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Galactic accretion and angular momentum re-orientation

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SUMMARY

The generation of galactic angular momentum by tidal forces is re-examined. We calculate the tidal torques in linear theory on a spherical shell centred either on a random point or on a peak in the smoothed density field. We show that when the torque is decomposed into contributions from multipoles, the dipole term is more important than the quadrupole, and higher multipoles are unimportant. When the dipole contribution is taken into account, the torques of thin shells of radii r and r' are anticorrelated for $r/r' \gtrsim 2$. Significant contributions to the torque on any shell come from shells that are more than five times as big. The magnitude of the torque on a shell does not depend greatly on whether the shell is centred on a peak in the smoothed density field rather than on a random point. The dimensionless angular momentum parameter λ of virialized objects remains fairly constant at $\lambda \simeq 0.05$ as the object grows as a result of cosmic infall.

1 INTRODUCTION

It has often been argued that galaxies do not form suddenly, but by the more or less steady accumulation of material as it rains in on them (e.g. Gunn 1982). At the present epoch most of the infalling material will be dark matter and thus hard to detect directly. However, any material that falls in now is expected to have very high specific angular momentum, so that even a relatively small accumulation of mass could significantly change the angular momentum content of the recipient galaxy. If the angular momentum vector of the infalling material is not parallel to that of the existing, visible galaxy, angular momentum will be exchanged between the visible galaxy and recently acquired dark matter, and this exchange may manifest itself by warping the outer, and possibly even inner, portions of the galaxy's disc (Ostriker & Binney 1989; Binney 1990). Hence it is important to know the degree of misalignment between a galaxy's existing stock of angular momentum and the angular momentum it is acquiring at the present epoch.

Ryden & Gunn (1987) and Ryden (1988) have studied in detail cosmic infall in a universe dominated by cold dark matter (CDM). In particular, Ryden (1988) calculated the specific angular momenta of successively infalling shells of material, including an estimate of the rate at which the directions of these shells' angular momentum vectors change. Here we extend and refine Ryden's calculations in the following respects.

(i) The torques which spin shells up are due to fluctuations in the cosmic density field. Galaxies are expected to form around peaks in the density field, so to calculate the torques on shells around a protogalaxy, one requires a knowledge of the fluctuations in the neighbourhood of a peak. These are actually smaller than those in the field as a whole, but Ryden's results were calculated in the approximation that the reduction in the amplitude of fluctuations near a peak could be neglected. Here we work with the fluctuations appropriate to the neighbourhood of a peak.

(ii) It is convenient to decompose the density and potential around the peak into multipoles. Although every multipole but the monopole contributes to the torque on a shell, Ryden estimated the torque from the quadrupole alone. Here we calculate the contributions from every multipole and show that while multipoles higher than the quadrupole are unimportant, the largest contribution actually comes from the dipole term.

(iii) From the point of view of studies of galactic warps, the most important result of the present theory is the normalized correlation:

$$\cos\gamma \equiv \frac{\overline{\boldsymbol{\Gamma}(r)\cdot\boldsymbol{\Gamma}(r')}}{\overline{\Gamma^2}(r)^{1/2}\,\overline{\Gamma^2}(r')^{1/2}}, \tag{1}$$

between the torques $\boldsymbol{\Gamma}(r)$ and $\boldsymbol{\Gamma}(r')$ on spheres of radius r and r'. We recalculate this correlation and obtain a plot rather different from that published by Ryden.

2 TORQUE ON A SHELL

The formulae of Binney & Quinn (1991; hereafter Paper I) enable one to sample a Gaussian random field in such a way that a peak in the smoothed field of specified magnitude, scale and structure is centred on the origin of coordinates. We now calculate the gravitational torque on a spherical shell around such a peak, using the notation and many of the formulae of Paper I.

The gravitational force density is $\boldsymbol{f} = -\rho\nabla\Phi$, so the torque about the origin on the spherical shell bounded by the spheres of radii r/a and ra is:

$$\boldsymbol{\Gamma}^{(0)}(r, a) = -\int_{r/a}^{ra} \rho(\boldsymbol{x}\times\nabla)\Phi\, d^3\boldsymbol{x}$$

$$= -i\int \rho\boldsymbol{L}\Phi\, d\boldsymbol{x}, \qquad (\boldsymbol{L} \equiv -i\boldsymbol{x}\times\nabla), \tag{2}$$

where ρ is the difference between the actual density and the mean cosmic density ρ_b, and Φ is the perturbed gravitational potential to which ρ gives rise. Following Paper I we expand ρ and Φ as

$$\rho(\boldsymbol{x}) = \sqrt{\frac{2}{\pi}}\sum_{lm} Y_l^m(\theta, \phi)\int \rho_{lm}(k)\, j_l(kr)\, k\, dk;$$

$$\Phi(\boldsymbol{x}) = -4\pi G\sqrt{\frac{2}{\pi}}\sum_{lm} Y_l^m(\theta, \phi)\int \rho_{lm}(k)\, j_l(kr)\frac{dk}{k}, \tag{3}$$

where the $\rho_{lm}(k)$ are random functions describing the density field. Inserting (3) into (2) yields

$$\boldsymbol{\Gamma}^{(0)}(r, a) = 8Gi\int r'^2\, dr' \sum_{lml'm'}\int \rho_{lm}(k)\, j_l(kr')\, k\, dk$$

$$\times\int d\Omega\, Y_l^{m*}(\theta, \phi)\,\boldsymbol{L}Y_{l'}^{m'}(\theta, \phi)\int \rho_{l'm'}(k')\, j_{l'}(k'r')\frac{dk'}{k'}. \tag{4}$$

Since $L_z Y_l^m = mY_l^m$, this leads to

$$\Gamma_z^{(0)}(r, a) = -8Gi\int_{r/a}^{ra} r'^2\, dr' \sum_{lm} m\int \rho_{lm}(k)\, j_l(kr')\, k\, dk$$

$$\times\int \rho_{lm}^*(k')\, j_l(k'r')\frac{dk'}{k'}$$

$$= 8Gi\sum_l \sum_{m=1}^{l} m\int dk\, dk'\, \rho_{lm}(k)\,\rho_{lm}^*(k')\, h_l^{(0)}(r, a, k, k'), \tag{5a}$$

where

$$h_l^{(0)}(r, a, k, k') \equiv \left(\frac{k'}{k} - \frac{k}{k'}\right)\int_{r/a}^{cr} j_l(kr')\, j_l(k'r')\, r'^2\, dr'. \tag{5b}$$

As $a\to\infty$, $h_l^{(0)} \to \frac{1}{2}\pi(k'^{-2} - k^{-2})\,\delta(k - k')$ and $\Gamma_z^{(0)} \to 0$ as expected.

2.1 Correction for displacement of the centre of mass

From the point of view of galaxy formation, the physically interesting quantity is not so much the angular momentum of a shell about $\boldsymbol{x}=0$ but its angular momentum about the centre of mass $\bar{\boldsymbol{x}}$ of the material interior to it. Thus we wish to calculate

$$\boldsymbol{\Gamma}(r, a) = -\int_{r/a}^{ra} d^3\boldsymbol{x}\,\rho(\boldsymbol{x})(\boldsymbol{x} - \bar{\boldsymbol{x}})\times\nabla\Phi = \boldsymbol{\Gamma}^{(0)} - \bar{\boldsymbol{x}}\times\bar{\boldsymbol{f}}, \tag{6}$$

where the mean force $\bar{\boldsymbol{f}}$ is given by

$$\bar{\boldsymbol{f}} = -\int_{r/a}^{ra} r^2\, dr\int d\Omega\,\rho\nabla\Phi.$$

To first order in the fluctuations we have

$$\bar{z} = \frac{1}{M(r)}\sqrt{\frac{2}{\pi}}\int_0^r r^3\, dr\sum_{lm}\int k\, dk\,\rho_{lm}(k)\, j_l(kr)$$

$$\times\int d\Omega\sqrt{\frac{4\pi}{3}}\, Y_1^0 Y_l^m(\theta, \phi)$$

$$= \frac{3}{4\pi\rho_b}\sqrt{\frac{8}{3}}\int dk\,\rho_{10}(k)\, j_2(kr). \tag{7}$$

$\bar{x}$ can be calculated from (7) by relating the coefficients in the expansion of ρ with the line $\theta=0$ identified with the x-axis to the ρ_{lm} used here, and similarly for $\bar{y}$. After a straightforward calculation one finds that:

$$\bar{x} = \sqrt{\frac{3}{4\pi^2}}\frac{1}{\rho_b}\int dk[\rho_{11}(k) + \rho_{1-1}(k)]\, j_2(kr),$$

$$\bar{y} = i\sqrt{\frac{3}{4\pi^2}}\frac{1}{\rho_b}\int dk[\rho_{11}(k) - \rho_{1-1}(k)]\, j_2(kr). \tag{8}$$

The mean force $\bar{\boldsymbol{f}}$ is rather trickier to calculate. Since $\bar{\boldsymbol{x}}$ is a first-order quantity, it is sufficient to obtain $\bar{\boldsymbol{f}}$ to first order in the fluctuations. Now

$$\bar{f}_z = 4\pi G\rho_b\sqrt{\frac{2}{\pi}}\sum_{lm}\int k\, dk\frac{\rho_{lm}(k)}{k^2}\int_{r/a}^{ra} r^2\, dr\int d\Omega\frac{\partial}{\partial z}Y_l^m j_l(kr)$$

$$= 4\pi G\rho_b\sqrt{\frac{2}{\pi}}\sum_{lm}\int dk\frac{\rho_{lm}(k)}{k}\int_{r/a}^{ra} r^2\, dr$$

$$\times\int d\Omega\left(\frac{z}{r}\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)Y_l^m j_l(kr). \tag{9}$$

The integral over solid angle manifestly vanishes for $m\neq 0$. Also

$$\int d\Omega\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}Y_l^0 = -\frac{2}{r}\sqrt{\frac{4\pi}{3}}\,\delta_{l1}. \tag{10}$$

Similarly,

$$\int d\Omega\, Y_l^0\frac{z}{r} = \sqrt{\frac{4\pi}{3}}\,\delta_{l1}. \tag{11}$$

Thus

$$\bar{f}_z = 4\pi G\rho_b\sqrt{\frac{8}{3}}\int dk\,\rho_{10}(k)\, D(r, a, k), \tag{12a}$$

where

$$D(r\,a,k)\equiv\frac{1}{k}\int_{r/a}^{ra}r^2\,dr\left(\frac{\partial}{\partial r}+\frac{2}{r}\right)j_1(kr)$$

$$=\int_{r/a}^{ra}r^2\,dr j_0(kr). \tag{12b}$$

By analogy with the transformation from $\bar{z}$ to $\bar{x}$ (12a) yields

$$\bar{f}_x=4\pi G\rho_b\sqrt{\frac{4}{3}}\int dk[\rho_{11}(k)+\rho_{1-1}(k)]\,D(r,a,k),$$

$$\bar{f}_y=4\pi G\rho_b i\sqrt{\frac{4}{3}}\int dk[\rho_{11}(k)-\rho_{1-1}(k)]\,D(r,a,k).$$

Finally,

$$(\bar{\boldsymbol{x}}\times\bar{\boldsymbol{f}})_z=4iG\int dk\,dk'j_2(kr)\,D(r,a,k')$$
$$\times\{[\rho_{11}(k)+\rho^*_{11}(k)]\,[\rho_{11}(k')-\rho^*_{11}(k')]$$
$$-[\rho_{11}(k)-\rho^*_{11}(k)]\,[\rho_{11}(k')+\rho^*_{11}(k')]\}$$

$$=-8iG\int dk\,dk'\rho_{11}(k)\,\rho^*_{11}(k')\,g(r,a,k,k'), \tag{13a}$$

where

$$g(r,a,k,k')\equiv[j_2(kr)\,D(r,a,k')-j_2(k'r)\,D(r,a,k)]. \tag{13b}$$

Comparing equations (5a), (6) and (13a), we see that the required torque is given by (5a) but with coefficients h_l defined by

$$h_l=\begin{cases}h_l^{(0)} & \text{for } l\neq 1,\\ h_1^{(0)}+g & \text{for } l=1.\end{cases} \tag{14}$$

3 AUTOCORRELATION OF THE TORQUE

By equations (5), (6) and (13a), the autocorrelation of Γ_z is

$$\overline{\Gamma_z^2}(r_1,r_2,a)=-(8G)^2\sum_{l,l'}\sum_{m,m'=1}^{l}mm'\int dk\,dk'\,d\tilde{k}\,d\tilde{k}'$$
$$\times G^{(4)}_{lml'm'}(k,k',\tilde{k},\tilde{k}')\,h_l(r_1,a,k,k')\,h_{l'}(r_2,a,\tilde{k},\tilde{k}'), \tag{15}$$

where

$$G^{(4)}_{lml'm'}(k,k',\tilde{k},\tilde{k}')\equiv\overline{\rho_{lm}(k)\,\rho^*_{lm}(k')\,\rho_{l'm'}(\tilde{k})\,\rho^*_{l'm'}(\tilde{k}')}. \tag{16}$$

When $\boldsymbol{x}=0$ is an arbitrary point in the density field, each value $\rho_{lm}(k)$ is a Gaussian random variable independent of all others – see Paper I for details. That is,

$$G^{(2)}_{lm}(k,k')\equiv\overline{\rho_{lm}(k)\,\rho^*_{l'm'}(k')}$$
$$=P(k)\,\delta_{ll'}\delta_{mm'}\delta(k-k'), \tag{17}$$

where $P(k)$ is the power spectrum of the fluctuations. So long as the $\rho_{lm}(k)$ are independent Gaussian variables, $G^{(4)}_{lml'm'}$ is related to $G^{(2)}_{lm}$ by

$$G^{(4)}_{lml'm'}(k,k',\tilde{k},\tilde{k}')=G^{(2)}_{lm}(k,k')\,G^{(2)}_{l'm'}(\tilde{k},\tilde{k}')$$
$$+\delta_{ll'}[\delta_{mm'}G^{(2)}_{lm}(k,\tilde{k}')\,G^{(2)}_{lm}(\tilde{k},k')$$
$$+\delta_{m,-m'}G^{(2)}_{lm}(k,\tilde{k})\,G^{(2)}_{lm}(\tilde{k}',k')]. \tag{18}$$

When (18) is multiplied by $h_l(r,a,k,k')$, which is antisymmetric in k, k', the first term in the expansion (18) of $G^{(4)}_{lm}$ makes no contribution to the integral of (15). The last term in $G^{(4)}_{lm}$ cannot contribute since m, $m'\geq 0$. Also $\sum_1^l m^2=\frac{1}{6}l(l+1)(2l+1)$. So the contribution to $\Gamma_z^2(r_1,r_2,a)$ from any l is just

$$\Delta_l\overline{\Gamma_z^2}=-(8G)^2\tfrac{1}{6}l(l+1)(2l+1)$$
$$\times\int dk\,dk'\,P(k)\,P(k')\,h_l(r_1,a,k,k')\,h_l(r_2,a,k',k). \tag{19}$$

When the density field is required to peak at $\boldsymbol{x}=0$, the values of $\rho_{lm}(k)$ at different k are not independent of one another for $l\leq 2$. [However, $\rho_{lm}(k)$ is always independent of $\rho_{l'm'}(k')$ for $l\neq l'$ or $|m|\neq|m'|$.] Consequently the contributions from $l\leq 2$ to the torque Γ_z on a sphere centred on a peak are not given by equation (19). In the appendix we show that the contributions from $l\leq 2$ to the torque on a sphere centred on a peak are given by

$$\Delta_l\overline{\Gamma_z^2}=-(8G)^2\sum_{l=1}^{2}\sum_{m=1}^{l}m^2\int dk\,dk'\,d\tilde{k}\,d\tilde{k}'$$
$$\times[\tilde{\rho}(k)\,\tilde{\rho}(\tilde{k}')\,C(k',\tilde{k})+\tilde{\rho}(k')\,\tilde{\rho}(\tilde{k})\,C(k,\tilde{k}')$$
$$+C(k,\tilde{k}')\,C(k',\tilde{k})]\,h_l(r_1,a,k,k')\,h_l(r_2,a,\tilde{k},\tilde{k}'), \tag{20}$$

where $\tilde{\rho}(k)$ and $C(k,k')$ are defined by equations (A1) and (A2b) in the appendix; both carry suppressed subscripts (l,m). The mean-square torque (20) is entirely made up of contributions from multipoles higher than the monopole. Interestingly, the height ν of the peak one specifies to exist at the origin does not enter the equations that constrain these amplitudes – ν only appears in the equations constraining the monopole amplitudes $\rho_0(k)$. Consequently, *the rms torque on the material of a peak is strictly independent of the height of that peak.* In particular, the torque is unchanged if ν is negative, that is, if a trough rather than a peak is located at the origin.

4 RESULTS

Fig. 1 shows the rms torque per unit mass on a shell of radius r centred on an arbitrary point calculated by summing the contributions $\Delta_l\Gamma_z^2$ given by (19) for: (i) $l=2$ only (dashed curve); (ii) $l=1$, 2 (full curve), and (iii) $l=1,\ldots,4$ (dot-dashed curve – almost coincident with full curve). The dashed curve agrees excellently with Ryden's plot of Γ_z^2; this was obtained for the same spectrum and normalization as we have used, and from just the quadrupole induced by unconstrained fluctuations. The interesting point about Fig. 1 is its demonstration that the dipole term, $l=1$, makes a larger contribution than the quadrupole, while the higher multipoles are unimportant.

Fig. 2 shows the quantity $\cos\gamma$ defined by equation (1) for concentric spheres centred on an arbitrary point. Again we plot separately results obtained by including only quadrupole terms ($l=2$; dashed curves) and both dipole and quadrupole terms ($l=1$, 2; full curves). The dashed curves differ from their equivalents in Ryden's fig. 10 in that they do not approach the line $\gamma=1$ with a large gradient, but have smooth maxima at $r=r'$ as they must on both physical and mathematical grounds. When the dipole term is included, the torques on shells that differ in radius by more than a factor $\simeq 2$ are *anti*correlated. This occurs because shells pull each other in opposite directions along a line that in general does not pass through the centre of mass of the enclosed material.

The full curve in Fig. 3 shows the torque on shells centred on a peak in the smoothed density field, while the dashed curve in that figure shows the torque on shells centred on an arbitrary point. It will be seen that the torque is not greatly affected by the presence of a peak at the origin – the only significant change is a slight decrease in the torque at small radii. This is a consequence of the fluctuations becoming smaller near the centre of the peak. Fig. 4 shows the equivalent of Fig. 2 for shells centred on a peak. The anticorrelation of the torques is now more pronounced for shells whose radii are in the ratio $2 \lesssim r_2/r_1 \lesssim 4$, but disappears for larger values of r_2/r_1. The results shown in Figs 3 and 4 were obtained with 160 values of k.

It is interesting to calculate the total angular momentum $J(M)$ imparted to a sphere of mass M. The square of the total

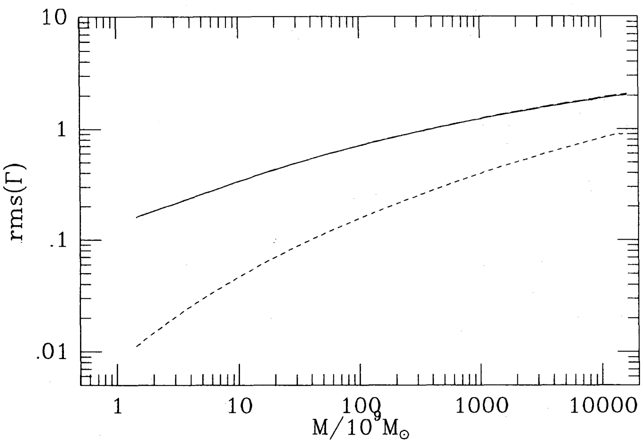

Figure 1. The rms torque per unit mass on a shell of radius r around an arbitrary point in the density field. The dashed curve shows $\sqrt{3}$ times the ($l=2$) quadrupole contribution $\Delta_2\overline{\Gamma_z^2}$ after division by the mass of the shell. The cold dark matter spectrum having been normalized as in Ryden (1988), this curve agrees well with Ryden's equivalent. (The units are 10^{15} cm^2 s^{-2}; a torque of magnitude 0.1 in these units acting on a particle through a radian imparts a tangential velocity of 141 km s^{-1}.) The full curve shows the result of including the dipole ($l=1$) contribution. The dot–dashed curve that almost coincides with the full curve shows that adding further multipoles, specifically $l=3$, 4, does not significantly change the torque obtained from the first two non-trivial multipoles. For computational convenience the integrals over k have been truncated by smoothing the spectrum on a scale of 50 kpc ($\sim 3\times10^8$ $M_\odot$).

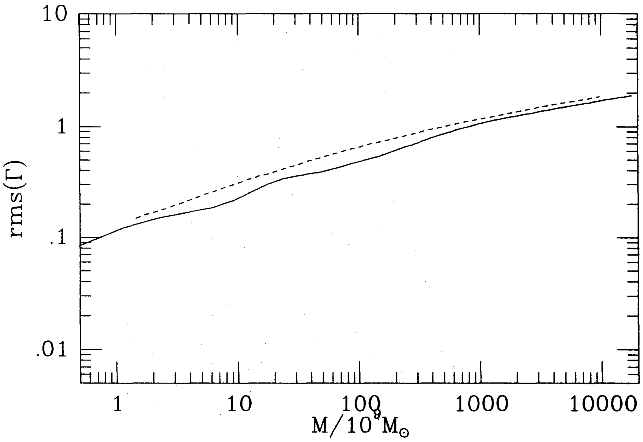

Figure 3. The same as Fig. 1 except for shells centred on a peak in the density field when smoothed with a Gaussian filter $w(k)=\exp(-k^2/2k_0^2)$ with $k_0=1/0.43$ Mpc. The dashed curve shows the torque on a randomly centred shell. Both curves include both dipole and quadrupole contributions.

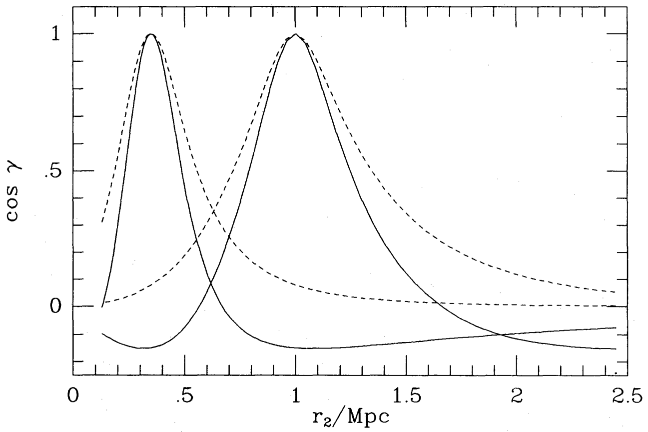

Figure 2. The quantity $\cos\gamma$ defined by equation (1) for shells of radius 0.35 and 1 Mpc centred on an arbitrary point. The magnitude of $\cos\gamma$ is a measure of the degree of alignment of the angular momenta of concentric shells. The dashed curves show the contribution of the quadrupole alone, while the full curves also include the dipole's contribution.

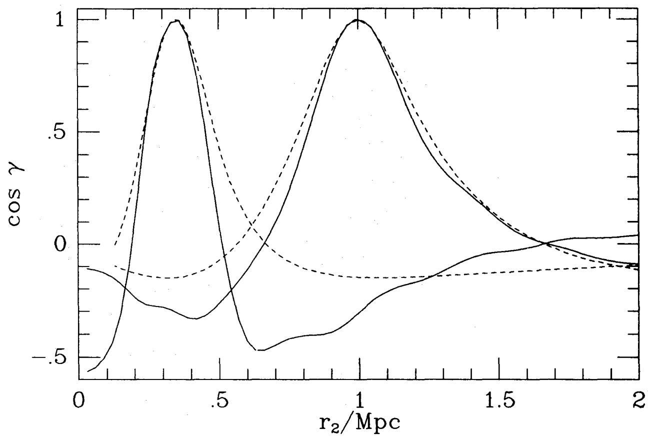

Figure 4. The same as Fig. 2 except for shells of radius 0.35 and 1 Mpc centred on a peak in the smoothed field. The dashed curves show $\cos\gamma$ for randomly centred shells. The smoothing scale is as in Fig. 3 and all curves include both dipole and quadrupole contributions.

torque on such a sphere predicted by linear theory for the present epoch is given by equation (15) with $r_1=r_2=r(M)$ and the limits r/a and ar of the integrals appearing in the definitions (5b) and (12b) of $h_l^{(0)}$ and D replaced by 0 and r, respectively. Equation (35) of Ryden (1988) gives an approximate prescription for deriving the time dependence of the actual torque from this linear result: one calculates the trajectory of the shell from the standard Newtonian closed universe and assumes that the shells which provide most of the torque expand according to a flat universe. Integrating up the scaling factors furnished by this model for each time, one obtains quantities by which the linear torque must be multiplied to yield the angular momentum J of each shell. On the other hand, the binding energy of the body formed by everything interior to a shell is

$$E=-\int_0^M \frac{GM'}{r_{\max}}\, dM', \tag{21}$$

where $r_{\max}(M)$ is the maximum radius reached by the shell containing mass M. Fig. 5 is a plot of the dimensionless quantity

$$\lambda \equiv \frac{J\,|E|^{1/2}}{GM^{5/2}}, \tag{22}$$

calculated for bodies formed by the collapse of a peak in the smoothed density field. (The collapse time-scales were evaluated by assuming that the peak was a 2.5σ fluctuation in the density field obtained by multiplying the raw CDM amplitudes ρ_{lm} by $w(k)\propto \exp(-k^2/2k_0^2)$ with $k_0^{-1}=0.43$ Mpc. Thus the amplitudes were those appropriate to the unconstrained field rather than one constrained to have a peak at the origin.) For r less than the smoothing length, Fig. 5 shows λ to be a rapidly varying function of r. However, this portion of the graph is of little interest since infall is unlikely to cease before shells that are at least a smoothing length in radius have fallen in. Subsequently, λ rises gently from ~ 0.04 to ~ 0.06 as further shells are accreted.

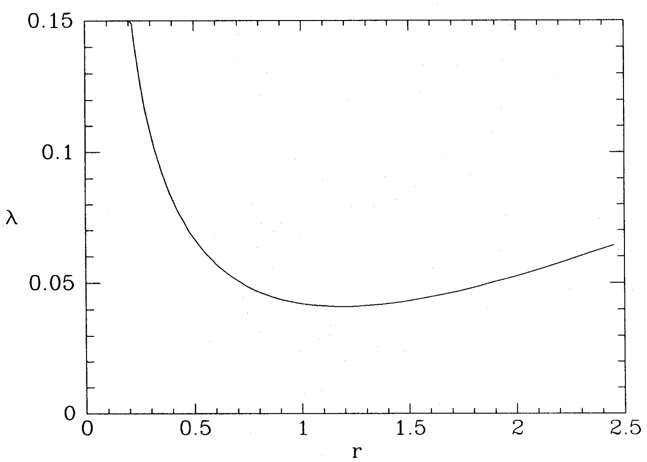

Figure 5. The dimensionless angular momentum (22) for an object formed by the infall and virialization of shells which at the present have unperturbed radii less than r. The time-scales over which the torques on shells act were calculated from equation (35) of Ryden (1988) for a 2.5σ peak in the field smoothed on a 0.43-Mpc scale. The binding energy of the body formed to radius r was obtained from (21).

Fig. 6 shows the equivalent of the full curves in Fig. 2, but for the angular momenta of solid spheres rather than the torques on shells. Since the great majority of any solid sphere's angular momentum is concentrated in its outermost shells, Figs 2 and 6 are remarkably similar.

Which shells contribute most to the torque on any given shell? Let the inner shell have radius r and the outer shell have radius r'. Then the potential at points on the inner shell due to the outer shell is by the first of equations (3) and equation (2-120) of Binney & Tremaine (1987):

$$\delta\Phi(\boldsymbol{x},r')=-4\pi G\sqrt{\frac{2}{\pi}}\,r'\delta r'\sum_{lm}\frac{Y_l^m(\theta,\phi)}{2l+1}\times\left(\frac{r}{r'}\right)^l\int k\,dk\,\rho_{lm}(k)\,j_l(kr'). \tag{23}$$

The potential energy of a shell through $\boldsymbol{x}$ is

$$\delta W=r^2\delta r\int \rho(\boldsymbol{x})\,\delta\Phi(\boldsymbol{x},r')\,d\Omega. \tag{24}$$

We obtain the torque about the origin on this shell by expressing δW as a function of ϕ_0, the angle through which the shell has been rotated from its original position. This we do by replacing ϕ in (23) by $(\phi-\phi_0)$, differentiating with respect to ϕ_0, and setting $\phi_0=0$. From (23) and (24) we then have

$$\delta W(\phi_0)=-8Gr^2\delta rr'\delta r'\sum_{lm}\frac{e^{im\phi_0}}{2l+1}\left(\frac{r}{r'}\right)^l\times\int k\,dk\int k'\,dk'\,\rho_{lm}^*(k')\,\rho_{lm}(k)\,j_l(k'r)\,j_l(kr'), \tag{25}$$

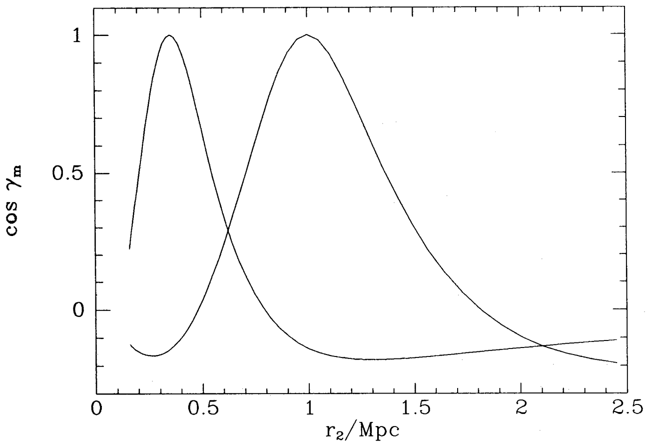

Figure 6. The quantity $\cos\gamma$ defined as in equation (1) but with $\boldsymbol{\Gamma}(r)$ having the meaning of the angular momentum acquired prior to the collapse of its outermost shell by a solid sphere of radius r. The two curves correspond to $r_1=0.35$ and 1 Mpc. The shells are centred on a random point.

so the torque on the inner shell is

$$\delta\Gamma_z^{(0)} = 8iGr^2\,\delta rr'\,\delta r' \sum_l \frac{(r/r')^l}{2l+1} \sum_{m>0} m$$

$$\times \int dk \int dk'\,\rho^*_{lm}(k')\,\rho_{lm}(k)\,f_l^{(0)}(k, k', r, r'), \tag{26a}$$

where

$$f_l^{(0)}(k, k', r, r') \equiv kk'[j_l(kr')\,j_l(k'r) - j_l(kr)\,j_l(k'r')]. \tag{26b}$$

On squaring this and taking the expectation value for an unconstrained field, one finds for the mean-square torque about the origin:

$$\overline{(\delta\Gamma_z^{(0)})^2} = (8G)^2(r^2\,\delta rr'\,\delta r')^2 \sum_l \frac{l(l+1)}{6(2l+1)} (r/r')^{2l}$$

$$\times \int dk\,dk'\,P(k)\,P(k')[f_l^{(0)}k, k', r, r')]^2. \tag{27a}$$

By analogy with equation (20), the corresponding expression for amplitudes which are constrained to produce a peak is

$$\overline{(\delta\Gamma_z^{(0)})^2} = (8G)^2(r^2\,\delta rr'\,\delta r')^2 \sum_{l=1}^{2} \sum_{m=1}^{l} \frac{m^2}{(2l+1)^2} (r/r')^{2l}$$

$$\times [\tilde{\rho}(k)\,\tilde{\rho}(\tilde{k}')\,C(k', \tilde{k}) + \tilde{\rho}(k')\,\tilde{\rho}(\tilde{k})\,C(k, \tilde{k}')$$
$$+ C(k, \tilde{k}')\,C(k', \tilde{k})]\,f_l^{(0)}(k, k', r, r')\,f_l^{(0)}(\tilde{k}, \tilde{k}', r, r'). \tag{27b}$$

The torque $\delta\boldsymbol{\Gamma}$ around the centre of mass of the material interior to r is $\delta\boldsymbol{\Gamma}^{(0)} - \bar{\boldsymbol{x}} \times \delta\bar{\boldsymbol{f}}$, where $\delta\bar{\boldsymbol{f}}$ is the mean force exerted on the inner shell by the outer. One finds:

$$\delta\bar{f}_x = 4\pi G\rho_{\rm b} \sqrt{\frac{4}{3}}\, r^2\,\delta r\,\delta r' \int k\,dk[\rho_{11}(k) + \rho^*_{11}(k)]\,j_1(kr'),$$

$$\delta\bar{f}_y = 4\pi G\rho_{\rm b}\, i \sqrt{\frac{4}{3}}\, r^2\,\delta r\,\delta r' \int k\,dk[\rho_{11}(k) - \rho^*_{11}(k)]\,j_1(kr'), \tag{28}$$

so

$$(\bar{\boldsymbol{x}} \times \delta\bar{\boldsymbol{f}})_z = -8iG(r^2\,\delta r\,\delta r')$$

$$\times \int dk\,dk'\,\rho_{11}(k)\,\rho^*_{11}(k')\,f(k, k', r, r'), \tag{29a}$$

where

$$f(k, k', r, r') \equiv j_2(kr)\,k'j_1(k'r') - j_2(k'r)\,kj_1(kr'). \tag{29b}$$

Comparing (29a) with (26) we see that the mean-square torque about the centre of mass is given by (27) but with $f_1^{(0)}$ replaced by

$$f_1^{(0)} + \frac{3f}{r}. \tag{30}$$

Fig. 7 shows $\overline{(\delta\Gamma_z)^2}^{1/2}/(4\pi\rho_{\rm b} r^2\,\delta r\,\delta r')$ for shells of radii 0.35 and 1 Mpc around a peak in the density field smoothed on a 0.43-Mpc scale. The torque on a shell of radius r due to one of radius $r' > r$ first rises as r' increases away from r, due to the increasing misalignment of the two shells. When $r' \sim 2r$ the torque begins to fall as a result of the difficulty which the multipole fields experience in propagating between the shells. For $r' \gg r$ the torque declines roughly as $1/r'$. This can be understood as follows. The asymptotic behaviour is governed by the dipole contribution to (27). The mass of a shell of constant thickness $\delta r'$ increases as r'^2, *so similar shells of increasing radii* r' would make equal contributions to the coefficients of Y_1^m at r. However, once a shell is larger than the largest structures predicted by the spectrum, the amplitude of a shell's global irregularity declines as the square root of its area. Hence the torque between shells of radii $r \ll r'$ eventually declines as $1/r'$.

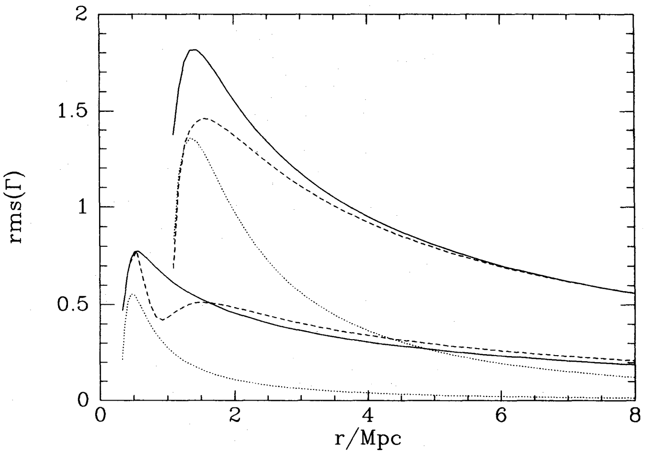

Figure 7. $\overline{(\delta\Gamma_z)^2}^{1/2}/(4\pi\rho_{\rm b} r^2\,\delta r\,\delta r')$ for concentric shells of radii 0.35 and 1 Mpc. (These shells contain $M = 0.29$, $6.7 \times 10^{11}\ M_\odot$ of material respectively.) The full and dashed curves include the contributions from the $l = 1, 2$ harmonics, while the dotted curves include the quadrupole contributions only. The dashed curves are for shells centred on a 2.5σ peak in the field smoothed on a 0.43-Mpc scale, while the full and dotted curves are for shells centred on random points.

It is not so easy to predict the speed with which the net torque of a shell of radius r converges to its final value as contributions from successively larger shells r' are added in, since this depends on the range in r' over which shells are coherently aligned. If the coherence length remained $\sim r'$ as is the case with a CDM spectrum for $r' \lesssim 1$ Mpc, the torque would never converge since it would be the distance travelled after an infinite number of random steps. Clearly the torque does converge if the coherence length $\delta r'$ tends to a constant, but for spectra of interest we cannot expect the convergence to be rapid.

From equation (12a) it follows that the autocorrelation of the net force $\boldsymbol{F}(r)$ on the solid sphere of radius r is

$$f(r_1, r_2) \equiv \overline{\boldsymbol{F}(r_1) \cdot \boldsymbol{F}(r_2)} = 8(4\pi G\rho_{\rm b})^2$$

$$\times \int dk\,P(k)\,D(r_1, k)\,D(r_2, k), \tag{31}$$

where $D(r, k)$ is defined by (12b) with the integration limits set to 0 and r. The mean velocity of a sphere will be proportional to $\boldsymbol{F}(r)/M_r$, where M_r is its mass. Thus the rms difference between the mean velocity of a sphere of radius r_2 and an embedded sphere of radius r_1 is

$$\overline{[(v_1-v_2)^2]}^{1/2}=\left[\frac{f(r_1,r_1)}{M_{r_1}^2}+\frac{f(r_2,r_2)}{M_{r_2}^2}-2\frac{f(r_1,r_2)}{M_{r_1}M_{r_2}}\right]^{1/2}. \quad (32)$$

In Fig. 8 this is plotted as a fraction of v_1 for $r_1=0.35$ Mpc. The autocorrelation $f(r_1, r_2)$ of $\boldsymbol{F}$, unlike that of $\boldsymbol{\Gamma}$, remains positive even for large r_2-r_1, and thus the rms fractional velocity difference plotted in Fig. 8 is smaller than unity throughout the plotted range of r_2. On the other hand, f is second order in the fluctuations, while the autocorrelation of $\boldsymbol{\Gamma}$ is a fourth-order quantity. So while $\overline{\boldsymbol{\Gamma}\cdot\boldsymbol{\Gamma}}$ is dominated by *relative* velocities of shells, in f such contributions are swamped by much larger, positively correlated velocities associated with large-scale streaming. For example, measurements of the microwave background show that our own Galaxy has $v_1\sim600$ km s^{-1}. Hence a fractional velocity difference of order 0.15 corresponds to an actual velocity difference ~90 km s^{-1}. This then gives the order of the momentum exchange that must have taken place between the inner Galaxy and the bulk of the dark halo over a Hubble time. To effect this exchange the inner Galaxy must have been displaced from the centre of mass of the halo as a whole. We defer investigation of the distortions of the inner Galaxy to which this displacement gives rise to another paper.

5 CONCLUSIONS

We have calculated the rms torques on shells surrounding both arbitrary points and peaks in the smoothed cosmic density field. We find that significant contributions to the torque arise from both the dipole and quadrupole terms in a spherical harmonic decomposition of the cosmic density and potential. Multipoles higher than the quadrupole make negligible contributions to the torque. The dipole contribution causes the overall torques on shells that differ in radius by more than a factor 2 to be anticorrelated.

The torque on a shell depends significantly on whether the shell is centred on a peak or a random point only when the shell's radius is comparable to the scale size of the peak – then the torque is smaller when the shell is centred on a peak. Furthermore, the tendency for shells to have anticorrelated torques is enhanced when the shells are centred on a peak.

The dimensionless angular momentum λ of the virialized body that forms from a peak in the smoothed density profile quickly reaches a value $\simeq0.05$ and then remains fairly constant as further shells are accreted by the body.

Any individual shell receives non-negligible contributions to the torque on it from shells larger by a factor of 5 or more. Thus it is important that n-body simulations of the virialization of peaks in the cosmic density field represent a region around the peak that contains considerably more mass than the peak itself.

Since this work is based on linear theory, its conclusions should be treated with caution pending confirmation by appropriate n-body simulations. In particular, we have taken no account of angular momentum exchange *within* a shell. In hierarchical clustering models, such exchanges become important at even modest distances from a peak, as the shell fragments into subclumps (e.g. Katz 1991). However, this work does represent an advance on that of Ryden & Gunn (1987) and Ryden (1988) in two respects. (i) We have clarified the significance for infall of galaxies forming at peaks rather than random points in the cosmic density field. These effects are not large but may be significant for certain applications. (ii) We find that the dipole term, which was discounted in previous work, is important in two respects.

First, it gives rise to an anticorrelation in the spins acquired by neighbouring shells. This anticorrelation must make it harder for galaxies to maintain a common rotation axis from their centres to their edges, since it accelerates the rate at which the angular momentum of infalling matter slews. In particular, our calculations suggest that a typical galactic spin axis now is on the average antiparallel to its orientation at $z\simeq0.3$. It remains to be shown that galaxies can in fact accomplish this re-orientation. If they can, the flux of off-axis angular momentum through the disc from large radii to the centre will manifest itself in warping of the disc wherever the latter's self-gravity is weak (Ostriker & Binney 1989). Polar rings, counter-rotating cores and other features frequently observed in early-type galaxies may arise when a galaxy cannot keep up with the rate at which the angular momentum of accreting material slews, and simply incorporates accreted material in something close to its original state of spin.

The second way in which the dipole term may have observable consequences is more direct: this term causes successive shells of infalling material to have differing net momenta. In effect the galaxy is being pushed in one direction or another by the downpour of accreting material to which it is exposed. We need to understand how the underlying galaxy responds to this impulse. An intriguing possibility is that the $m=1$ distortions seen in galaxies as diverse as M101 and NGC 1275 arise from this cause.

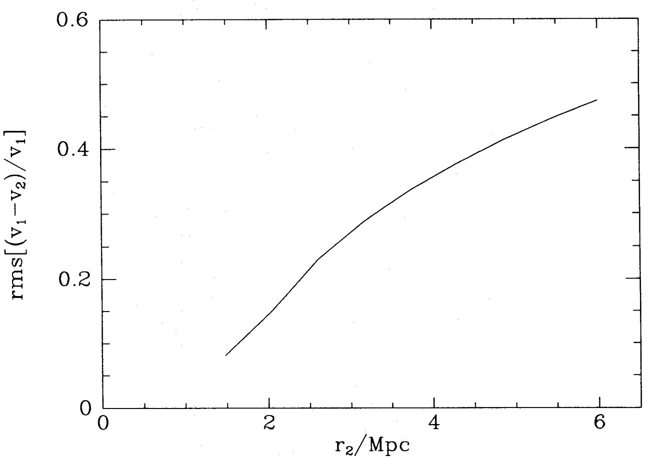

Figure 8. The rms fractional difference between the velocity acquired gravitationally by the material inside 0.35 Mpc and that of all the material inside the sphere of radius r_2.

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APPENDIX: THE CONSTRAINED TWO- AND FOUR-k CORRELATIONS

Equation (11) of Paper I expresses $\rho_{lm}(k)$ as a sum of a mean field $\tilde{\rho}(k)$ characteristic of the peak being studied and a fluctuating component $\sum_\alpha a_\alpha w^{(\alpha)}(k)$ which is proportional to independent, Gaussianly distributed random variables a_α:

$$\rho(k_j)=u(k_j)+\sum_{\alpha>0}\left[\tilde{b}_\alpha v^{(\alpha)}(k_j)+a_\alpha\sum_{\beta>0}k_\beta^{(\alpha)}v^{(\alpha)}(k_j)\right]$$

$$\equiv\tilde{\rho}(k_j)+\sum_{\alpha>0}a_\alpha w^{(\alpha)}(k_j). \tag{A1}$$

In this equation the lm subscripts on ρ_{lm} have been suppressed, the continuous variable k is represented by a discrete set of N values k_j and Greek indices also run over N values. The quantities u, $v^{(\alpha)}$, $\tilde{b}_\alpha$ and $k_\beta^{(\alpha)}$ are all uniquely determined by the nature of the peak and the power spectrum. With ρ expressed in the form (A1) the two-k correlation is readily obtained:

$$\begin{aligned}G_{lm}^{(2)}(k_i,k_j)&=\overline{\rho(k_i)\,\rho^*(k_j)}=\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)+w_i^{(\alpha)}w_j^{(\beta)}\overline{a_\alpha a_\beta^*}\\&=\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)+w_i^{(\alpha)}w_j^{(\beta)}\sigma_\alpha^2\\&=\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)+C(k_i,k_j),\end{aligned} \tag{A2a}$$

where $w_i\equiv w(k_i)$,

$$C(k_i,k_j)\equiv\sum_{\alpha>0}w_i^{(\alpha)}w_j^{(\alpha)}\sigma_\alpha^2, \tag{A2b}$$

and the dispersion σ_α of a_α is defined in Paper I. [The relevant equation in Paper I, (A25), could be misleading in that it suppresses the summation over (l, m). One has to remember that when $m\neq0$ the same quantity $|a_\alpha|^2$ occurs in the action S twice, once for positive m and once for negative m. Hence its overall factor in S is $1/\sigma_\alpha^2$ rather than the value $1/2\sigma_\alpha^2$ shown by equation (A25) of Paper I.]

We require $G_{lml'm'}^{(4)}(k_i,k_j,k_k,k_l)$ only in the case $m, m'>0$. There are two subcases to consider.

(i) $(l,m)\neq(l',m')$. Given that $m, m'>0$, ρ_{lm} and $\rho_{l'm'}$ cannot be correlated. So

$$G_{lml'm'}^{(4)}(k_i,k_j,k_k,k_l)=G_{lm}^{(2)}(k_i,k_j)\,G_{l'm'}^{(2)}(k_k,k_l)\qquad(|m|\neq|m'|). \tag{A3}$$

(ii) $(l,m)=(l',m')$. From (A1) we have

$$\begin{aligned}G_{lmlm}^{(4)}(k_i,k_j,k_k,k_l)&=\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)\,\tilde{\rho}(k_k)\,\tilde{\rho}(k_l)\\&+\sum_{\alpha>0}[\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)\,w_k^{(\alpha)}w_l^{(\alpha)}\sigma_\alpha^2+\tilde{\rho}(k_k)\,\tilde{\rho}(k_l)\,w_i^{(\alpha)}w_j^{(\alpha)}\sigma_\alpha^2\\&+\tilde{\rho}(k_i)\,\tilde{\rho}(k_l)\,w_k^{(\alpha)}w_j^{(\alpha)}\sigma_\alpha^2+\tilde{\rho}(k_k)\,\tilde{\rho}(k_j)\,w_i^{(\alpha)}w_l^{(\alpha)}\sigma_\alpha^2]\\&+\sum_{\alpha\beta\gamma\delta>0}w_i^{(\alpha)}w_j^{(\beta)}w_k^{(\gamma)}w_k^{(\gamma)}w_l^{(\delta)}\overline{a_\alpha a_\beta^* a_\gamma a_\delta^*}.\end{aligned} \tag{A4}$$

With the definition (A2b) and the standard result that for a complex Gaussian variable

$$\overline{a_\alpha a_\beta^* a_\gamma a_\delta^*}=\sigma_\alpha^2\delta_{\alpha\beta}\sigma_\gamma^2\delta_{\gamma\delta}+\sigma_\alpha^2\delta_{\alpha\delta}\sigma_\beta^2\delta_{\beta\gamma}, \tag{A5}$$

this becomes

$$\begin{aligned}G_{lmlm}^{(4)}(k_i,k_j,k_k,k_l)&=\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)\,\tilde{\rho}(k_k)\,\tilde{\rho}(k_l)\\&+\tilde{\rho}(k_i)\,\tilde{\rho}(k_j)\,C(k_k,k_l)+\tilde{\rho}(k_k)\,\tilde{\rho}(k_l)\,C(k_i,k_j)\\&+\tilde{\rho}(k_i)\,\tilde{\rho}(k_l)\,C(k_k,k_j)+\tilde{\rho}(k_k)\,\tilde{\rho}(k_j)\,C(k_i,k_l)\\&+C(k_i,k_j)\,C(k_k,k_l)+C(k_i,k_l)\,C(k_j,k_k).\end{aligned} \tag{A6}$$

When this expression for $G_{lmlm}^{(4)}$ is multiplied by $h_l(r_1,a,k_i,k_j)\,h_l(r_2,a,k_k,k_l)$, which is symmetric in (ij) and (kl), only three of its seven terms survive and we obtain equation (20).