International Journal of Innovative Computing, Information and Control
Volume **9**, Number **7**, July **2013**
 ISSN 1349-4198
pp. 2713–2726

FUZZY SLIDING MODE CONTROL DESIGN FOR A CLASS OF NONLINEAR SYSTEMS WITH STRUCTURED AND UNSTRUCTURED UNCERTAINTIES

Mohammad Reza Soltanpour[1], Behrouz Zolfaghari[2]
Mahmoodreza Soltani[3] and Mohammad Hassan Khooban[4]

[1]Department of Electrical Engineering
Aeronautical University of Science and Technology
Tehran, Iran
m_r_soltanpour@yahoo.com

[2]Department of Computer Engineering and Information Technology
Amirkabir University of Technology
No. 424, Hafez Ave., Tehran, Iran
zolfaghari@iust.ac.ir

[3]Department of Civil Engineering
University of Texas at El Paso
No. 500, West University Avenue, El Paso, Texas 79968, U.S.A.
mah.soltani@gmail.com

[4]Department of Electrical and Robotic Engineering
Garmsar Branch Islamic Azad University of Iran
Garmsar, Iran
mhkhoban@gmail.com

Received March 2012; revised July 2012

Abstract. *This paper presents controlling of a class of nonlinear systems with structured and unstructured uncertainties using fuzzy sliding mode control. First known dynamics of the system are eliminated through feedback linearization and then fuzzy sliding mode controller is designed using TS method, based on the Lyapunov method, which is capable of handling uncertainties. There are no signs of the undesired chattering phenomenon in the proposed method. The globally asymptotic stability of the closed-loop system is mathematically proved. Finally, this method of control is applied to the inverted pendulum system as a case study. Simulation results show the system performance is desirable.*
Keywords: Nonlinear systems, Structure uncertainties, Unstructured uncertainties, Fuzzy, Sliding mode control

1. **Introduction.** Most of plants in the industry have severe nonlinearity, which causes researches to develop nonlinear control systems. Thus, great attention has been attracted from both the academic and industrial communities. To overcome certain difficulties in the design of a controller for a nonlinear system, various schemes have been developed in the last two decades, among which a successful approach is fuzzy control.

The fuzzy logic control (FLC) has been an active research topic in automation and control theory since the work of Mandani (1974) based on the fuzzy sets theory of Zadeh (1965) to deal with the system control problems which are not easy to be modeled and/or to be modeled accurately. The concept of FLC is to utilize the qualitative knowledge of a system to design a practical controller. It is particularly suitable for those systems with uncertain or complex dynamics. In general, a fuzzy control algorithm consists of a set of heuristic decision rules and can be regarded as a nonmathematical control algorithm, in contrast to a conventional feedback control algorithm.

In recent years, there have been significant advances in the study of the stability analysis and controller synthesis for the so-called Takagi-Sugeno (T-S) fuzzy systems, which have been used to represent certain complex nonlinear systems.

For a nonlinear system the overall model is obtained by the fuzzy blending of local models. For each local linear model, a linear feedback control is designed. The resulting overall controller is again a fuzzy blending of the individual linear controllers. Originally, Tanaka and his colleagues have provided certain conditions that are sufficient for the stability of the T-S fuzzy systems in the sense of Lyapunov [2].

Sliding mode control (SMC) is a robust nonlinear feedback control technique whose structure is intentionally changed to achieve the desired performance [3-5]. In the design of SMC, it is assumed that the control can be switched from one structure to another infinitely fast. However, because of the switching delay computation and the limitation of the physical actuators which cannot handle the switching of control signal at an infinite rate, it is practically impossible to achieve high-speed switching control. As a result of this imperfect control switching between structures, the system trajectory appears to chatter instead of sliding along the sliding surface. There are essentially two ways to counter the chattering phenomenon. One way is to use higher order sliding mode [6-8], and the other most common way for chattering reduction involves introducing a boundary layer around the sliding surface and using a continuous control within the boundary layer [9,10]. The width of the boundary is normally constant, and the larger the boundary width, the smoother the control signal. Even though the boundary design alleviates the chattering phenomenon, it no longer drives the system state to the origin, and steady-state error will appear. The larger the width of the boundary layer, the larger of the steady-state error.

Some researchers applied fuzzy logic systems to sliding mode control to improve the performance of SMC. Hence, there are the so-called fuzzy sliding mode control (FSMC) [11-18] and sliding mode fuzzy logic control (SMFC) [19-26]. The first substantially is fuzzy adaptive sliding mode control algorithm in which unknown system dynamics are identified by FLS to form the equivalent control of SMC. The latter is FLC based on sliding mode or sliding mode control.

In FSMC, continuous part of SMC controller is obtained by using FLS to approximate the unknown nonlinear dynamics, and the closed-loop system is verified to be stable based on Lyapunov stability theory. Thereby adaptive law is found directly, which tunes parameters of the fuzzy set supports, i.e., rule base functions. Temeltas presented a class of such FSMC based on a kind of special fuzzy reasoning principle for nonlinear systems [11]. The others proposed such an FSMC controller based on feedback linearization [12], and then researchers studied this method for MIMO nonlinear systems [13]. Other previous proposed works can be found in [16-18]. For SMFC, there are also many investigations and reports including some direct or indirect adaptive fuzzy control [25,26] and practical applications [24]. Palm originally presented sliding mode fuzzy control [19], in which the main idea is that SMC is acquired firstly, then fuzzy controller is constructed such that the two controllers are equivalent. Afterwards based on this main idea, researchers put forward sliding-like fuzzy logic control (SMLFC) [20,21]. In these methods, fuzzy controller is equivalent to the sliding mode controller pre-designed and the boundary layer thickness can be self-tuned online after introducing appropriate adaptive laws, wherefore the controller has no chattering [20,21]. However, in papers [19-21], the methods are only applicable into systems which represented through second-order dynamics. Furthermore, as the dead zone parameter converges to zero, input control also has a little chattering.

For nonlinear systems with high-order dynamics, the controller design should be investigated in different way to ensure its desired performance. In addition, due to adaptive law in the control input, computation of control input will be increased. So, the practical implementation of these methods is difficult.

In recent years, for controlling of nonlinear systems, sliding mode control technique has been used in different ways [27-30]. In these methods, there are interesting solutions to overcome on some issues such as mismatched uncertainties, nonlinear jump systems and uncertain systems with time delay have been presented. However, these methods are too complicated and have large computation volume. They also have undesirable chattering phenomena in overcoming the existing challenges.

In this paper a fuzzy sliding mode control is presented. It is very simple and does not have complications of the methods which were already mentioned. According to the recent method first known dynamics of the system are eliminated through feedback linearization. In the next step TS fuzzy model-based approach is used to design the fuzzy sliding mode controller. This method can be applied into nonlinear systems of order n. Furthermore, the globally asymptotic stability of the closed-loop system is mathematically proved.

Simulation results show the control action is free of chattering and the closed-loop system has good performance.

This paper is organized as follows. Section 2 presents the problem statement. Section 3 provides required bases for the controller design. Sliding mode control design steps are presented in Section 4. TS fuzzy model-based method is explained in Section 5 and fuzzy sliding mode control design is illustrated in Section 6. Section 7 deals with key features of the proposed method. Section 8 explains the simulation results in 4 steps. Finally conclusions are discussed in Section 9.

2. **Problem Statement.**

In this paper, the single-input single-output (SISO) nonlinear system is considered. It is supposed that the system can be explained as follows:

$$\begin{aligned} X^{(n)}(t) &= f(X(t),t) + g(X(t),t)u(t) + d(t) \\ y(t) &= x(t) \end{aligned} \tag{1}$$

where $X(t) = [x(t), \dot{x}(t), \ldots, x^{(n)}(t)]^T$ is n^{th} order state vector of the system, $y(t) \in R$ is the output of the system, $f(X(t),t)$ and $g(X(t),t)$ are not exactly known but smooth functions, $u(t) \in R$ is the control input and $d(t) \in R$ is an external disturbance.

The control problem is to force the output $y(t)$ to follow a given bounded reference input signal $y_d(t)$. Let $e(t) = y(t) - y_d(t)$ be the tracking error and its forward shifted values, defined as $e^{(i)} = y^{(i)}(t) - y_d^{(i)}(t) \quad (i = 1, 2, \ldots, n-1)$. So the error vector is defined as $e(t) = [e(t), \dot{e}(t), \ldots, e^{(n-1)}(t)]$.

In order to design the control input, the following assumptions are necessary:

1. The states of the system $X(t)$ are measurable.
2. The extent of the imprecision on $f(X(t),t)$ is upper bounded by a known continuous function of $X(t)$.
3. $g(X(t),t)$ is lower and upper bounded such as $0 < \underline{g} < g(X(t),t) < \overline{g}$ where $\underline{g}$ and $\overline{g}$ are positive constants.
4. $d(t)$ is unknown, but it is bounded, i.e., $|d(t)| < D$. Where the "D" is an known positive constant.

3. **System Model in State Space and Selecting of Sliding Surface.**

To have the system described by Equation (1) in the state space form we define

$$x(t) = x_1(t), \quad \dot{x}(t) = x_2(t), \ldots, x^{(n-1)}(t) = x_n(t) \tag{2}$$

By substituting Equation (2) into Equation (1), we have

$$\begin{aligned}&\dot{x}_1(t) = x_2(t)\\&\dot{x}_2(t) = x_3(t)\\&\vdots\\&\dot{x}_n(t) = f(X(t),t) + g(X(t),t)u(t) + d(t)\\&y(t) = x_i(t), \quad i = 1,2,\ldots,n\end{aligned} \tag{3}$$

By transferring the nonlinear system equations to the state space domain, the following error equations will appear

$$x_1(t) - x_{1_d}(t) = e_1(t), \quad x_2(t) - x_{2_d}(t) = e_2(t), \ldots, x_n(t) - x_{n_d}(t) = e_n(t) \tag{4}$$

In the above equations, $x_{i_d}(t)$ is the $(i-1)$th derivative of the desired path which should be tracked by the input control. From Equations (2) and (4) we can conclude that

$$\dot{e}_1(t) = e_2(t), \quad \dot{e}_2(t) = e_3(t), \ldots, \dot{e}_{n-1}(t) = e_n(t) \tag{5}$$

Now we define the sliding surface as

$$S(t) = c_1e_1(t) + c_2e_2(t) + \ldots + c_{n-1}e_{n-1}(t) + e_n(t) \tag{6}$$

where c_i, $(i = 1, \ldots, n-1)$ are constant positive factors.

4. Sliding Mode Control Design.

In this section, the control action $u(t)$ is designed in a way that the output is capable of tracking a desired path. Moreover, the tracking error and all its derivatives will tend to zero. Thus, the control action is defined as follows:

$$u(t) = \hat{g}^{-1}(X(t),t)\left\{-\hat{f}(X(t),t) + \dot{x}_{n_d}(t) + u_f(t)\right\} \tag{7}$$

$\hat{g}(X(t),t)$ and $\hat{f}(X(t),t)$ are known parts of $g(X(t),t)$ and $f(X(t),t)$ respectively. $\dot{x}_{n_d}(t)$ is derivative of the desirable path $x_{n_d}(t)$ with respect to time and $u_f(t)$ is the sliding mode control input which is designed to handle structured and unstructured uncertainties. From now on this section for the sake of brevity, $\hat{f}$, $\hat{g}$, f and g will be used instead of $\hat{f}(X(t),t)$, $\hat{g}(X(t),t)$, $f(X(t),t)$ and $g(X(t),t)$ respectively. Equation (7) is substituted in Equation (3):

$$\dot{x}_n(t) = f + g\left[\hat{g}^{-1}\left\{-\hat{f} + \dot{x}_{n_d}(t) + u_f(t)\right\}\right] + d(t) \tag{8}$$

To Equation (8), $\dot{x}_{n_d}(t)$ and $u_f(t)$ are added and subtracted.

$$\dot{x}_n(t) = f + g\left[\hat{g}^{-1}\left\{-\hat{f} + \dot{x}_{n_d}(t) + u_f(t)\right\}\right] + d(t) + \dot{x}_{n_d}(t) - \dot{x}_{n_d}(t) + u_f(t) - u_f(t) \tag{9}$$

Equation (9) can be rewritten as follows:

$$\dot{x}_n(t) - \dot{x}_{n_d}(t) = f - g\,\hat{g}^{-1}\hat{f} + (g\,\hat{g}^{-1} - 1)\dot{x}_{n_d}(t) + (g\,\hat{g}^{-1} + 1)\,u_f(t) + d(t) - u_f(t) \tag{10}$$

To simplify Equation (10), the following equations are used

$$\begin{aligned}&\dot{e}_n(t) = \dot{x}_n(t) - \dot{x}_{n_d}(t)\\&\eta = f - g\,\hat{g}^{-1}\hat{f} + (g\,\hat{g}^{-1} - 1)\dot{x}_{n_d}(t) + (g\,\hat{g}^{-1} + 1)\,u_f(t) + d(t)\end{aligned} \tag{11}$$

According to Equation (11), η includes all existing uncertainties. That is, if the system does not have structured and unstructured uncertainties then $\eta = 0$. By substituting Equation (11) in (10)

$$\dot{e}_n(t) = \eta - u_f(t) \tag{12}$$

In sliding mode control design $u_f(t)$ consists of two parts, $u_{eq}(t)$ equivalent control and $u_s(t)$ switching control [9]:

$$u_f(t) = u_{eq}(t) + u_s(t) \tag{13}$$

In the sliding phase, where $S(t) = 0$ and $\dot{S}(t) = 0$, the equivalent term $u_{eq}(t)$ is designed to keep the system on the sliding surface. In the approaching phase, where $S(t) \neq 0$, the switching term $u_s(t)$ is designed to satisfy the reaching condition $S(t)\dot{S}(t) < 0$.

For desigining the part $u_{eq}(t)$ the derivative of Equation (6) is supposed to be equal to zero:

$$\dot{S}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \dot{e}_n(t) = 0 \tag{14}$$

Equation (12) is substituted in Equation (14):

$$c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \eta - u_f(t) = 0 \tag{15}$$

In design of $u_{eq}(t)$, it is assumed that the sliding surface is zero. So, the task of $u_{eq}(t)$ is preventing the sliding surface from changes. According to this assumption, $u_s(t)$ in this part of the design may be considered as zero. By considering the above points and substituting Equation (13) in (15):

$$c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \eta - u_{eq}(t) = 0 \tag{16}$$

Finally $u_{eq}(t)$ is derived from the above equation:

$$u_{eq}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \eta \tag{17}$$

Concerning Equation (17) we can conclude that

$$\|u_{eq}(t)\| \leq \|c_1\dot{e}_1(t)\| + \|c_2\dot{e}_2(t)\| + \ldots + \|c_{n-1}\dot{e}_{n-1}(t)\| + \|\eta\| \tag{18}$$

Where the $\|\circ\|$ symbol is norm. According to Equation (18), $u_{eq}(t)$ can be set as

$$u_{eq}(t) = \|c_1\dot{e}_1(t)\| + \|c_2\dot{e}_2(t)\| + \ldots + \|c_{n-1}\dot{e}_{n-1}(t)\| + \|\eta\| \tag{19}$$

Now, the $u_s(t)$ is designed in a way that the sliding surface tends to zero. Therefore, the following Lyapunov candidate function is introduced:

$$V(S(t)) = \frac{1}{2}S^2(t) \tag{20}$$

Derivative of the Lyapunov candidate function with respect to time is

$$\dot{V}(S(t)) = \dot{S}(t)S(t) \tag{21}$$

From Equations (6) and (21), we conclude that

$$\dot{V}(S(t)) = (c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \dot{e}_n(t))S(t) \tag{22}$$

From Equations (12), (13) and (22), it results that

$$\dot{V}(S(t)) = (c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t) + \eta - (u_{eq}(t) + u_s(t)))S(t) \tag{23}$$

We substitute Equation (19) in Equation (23):

$$\begin{aligned} \dot{V}(S(t)) = {} & c_1\dot{e}_1(t)S(t) + c_2\dot{e}_2(t)S(t) + \ldots + c_{n-1}\dot{e}_{n-1}(t)S(t) \\ & + \eta S(t) - \|c_1\dot{e}_1(t)\| S(t) - \|c_2\dot{e}_2(t)\| S(t) - \ldots - \|c_{n-1}\dot{e}_{n-1}(t)\| S(t) \\ & - \|\eta\| S(t) - u_s(t)S(t) \end{aligned} \tag{24}$$

It is resulted from Equation (24) to have the inequality $\dot{V}(S(t)) < 0$ satisfied, the following condition has to be met:

$$\begin{cases} u_s(t) = \rho & \text{if} \quad S(t) > 0 \\ u_s(t) = -\rho & \text{if} \quad S(t) < 0 \end{cases} \tag{25}$$

where ρ is a constant positive factor. Concerning (13), (19) and (25) we have

$$\begin{cases} u_f(t) = u^+(t) = u_{eq}(t) + \rho & \text{if} \quad S(t) > 0 \\ u_f(t) = u^-(t) = u_{eq}(t) - \rho & \text{if} \quad S(t) < 0 \end{cases} \tag{26}$$

5. **T-S Fuzzy Model.** T-S fuzzy logic system is given in the following form of IF-THEN rules:

$$R^i: \text{ IF } x_1(t) \text{ is } A_{1i} \text{ and } \dots \text{ and } x_q(t) \text{ is } A_{qi} \text{ THEN } u_i(t) = f_i(X(t), t), \quad i = 1, \dots, r \tag{27}$$

where R^i represents the i^{th} fuzzy inference rule, x_j and A_{ij} ($i = 1, \dots, r$ and $j = 1, \dots, q$) are the premise variables and fuzzy sets, and r is the number of fuzzy IF-THEN rules.

Following the fuzzy inference method of T-S fuzzy system, the control input $U(t)$ of the overall system can be obtained in the weighted average form along the trajectories $X(t)$:

$$U(t) = \frac{\sum_{i=1}^{r} w_i(X(t)) f_i(X(t), t)}{\sum_{i=1}^{r} w_i(X(t))} \tag{28}$$

where $U(t) \in R^p$, and $X(t) \in R^q$, the weight functions are defined as

$$w_i(X(t)) = \prod_{j=1}^{q} A_{ij}(x_j(t)) \tag{29}$$

where $A_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in the fuzzy set A_{ij}. The weight functions $w_i(X(t))$ are nonnegative and measurable, and usually satisfy

$$\sum_{i=1}^{r} w_i(X(t)) > 0, \quad \text{for all } t > 0 \tag{30}$$

6. **Fuzzy Sliding Mode Control Design.** The proposed T-S fuzzy model-based sliding mode control is based on the intuitive feedback control strategy. Thus, the fuzzy inference rule base is established as

$$\begin{array}{l} R^1: \text{ IF } S(t) \text{ is positive THEN } u_f(t) = u^1(t) = u^+(t) \\ R^2: \text{ IF } S(t) \text{ is negative THEN } u_f(t) = u^2(t) = u^-(t) \end{array} \tag{31}$$

Finally, the system control $u_f(t)$ can be obtained through centre of gravity defuzzification method

$$u_f(t) = \frac{\sum_{i=1}^{2} w_i(S(t)) u^i(t)}{\sum_{i=1}^{2} w_i(S(t))} \tag{32}$$

where $w_i(S(t))$ is as same as the one defined in (29). Therefore, the designed control input in its general form is as follows:

$$\begin{cases} u(t) = \hat{g}^{-1}(X(t), t) \left\{ -\hat{f}(X(t), t) + \dot{x}_{n_d}(t) + u_f(t) \right\} \\ u_f(t) = \frac{\sum_{i=1}^{2} w_i(S(t)) u^i(t)}{\sum_{i=1}^{2} w_i(S(t))} \\ u^1 = u_{eq}(t) + \rho \\ u^2 = u_{eq}(t) - \rho \\ u_{eq} = \|c_1 \dot{e}_1(t)\| + \|c_2 \dot{e}_2(t)\| + \dots + \|c_{n-1} \dot{e}_{n-1}(t)\| + \|\eta\| \end{cases} \tag{33}$$

Note 1. To calculate $\|\eta\|$, the assumptions of Section 2 and Equation (11) can be used:

$$\|\eta\| = \bar{f} + \underline{g}\, \hat{g}^{-1} \hat{f} + \left(\bar{g}\, \hat{g}^{-1} - 1\right) \|\dot{x}_{n_d}(t)\| + \left(\bar{g}\, \hat{g}^{-1} + 1\right) \|u_f(t)\| + D \tag{34}$$

where $\bar{f}$ is an upper bound of f. Considering (34) reveals that $\|\eta\|$ is directly related to $\|u_f(t)\|$. So, for choosing the positive constant ρ, there must be appropriate watchfulness. It is because of the reason that if the coefficient of ρ was chosen very large, consequently, the boundaries of uncertainties would be increased literally. Thus, the control input increases and the actuators may be leaded to saturation.

Note 2. According to the above discussion, the procedure for designing a fuzzy sliding mode controller is described as:

1. Define the tracking error variable $e_i(t)$s.
2. Specify $S(t)$ sliding surface by selecting $c_1, c_2, \ldots, c_{n-1}$ as positive constant coefficients.
3. Specify $\hat{f}$ and $\hat{g}$ as known parts of f and g. Then specify upper and lower bounds of f and g.
4. Design the T-S fuzzy model control law $u^1(t)$ and $u^2(t)$, and the related fuzzy membership functions.
5. Build fuzzy inference rule base.
6. Defuzzify the fuzzy variables through centre of gravity method to get the crisp control law $u_f(t)$.
7. Implementation.

7. **Discussion.** In the proposed control design, there are some points have been considered which have a prominent role in its practical implementation. These points respectively are as follows:

1. In this control method, the bound of uncertainties can be decreased as much as possible because of using the feedback linearization. Then the fuzzy sliding mode controller is designed through applying the TS fuzzy model and an inference engine composed of a very brief rule base (only two rules).

2. This control method is free of undesirable chattering phenomenon. Moreover, it can handle structured and unstructured uncertainties while adaptive methods are weak in coping with unstructured uncertainties [31-33].

3. Another benefit of the designed controller is its light burden of computations which is an important figure in practical implementation and online control cases. In the control of industrial complex systems the computational burden of the control action is very significant as the heavy computational burden of the control action is not only expensive but also can cause instability of the closed-loop system [34-35].

4. In the design step of controller design in many industrial nonlinear systems such as robot manipulator, there are a lot of challenges due to existence of the structured and unstructured uncertainties in dynamics equations [31-35]. On the other hand, if we also want to consider the actuator dynamics in the design of controllers for these industrial systems, the role of unstructured uncertainties would be highlighted. Beside of these existing issues, we should point to the volume of the calculations and being soft and smooth of the control input, as well. Hence, it is observed that the controller design for these nonlinear systems is very difficult and it can be a part of the research issues nowadays. All above mentioned points are considered in proposed design controller. Thus, the proposed method can be very powerful in controlling of theses nonlinear systems.

8. **Simulation Results.** In this section, illustrative numerical simulation examples are provided to demonstrate the effectiveness and robustness of the proposed approach. The problem to be considered is a pole-balancing of an inverted pendulum as shown in Figure

1. The system is represented by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{mlx_2^2\sin(x_1)\cos(x_1)-(M+m)g\sin(x_1)}{ml\cos^2(x_1)-(\frac{4}{3})l(M+m)} + \frac{-\cos(x_1)}{ml\cos^2(x_1)-(\frac{4}{3})l(M+m)}u(t) + d(t) \end{cases} \tag{35}$$

where x_1 angle θ (in radians) of the pendulum from the vertical, M mass of cart, m mass of the pole, $u(t)$ force applied to the cart and $d(t)$ is an external disturbance. The parameters employed in this simulation are given as follows: $M = 1$kg, $m = 0.3$kg, $l = 0.5$m and $g = 9.8\text{m/s}^2$

In this simulation, the known parts of $f(X(t))$ and $g(X(t))$ are listed as follows:

$$\begin{cases} \hat{f}(X(t)) = \frac{\hat{m}\hat{l}x_2^2-(\hat{M}+\hat{m})\hat{g}}{\hat{m}\hat{l}-\frac{4}{3}\hat{l}(\hat{M}+\hat{m})} \\ \hat{g}(X(t)) = \frac{-1}{\hat{m}\hat{l}-\frac{4}{3}\hat{l}(\hat{M}+\hat{m})} \end{cases} \tag{36}$$

The values of parameters $\hat{m}$, $\hat{l}$, $\hat{M}$ and $\hat{g}$ are considered to be 90 percent of their real values. The error and the sliding surface equations are defined as $e_1(t) = x_1(t) - x_{1_d}(t)$, $e_2(t) = x_2(t) - x_{2_d}(t)$ and $S(t) = 20\,e_1(t) + e_2(t)$ respectively. The equivalent control input is $u_{eq}(t) = \|20\,\dot{e}_1(t)\| + \|\eta\|$. $\|\eta\|$ and it would be calculated as follows:

$$\|\eta\| = 0.01\,x_2^2 + 0.8 + 0.01\,\|\dot{x}_{2_d}(t)\| + 0.01\,\|u_f(t)\| \tag{37}$$

Having calculated $u_{eq}(t)$, control inputs $u^1(t)$ and $u^2(t)$ are:

$$\begin{cases} u^1(t) = u_{eq}(t) + \rho & \text{if } S(t) > 0 \\ u^2(t) = u_{eq}(t) - \rho & \text{if } S(t) < 0 \end{cases} \tag{38}$$

$S(t)$ is introduced as premise variable and the fuzzy membership functions are defined as:

$$\begin{cases} w^-(S(t)) = \frac{1}{1+\exp(rS(t))} & \text{if } S(t) \text{ is negative} \\ w^+(S(t)) = \frac{1}{1+\exp(-rS(t))} & \text{if } S(t) \text{ is positive} \end{cases} \tag{39}$$

where r is equal to 100. In this simulation, initial condition of $x_1(t)$ is 0.1 which is equivalently 5 degrees. In this paper to highlight the good performance of the system, simulations are presented in 4 steps.

Simulation 1. In this stage of the simulation, the $x_{1_d}(t) = 0$ and the system is considered without external disturbance $d(t)$. To apply the proposed control method, the coefficient ρ is chosen equal to 23. Simulation results are depicted in Figures 2 and 3. It is resulted from Figure 2 that during the simulation the system performed well. In presence of the parametric uncertainties in less than 0.5 seconds the error converges into “0” and

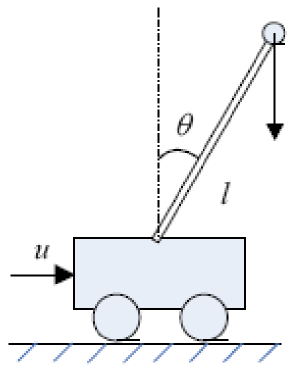

FIGURE 1. Inverted pendulum system

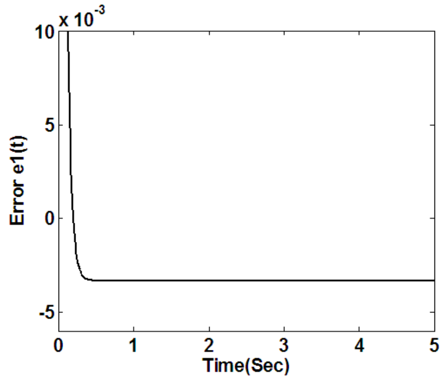

FIGURE 2. Error signals with no external disturbance

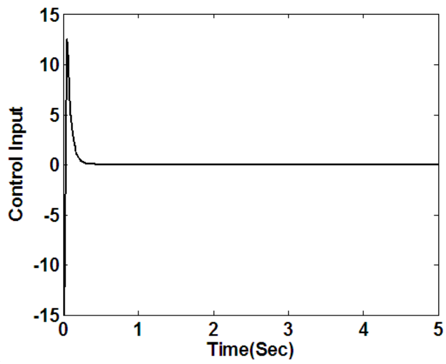

FIGURE 3. Control input with no external disturbance

finally it has persistent error of 0.003 radian. As it appears in Figure 3, control input is free of chattering and also it is in acceptable range.

Simulation 2. In this stage of the simulation, the proposed control coefficients are set just like the previous section and the external disturbance $d(t)$ is chosen equal to 3.5. Thus, the inverted pendulum system apart from the parametric uncertainty has unstructured uncertainties as well. By applying the control input to Inverted pendulum system (Figure 4), it is resulted that error signal in less than 0.5 seconds converges to zero and the steady-state error would be equal to 0.004 radian. Figure 5 illustrates that the control input is without chattering and it is in the feasible range. In the next step let $d(t) = -4\ \sin(0.3t)$ and $\rho = 25$. That is, the system has varying external disturbance with respect to time. Having applied the proposed control method to the system, it is revealed that the maximum error would be 0.004 radian which shows the good performance of the designed control input (Figure 6). In Figure 7, there is control input. In this figure the input control is free of chattering. It is concluded from Figure 8 choosing values of 25, 30 and 40 for coefficient ρ, the maximum error will be decreased massively. This issue is also another advantage of the proposed control method.

Simulation 3. In this stage of the simulation, for making further challenging onto the proposed control method, not only the external disturbance is exerted $d(t) = -4\ \sin(0.3t)$, but also there will be a square signal with an amplitude of 15 in the time interval from 4 to 5 second which will be added to the inverted pendulum system. In the other words, we intend to demonstrate the sophisticated control ability and performance by applying an impulsive disturbance between second 4 and 5. By choosing $\rho = 50$ control input was exerted to inverted pendulum system. By considering, Figure 9 shows that the control

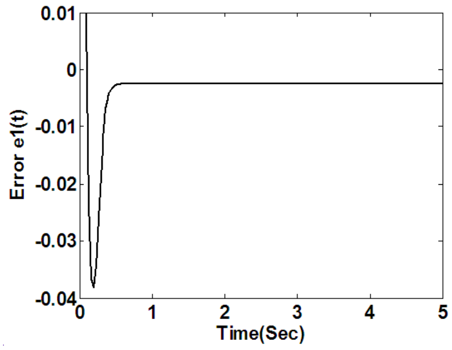

Figure 4. The error signal in presence of external disturbance 3.5

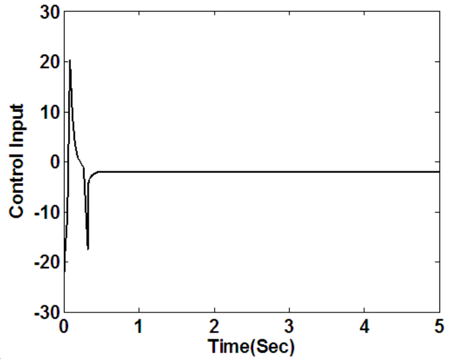

Figure 5. The control input in presence of the external disturbance 3.5

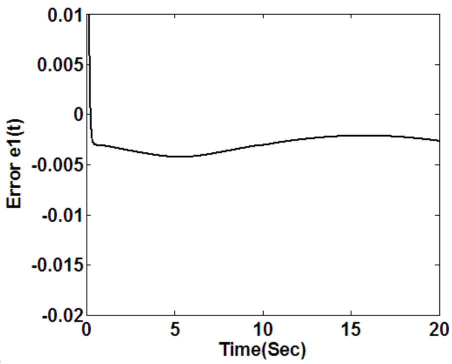

Figure 6. The error signal in presence of external time-varying disturbance

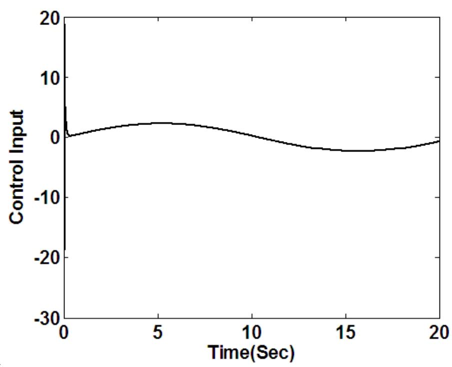

Figure 7. The control input in presence of external time-varying disturbance

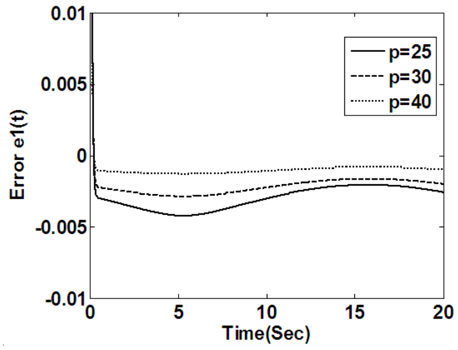

FIGURE 8. Decreasing the error signal by means of ρ increasing

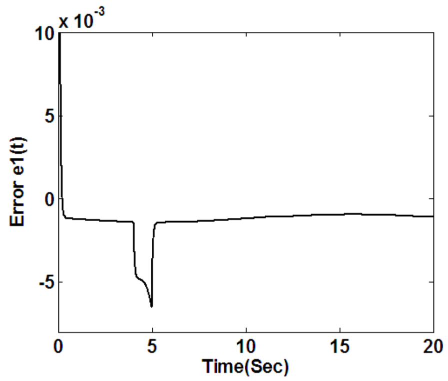

FIGURE 9. The error signal in presence of square disturbance signal

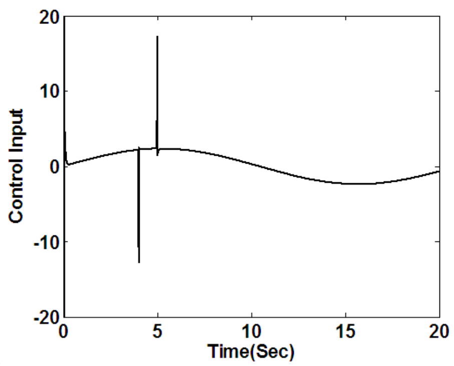

FIGURE 10. The control input in presence of square disturbance signal

input in presence of all existing uncertainties can handle the recent disturbance and it will keep the closed-loop system stable. The maximum error of 0.006 radian happened at the second 5. The effect of the square signal is visible in the 5th second in Figure 10. In Figure 10 we can see that the control action stays in the legitimate range. Having selected values of 50 and 60 for ρ, Figure 11 illustrates that the maximum error decreases from 0.006 radian to 0.0025 radian.

Simulation 4. In this stage of simulation, the proposed control method will be challenged in tracking the desirable path varying with respect to time. To do this, let $x_{1_d}(t) = 0.2\sin(t)$, $d(t) = -4\sin(0.3t)$ and $\rho = 29$ are chosen. Considering Figure 12, reveals that the control input performed well and with the maximum error of 0.004 radian, the desirable path was tracked. The control signal which is shown in Figure 13,

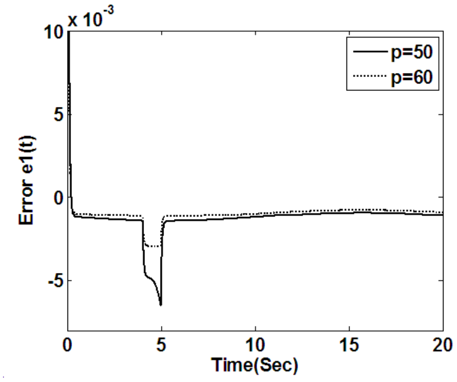

Figure 11. Decreasing the error signal by means of ρ increasing

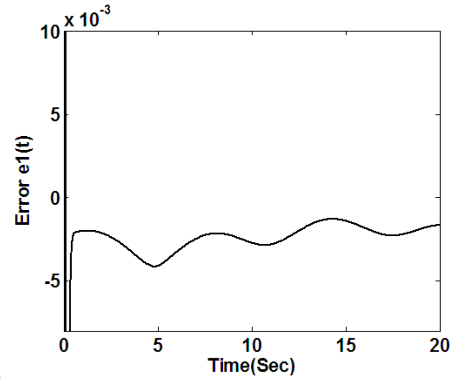

Figure 12. The signal error of tracking for the time-varying desirable path

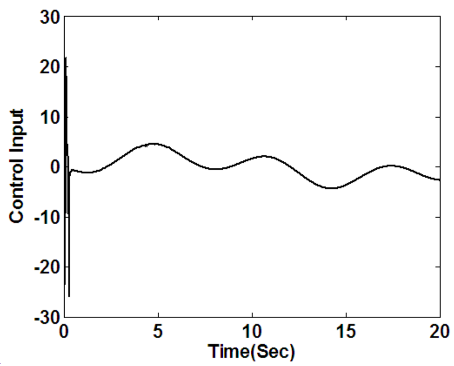

Figure 13. The control input for tracking the time-varying desirable path

there is no sign of any chattering. Considering the amplitude of this control input, ensures that there is no room for concerns regarding the probable saturation of the actuator.

9. **Conclusion.** In this paper, by combining the feedback linearization method and the fuzzy TS model-based method, the fuzzy sliding mode control was presented. The new method was very simple and free of chattering. The feedback linearization method let the designer decrease the boundaries of the existing uncertainties. Moreover, the fuzzy TS model-based part not only compensated lack of information about the remaining of structured and unstructured uncertainties, but also it decreased the burden of control input calculation. The mathematical proof illustrated that the closed-loop system along with the proposed control scheme, was global asymptotic stable. For better demonstration of

the proposed control system, 4 steps of simulation were presented. In those simulations it was tried to challenge the control system from different aspects. The results of simulations demonstrated that the control system in presence of various types of uncertainties overcame the challenges and it showed good performance. The results of simulations presented that with increasing proposed control coefficients, the error of tracking would decrease. Then, the desirable path would be tracked more accurately.

REFERENCES

[1] T. Takagi and M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions of Systems, Man and Cybernetic*, vol.15, pp.116-132, 1985.
[2] K. Tanaka and M. Sugeno, Stability analysis and design of fuzzy control systems, *Fuzzy Sets and Systems*, vol.45, no.2, pp.135-156, 1992.
[3] V. I. Utkin, *Sliding Modes and Their Application to Variable Structure Systems*, MIR Publishing Inc, Moscow, 1978.
[4] W. Gao and J. C. Hung, Variable structure control of nonlinear system, *IEEE Transactions on Industrial Electronics*, vol.40, no.3, pp.45-55, 1993.
[5] R. A. Decarlo, Variable structure system with sliding modes, *IEEE Transaction, Automation and Control*, vol. 22, no.4, pp.212-222, 1977.
[6] G. Bartolini, A. Ferrara and E. Usai, Chattering avoidance by second order sliding mode control, *IEEE Transaction, Automation and Control*, vol.43, no.2, pp.241-246, 1998.
[7] G. Bartolini and P. Pydynowski, Asymptotic linearization of uncertain nonlinear systems by means of continuous control, *International Journal of Robust Nonlinear Control*, vol.3, no.6, pp.87-103, 1993.
[8] G. Bartolini, An improved chattering free VSC scheme for uncertain dynamical systems, *IEEE Transaction, Automation and Control*, vol.41, no.7, pp.1220-1226, 1996.
[9] M. R. Soltanpour and M. M. Fateh, Sliding mode robust control of robot manipulators in the task space by support of feedback linearization and backstepping control, *World Applied Sciences Journal*, vol.6, no.1, pp.70-76, 2009.
[10] J. J. Slotine and W. Li, *Applied Nonlinear Control*, Publishing Inc., New York, 1991.
[11] H. Temeltas, A fuzzy adaptation technique for sliding mode controllers, *Proc. of IEEE International Symposium on Industrial Electronics*, vol.1, pp.110-115, 1998.
[12] C. C. Chiang and C. C. Hu, Adaptive fuzzy controller for nonlinear uncertain systems, *Proc. of the 2nd International Conference on Intelligent Processing and Manufacturing of Materials*, vol.2, pp.1131-1136, 1999.
[13] C. Hsu and A. M. Heidar, Fuzzy variable structure control for MIMO systems, *Proc. of Fuzzy Systems Proceedings, IEEE World Congress on Computational Intelligence*, vol.1, pp.280-285, 1998.
[14] S. Y. Li and Y. G. Xi, Adaptive fuzzy sliding mode control for a class of uncertain dynamic systems, *Proc. of the 3rd World Congress on Intelligent Control and Automation*, vol.2, pp.1849-1853, 2000.
[15] H. R. Lin and W. J. Wang, Fuzzy control design for the pre-specified tracking with sliding mode, *Proc. of Fuzzy Systems Proceedings, IEEE World Congress on Computational Intelligence*, vol.1, pp.292-295, 1998.
[16] L. K. Wong, A fuzzy sliding controller for nonlinear systems, *IEEE Transactions on Industrial Electronics*, vol.48, pp.32-37, 2001.
[17] F. Barrero, Speed control of induction motors using a novel fuzzy sliding mode structure, *IEEE Transactions on Fuzzy Systems*, vol.10, pp.375-383, 2002.
[18] S. H. Ryu and J.-H. Park, Auto-tuning of sliding mode control parameters using fuzzy logic, *Proc. of the American Control Conference*, vol.1, pp.618-623, 2001.
[19] R. Palm, Sliding mode fuzzy control, *Proc. of IEEE International Conference on Fuzzy Systems*, vol.1, pp.519-526, 1992.
[20] C. G. Lhee, Sliding-like fuzzy logic control with self tuning the dead zone parameters, *Proc. of IEEE International Fuzzy Systems Conference*, vol.1, pp.544-549, 1999.
[21] C. G. Lhee, Sliding-like fuzzy logic control with self tuning the dead zone parameters, *IEEE Transaction on Fuzzy Systems*, vol.9, pp.343-348, 2001.
[22] J. Y. Chen and Y. H. Lin, A self-tuning fuzzy controller design, *Proc. of IEEE International Conference on Neural Networks*, vol.3, pp.1358-1362, 1995.

[23] X. Y. Zhang, H. Y. Su and J. Chu, Adaptive sliding mode like fuzzy logic control for high-order nonlinear systems, *Proc. of the IEEE International Symposium on Intelligent Control*, vol.1, pp.788-792, 2003.

[24] R. J. Wai, C. M. Lin and C. F. Hsu, Self-organizing fuzzy control for motor-toggle servomechanism via sliding mode technique, *Fuzzy sets and systems*, vol.131, pp.235-249, 2002.

[25] K. Y. Tu, T. T. Lee and W. J. Wang, Design of a multi-layer fuzzy logic controller for multi-input multi-output systems, *Fuzzy sets and systems*, vol.111, pp.199-214, 2000.

[26] W. S. Lin and C. S. Chen, Sliding-mode-based direct adaptive fuzzy controller design for a class of uncertain multivariable nonlinear systems, *Proc. of the American Control Conference*, vol.3, pp.2955-2960, 2002.

[27] J. Zhang, P. Shi and Y. Xia, Robust adaptive sliding-mode control for fuzzy systems with mismatched uncertainties, *IEEE Transactions on Fuzzy Systems*, vol.18, no.4, pp.700-711, 2010.

[28] M. Liu, P. Shi, L. Zhang and X. Zhao, Fault-tolerant control for nonlinear Markovian jump systems via proportional and derivative sliding mode observer technique, *IEEE Transactions on Circuits and Systems-I*, vol.58, no.11, pp.2755-2764, 2011.

[29] L. Wu, P. Shi and H. Gao, State estimation and sliding-mode control of Markovian jump singular systems, *IEEE Transactions on Automatic Control*, vol.55, no.5, pp.1213-1219, 2010.

[30] P. Shi, Y. Xia, G. P. Liu and D. Rees, On designing of sliding-mode control for stochastic jump systems, *IEEE Transactions on Automatic Control*, vol.51, no.1, pp.97-103, 2006.

[31] M. M. Fateh and M. R. Soltanpour, Robust task-space control of robot manipulators under imperfect transformation of control space, *International Journal of Innovative Computing, Information and Control*, vol.5, no.11(A), pp.3949-3960, 2009.

[32] M. R. Soltanpour and S. E. Shafiei, Robust backstepping control of robot manipulator in task space with uncertainties in kinematics and dynamics, *Electronics and Electrical Engineering Journal, Automation and Robotics*, vol.96, no.8, pp.75-80, 2009.

[33] M. R. Soltanpour and M. Siahi, Robust control of robot manipulator in task space, *The International Journal of Applied and Computational Mathematics*, vol.8, no.2, pp.227-238, 2009.

[34] S. E. Shafiei and M. R. Soltanpour, Robust neural network control of electrically driven robot manipulator using backstepping approach, *International Journal of Advanced Robotic Systems*, vol.6, no.4, pp.285-292, 2009.

[35] M. R. Soltanpour and S. E. Shafiei, Design and stability analysis of a robust impedance control system for a robot manipulator, *Studies in Informatics and Control*, vol.17, no.1, pp.34-40, 2010.