset \neq U \in FO(X)$. Then $\langle U \rangle \cap K(X)$ is open in $K(X)$. Since β is dense in $K(X)$, $\langle U \rangle \cap K(X) \neq \emptyset$. Let $A_n \in \langle U \rangle \cap K(X)$. Then $A_n q U$. Let $a_{n, \lambda_n} q U$. Thus D is Q-dense in X. So D is countable Q-dense in X. Hence, by Result 1.A, X is F-separable.

Theorem 2.9. Let X be a FT_1-space. If $D = \{x_{n, \lambda_n}\}_{n \in N}$ is Q-dense(resp. dense) in X, then $D^n = D \times, \cdots, \times D$(n factors) is Q-dense (resp. dense) in $X^n = X \times, \cdots, \times X$.

Proof. Let U be a nonempty F-open set in X^n, where $U = U_1 \times U_2 \times \cdots \times U_n$ and $U_i \in FO(X)$ for each $i = 1, \cdots, n$. Since $U \neq \emptyset$, $U_i \neq \emptyset$ for each $i = 1, \cdots, n$. Since D is Q-dense(resp. dense) in X, there exists $m_i \in N$ such that $x_{n_i, \lambda_n} q D$(resp. $x_{m_i, \lambda_n} \in D$) for each $i = 1, \cdots, n$. Thus $(x_{m_1, \lambda_{m_1}}, \cdots, x_{m_n, \lambda_{m_n}}) q D^n$ (resp. $(x_{m_1, \lambda_{m_1}}, \cdots, x_{m_n, \lambda_{m_n}}) \in D^n$. Hence D^n is Q-dense(resp. dense) in X^n.

Theorem 2.10. Let X and Y be fts's and let f: $X \to Y$ a F-continuous surjection. If $D = \{x_{n, \lambda_n}\}_{n \in N}$ is Q-dense(resp. dense) in X, then $f(D)$ is Q-dense(resp. dense) in Y.

Proof. It is obvious.

Theorem 2.11. Let X be a FT_1-space. If $F_n(X)$ is separable, then X is F-separable.

Proof. Suppose $F_n(X)$ is separable. Let $\beta = \{A_n\}_{n \in N}$ be a countable dense subset of $F_n(X)$. For each $n \in N$, choose $a_{n, \lambda_n} \in A_n$. Let $D = \{a_{n, \lambda_n}\}_{n \in N}$. Then D is countable. Let $\emptyset \neq U \in FO(X)$. Then $\mathrm{U} = \langle U \rangle \cap F_n(X)$ is open in $F_n(X)$. B is dense in $F_n(X)$, $\beta \cap \mathrm{U} \neq \emptyset$. Let $A_n \in \beta \cap \mathrm{U}$. Then $A_n \in \langle U \rangle$. Thus

$A_n q U$. Let $a_{n,\lambda_n} \in A$ such that $a_{n,\lambda_n} q U$. Then a_n, $\lambda_n \in D$ such that $a_{n,\lambda_n} q U$. Thus D is Q-dense in X. So X is Q-separable. Hence, by Result 1.A, X is F-separable.

Theorem 2.12. Let X be a finitely F-compact space. If $K(X)$ is first countable, then X is C_I.

Proof. Let $x_\lambda \in F_p(X)$. Since each finite fuzzy set is F-compact in X, $\{x_\lambda\} \in K(X)$. Since $K(X)$ is first countable, there exists a countable local base U at $\{x_\lambda\}$. Without loss of generality, let U $= \{\langle U_n \rangle\}_{n \in N}$ be a countable local base at $\{x_\lambda\}$. Then clearly $\{U_n\}_{n \in N}$ is a countable local base at x_λ. Hence X is C_I.

From Theorem 2.12 and Result 1.B, we can easily obtain the following.

Corollary 2.12. Let X be a finitely F- compact space. If $K(X)$ is finite countable, then X is $Q-C_I$.

Theorem 2.13. If I_0^X is first countable, then each one of the subspace of I_0^X is first countable.

Theorem 2.14. Let X be a $(q, \in)-FT_1$- space. If X is $Q-C_I$, then $F(X)$ is first countable.

Proof. Suppose X is $Q-C_I$ and let $E = \{x_{1,\lambda_1}, \cdots, x_{n,\lambda_n}\} \in F(X)$, where $x_{i,\lambda_i} \in F(X)$ for each $i = 1, \cdots, n$. Since X is $Q-C_I$, for each $i = 1, \cdots, n$, there exists a countable Q-local base B $_i(x_{i,\lambda_i})$ at x_{i,λ_i}. Let β be the collection of all open sets of the form $\langle V_1, \cdots, V_n \rangle \cap F(X)$, where $V_i \in$ B $_i(x_{i,\lambda_i})$ for each $i = 1, \cdots, n$. Then clearly β is countable. We show that β is a local base at E. Let $B \in$ B. Then $B = \langle V_1, \cdots, V_n \rangle \cap F(X)$, where $V_i \in B_i(x_{i,\lambda_i})$. Since $B_i(x_{i,\lambda_i})$ is a Q-local base at x_{i,λ_i}, $x_{i,\lambda_i} q V_i$ for each $i = 1, \cdots, n$. Then $E \subset \bigcup_{i=1}^{n} V_i$. So $E \in \beta$. Now let U $= \langle U_1, \cdots, U_m \rangle \cap F(X)$ such that $E \in$ U. Then $E \subset \bigcup_{j=1}^{m} U_j$ and $E q U_j$ for each $j = 1, \cdots, m$. Thus for each $i = 1, \cdots, n$, there exists $j \in \{1, \cdots, m\}$ such that $x_{i,\lambda_i} q U_j$. Let $U = \cap_{j=1}^{m}\{U_j : x_{i,\lambda_i} q U_j\}$. Then clearly, $x_{i,\lambda_i} q U$ and $U \in FO(X)$. Since B $_i(x_{i,\lambda_i})$ is a Q-localbase at x_{i,λ_i}, there exists $B_i \in \beta_i(x_{i,\lambda_i})$ such that $x_{i,\lambda_i} q B_i \subset U$. Since X is a $(q, \in)$-fts, $x_{i,\lambda_i} \in B_i$, $x_{i,\lambda_i} \in U$ and $x_{i,\lambda_i} \in U_j$ for each $i = 1, \cdots, n$. Then $\bigcup_{i=1}^{n} B_i \subset \bigcup_{j=1}^{m} U_j$ and for each U_j there exists a B_i such that $B_i \subset U_j$. Thus, by Result 2.A, $\langle B_1, \cdots, B_n \rangle \cap F(X) \subset \langle U_1, \cdots, U_m \rangle \cap F(X)$. Moreover, $E \subset \bigcup_{i=1}^{n} B_i$ and $x_{i,\lambda_i} q B_i$ for each $i = 1, \cdots, n$. Thus $E \in \langle B_1, \cdots, B_n \rangle$. So β is a countable local base at E. Hence $F(X)$ is first countable.

From Result 1.B and Theorem 2.14, we can easily obtain the following.

Corollary 2.14-1. Let X be a $(q, \in)-FT_1$ space. If X is C_I, then $F(X)$ is first countable.

From Theorem 2.14 and Theorem 2.13, we can easily obtain the following.

Corollary 2.14-2. Let X be a $(q, \in)-FT_1$ space. If X is $Q-C_I$, then $F_n(X)$ is first countable.

From Theorem 2.14 and Result 1.B, we can easily obtain the following.

Corollary 2.14-3. Let X be a $(q, \in)-FT_1$-space. If X is C_I, then $F_n(X)$ is first countable.

Theorem 2.15. Let X be a FT_1 space. If X is C_{II}, then $K(X)$ is second countable.

Proof. Suppose X is C_{II}. Let $\beta = \{U_n\}_{n \in Z^*}$ be a countable base for X. Let $\beta^\wedge = \{\langle U_{a_1}, \cdots, U_{a_n} \rangle : U_{a_i} \in \beta\}$. Then clearly $\beta^\wedge$ is a countable base for $K(X)$. Hence $K(X)$ is second countable.

Definition 2.16. Let X be a fts and let $A \in I^X$,

(1) A subcollection U of I^X is called a proper cover of A if
 (i) U is a cover of A
 (ii) for each $U \in$ U, $U q A$.

(2) X is said to be compactly C_{II} if for each F-compact set K in X there exists a countable collection β of F-open sets in X such that if $\{U_1, \cdots, U_n\}$ is a proper cover of K then there exists a proper cover $\{V_1, \cdots, V_m\} \subset \beta$ of K such that $\langle V_1, \cdots, V_m \rangle \subset \langle U_1, \cdots, U_n \rangle$.

Theorem 2.17. Let X be a fts. Then X is compactly C_{II} if and only if $K(X)$ is first countable.

Proof. ($\Rightarrow$): Suppose X is compactly C_{II} and let $K \in K(X)$. Let β be the countable collection of F-open sets in X satisfying Definition 2.16. Let $B(K)$ be the collection of open sets in $K(X)$ which are constructed from the finite proper covers of K contained in β. Then clearly $B(K)$ is countable. We shall show that $B(K)$ is a local base at K. Let $K \in \langle U_1, \cdots, U_n \rangle \cap K(X)$, where $\langle U_1, \cdots, U_n \rangle$ is a base member for T_v. Then clearly $\{U_1, \cdots, U_n\}$ is a proper cover of K. By Definition 2.16, there exists a proper cover $\{V_1, \cdots, V_m\}$ of K in β such that $\langle V_1, \cdots, V_m \rangle \cap K(X) \subset \langle U_1, \cdots, U_n \rangle \cap K(X)$. Hence $K(X)$ is first countable.

($\Leftarrow$): Suppose $K(X)$ is first countable and let $K \in K(X)$. Then there exists a countable local base $B(K)$ at K. We may assume that each member B_k of $B(K)$ is of the form $B_k = \langle U_1^k, \cdots, U_{n_k}^k \rangle \cap K(X)$. Let $\beta = \{U_i^k : k \geq 1 \text{ and } 1 \leq i \leq n_k\}$. If $\{U_1, \cdots, U_n\}$ is a proper cover of K, then $K \subset \bigcup_{i=1}^{n} U_i$ and $K q U_i$ for each $i = 1, \cdots, n$. Thus $K \in \langle U_1, \cdots, U_n \rangle \cap K(X)$. So there exists $B_k \in B(K)$ such that $B_k \subset \langle U_1, \cdots, U_n \rangle \cap K(X)$ and $\{U_1^k, \cdots, U_{n_k}^k\}$ is a desired proper cover in β. Hence X is compactly C_{II}.

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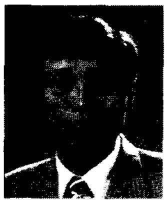

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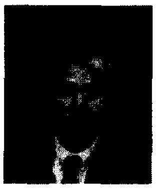

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