Fuzzy Sliding-Mode Consensus Control for Multi-Agent Systems

Yeong-Hwa Chang, Chia-Wen Chang, and Wei-Chou Chan

Abstract— A fuzzy sliding-mode consensus control is investigated for multi-agent systems in this paper. The consensus problem is discussed for networks of dynamic agents with external disturbances and model uncertainty. To deal with the consensus problems of the multi-agent system, a novel consensus algorithm combining the concepts of graph theory and fuzzy sliding-model control is proposed. According to the communication topology, the consensus stability conditions can be determined so that the fuzzy sliding-mode consensus controller (FSMCC) can be derived. Simulation results are provided to illustrate the effectiveness of the provided control scheme. Compared to the conventional consensus algorithms, the simulation results empirically support the promising performance of desire.

I. INTRODUCTION

During the last few years, the investigation of the co-ordination of multiple agents has attracted much attention. Taking the advantages of distributed sensing and actuation, a multi-agent system (MAS) can perform some cooperative tasks such as moving a large object that is usually not executable by a single-agent. Applications of this research include autonomous underwater vehicles [1], autonomous formation flight [2], congestion control of communication networks [3], and distributed sensor networks [4].

Among the cooperative control strategies, consensus algorithm is a relatively new development that combines the graph theory with system control theories for the distributed networked control systems [5]. In the networked MAS, consensus means to drive the information states of all agents to a common value. Due to recent technological advances in communication and computation, consensus problems of multi-agent systems have drawn substantial research efforts to many practical applications such as cooperative control of unmanned vehicles [6], [7], sensor networks [8] and mobile robots [9]–[11].

In a leader-follower multi-agent system, the behaviors of followers will be influenced by the leader, where the leader is usually independent of their followers. In this case, the leader is preprogrammed or provided by an external source which means that only the leader has the knowledge of group trajectory information. Then, the cooperative task is built on the reaction of the other agents to the motion of the leader. A typical leader-follower formation control approach assumes that there is only one group leader within the team. The control scheme of a leader-follower system not only simplifies the design and implementation, but also saves the control energy and cost. In the literature, there are

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many researches [12]–[14] that are based on the consensus algorithm to overcome the multi-robot formation problem. In [12], a unified formation control architecture being allowable for an arbitrary number of group leaders and arbitrary information flow among vehicles was proposed. To avoid and adapt to obstacles in an environment, the dynamically changing or time varying formation shape was considered [13]. In [14], the formation stabilization with linear dynamics was investigated, and the role of Laplacian eigenvalues was clearly discussed.

In this paper, the graph theory is used to model the communication topology between agents. For each agent, the single-integrator dynamic model with uncertainty is considered. Particularly, a novel consensus algorithm, fuzzy sliding-mode consensus controller (FSMCC), is proposed to investigate the consensus problem of multi-agent systems in directed graphs. The advantages of fuzzy sliding-mode control are that can alleviate the chattering effects with only using sliding-model control and can reduce the fuzzy rules complexity with only using fuzzy control. The consensus stability condition of the controlled multi-agent system can be determined by utilizing the Lyapunov stability theorem. Furthermore, the parameters of fuzzy membership functions are shown to be dependent with the communication topology.

The paper is organised as follows. In Section II, some preliminary knowledge related to graph theory are presented. The proposed fuzzy sliding-mode consensus controller for the single-integral dynamic model is discussed in Sections III. Also, the selection conditions for the controller parameters are derived to preserve the system consensus stability. To validate the proposed works, some numerical examples are given in Section IV. Finally, the concluding remarks are given in Section V.

II. PRELIMINARIES

In this section, some fundamental definitions in algebraic graph theory used for multiagent systems will be introduced. Considering a multiagent system consisting of n agents, the graph theory is utilized to model the information exchange among agents. Let $\mathcal{G} = (V, E)$ be a directed graph (digraph), which consists of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and an edge set $\mathcal{E} \subset V \times V$. The vertexes v_i and v_j represent the i-th and j-th agents, respectively. In the digraph, an edge of $\mathcal{G}$ is an ordered pair of distinct nodes of $\mathcal{V}$, $(v_i, v_j) \in \mathcal{E}$. It means that agent i can receive information to agent j, but not necessarily vice versa. If all the adjacent nodes v_i and v_j can obtain information from each other in a graph, i.e. the edge $(v_i, v_j) \in \mathcal{E}$ and $(v_j, v_i) \in \mathcal{E}$, the associated communication

topology can be denoted as an undirected graph, a special case of a digraph.

The weighted adjacency matrix of a digraph $\mathcal{G}$ is denoted as

$$\mathcal{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \in \mathcal{R}^{n \times n} \qquad (1)$$

where a_{ij} is said to be the weight of the link (v_i, v_j) and

$$\begin{cases} a_{ij} = 1, & (v_i, v_j) \in \mathcal{E} \\ a_{ij} = 0, & (v_i, v_j) \notin \mathcal{E} \\ a_{ii} = 0 \end{cases}$$

The degree matrix of the digraph $\mathcal{G}$ is a diagonal matrix, $\mathcal{D} = [d_{ij}] \in \mathcal{R}^{n \times n}$, where

$$d_i = \begin{cases} 0, & i \neq j \\ \sum_{j=1}^{n} a_{ij}, & i = j \end{cases}$$

and d_i is called the in-degree of node v_i.

Then the Laplacian matrix associated with the digraph $\mathcal{G}$ can be defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathcal{R}^{n \times n}$$

In this paper, we will focus on the leader-follower multi-tiagent system that consist of one leader agent and $n-1$ follower agents. Based on previous discussion, the agents indexed by $1, 2, \ldots, n-1$ are denoted as the followers and n is the leader. Assume that the leader agent only has the ability of transmission. In other words, the leader cannot obtain any information from other follower agents, $a_{nj} = 0, j = 1, \ldots, n$. The related graph representations can be re-modelled as follows. The topology relationships among all the follower agents are described by a directed graph $\bar{\mathcal{G}}$ that is the subgraph of $\mathcal{G}$. Then, the adjacency matrix of digraph $\bar{\mathcal{G}}$ is rewritten as

$$\bar{\mathcal{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(n-1)} \\ a_{21} & a_{22} & \cdots & a_{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(n-1)1} & a_{(n-1)2} & \cdots & a_{(n-1)(n-1)} \end{bmatrix} \qquad (2)$$

Furthermore, let $\bar{\mathcal{D}} = \text{diag}\{\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_{n-1}\}$ be a degree matrix of digraph $\bar{\mathcal{G}}$, where $\bar{d}_i = \sum_{j=1, j \neq i}^{n-1} a_{i,j}$, $i = 1, 2, \ldots, n-1$. The Laplacian matrix of the digraph $\bar{\mathcal{G}}$ can be redefined as follow

$$\bar{\mathcal{L}} = \bar{\mathcal{D}} - \bar{\mathcal{A}} \qquad (3)$$

The connection relationship between leader and followers can be described as $\bar{\mathcal{B}} = \text{diag}\{\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_{n-1}\}$, where $\bar{b}_i = a_{in}, i = 1, 2, \ldots, n-1$.

III. CONSENSUS CONTROL

Considering possible uncertainties, the dynamic model of a single integrator system is described as follows

$$\dot{x}_i = u_i + \delta_i, \quad i = 1, 2, \ldots, n \qquad (4)$$

where x_i is the position of x-axis of the i-th agent, u_i is the control input, and δ_i represents the uncertainty term.

In the leader-follower control structure that has $n-1$ followers and one leader, the objective of the consensus control is that all of follower agents will converge to the leader agent. Therefore, based on the consensus algorithm, the error function can be denoted as

$$e_i = \sum_{j=1, j \neq i}^{n-1} a_{ij}(x_i - x_j) + b_i(x_i - x_n) \qquad (5)$$

The derivative of error function can be obtained as

$$\begin{aligned} \dot{e}_i = \sum_{j=1, j \neq i}^{n-1} a_{ij}(u_i - u_j) + b_i(u_i - u_n) \\ + \sum_{j=1, j \neq i}^{n-1} a_{ij}(\delta_i - \delta_j) + b_i(\delta_i - \delta_n) \end{aligned} \qquad (6)$$

Let an integral sliding function for the i-th agent be given as

$$s_i = e_i + c_i \int e_i dt \qquad (7)$$

where c_i is a positive constant. The derivative of s_i is shown as

$$\begin{aligned} \dot{s}_i &= \dot{e}_i + c_i e_i \\ &= c_i \sum_{j=1, j \neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \\ &+ \sum_{j=1, j \neq i}^{n-1} a_{ij}(\delta_i - \delta_j) + b_i(\delta_i - \delta_n) \\ &+ \sum_{j=1, j \neq i}^{n-1} a_{ij}(u_i - u_j) + b_i(u_i - u_n) \end{aligned} \qquad (8)$$

The output of the FSMCC is designed to be

$$u_i = u_{ieq} + \left(\sum_{j=1, j \neq i}^{n-1} a_{ij} + b_i \right)^{-1} u_{ifs} \qquad (9)$$

where u_{ieq} is the equivalent control action and u_{ifs} is the output of the fuzzy switching mechanism for the i-th agent. The derivative of s_i is required to be zero so that the states of the fuzzy control system can remain on the sliding surface when $s_i = 0$. From (8), considering the uncertainty-free case, it can be obtained that

$$\begin{aligned} \dot{s}_i &= \dot{e}_i + c_i e_i \\ &= c_i \sum_{j=1, j \neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \\ &+ \sum_{j=1, j \neq i}^{n-1} a_{ij}(u_i - u_j) + b_i(u_i - u_n) \end{aligned} \qquad (10)$$

Thus, the equivalent control, u_{ieq}, can be designed as

$$\begin{aligned} u_{ieq} = -\Big(c_i \sum_{j=1, j \neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \\ - \sum_{j=1, j \neq i}^{n} a_{ij} u_j - b_i u_n \Big) \Big(\sum_{j=1, j \neq i}^{n-1} a_{ij} + b_i \Big)^{-1} \end{aligned} \qquad (11)$$

to have $\dot{s}_i = 0$.

For the FSMCC, a fuzzy switching mechanism is used to assure that the fuzzy sliding control system can approach the sliding surface. Let the s_i and the u_{ifs} be the input and the output variables of the fuzzy switching control (FSC), respectively. With the common knowledge of making the system approach the sliding surface, the following instinctive rules can be obtained as

$$\begin{cases} u_{ifs} > 0, & if \ s_i < 0 \\ u_{ifs} = 0, & if \ s_i = 0 \\ u_{ifs} < 0, & if \ s_i > 0 \end{cases} \qquad (12)$$
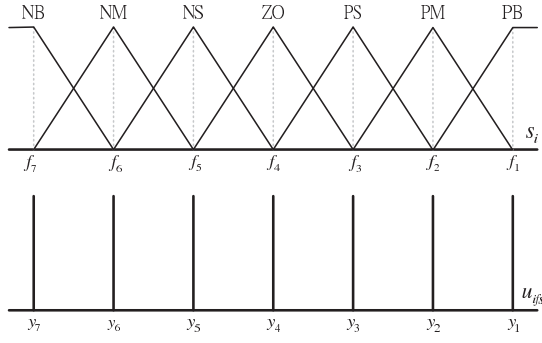
Fig. 1. Membership functions of the fuzzy switch controller.

TABLE I
FUZZY RULE BASE.

Input(s_i)	NB	NM	NS	ZO	PS	PM	PB
Output(u_{ifs})	PB	PM	PS	ZO	NS	NM	NB

The input and output spaces are fuzzily partitioned into seven fuzzy sets, Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZO), Positive Small (PS), Positive Medium (PM) and Positive Big (PB). The input and output membership functions are shown in Fig. 1. Then the fuzzy rules for the i-th agent have the following form:

$$R_{ik}:\ IF\ s_i\ is\ M_{ik}\ THEN\ u_{ifs}\ is\ G_{ik},\ k=1,2,\ldots,7, \tag{13}$$

where M_{ik} and G_{ik} are the corresponding fuzzy sets of antecedent and consequence. Based on (12), complete fuzzy rules in the fuzzy switching mechanism of the FSMCC are provided in Table I. By using the centroid defuzzification technique, the switching output u_{ifs} is calculated as

$$u_{ifs} = \frac{\sum_{k=1}^{7} g_{ik} M_{ik}(s_i)}{\sum_{k=1}^{7} M_{ik}(s_i)} \tag{14}$$

where $g_{ik} \in \{y_{i1}, y_{i2}, \ldots, y_{i7}\}$, $k = 1, 2, \ldots, 7$, are the values of the corresponding output fuzzy singletons. From the triangular membership functions depicted in Fig. 1, it is easy to see that (14) can be simplified to be

$$u_{ifs} = \sum_{k=1}^{7} g_{ik} M_{ik}(s_i) = -\sum_{k=1}^{7} |g_{ik}|sign(s_i) M_{ik}(s_i) \tag{15}$$

where

$$sign(s_i) = \begin{cases} 1, & if\ s_i > 0 \\ 0, & if\ s_i = 0 \\ -1, & if\ s_i < 0 \end{cases}$$

Substituting u_{ieq} and u_{ifs} into (9), the output of the fuzzy sliding consensus controller FSMCC is,

$$u_i = -\left(\sum_{k=1}^{7} |g_{ik}|sign(s_i) M_{ik}(s_i) \right. \\ + c_i \sum_{j=1,j\neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \\ \left. - \sum_{j=1,j\neq i}^{n} a_{ij} u_j - b_i u_n \right) \left(\sum_{j=1,j\neq i}^{n-1} a_{ij} + b_i\right)^{-1} \tag{16}$$

Let $V_i = s_i^2/2 > 0$ be a Lyapunov function candidate. The derivative of V_i can be obtained as

$$\dot{V}_i = s_i \dot{s}_i \\ = s_i \left(c_i \sum_{j=1,j\neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \right. \\ + \sum_{j=1,j\neq i}^{n-1} a_{ij}(u_i - u_j) + b_i(u_i - u_n) \\ \left. + \sum_{j=1,j\neq i}^{n-1} a_{ij}(\delta_i - \delta_j) + b_i(\delta_i - \delta_n)\right) \tag{17}$$

Assume that $|\delta_i| < Q < \infty$, $i = 1, \ldots, n$, where $Q > 0$. It denotes that the uncertainty term of each agent has the same bounded value. Therefore, the following assumption can be obtained,

$$\sum_{j=1,j\neq i}^{n-1} a_{ij}(\delta_i - \delta_j) + b_i(\delta_i - \delta_n) \\ < \sum_{j=1,j\neq i}^{n-1} a_{ij}(Q + Q) + b_i(Q + Q) \\ = 2(\bar{d}_i + \bar{b}_i)Q \tag{18}$$

Define $Q_i = 2(\bar{d}_i + \bar{b}_i)Q > 0$. Thus, from (17), it leads to

$$\dot{V}_i < s_i \left(c_i \sum_{j=1,j\neq i}^{n-1} a_{ij}(x_i - x_j) + c_i b_i(x_i - x_n) \right. \\ + (\sum_{j=1,j\neq i}^{n-1} a_{ij} + b_i)u_i \\ \left. - \sum_{j=1,j\neq i}^{n-1} a_{ij} u_j - b_i u_n + Q_i \right) \tag{19}$$

Substituting u_i in (16) into (19), $\dot{V}_i$ becomes

$$\dot{V}_i < s_i \left(Q_i - \sum_{k=1}^{7} |g_{ik}|sign(s_i) M_{ik}(s_i)\right) \tag{20}$$

Observing above inequality, we can easy find that if

$$\sum_{k=1}^{7} |g_{ik}| M_{ik}(s_i) > Q_i$$

then the sliding mode with the FSMCC is guaranteed. As shown in Fig.1, the input and output membership functions of the fuzzy mechanisms in the FSMCC is determined with

$$f_{ik} = -f_{i(8-k)},\ f_{ik} > 0,\ k = 1, 2, 3,\ f_{i4} = 0;$$

$$y_{ik} = -y_{i(8-k)},\ k = 1, 2, 3,\ y_{i4} = y_i sign(s_i).$$

Theorem 1: For the multi-agent consensus control system, the stability of the i-th agent with the fuzzy sliding-mode consensus controller (FSMCC) is guaranteed if there exists a bounded positive constant Q_i and the parameters of the membership functions in Fig. 1 are designed as follows:

$$\begin{cases} y_{ik} = \xi y_{k+1},\ \xi > 1,\ k = 1, 2 \\ |y_{ik}| = |y_{i(8-k)}|,\ k = 1, 2, 3 \\ y_i > Q_i \end{cases} \tag{21}$$

Proof. As shown in Fig. 1, the universe of the sliding variable s_i can be divided into two types of regions, $s_i \geq 0$ and $s_i < 0$. When $s_i \geq 0$, we have

$$\sum_{k=1}^{7} |g_{ik}|sign(s_i) M_{ik}(s_i) \\ = \left(\xi^3 M_{i1}(s_i) + \xi^2 M_{i2}(s_i) + \xi M_{i3}(s_i) + M_{i4}(s_i)\right) y_i \tag{22}$$

Substituting (22) into (20), the derivative of V_i can be obtained as

$$\dot{V}_i < s_i \left(Q_i - \left(\xi^3 M_{i1}(s_i) + \xi^2 M_{i2}(s_i) \right. \right. \\ \left. \left. + \xi M_{i3}(s_i) + M_{i4}(s_i)\right) y_i\right) \tag{23}$$

Since $y_i > Q_i$ and $\xi > 1$, we can know that

$$\left(\xi^3 M_{i1}(s_i) + \xi^2 M_{i2}(s_i) + \xi M_{i3}(s_i) + M_{i4}(s_i)\right) y_i > Q_i.$$

Therefore, in the region $s_i \geq 0$ the derivative of V_i can be shown as follows

$$\dot{V}_i = s_i \dot{s}_i < 0$$

Similarly, the negative definiteness of $\dot{V}$ can be shown for the case $s_i < 0$. From the aforementioned discussion, it can be concluded that the stability of i-th follower in multi-agent consensus control system with the proposed fuzzy sliding consensus controller is guaranteed. ■

Remark 1 Due to $y_i > Q_i = 2(\bar{d}_i + \bar{b}_i)Q > 0$, it can be known that the parameter y_i is dependedt on communication topology, i.e. $\bar{d}_i$ and $\bar{b}_i$.

The previous discussion about the consensus control is only for one particular agent, the i-th agent. In fact, the proposed FSMCC can be applied to a general multi-agent system with $n - 1$ followers. Let X be the state vector of $n - 1$ followers, $X = [x_1 \ x_2 \ \ldots \ x_{n-1}]^T$, $X \in \mathbf{R}^{n-1}$, $U_{eq} = [u_{1eq} \ u_{2eq} \ \ldots \ u_{(n-1)eq}]^T$, $U_{fs} = [u_{1fs} \ u_{2fs} \ \ldots \ u_{(n-1)fs}]^T$, and $U = [u_1 \ u_2 \ \ldots \ u_{n-1}]^T = U_{eq} + \left(\bar{\mathcal{D}} + \bar{\mathcal{B}}\right)^{-1} U_{fs}$, $U \in \mathbf{R}^{(n-1)}$. From (11) and (15), the equivalent control action, U_{eq}, and the output of fuzzy switching mechanism, U_{fs}, for the multi-agent system, respectively, are represented as follows

$$U_{eq} = -\left(\bar{\mathcal{D}} + \bar{\mathcal{B}}\right)^{-1} \left(C\left(\bar{\mathcal{L}} + \bar{\mathcal{B}}\right) X - \bar{\mathcal{A}}U \\ -C\bar{\mathcal{B}}\mathbf{1}x_n - \bar{\mathcal{B}}\mathbf{1}u_n\right) \quad (24)$$

and

$$U_{fs} = \begin{bmatrix} -\sum_{k=1}^{7} |g_{1k}| sign(s_1) M_{1k}(s_1) \\ -\sum_{k=1}^{7} |g_{2k}| sign(s_2) M_{2k}(s_2) \\ \vdots \\ -\sum_{k=1}^{7} |g_{(n-1)k}| sign(s_{n-1}) M_{(n-1)k}(s_{n-1}) \end{bmatrix} \quad (25)$$

where $C = diag\{c_1, c_2, \ldots, c_{n-1}\}$. Therefore, the output of FSMCC in the multi-agent system can be given as

$$U = -\left(C((\bar{\mathcal{L}} + \bar{\mathcal{B}}) X - \bar{\mathcal{B}}\mathbf{1}x_n) - \bar{\mathcal{B}}\mathbf{1}u_n + U_{fs}\right)\left(\bar{\mathcal{L}} + \bar{\mathcal{B}}\right)^{-1} \quad (26)$$

IV. SIMULATION RESULTS

To validate the feasibility of proposed consensus protocol, an ideal single-integrator multi-agent system is considered. The addressed single-integrator multi-agent system contains one leader and four followers. Two cases of the consensus control will be investigated, i.e. the position of leader agent being fixed and time-varying, respectively. The communication graph with $(0, 1)$-weights for modeling the interactions among agents is shown in Fig. 2, where numbers 1-4 are follower agents and the number 5 is the leader agent. Thus the associate adjacency matrix can be determined as

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
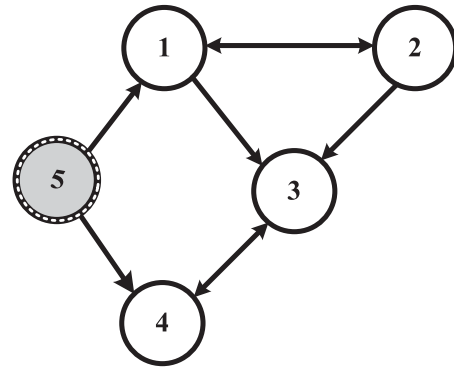

Fig. 2. Communication topology with multi-agent system.

Let the initial positions of the followers 1-4 be $[0, 0.5]$, $[0.1, 1]$, $[0.3, 2]$ and $[0.4, 2.5]$(m), respectively, and the initial position of leader be $[1, 2]$(m). For the consensus control with fixed leader, the velocity of the leader is $[0, 0]$. Accordingly, the simulation results are shown in Figs. 3-6, where Fig. 3 and Fig. 5 are the X-Y position responses, and the position errors in the x-axis and y-axis are shown in Fig. 4 and Fig. 6, respectively. Figs. 3-4 and Figs. 5-6 are the corresponding results with the conventional consensus algorithm and the proposed FSMCC, respectively. The lines $F1$-$F4$ are the trajectories of follower agents 1-4 and the symbol 'o' is the position plotted in 1.25 second interval. From Figs. 3-6, it can be seen that the required consensus can be achieved by utilizing the conventional consensus algorithm, and the proposed FSMCC. It can be also observed that the transient responses of FSMCC is better than the responses of the conventional algorithm. Furthermore, the case of time-varying leader is considered, where the initial position and the velocity of the leader are given as $[0.2, 1.5]$(m) and $[0.2, 0.2]$ (m/sec), respectively. The initial positions of followers are the same as the fixed-leader case. The trajectories of the leader and followers are shown in Figs. 7-10. It is interested to point out that the consensus requirement can not be achieved by using the conventional consensus algorithm. As for he proposed FSMCC, not only the consensus can be achieved, but also the transient and steady-state responses can be performed quite well.

V. CONCLUSIONS

This paper presents a fuzzy sliding-mode consensus controller for a multi-agent system. The multi-agent system with the proposed FSMCC can reach the desired position asymptotically. Moreover, the stability of the control system is guaranteed by the Lyapunov theorem. From the simulation results, the effectiveness of the designed FSMCC for multi-agent system can be provided. The simulation results empirically support the promising performance of the presented FSMCC controller.
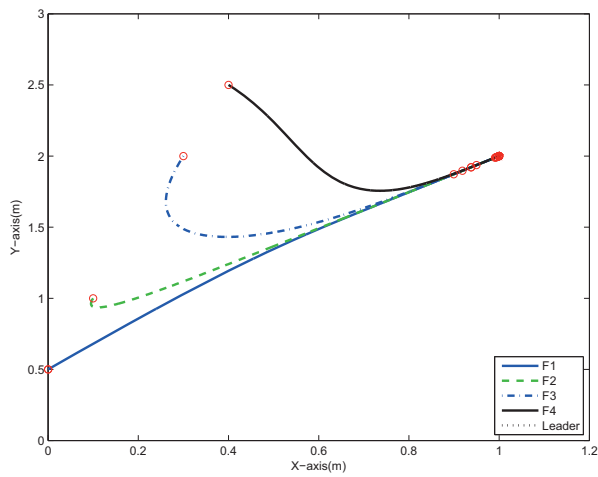
Fig. 3. Multi-agent consensus control with fixed position: conventional consensus algorithm [5]
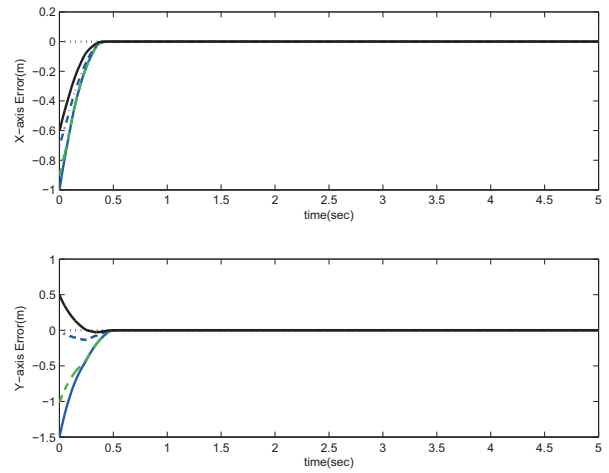

Fig. 4. Multi-agent consensus control with fixed position: conventional consensus algorithm [5]

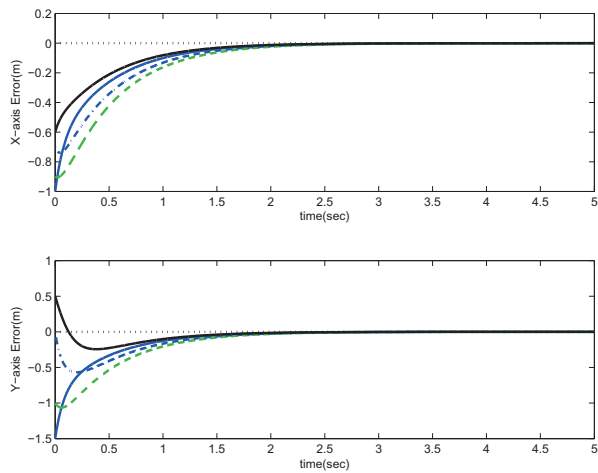Fig. 5. Multi-agent consensus control with fixed position: FSMCC

Fig. 6. Multi-agent consensus control with fixed position: FSMCC

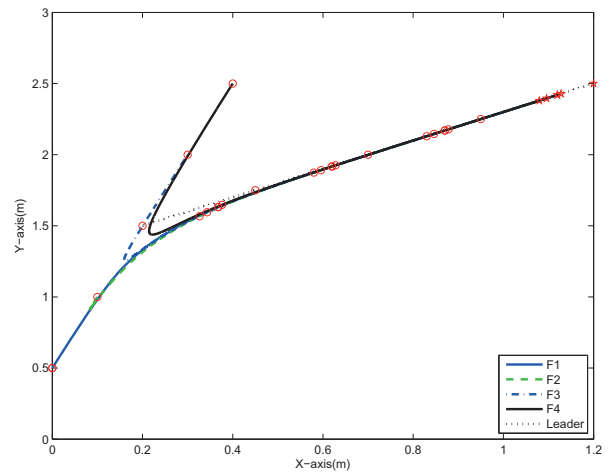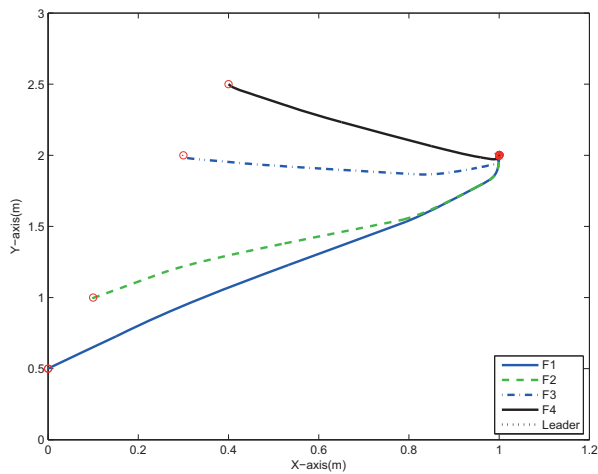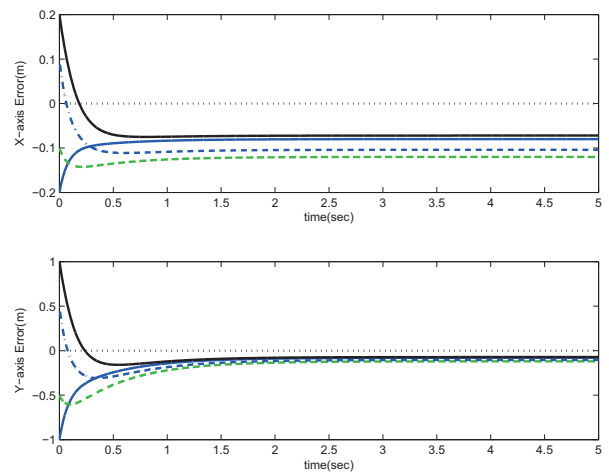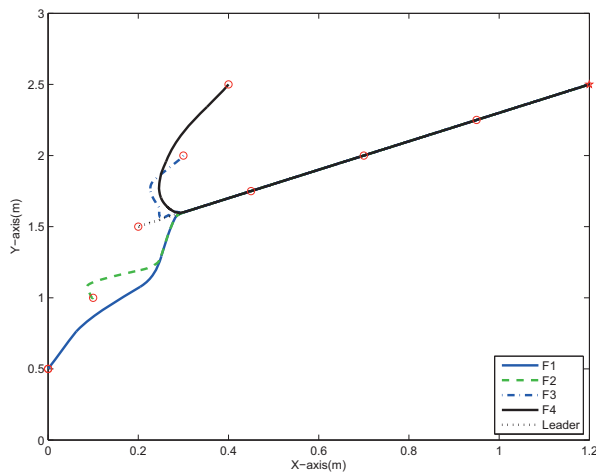Fig. 7. Multi-agent consensus control with moving leader: conventional consensus algorithm [5]

Fig. 8. Multi-agent consensus control with moving leader: conventional consensus algorithm [5]

Fig. 9.   Multi-agent consensus control with moving leader: FSMCC
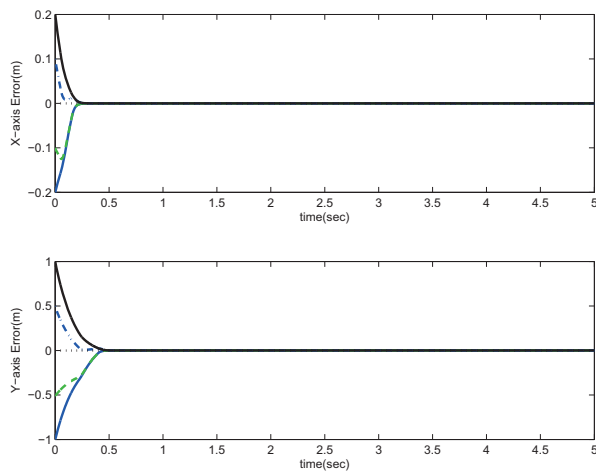

Fig. 10.   Multi-agent consensus control with moving leader: FSMCC

VI. ACKNOWLEDGMENTS

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