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FUZZY SETS IN PATTERN RECOGNITION, IMAGE ANALYSIS AND AUTOMATIC SPEECH RECOGNITION

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Fuzzy set theory, a recent generalization of classical set theory, has attracted the attention of researchers working in various areas including pattern recognition, which has had a seminal influence in the development of this new theory. This paper attempts to discuss some of the methodologies that have been suggested for pattern recognition, and techniques for image processing and speech recognition.

INTRODUCTION

Recently propounded [1] theory of fuzzy sets has attracted the attention of researchers in various disciplines because this theory is apparently a generalisation of the classical set theory. Since 1965, a great deal of work has been done on the development of this theory and on its applications [32]. Pattern recognition has had a seminal influence [2] on the development of this theory, and this paper discusses some of various techniques and methodologies suggested. In some cases, the techniques are equivalent to conventional ones, in others, the approach is refreshingly new. Interestingly enough, Tamuta et al. [35] suggested a classification method based on fuzzy relations, which was found to provide a mathematical basis for a hierarchical clustering scheme known since long.

The first section deals with methodologies suggested in pattern recognition. The second takes up a central problem in applications — the determination of a membership function in real-life cases.

Next two sections deal with image processing and speech recognition — two important areas of pattern recognition in practice. An incomplete bibliography of the work done by the present author and his colleagues [21], [23], [24], [28], [29], and [36] to [68] in the field of fuzzy mathematics and its application in pattern recognition, image analysis, and automatic speech recognition is included in the reference list.

I. FUZZY SETS IN PATTERN RECOGNITION

Pattern recognition is said to be supervised when a machine is given a set of objects with known classifications and is asked to classify an unknown object based on the information acquired by it during its training. Supervised pattern recognition in all its various forms has been treated in a unified way [3] using the framework of fuzzy set theory. Supervised pattern recognition is dissected into two basic operations "abstraction" and "generalization". Let the objects be completely described by a set of measurements written as a vector $\mathbf{x}$. A training set (a collection of samples or observations) from a fuzzy class A of objects is given by

$$\{(\mathbf{x}_1, f_1), (\mathbf{x}_2, f_2), \ldots, (\mathbf{x}_n, f_n)\}$$

where $f_i \in [0, 1]$ is the grade of membership of the ith object whose measurements are written as $\mathbf{x}_i$.

Abstraction on this collection means, in informal terms, the identification of those properties of the samples which they have in common and which, in aggregate, define the set A. More formally, abstraction on this collection means the estimation of the membership function f on A from the samples. Having obtained the estimate $\bar{f}$ of f, generalization is performed when this estimate is used to compute the values of f at points other than $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$.

Thus, given two fuzzy sets A, B in Q with the corresponding two unknown membership functions f^A, f^B and also given a training-set

$$\{(\mathbf{x}_1, f_1^A, f_1^B), (\mathbf{x}_2, f_2^A, f_2^B), \ldots, (\mathbf{x}_i, f_i^A, f_i^B), \ldots, (\mathbf{x}_n, f_n^A, f_n^B)\},$$

abstraction involves the estimation of f^A, f^B, generalization involves the use of the obtained $\bar{f}^A, \bar{f}^B$ for an unknown object $\mathbf{x}$ not contained in the training set.

This can easily be extended to the case of $m(m > 2)$ classes.

It is evident that the problem of nonfuzzy classes is included as a special case where the membership function is a map $f\colon Q \to \{0, 1\}$.

Zadeh [4] has suggested that features be linguistically valued, e.g., the feature "size" can have values "small", "very small", etc. If there are r such features, there is a fuzzy relation on the universe $X_1 \times X_2 \times \ldots \times X_r$, where X_j is the universe of the jth feature m_j (which takes linquistic values). A fuzzy pattern class is a fuzzy relation on $X_1 \times X_2 \times \ldots \times X_r \times [0, 1]$, i.e., a type 2 fuzzy class. Thus we can visualize a table of the form:

m_1	$\ldots$	m_j	$\ldots$	m_r	
p_1^1	$\ldots$	p_1^j	$\ldots$	p_1^r	p_1^{r+1}
$\vdots$		$\vdots$		$\vdots$	$\vdots$
p_i^1	$\ldots$	p_i^j	$\ldots$	p_i^r	p_i^{r+1}
$\vdots$		$\vdots$		$\vdots$	$\vdots$
p_n^1	$\ldots$	p_n^j	$\ldots$	p_n^r	p_n^{r+1}

where for $i = 1, \ldots, n$, $j = 1, \ldots, r$,

p_i^j represents the linguistic value of the jth feature corresponding to the ith pattern, p_i^{r+1} represents a linguistic truth value: a fuzzy set on $[0, 1]$.

One can interpret the table as answers for r questions. The ith row of this table is a fuzzy rule that says: if for a pattern p_i, the first feature has linguistic value p_i^1, the second has $p_i^2, \ldots$, the jth has p_i^j, etc., then the membership of that pattern is p_i^{r+1}.

Then the fuzzy pattern class is given by the relation

$$R = \bigcup_{i=1}^{n} \bigcap_{j=1}^{r+1} p_i^j .$$

Given an unknown object x, whose feature values are $m_1(x), \ldots, m_r(x)$, its membership to the pattern class is obtained by max – min composition with R.

When there are m pattern classes, there would be m such relational tables.

A somewhat analogous approach is that of Chang and Pavlidis [5]. They look at each column of the relation table as a set of possible answers to a question concerning feature j and thus arrive at a fuzzy decision tree for a given pattern. A fuzzy decision tree is a tree such that each nonleaf node i has a k-tuple decision function

$$f_i : Q \to [0, 1]^k$$

and k ordered sons. Each non-leaf son corresponds to possible answer to a previous question and the son is also associated with a question determined by the previous answer. To each branch in the tree there is associated a value in the interval $[0, 1]$. Each leaf corresponds to a pattern class. Each path from the root to a leaf represents a decision assignment of the sample to the class corresponding to the leaf. Each decision is valued by the minimum (or product) of the values corresponding to the branches composing the path. The object is usually assigned to the class corresponding to the leaf ending the best-valued path. Figure 1 shows such a tree.

It has been pointed out in [6] that owing to some inherent structure, sometimes a set of objects possesses to some degree the same property that the individual objects possess. Such a set is called a property-system. For example, consider

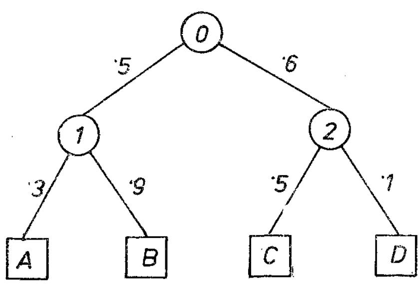

Fig. 1. Fuzzy decision tree.

a team of acrobats each of whom depends on the others for his balance, so that the degree of balance is a property that both the team as a whole and the individuals possess. The property values of the system is given by AND (or OR) of the values of the individual elements if these values are boolean, i.e., the values 0 or 1 (consider a set of switches in series or parallel); and, in a more general case it is given by MIN (or MAX) if the individual values are formal, i.e., numbers from [0, 1]. Another example of such a system is called a "Collective", where the property value of the set as a whole is given by a measure such as arithmetic average of the formal property values of the individual elements. In such a case the property value of the set increases with increasing number of elements and with increasing property values.

In some cases, where a number of prototypes for a class are given, it may be meaningful to compute the collective property of the prototypes and consider that as the reference for the class. Such use of fuzzy sets and property sets have been suggested in [30] and is also one of the motives behind [28].

The syntactic approach to pattern recognition involves the representation of a pattern by a string of concatenated subpatterns called primitives. These primitives are considered to be the terminal alphabets of a formal grammar whose language is the set of patterns belonging to the same class. Recognition therefore involves a parsing of the string.

The syntactic approach has incorporated fuzzy sets at two levels. Firstly, the pattern primitives are themselves considered to be labels of fuzzy sets, i.e., such subpatterns as "almost circular arcs" are considered. Secondly, the structural relations among the subpatterns may be fuzzy, so that the formal grammar has weighted production rules and the grade of membership of a string is obtained by min – max composition of the grades of the productions used in the derivations.

A practical application of a fuzzy syntactic approach has been made in [7] where the goal is the recognition of handwritten capitals. High variability in handwrittings motivated the use of fuzzy set concepts. A nonfuzzy context-free grammar with eleven production rules is used with six fuzzy primitives. The membership of the entire pattern is given by the minimum of the grades of memberships of the pattern segments to the respective fuzzy classes of primitives. Thus dubious assumptions about the existence and shape of probability density functions are avoided.

This approach was criticized by Stallings in [8] who developed a Bayesian hypothesis testing scheme for the same problem. Given a pattern S_r the hypothesis H_k is that the writer intended the letter H_k. Associated with each decision is a cost C_{ij} which is the cost of choosing H_i when H_j is true. The parsing of the pattern is performed as before. Only a probability is associated with each segment for a given letter. Regarding unknown densities the author suggests the use of maximum likelihood tests.

Since both membership functions and probability density functions are maps into the interal [0, 1], the only difference is the use of min/max operators, where, the author argues, the "min" operator loses a lot of information and is drastically affected by one low value. The author argues that though frequentistic probability is not appropriate in dealing with pattern variability, subjective probability is perfectly suitable and more intuitively obvious than "grade of membership".

In a rejoinder [9], it is argued that fuzzy set theory is more flexible than it is assumed in [8], where all arguments are directed against a particular case treated in [7]. Recalling the idea of collectives (from property sets), where the arithmetic average replaces "min", there remains little difference between the schemes of [7] and of [8].

In a reply, Stallings insisted that the Bayesian approach is superior since it offers a convenient way for assignment of costs to errors and gains to correct answers.

Inference of a fuzzy grammar is an interesting problem. The problem is to infer from a specified fuzzy language, the productions as well as the weights of these rules.

One algorithm [10] is a reinforcement algorithm where a learning process consists of decreasing the weights of some productions while leaving those of others unchanged. The ones whose weights are unchanged are those that yield the patterns given as samples. In the case of ambiguity, only one of the possible derivations is taken.

Non-supervised pattern classification as well as problems in various disciplines may be solved by a method of data analysis known as clustering. The aim of clustering is to partition a set of data points into a number of neutral and homogeneous clusters. The term homogeneous is used in the sense that all data points contained in the same cluster are more similar to each other than they are to data points in other clusters. However, there is rarely a unique non-trivial solution to the clustering problem. Further, clusters are rarely compact and well-separated. So an object may be assigned to a cluster with a degree of cluster membership. Thus non-uniqueness, when it exists, may be identified as such, while the corresponding classical partition still remains meaningful.

Gitman and Levine [11] presented a method of converting multimodal fuzzy sets into unimodal ones and thereby deriving clusters. Each data point is associated with a membership value. The algorithm makes use of the order of the points according to the distance as well as the order of the points according to the grade of membership. Given a finite set X of vectors ($|X| = n$), and a metric d, let

$$C_{i\theta} = \{x \in X \mid d(x_i, x) \leqq \theta\},$$

where θ is a chosen threshold $\in \mathbb{R}^+$ for every $x_i \in X$, $i = 1, \ldots, n$. Let A be a fuzzy set on X where the membership function is given by

$$\mu_A(x_i) = \frac{|C_{i\theta}|}{|X|}.$$

The maxima of μ_A are the 'centers' of the clusters existing in X. This multimodal membership function is decomposed into unimodal fuzzy sets and maximum separation between these is obtained. The number of clusters is equal to the number of the local maxima of μ_A. In the experiment, a reasonable guess is to be made of θ.

The notion of a fuzzy partition has been introduced in [12]. A fuzy partition is defined as a family of fuzzy sets $S_1, S_2, \ldots, S_m$ on X such that for every $x \in X$, $\sum_{i=1}^{m} \mu_{S_i} = 1$.

The problem is to find $S_1, \ldots, S_m$ for known m, and a given metric d so that elements which do not have very large mutual distances belong to the same cluster.

The problem of classification is then for a given x to find the classification vector $C[x] = [\mu_{S_1}(x), \ldots, \mu_{S_m}(x)]$.

Let v be a function from $[0, 1]^m \times [0, 1]^m$ to $\mathbb{R}^+$ such that $v(a, a) = 0$, $v(a, b) = v(b, a)$. Usually this is the Euclidean distance. Let f be a positive nondecreasing, not identically zero real function of one real variable such that $f(0) = 0$. Then C should satisfy

$$v(C(x), C(y)) = f(d(x, y)) \quad \text{for every} \quad x, y \in X .$$

It is usually relaxed into the following minimization problem: minimize

$$\sum_{x,y \in X} w(x)\, w(y)\, [v(C(x), C(y)) - f(d(x, y))]^2$$

where w is a weighting function.

A clustering algorithm which also gives the cluster centers (most representative elements) is the ISODATA algorithm [13]. This algorithm has been improved by allowing fuzzy clusters to be generated [14], [15].

An elaborate algorithm has been developed by Backer [16]. The goal is to decompose the original set of objects

$$C = \{u_r\}, \quad r = 1, \ldots, N ,$$

into m disjoint subsets

$$C = \{C_i\}, \quad i = 1, \ldots, m ,$$

in such a way that the data points assigned to one subset be as similar as possible to each other and as dissimilar as possible to points assigned to other subsets. These subsets are the clusters. It is assumed that there exists an underlying structural property of the samples expressed by a set of so-called point-to-subset affinity values representing some sort of relationship between an element and a group of elements. The affinity may be expressed by a distance function, or by some neighbourhood relations. Using a certain measure of affinity, the set-membership of a certain point is decomposed into degrees of memberships of that point to m subsets given by the m-partition.

The algorithm starts with an initial guess of m disjoint subsets

$$C = \{C_i\}^{(0)}, \quad i = 1, \ldots, m .$$

For all $u \in C$, a membership value is determined representing an m-collection of induced fuzzy sets

$$\{(f_i, u) \mid f_i \in [0, 1]\} \quad \text{for every} \quad u \in C\,, \quad i = 1, \ldots, m\,.$$

Then relocation takes place in attempt to optimize some criterion function. Thus we obtain the next partition

$$C = \{C_i\}^{(1)}\,.$$

This is continued until no further move causes an improvement.

Tamura et al. [35] have developed a hierarchical clustering scheme from the standpoint of a fuzzy similarity relation, by invoking a theorem that such a relation can be resolved into a nested sequence of equivalence relations, i.e., nonfuzzy partitions.

II. EVALUATION OF MEMBERSHIP VALUES IN FUZZY SETS

One of the main difficulties in the application of the theory of fuzzy sets to real-life problems in the hard sciences, such as engineering, is that of finding a unique and correct membership function that characterizes an "intuitively obvious" fuzzy set.

One common hypothesis is that there is an ideal prototype for each fuzzy set, and thus the membership value for any element is related to its similarity to the ideal prototype.

Bremermann [31] has suggested that the degree of dissimilarity between an object and the ideal prototype may be given by the energy necessary to deform the latter so as to match it with the former. This dissimilarity normalized to [0, 1] directly yields the degree of membership.

The conceptually simplest approach is to reconstruct the membership function from the knowledge of a finite number of its samples. This is the method of exemplifications as suggested by Zadeh [33].

An analytical justification of the S-shape of the membership curve suggested by Zadeh has been given by Kochen and Badre [17]. Consider a fuzzy variable $A =$ = large. The marginal increase rate of the strength of belief in "x is A" is assumed proportional to the strength of this belief. This argument is repeated for the complement.

Then if the membership function is assumed to be continuous and differentiable, we must have

$$\frac{\mathrm{d}\mu_A}{\mathrm{d}x}(x) \propto \mu_A(x)\,[1 - \mu_A(x)]\,,$$

whose solution is

$$\mu_A(x) = \frac{1}{1 + \mathrm{e}^{a-bx}}\,,$$

where a and b are constants.

For small sets, a comparison of subsets [18] yields a system of inequalities from which the membership function may be obtained. Given a fuzzy set A of X with membership function μ_A, a fuzzy set $\bar{A}$ on $\mathbb{P}(X)$ is induced as an average membership value of the elements, i.e.

$$\mu_{\bar{A}}(S) = \mu_{\bar{A}}[\{x_1, \ldots, x_k\}] = \frac{1}{k} \sum_{i=1}^{k} \mu_A(x_i)$$

where S is any subset of X, $S \in \mathbb{P}(X)$.

Data is obtained in the form of comparison of subsets through a preference relation $\geqq$ defined in $\mathbb{P}(X)$,

$$S_1 \geqq S_2 \quad \text{iff} \quad \mu_{\bar{A}}(S_1) \geqq \mu_{\bar{A}}(S_2)$$

for any pair of subsets S_1, S_2. Namely, $S_1 \geqq S_2$ means informally that S_1 matches A better than S_2. Thus from this preference data, a system of inequalities is obtained relating $\mu_A(x_i)$ for all i.

Saaty (19) has suggested a relative preference method for a discrete universe by comparing every pair of elements as a matrix. This becomes difficult for large sets.

Recently, Dishkant [20] has discussed the derivation of the membership of a fuzzy variable which is the sum of many fuzzy variables with known memberships under some conditions.

More recently, Chaudhuri and Dutta Majumder [21] discussing the problem of learning the membership of a fuzzy set from the knowledge of a subset of training samples, have subdivided it into two cases – in one a mathematical expression for the membership function being available, and in the other, not available.

In cases where a mathematical expression may be assumed, those involving a distance or dissimilarity measure have been found to be popular. Two simple forms of such functions are

(1) $$\mu_A(\mathbf{x}) = \frac{1}{1 + D(\mathbf{x}, \mathbf{c})},$$

(2) $$\mu_A(x) = \exp\left[-D(\mathbf{x}, \mathbf{c})\right],$$

where $\mathbf{x}$ = the measurement vector of the sample, D = the distance of $\mathbf{x}$ from $\mathbf{c}$, $\mathbf{c}$ = the core point, i.e., $\mu_A(\mathbf{c}) = 1$.

If D denotes a weighted distance, its explicit form may be

$$D(\mathbf{x}, \mathbf{c}) = [\mathbf{x} - \mathbf{c}]' \, \mathbf{W}[\mathbf{x} - \mathbf{c}],$$

where $\mathbf{W}$ = the weight matrix. Thus, from a set of M prototypes, it is necessary to estimate $\mathbf{W}$ and $\mathbf{c}$. Fisrtly, when M is large, a statistical approach may be used, such as

$$\mathbf{c} = E(\mathbf{y}),$$

$$w_{ij} = \begin{cases} 1/\sigma_i(\mathbf{y}), & i = j, \\ 0 & i \neq j. \end{cases}$$

Secondly, when M is not large, a non-statistical approach is suggested. If the membership values of the prototypes are known, then the equation (1) or (2) are used to get consistent (if any) solution for **W**, **c** from M equations. If the numerical membership values for the prototypes are known up to a tolerance interval, then certain inequalities have to be solved. Five cases are considered depending on the nature of **W** and whether **c** is known or not. Thirdly, when the prototypes are only rank-ordered, e.g.

$$\mu_A(\mathbf{y}_1) \leqq \mu_A(\mathbf{y}_2) \leqq \cdots \leqq \mu_A(\mathbf{y}_m),$$

c and **W** are to be estimated within some range.

A mathematical expression for the membership is however not available in most cases. Here three cases are treated. In the first case, the feature vectors of the prototypes are accurately measurable, and their grades of membership are known. In the second case, the prototypes are measurable but their membership values are unknown. In the third case, for each feature, the prototypes are rank-ordered, but the rank ordering considering the aggregation of the features is not known.

III. FUZZY SETS AND IMAGE PROCESSING

Although the shape is a very important cue in the analysis of images, it has no unique mathematical definition. Conventional methods often give rise to a large number of Fourier descriptor coefficients, or force a fit with an n-degree polynomial. However, human analysis of shapes often makes use of imprecise concepts: for example, we are often quite satisfied with a description "almost symmetric". A fuzzy approach to the shape analysis accommodating imprecise concepts therefore merits consideration.

The use of fuzzy sets has been suggested for image enhancement. However, the application of the fuzzy operator "INT" is equivalent to the well-known technique of nonlinear gray-scale transformation. The "min" and "max" operators have been used in the neighborhood of a pixel in a gray-scale picture to remove salt-and-pepper noise.

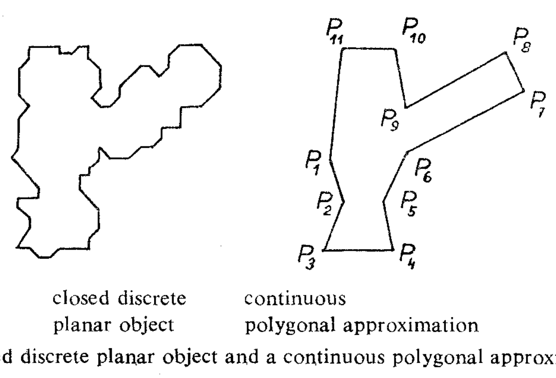

Fig. 2. A closed discrete planar object and a continuous polygonal approximation to it.

A. Shape analysis

Lee [34] has used the fuzzy set theory for classification of chromosomes through shape. Vanderheydt [22] considers the problem of decomposing a polygonal shape into its meaningful parts using the fuzzy set theory. In particular, they examine an object which is actually two touching chromosomes.

Consider Figure 2. It is necessary to decompose the polygonal approximation meaningfully by a line joining a pair of vertices p_i, p_j. Human beings assign a greater grade of membership to the decomposition generated by p_6, p_9 than to p_1, p_9. Apparently, this is because humans optimize a double aim in the decomposition strategy. Firstly, the degree of fit of the line $p_i p_j$ to the concept "decomposition line" is checked, and secondly, the "meaningfulness" of the generated subparts are examined. Fuzzy sets are used to model this decision process. For a pair of vertices, a measure of intrusion is given by the largest Euclidean distance from the boundary to the line joining the pair. Normalizing this distance, we have the grade of membership for the given pair of vertices. This defines a fuzzy relation and thus a symmetric fuzzy graph. The exactness of the decomposition by p_i, p_j, creating a subobject O_s, is given by

$$\mu_{\mathrm{Ex}}(O_s) = \mu_{\mathrm{GC}}(p_i) \cdot \mu_{\mathrm{GC}}(p_j) \cdot \mu_{\mathrm{SD}}(p_i p_j) \cdot \mu_{\mathrm{NC}}(O_s) \cdot \mu_{\mathrm{RL}}(O_s)\,,$$

where $\mu_{\mathrm{GC}}(p_i)$ = the grade of membership of curvature at p_i to the description "great concavity", $\mu_{\mathrm{SD}}(p_i p_j)$ = the grade of membership of $p_i p_j$ to the description "small distance", $\mu_{\mathrm{NC}}(O_s)$ = the grade of membership of O_s to the description "nearly chromosome", $\mu_{RL}(O_s)$ = the grade of membership of O_s to the description "relatively large", the first two fuzzy sets GC being defined using curvature, and the last two NC, RL being defined using the fuzzy relation defined earlier.

Chaudhuri and Dutta Majumder [23] have suggested that the shape be described using fuzzy properties such as "sidedness", "cornerity", "symmetry", by a linguistic description such as "many-sided", "sharp-cornered", "very symmetric", and by grades of membership

$$\mu_{\mathrm{fig}} = \max\{\min_i(\mu_{\mathrm{Sid}}(i)), \min_i(\mu_{\mathrm{cor}}(i)), \mu_{\mathrm{sym}}\}\,.$$

Hough transform is taken, and if for any set of successive points the tolerance is θ, then they belong to one side with membership

$$\mu_{\mathrm{Sid}} = \left(1 - \frac{\theta}{\pi F_{\mathrm{d,si}}}\right) F_{\mathrm{e,si}}\,,$$

where $F_{\mathrm{d,si}}$, $F_{\mathrm{e,si}}$ are normalizing constants. Since in the Hough transform plane all information regarding cornerity is lost, another fuzzy set "sharp corner" is used, and membership for a segment ABC (Fig. 3) is given by

$$\mu_{\text{cor}} = \left(1 - \frac{\min\{\text{curve length } AB, \overline{AC} + \overline{BC}\}}{F_{\text{d,cor}} \max\{\text{curve length } AB, \overline{AC} + \overline{BC}\}}\right)^{F_{\text{e,cor}}},$$

where $\overline{AC}$ means the linear distance between A and C.

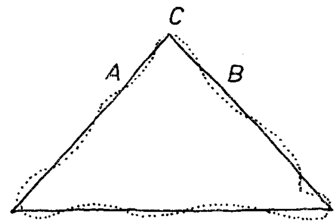

Fig. 3. A man-made triangle and its hough transform approximation.

Similarly the membership of the figure to the fuzzy set "symmetric" is give by

$$\mu_{\text{sym}} = \left(1 - \frac{4 \max R_{j,n/2+j}}{F_{\text{d,sym}} \sum_{i=1}^{n} C_i^2}\right)^{F_{\text{e,sym}}},$$

where C_i = curvature at the point i,

$$R_{j,n/2+j} = \sum_{i=2}^{n/2} C_i C_{n+2-i}$$

is a correlation measure of the curvature values between pairs of points equidistant from the axis of symmetry in question.

B. Image enhancement

Pal and King [24] have considered the pixels as fuzzy singletons and have applied the fuzzy operator "contrast intensifier", and, considering some gray scale pictures, have demonstrated that the index of fuzziness decreases. Let us have a picture

$$X = (x_{mn}), \quad m = 1, \ldots, M; \quad n = 1, \ldots, N,$$

where x_{mn} is the pixel corresponding to the mth row and nth column. Entering a property plane by a transformation

$$P_{mn} = G(x_{mn}) = \left[1 + \frac{(x_{\max} - x_{mn})}{F_{\text{d}}}\right]^{-F_{\text{e}}},$$

$m = 1, \ldots, M$; $n = 1, \ldots, N$, where $x_{\max}$ is the maximum gray level, $P = (p_{mn})$ is an $M \times N$ matrix where p_{mn} lies in the real interval $[0, 1]$. Regarding this as $M \times N$

fuzzy singletons, the "intensification" transform gives on the rth application

$$p'_{mn} = T_r(p_{mn}) = \begin{cases} T'_r(p_{mn}), & 0 \leqq p_{mn} \leqq 0{\cdot}5, \\ T''_r(p_{mn}), & 0{\cdot}5 \leqq p_{mn} \leqq 1{\cdot}0, \end{cases}$$

for $r = 1, 2, \ldots$ and the iterative application rule is

$$T_s(p_{mn}) = T_1\{T_{s-1}(p_{mn})\} \quad \text{for} \quad s = 1, 2, \ldots,$$

with T_1 the usual fuzzy INT operator,

$$T_1(p_{mn}) = \begin{cases} 2(p_{mn})^2, & 0 \leqq p_{mn} \leqq 0{\cdot}5, \\ 1-2(1 - p_{mn})^2, & 0{\cdot}5 \leqq p_{mn} \leqq 1{\cdot}0. \end{cases}$$

Figure 4 shows graphically the result of the iterative application of this INT operator.

Rosenfeld and Nakagawa [25] have discussed the application of "min" and "max" operators on the neighborhood of a pixel to smooth salt-and-pepper noise on gray-scale pictures.

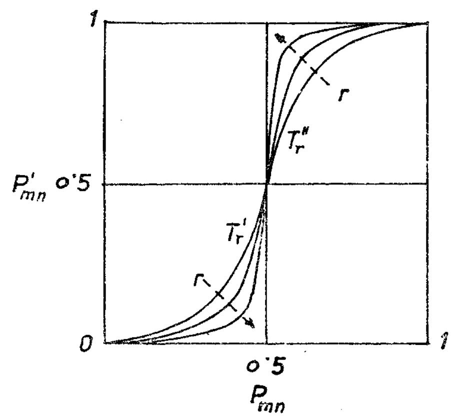

Fig. 4. INT transformation function for contrast enhancement in a property plane.

For a 2-tone or binary picture with salt-and-pepper noise (sprinklings of isolated 0's in a background of 1's and vice-versa), replacing a pixel by the AND of the neighborhood pixels is equivalent to shrinking a mass of 1's; and subsequent use of OR expands masses of 1's, and also cleans up salt-and-pepper noise.

Given a gray-scale picture with salt-and-pepper noise, it is possible to threshold the picture and perform the above binary operation, but early thresholding may be undesirable. Since the operators AND and OR on $\{0, 1\}$ generalize to MIN and MAX on $[0, 1]$, the latter set of operatots are used on the gray-scale picture (after normalizing the pixel gray scale values to $[0, 1]$). It has been shown that this produces cleaning of salt-and-pepper noise as well as what is described as "fuzzy thinning".

C. Scene labeling

Rosenfeld et al. [26] have proposed a fuzzy model for labeling a scene by a relaxation procedure. The problem is to identify a set of objects in a scene with a set of labels. Since identifications are ambiguous to start with, the relationships among the objects are used to reduce or eliminate the ambiguity. The fuzzy model accounts for preferences among the interpretations.

IV. FUZZY SETS IN SPEECH RECOGNITION

Since speech is a pattern of biological origin, and is influenced by the message, the speaker, the latter's health and mood, as well as the environment, it is found to be fuzzy in nature to a considerable extent. Again since conditional densities of classes are often not known, stochastic techniques are not very appealing. Available acoustic-phonetic knowledge is mostly non-numerical involving imprecise relations between acoustic features and their phonetic or phonemic interpretation. Thus fuzzy set theory appears to be a useful alternative as a tool in the problem of recognition of speech.

DeMori et al. [27] have used fuzzy algorithms in a speech understanding system, organized with several levels of knowledge sources, each knowledge source being a set of syntactic rules. Fuzzy algorithms are used to model the fact that acoustic-phonetic properties of speech sounds are known with a degree of vagueness, e.g., the signal energy is high for vowels, nonsonorant consonants have high frequency components, and in univoiced stops there is an interval of silence followed by some noise.

First, phonetic features like vocalic-nonvocalic, sonorant-nonsonorant, etc. are extracted by answering a branching questionaire or relational table in the manner of Zadeh, as discussed. Next, fuzzy linguistic variables are used to represent subjective judgment after inspecting some acoustic parameters. Given x, a value of an acoustic parameter, A a fuzzy label (e.g. "high consonant durations") defined on the universe of x, a membership function $\mu_A(x)$ is used to compute the compatibility of x with the judgment A. These membership functions were established subjectively after inspection of the distribution of acoustic measurements made on a large number of sound samples. Thus B, a string of linguistic variables is obtained. If p is an acoustic pattern represented by its description in terms of acoustic features, then the possibility that a phonetic feature (FF) is present in p is given by

$$\mu_p(\mathrm{FF}) = \sup_B (\min (\mu_p(B), \mu_B(\mathrm{FF}))$$

and is considered as the result of the invocation of a knowledge source to verify the hypothesis FF is in p.

Thus the use of fuzzy sets attempts to blend the non-numerical nature of acoustic-phoentic knowledge with the numerical formalism required for computer processing.

Pal et al. [28] have applied fuzzy set theory for machine recognition of speech and informants using the first three formants only. The problem is to answer, with minimum of errors, the question; "What is the vowel contained in, and who is the informant of an unknown utterance in a large number of spoken CNC (consonant – vowel nucleus – consonant) words?" They worked with Telugu (a major Indian language) words using two recognition schemes.

In the first scheme, the membership of a measurement vector $\mathbf{x}$ to the jth class is a normalised ($\in [0, 1]$) value computed in terms of the distance $d(\mathbf{x}, R_j)$ to a reference prototype R_j,

$$\mu_{C_j}(\mathbf{x}) = \operatorname*{Min}_{R_j} \left\{ \frac{1}{1 + \left(\frac{d(\mathbf{x}, R_j)}{E} \right)^F} \right\}$$

where E, F are constants.

In the second scheme, n dimensions of the measurement vector $\mathbf{x}$ are considered as n properties, and the idea of property sets is used, as discussed.

The results are given in Tables I and II.

Tab. I. Confusion matrix showing machine's performance on vowel recognition.

	Recognized as					
Spoken	i	e	ə	a:	o	u
i	155	17				
e	25	154	20	8		
ə		9	52	8	2	1
a:			17	70	2	
o		1	7		138	34
u			1		6	144

Tab. II. Accuracy rate of speaker identification for each vowel formant.

	Recognition score					
Speaker	i	e	ə	a:	o	u
X	89·47	92·6	100·0	82·76	95·84	91·3
Y	90·9	90·63	83·4	78·95	94·45	90·63
Z	100·0	100·0	96·43	95·13	83·34	92·0
Total	94·12	94·85	93·06	87·64	90·63	91·25

A similar study was undertaken for recognition of unaspirated plosives in CVC context. Again Telugu words were used. The velars k, g, the alveolar ṫ, ḋ, the dentals t, d, and the bilabials p, b in combination with ten vowels ə, a:, i*, i:, u*, u:, e*, e:, o* and o: including shorter and longer categories had been selected. The results are shown in Table III (cf. [29]).

Tab. III. Percentage of correct classification of plosives.

Target vowel	k	ṫ	t	p	g	ḋ	d	b
ə	31·58	88·89	38·10	100·00	33·34	50·00	83·34	100·00
a:	48·14	60·00	37·50	100·00	38·46	76·92	40·00	100·00
e	100·00	75·00	75·00	22·23	100·00	85·71	80·00	40·00
o	100·00	62·50	72·73	100·00	100·00	100·00	100·00	66·67
u	100·00	66·62	88·89	90·90	100·00	58·82	15·00	93·75
i	91·67	25·00	70·00	11·12	100·00	64·70	13·34	66·67

V. CONCLUSIONS

Fuzzy set theory, a recent generalization of classical set theory, has attracted the attention of researchers. A large number of techniques and methodologies that have evolved in the area of pattern recognition are sampled in this paper, and the problem of determination of the membership function is also discussed.

The principal motives for invocation of fuzzy set theory is that of accounting for variability in patterns of the same class, the non-uniqueness of some cues used by human beings, the artificiality and arbitrariness in assuming probability densities, and the generalization of the Boolean AND/OR offered by the min/max operators.

The principal drawbacks are that firstly some of the "fuzzy" methods are equivalent to standard ones; secondly, arbitrary membership functions are often assumed.

Though there is, as yet, no unified "fuzzy" approach to pattern recognition, there is a promise of interesting developments.

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Souhrn

NEOSTRÉ MNOŽINY V ROZPOZNÁVÁNÍ OBRAZCŮ, ANALYSE OBRAZŮ A AUTOMATICKÉM ROZPOZNÁVÁNÍ ŘEČI

D. Dutta Majumder

Teorie neostrých množin, nedávné zobecnění klasické teorie množin, připoutala pozornost výzkumníků pracujících v rozličných oblastech včetně rozpoznávání obrazců, což mělo zásadní význam v rozvoji této nové teorie. V tomto článku se podává přehled některých metod, jež byly navrženy pro rozpoznávání obrazců, analysu obrazů a rozpoznávání řeči.

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