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Galaxy Formation and the Primordial Turbulence in the Expanding Hot Universe

Humitaka SATŌ, Takuya MATSUDA and Hidenori TAKEDA

Department of Physics
Kyoto University, Kyoto

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Once, Gamow[1] and Weizsäcker[2] emphasized the importance of the cosmic turbulence in a formation of astronomical objects. Recently, Ozernoi and Chernin[3] have revitalized this idea in connection with the galaxy formation following the hot universe model based by the discovery of the cosmic black-body radiation.[4] In this letter, we expand their idea to explain the observed mass of a galaxy (or a cluster of galaxies) from a consideration of the scale of the turbulent eddies.

As the primordial turbulence, we consider the superposition of turbulent eddies of different sizes. Before the decoupling time t_D when the cosmic plasma is neutralized at the radiation temperature $T_r=4000°\mathrm{K}$,[5] the turbulent velocity v_t is subsonic and in rotational motions (div $v_t=0$). After t_D, however, v_t becomes supersonic because of a sudden decrease of the sound velocity and, then, potential motions (div $v_t \neq 0$) come into existence, which result in a generation of remarkable density fluctuations of a scale as large as the sizes of the eddies. These density fluctuations are seeds of galaxies (or clusters of galaxies) and evolve into the gravitationally found systems. Although the above mentioned theory is very vague, it is very noticeable, because the other theories such as the Jeans-Lifshitz gravitational instability[6] or the thermal instability[7] contain the unfounded assumptions such as the large initial density fluctuations and the pre-galactic heating of gas respectively.

Now, we consider the size of the turbulent eddy which leads to the generation of density fluctuations. In general, the viscous dissipation of the eddy motions is important only for the small eddies. For the eddy of the size λ, the decay time τ_d by the viscosity is given as

$$\tau_d=\lambda^2/\nu\,, \tag{1}$$

and the kinetic viscosity ν is given for the photon-plasma gas as[8]

$$\nu=\frac{4}{15}\frac{m_p}{\sigma_T\rho_m}\frac{c\rho_r}{\rho_r+\rho_m}\,, \tag{2}$$

where ρ_m and ρ_r are the matter denisty and the radiation density, and σ_T/m_p is the opacity. As τ_d/t increases with time in the expanding hot universe, the eddy motions with a size larger than $(\nu t)^{1/2}$ are survived until t without suffering a serious dissipation by the viscosity, where t is a time after the start of the cosmic expansion. Therefore, the survived eddies up to t must satisfy the condition such as

$$M_\lambda > M_{\rm vis} \equiv 4\pi\rho_m(\nu t)^{3/2}/3\,, \qquad (3)$$

where M_λ is the gaseous mass contained in λ, i.e. the size of the eddy. Taking the present matter density $\rho_{m0}=10^{-29}\Omega\,{\rm g/cm^3}$ and the present radiation temperature $T_{r0}=2.7°{\rm K}$, we have the variation of $M_{\rm vis}$ with time as shown in the Figure. At $t=t_D$, Eq. (3) becomes,

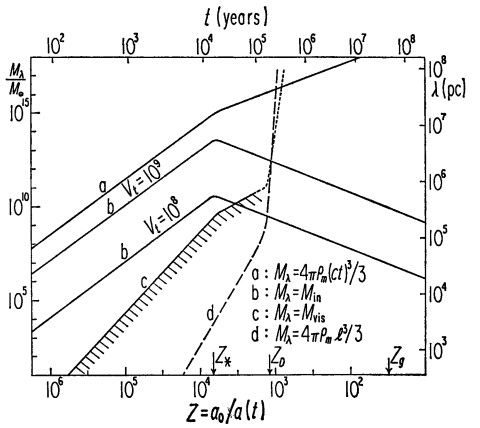

Fig. 1. For the model $\Omega=10^{-0.5}$, the variations of $M_{\rm vis}$, $M_{\rm in}$, the horizon and the mean free path are shown. The scale of the ordinate on the right represents the size of the eddy if it is expanded uniformly up to the present. $M_{\rm in}$ is given for $v_{ti}=10^{10}$ cm/sec and 10^9 cm/sec, for which cases v_t at $\rho_m=10^{-24}$ g/cm^3 are $10^{7.9}$ cm/sec and $10^{6.9}$ cm/sec respectively.

$$M_\lambda > 10^{9.8}\Omega^{-11/4}M_\odot \quad \text{for } \Omega > 10^{-1.1},$$
$$10^{12.2}\Omega^{-1/2}M_\odot \quad \text{for } \Omega < 10^{-1.1}. \quad (3')$$

For $t>t_D$, the mean free path $l=m_p/\sigma_T\rho_m$ becomes greater than the horizon given by ct, as shown in the Figure, and the viscous dissipation by the photon-plasma gas ceases. The viscous dissipation by the atomic or ionic collision is effective only for the absurdly small eddies such as $M_\lambda<10^{-8}M_\odot$.

A change of the eddy's velocity with size λ by the inertia term in the Navier-Stokes equation takes a time of the order of $t_i=\lambda/v_t$. The time scale for the occurrence of the potential motions and/or the shock waves from the rotational motions after t_D may also be given roughly by t_i. Therefore, the eddies available for the generation of the density fluctuation at t must satisfy the condition such as

$$M_\lambda < M_{\rm in} \equiv 4\pi\rho_m(v_t t)^3/3\,. \qquad (4)$$

The pattern of the eddy motions larger than $M_{\rm in}$ will remain undisturbed.

If we assume the characteristic velocity, v_g, observable in the galactic rotation or the galactic peculiar motion is the remains of the primordial turbulence, we can estimate the magnitude of v_t in the early stage. The velocity is decelerated by the uniform expansion following $(\rho_r+\rho_m)a^4v_t=\text{const}$ in the unbound system, a being the scale factor of the expansion, but the velocity remains constant in the bound system.[9] Therefore the velocity at t_D is given as

$$v_t = v_g Z_D/Z_g \quad \text{for } \Omega > 10^{-1.1},$$
$$v_g Z_*/Z_g \quad \text{for } \Omega < 10^{-1.1}, \qquad (5)$$

where $Z_D=4000°{\rm K}/2.7°{\rm K}$, $Z_*=\rho_{m0}/\rho_{r0}$ and $Z_g=[10^{-29}\Omega({\rm g/cm^3})/10^{-24}({\rm g/cm^3})]^{-1/3}$, assuming the bound system is completed at $\rho_m=10^{-24}$ g/cm^3. Taking $v_g=10^{7.5}$ cm/sec, v_t at t_D is $10^{9.0}\sim10^{7.6}$ cm/sec and the initial constant velocity v_{ti} at the stage $\rho_r>\rho_m$ is $10^{10.2}\sim10^{7.6}$ cm/sec for $\Omega=1\sim10^{-2}$, where v_{ti} must be subsonic, i.e. $v_{ti}<c/\sqrt{3}$. Referring to these values, Eq. (4) is rewritten for $t=t_D$ as follows:

$$M_\lambda < 10^{12.5}\Omega^{-1/2}(v_t/10^9)^3 M_\odot \quad \text{for } \Omega > 10^{-1.1},$$
$$10^{14.1}\Omega(v_t/10^9)^3 M_\odot \quad \text{for } \Omega < 10^{-1.1}, \qquad (4')$$

v_t being the value at t_D and in cm/sec.

Thus, we have got in Eqs. (3′) and (4′) the lower and the upper limits of the turbulent eddy available for the galaxy formation. For some choice of Ω and v_t, the lower limit may become larger than the upper, but we do not take it seriously. We rather notice that these two limits are very close and the observed mass of a galaxy (or cluster of galaxies) is limited in the range of $10^{10} \sim 10^{15} M_{\odot}$.

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