Fuzzy Thinking: The New Science of Fuzzy Logic

Bart Kosko, (New York: Hyperion, 1993).

This very strange book of fifteen chapters is divided into four parts. The first part, "The Fuzzy Principle: Everything is a Matter of Degree," comprises three chapters. The remaining twelve chapters are equally divided among "The Fuzzy Past," "The Fuzzy Present," and "The Fuzzy Future." A helpful glossary, index and bibliography are included.

Part I

This part is a preview of the parts to follow, and as the title suggests, the central thesis is that everything is a matter of degree. The world is grey, not black-and-white. But Western scientists and philosophers have refused to face up to this fact; they persist in describing the grey world in black-and-white language. Their doing so is what the author calls the "mismatch problem," a problem rooted in the uncritical acceptance of two-valued logic - "binary faith." Binary logic, says Kosko, sacrifices accuracy for simplicity. Bivalence is a rounding off that works fine at extremes but fails everywhere else. Indeed, the core principles of bivalent logic - the Law of Excluded Middle and the Principle of Non-Contradiction - are merely limiting cases of a more proper multi-valued logic.

Kosko at once illustrates the awesome power of multivalent logic by applying it to Russell's barber paradox. The barber claims to shave all and only those who do not shave themselves. Does the barber shave himself? Outmoded Aristotelian logic tells us that if he does shave himself then he does not, and if he does not, he does.

> If for some cultural reason we limit what we say to the two bivalent options of all or none, true or false, yes or no, then we pay the price and have a real contradiction on our hands, a case of A AND not-A. (26)

The fuzzy solution, says Kosko, is superior to the traditional solutions, for it recognizes that the barber's claim is literally a half-truth lying exactly midway between the limiting cases, midway between 100% truth and 100% falsity. The traditional solutions, of course, contend that any claim that leads to a contradiction must be false. Hence, there can be no barber who shaves all and only those who do not shave themselves. (In set theoretical terms: there is no set that, without qualification, contains all and only those sets that do not contain themselves.) I see no merit in Kosko's "solution" for it makes no provisions for preventing the deduction of any and every well-formed-formula. As for the many degrees of truth allowed by Kosko's logic, we can easily accomplish the same things without sacrificing the standards by which good reasoning is judged. For instance, Kosko thinks that a half-filled glass of water is problematic for bivalent logic ("The water is 50% in the glass."), and he requires that we have a degree of truth for every degree to which the glass is filled. All we need, of course, is a proposition for each degree to which the

glass is filled. It is 100% true that the glass is half full, not 50% true that the glass is entirely full.

Analogous to degree of truth with respect to propositions, are degrees of containment in sets. Traditionally, sets are thought of as consisting of elements contained entirely in the sets. But fuzzy sets have members that belong "somewhat" to the sets. The degree to which a set belongs to another set is calculated using Kosko's awkwardly-named Subsethood Theorem. For sets A and X, the degree to which X is a subset of A is simply the elements in their intersection divided by the elements in X. Kosko spends pages relating the circumstance under which he conceived this theorem (he was in a state of zazen in a waterbed after bathing in a hot tub after a workout in the gym . . .) and concludes that it is proof that the part contains the whole and that that is what probability is. The claim that the whole is contained in the part is not as mysterious as it sounds. As best I can tell, choosing which set is the part and which is the whole is arbitrary, and as long as the sets' elements are somehow quantifiable (countable) it makes perfectly good sense to say that the sets intersect to the degree described by the Subsethood Theorem. But I see nothing new or interesting about this.

Part II

The battle between defenders of the binary faith and the more enlightened enthusiasts of multivalence is a battle between Aristotle and the Buddah. Kosko reminds us that Aristotle.". . . argued for the legitimacy of aristocracy and slavery and may have kept slaves," and was.". . . politically correct by the standards of his day . . . " (74). Buddah, on the other hand, relinquished his wealth and lived the life of a beggar. And though he was not a mathematician or logician in the strictest sense of the words,

> . . . he had the shades-of-grey idea: He tolerated A AND not-A. He carefully avoided the artificial bivalence that arises from the negation term "not" in natural languages. Hence his famous line: "The no-mind not-thinks no-thoughts about no-things." (77)

Existentialism, it seems, has a longer history that we thought.

In a chapter titled "What Is Truth?" Kosko defends in earnest the multi-valued logic of Buddah, and it is here that the author's confusion comes into focus. The confusion is this: Kosko conflates certainty with truth, the subjective with the objective. He is by no means the first to err in this way, but it is surprising to see a person of his ability commit such a fundamental mistake. A few passages show this: "Each factual statement is an inaccurate description, an inexact sensory hypothesis *that the next experiment may knock down"* (85). Likewise, "Positivists and scientists and physicians and the like accept the truth mismatch but deny it in practice by making all statements certain, by giving factual statements the status of logic statements" (89). Criticizing Tarski's acceptance of two-valued logic Kosko says, "The problem with this lies in tacking on the 100% *certainty factors"* (90) [all emphases mine]. These certainty factors Kosko refers to are simply Tarski's claim that "Snow is white," is 100% true if and only if snow is white. And of Kant, Kosko says that he," . . . claimed that there were 'synthetic *a priori*' statements,

statements both factually true and logically true" (89). Kosko does not seem aware of the fact that *'a priori'* is an adverb, not an adjective. To know something *a priori* is not to know something that is itself *a priori*; rather, it is to know something in a particular manner. Analytic statements are know**able** *a priori*, and synthetic statements are know-**able** *a posteriori.*

Kosko, of course, believes that it is the scientists who are confused - a situation he largely blames on the logical positivists who.". . . swallow the mismatch whole and use high-pressure sales tactics to get everyone else to swallow it" (89). In spite of his contempt for positivists, Kosko does agree with them that the truth of mathematics are tautologies, limiting cases to which the Aristotelean laws of thought do apply. This would also explain scientists' bivalent faith, because they are so infatuated with mathematics:

> They call fact statements true or false and mean 100% true or 100% false as if they were logical tautologies or math theorems. And they cover up and cloud the whole issue with probability disclaimers that all such black-and-white talk holds with "some probability" (89).

No one, says Kosko, has ever met the "Hemingway Challenge" of producing one 100% true factual statement. I take it that since this is itself a factual claim it is not 100% true, and therefore someone has produced a 100% true synthetic proposition. Perhaps that 100% true proposition is the claim that no one has ever . . .

Heisenberg's uncertainty principle is inevitably mentioned in fuzzy discourses, but it is to Kosko's credit that he does not exaggerate its role in fuzzy theory. It is true, says Kosko, that Heisenberg makes us question binary logic, and it is true that he "made doubt scientific" (103). However, the principle is itself only an approximation, for it is a linear theory about a non-linear world. Indeed it is part of the "linear package" of still another mismatch problem: the attempts by scientists to fit linear models to a non-linear world. Thus, says the author, uncertainty is not restricted to the realm of quantum mechanics.

Part III

> Consider the number zero. 0. It took some ancient societies thousands of years to find zero. Now we can find it at a glance as a spike on a number line . . . The spike means the number zero belongs 100% to the set ZERO and no other number belongs to it. Every number is either in the ZERO set or out of it. All or none. In this set sense the number zero alone belongs to the set ZERO.

> But what about the numbers *close* to zero or *almost* zero or *nearly* zero? These numbers, like big numbers or medium numbers or very small numbers, are fuzzy numbers. They define a spectrum of numbers near zero and some belong more in the set than others belong. The closer a small number to zero, the more it belongs to the fuzzy set of small numbers. The number 1 is close to 0 than the number 2 is, and 2 is closer than 3 is, and so on. Likewise the negative number -1 is closer to 0 than -2 is, and -2 is closer than -3 is, and so on. The number 0 belongs 100%

> to the set ZERO but close numbers may belong only 80% or 50% or 10%. We might draw the fuzzy number zero as a bell curve or triangle centered at the exact number 0. (124)

Similar arguments apply to any set of objects having a property that admits of degrees (and this is everything, says Kosko). Furthermore, a triangular set allows us to compute the degree to which an element belongs to the set. The set ZERO, above, is centered about 0 with its base extending from -2 to +2. If we designate the height of the triangle (at its peak, above 0) as 1, then that represents the degree (100%) to which zero belongs to the set. The number +1 belongs 50% to the set because a line extended upward from +1 intersects the triangle at height (y)=0.5. As we shall see, these triangular sets are the key to the success of so-called fuzzy systems.

Control systems that make use of fuzzy logic do work in a wide variety of applications including air conditioners, automobiles, office equipment, home appliances, elevators, and so on. According to Kosko, as of 1990 the Japanese held over a thousand "fuzzy" patents in Japan and thirty in the US. Surely the success of these many products is evidence enough that fuzzy theorists are right and their detractors wrong. But this argument is not available to fuzzy theorists because they say that conventional science and technology are ill-founded in spite of their success. Kosko is anything but politic in his indictment of conventional science:

> One day I learned that science was not true. I do not recall the day but I recall the moment. The God of the twentieth century was no longer God.
>
> There was a mistake and everyone in science seemed to make it. They said that all things were true or false. They were not always sure which things were true and which were false. But they were sure that all the things were either true or false. (XV)

One critic of fuzzy logic has said that it is an application in search of a theory. That assessment is correct, I think, and the theory that will do the job is none other than the Devil's own two-valued logic.

Consider the control system Kosko describes that regulates the operation of an air conditioner. The idea is to link varying temperatures to varying blower motor speeds rather than have the blower run at just one speed or not at all. Imagine five temperature ranges represented by five overlapping triangles arranged along a horizontal line (x-axis). From 40 degrees to 50 degrees is the COLD range, from 45 to 65 is COOL, from 60 to 70 is JUST RIGHT, from 65 to 85 is WARM, and from 80 to 90 is HOT. Along a vertical line (y-axis) are overlapping triangles that represent motor speeds. From 0 rpm to 30 rpm is STOP, from 10 to 50 is SLOW, from 40 to 60 is MEDIUM, from 50 to 90 is FAST, and from 70 to 100 is BLAST. (Note that the JUST RIGHT and MEDIUM regions are narrower than the others. This is to prevent "overshoot".)

Now the temperature triangles are correlated with the motor speed triangles by rules such as: "If the temperature is cold, the motor stops." or "If the temperature is just right,

the motor speed is medium." These rules can be represented graphically as rectangular patches formed by the intersection of columns extending out from the bases of triangles on the temperature line with columns extended out from the bases of (corresponding) triangles on the motor speed line.

But what happens when a temperature "belongs" to two triangles? Using a scheme for computing sort of an average figure that represents the amount of overlap of triangles, Kosko can "defuzzify" (!) the triangles to get a discrete motor speed. And as far as I can tell, this control would indeed work as well as Kosko claims it would but not the *way* he claims it would. I see absolutely nothing fuzzy about this "fuzzy" system. What fuzzy theorists call "degrees of membership" aren't really that at all - they are simply ways of measuring how close to a given number (the center of the triangle) another number lies. It's not that, say, 68 degrees belongs to the JUST RIGHT set more than 69 degrees does, or -1 belongs to the above-mentioned set ZERO more than -2 does; rather, 68 degrees is simply closer to the ideal center temperature of 65 degrees (by a measurable amount).

A refinement of the basic fuzzy control system is the adaptive fuzzy system which is able to learn the correlation rules (e.g. If the temperature is just right, then the motor speed is medium), by keeping track of how the user operates the system. After a person has adjusted the system a number of times the system "knows" the user's preferences and can take over unassisted.

Because fuzzy control systems do work, they will continue to find their way into more and more products. But they will do so, I think, in spite of some of the fuzzy theorists who promote them.

Part IV

The very short section of *Fuzzy Thinking* is a departure from the somewhat technical previous parts and consists primarily of the author's musings about life and death, right and wrong, and God. About life and death Kosko says they are both a matter of degree, each shading into the other. Concerning right and wrong, Kosko takes a position I can sympathize with - ethical non-cognitivism. Moral judgments, he says, can be neither confirmed nor disconfirmed by experiment and are therefore without truth-values. They function as exhortations and commendations, not as descriptions of matters of fact. And what about God? Kosko begins with the question, "Why is there something rather than nothing?" After some intolerably fuzzy and convoluted ramblings, Kosko concludes that if there were nothing, then mathematics would not work. So,

> God is He who wrote the math. Or She who wrote the math. Or It who wrote the math. Or the Nothingness that wrote the math. The Mathmaker. (281)

The book jacket of *Fuzzy Thinking* shows a picture of Kosko sitting in what I think is called the lotus position. Superimposed over this picture are a couple of yin-yang symbols and some Rubik's Cubes. There is nothing fuzzy about the book jacket - it is 100% truth-in-advertising.

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