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Galaxy Formation by Cosmic Strings

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Cosmic string loops aggregate the expanding collisionless matter to form a galactic halo, in which the density falls as r^{-2} such as derived from the rotation curves of spiral galaxies. The total amount of the collected matter is estimated to be 10^{13} $(G\mu/10^{-7})^{3/2}M_\odot$ and a size of the loop which can aggregate the matter varies depending whether it is the cold matter or the hot matter.

§ 1. Introduction

In spite of large variety of the proposed scenarios of galaxy formation, most of them meet some difficulties if they try to explain both the large scale structure and the galactic or subgalactic structure by a single ingredient. In most of the scenarios, it is supposed that a small density fluctuation in the dominant component of the dark matter leads to form both the structures mentioned above. However, the observational upper limit of the cosmic microwave anisotropy has put a very stringent restriction on these scenarios. Although these single-ingredient scenarios would be economical and more desirable, there is no guarantee that this principle is true.[1] In this paper, we explore a scenario which contains a new ingredient, cosmic strings,[2)~7)] in addition to the collisionless dark matter.[8]

As briefly discussed in our previous paper,[9] an aggregation of the collisionless matter onto a string loop derives the density distribution of $\rho(r)=\mu r^{-2}$ in a virialized bound object, where μ is the line density of the string. This distribution explains the observed rotation curve for spiral galaxies. We have proposed that the velocity $v_{\rm rot}$ at the plateau of the rotation curve is related to the energy scale η of the unified theory as $v_{\rm rot}=\eta/m_{\rm Pl}$, $m_{\rm Pl}$ being the Planck mass. This result gives a right order of magnitude for μ and encourages us to explore this scenario further.

In the previous paper, we discussed only the cold matter case. The aggregation of the matter by the string loop works mainly to form the structure with sizes between galactic halos and subgalactic objects like galactic nuclei and quasars. Therefore, the large scale structure formation might be still attributed to the primordial fluctuation in the dark matter, although an alignment by long strings is also promising.[10),11)] In this paper, we explore the hot matter as well as the cold matter. The difficulty of the hot matter scenario such as the light neutrino scenario lies in the formation of galactic or subgalactic objects.[8] In our scenario, however, these smaller objects are formed by a different mechanism and the hot matter scenario could revive again.

Our scenario is not strongly coupled to a detailed property of the string. Essential point lies in that some bound objects with nonlinear energy fluctuation are formed in the early universe and keep their size in spite of the cosmic expansion. The nonlinear feature does not contradict with the microwave anisotropy limit because the average density of these objects ρ_s is assumed to be small, that is, $\delta\rho_s/\rho_{\rm total}<(\delta T/T)_{\rm obs}$ in spite of $\delta\rho_s/\rho_s \gg 1$, because of $\rho_s \ll \rho_{\rm total}$. The microwave anisotropies from the string loops have been estimated.[12] The Sachs-Wolfe effect is a dominant mechanism of it and the anisotropy is found to be much less than the observed upper limit. In order to form bound object in the expanding uniform medium with dominant energy density, some non-gravitational force is necessary, an example of which is a stress in the string.

In § 2, it is summarized how the loops are formed from the cosmic strings. In § 3, the aggregation of the collisionless matter onto the string loop is discussed and, in §4, sizes of the loops which can aggregate the cold matter and the hot matter are estimated. In § 5, the velocity of the loops, the baryonic matter and the average density of the bound objects thus formed are discussed.

§ 2. Cosmic strings and string loops

At phase transition of the vacuum in the early universe, one-dimensional topological defects of strings could be formed with a line density μ, which relates to energy scale of the symmetry breaking η as $G\mu \simeq (\eta/m_{\rm pl})^2$. Evolution of the cosmic strings in the expanding universe has been studied:[2),13),14)] The cosmological string-network was initially formed with a straight segment of a typical domain size. As the universe expands, the network is stretched conformally for the scales larger than the horizon t but, in the smaller scales, the strings tend to be straightened. The string begins to oscillate, crosses itself and intercommutes to form string loops. The loops keep oscillation but the self-intersection as well as mutual-intersection completes in a few expansion times. Therefore, the loops have remained without chopping themselves infinitely into the smaller loops. But, they disappear finally by losing their mass energy through the gravitational wave emission.

A loop of size $r_{\rm s}$ and mass $m_{\rm s}=\alpha r_{\rm s}$ is formed at $t \simeq r_{\rm s}$ and at a rate $1/r_{\rm s}^4$, that is, one loop generation per a horizon volume and an expansion time. Thus, the size spectrum of the loops is estimated as

$$n(r_{\rm s},\, t)=\nu r_{\rm s}^{-4}(a(r_{\rm s})/a(t))^3 \quad \text{for} \quad \gamma G\mu t < r_{\rm s} < t\,, \tag{2·1}$$

where $a(t)$ is the scale factor of the universe expansion and the lower cutoff is introduced because of the gravitational wave emission in a timescale $t_{\rm GW}=(\gamma G\mu)^{-1} r_{\rm s}$. Hence, after the time $t_{\rm eq}$ of equal matter and radiation, i.e., $\rho_{\rm r}=\rho_{\rm m}$,

$$n(r_{\rm s},\, t)=\nu r_{\rm s}^{-2.5} t_{\rm eq}^{0.5} t^{-2} \quad \text{for} \quad \gamma G\mu t < r_{\rm s} < t_{\rm eq} \tag{2·2}$$

and

$$n(r_{\rm s},t)=\nu r_{\rm s}^{-2} t^{-2} \qquad \text{for} \quad t_{\rm eq} < r_{\rm s} < t\,. \tag{2·3}$$

According to the numerical simulation of the loop formation in the expanding universe, $\alpha \sim 9$, $\nu \sim 0.01$ and $\gamma \sim 5$.[7),13),14)]

The center of mass of the newly formed loops may have a large velocity in general, because a recoil at the string reconnection could be strong due to a large stress such as $P=-\mu$. It would also be pointed out that the collision for the intersection occurs more frequently for a head-on collision and the smaller velocity loops might be more abundant. In the following sections, we neglect the velocity of the loops but, in the last section, the validity of this assumption will be discussed.

Gravitational effect of an infinite straight string is only to form a "conical "space, which is locally flat but has a deficient angle of $4\pi G\mu$ around the string.[15] However, the gravitational effect of a loop, which cannot stay in static state and must be in oscillatory motion, is more conventional. The loop looks like a point mass of m_s for $r \gg r_s$. Contributions from the higher multipole moments would be larger in the closer region to the loop. But, due to the rapid oscillations, the time averaged contribution of these higher multipole moments would be very small. Therefore, we replace the loop by a spherical shell with a mass $m_s = 2\pi\mu r_s$ and a radius r_s, which we call a string "shell". In this model, we can describe an intricate property of the loop's gravitational effect.[9]

§ 3. Aggregation of matter by the string "shell"

We consider aggregation of the collisionless matter onto the string shell in the matter dominant phase of $t > t_{eq}$.

We denote a radius of a spherical shell by $r_M(t)$, where M is a mass of the matter enclosed by this shell centered at the string shell's center. For $r_M(t) < r_s$, this shell expands as in the unperturbed universe, but once it gets out of r_s, it feels a gravity of the string shell. For $r_M(t) > r_s$, the motion of the mass shell obeys the equation

$$\frac{1}{2}\dot{r}_M{}^2 - \frac{G(M+m_s)}{r_M} = -\frac{Gm_s}{r_s}, \tag{3·1}$$

assuming that the background is the flat model of the density parameter $\Omega = 1$. If $\Omega \neq 1$, the additional term of $-k(r_M/a)^2/2$ appears on the right-hand side of Eq. (3·1), where a is a curvature radius and $k = \pm 1$. We can check that this new term is negligible under the condition

$$M < (4\pi G\mu / |\Omega_0 - 1|)^{1.5} M_H, \tag{3·2}$$

$M_H = 4\pi\rho(t_0)(2H_0{}^{-1})^3/3$ being the horizon mass and Ω_0 the density parameter at present t_0. As will be seen later in Eq. (3·10), our discussion will be verified even for $\Omega_0 \neq 1$.

As seen from Eq. (3·1), the mass shell reaches the maximum radius of

$$r_{M\max} = r_s(M + m_s)/m_s \tag{3·3}$$

and begins to collapse. In dissipationless gravitational contraction, the contraction is halted by "violent relaxation" to form a bound system with an effective radius of $r_{vir} = r_{M\max}/2$, which is derived from the virial theorem for the bound configuration.[16] Thus, for the virialized matter, the dissipation is derived as

$$M(r) = 2m_s r/r_s = 4\pi\mu r \quad \text{for} \quad M \gg m_s \tag{3·4}$$

and the local density is computed as

$$\rho(r) = \frac{1}{4\pi r^2}\frac{dM(r)}{dr} = \mu r^{-2}. \tag{3·5}$$

The above relation is understood also in the following manner; the fluctuation of $\delta(t_{cross}) = m_s/(M + m_s)$ is generated at the crossing time $t_{cross}(M)$, grows as $\delta \propto a(t)$ in the linear perturbation period and reaches $\delta \simeq 1$ at $r_M = r_{M\max}$ since

$$\delta(t_{\rm cross})\frac{a(t_{M\max})}{a(t_{\rm cross})}=\frac{m_s}{M+m_s}\frac{r_{M\max}}{r_s}\simeq 1\,, \tag{3·6}$$

which derives Eq. (3·3).

Equation (3·1) is solved as

$$r_M(t)=\frac{M+m_s}{2m_s}r_s(1-\cos\theta) \tag{3·7}$$

and

$$\sqrt{2Gm_s/r_s}(t-\Delta t)=\frac{M+m_s}{2m_s}r_s(\theta-\sin\theta)\,, \tag{3·8}$$

where Δt is given from $r_M(t_{\rm cross})=r_s$. For $M\gg m_s$, $\Delta t/t_{\rm cross}\ll 1$ and the virialized time $t_{\rm vir}$ defined by $r_M(t_{\rm vir})=r_{M\max}/2$ or $\theta=3\pi/2$ is

$$t_{\rm vir}=\frac{3\pi M}{4m_s}\sqrt{\frac{r_s}{2Gm_s}}r_s\,. \tag{3·9}$$

Therefore, the virialized mass upto the present epock t_0 is estimated from $t_{\rm vir}(M_{\rm vir})=t_0$ as

$$M_{\rm vir}=(8\sqrt{\pi}/9)(G\mu)^{1.5}M_{\rm H}\,, \tag{3·10}$$

where $M_{\rm H}=6t_0/G$ is the present horizon mass for $\Omega_0=1$.

The virialized mass has an extension of

$$r_{\rm vir}=(8/3\sqrt{\pi})(G\mu)^{0.5}t_0 \tag{3·11}$$

and the density $\rho(r)$ of Eq. (3·5) is rewritten as

$$\rho(r)=(27\pi^2/32)\rho(t_0)(r_{\rm vir}/r)^2 \quad \text{for } r_s<r<r_{\rm vir}\,. \tag{3·12}$$

In the region close to r_s, the string "shell" approximation will not be good and we simply assume that the mass shells of $M<m_s$ are kept within the string shell with density $\rho_c=3m_s/4\pi r_s^3$.

The mass distribution (3·4) gives a constant rotation velocity $v_{\rm rot}$ of

$$v_{\rm rot}^2=4\pi G\mu \tag{3·13}$$

or

$$G\mu=10^{-7.2}(v_{\rm rot}/250\ {\rm km\ sec^{-1}})^2\,. \tag{3·14}$$

The size and the mass of the virialized matter are rewritten as

$$r_{\rm vir}=10^{0.1}(G\mu/10^{-7})^{0.5}(2h)^{-1}{\rm Mpc} \tag{3·15}$$

and

$$M_{\rm vir}=10^{13.3}(G\mu/10^{-7})^{1.5}(2h)^{-1}M_\odot\,, \tag{3·16}$$

taking $H_0=h(100\ {\rm km\ sec^{-1}Mpc^{-1}})$.

As discussed in a previous paper,[9] we identify this virialized matter as the halo of galaxies. However, when we compare the above result with the observed rotation velocity, we have to take into account an infall of the baryonic matter toward the core after $t_{\rm vir}$. This redistribution of the baryonic component through the collisionless matter will occur as the cooling proceeds. The relation (3·13) will be modified as

$$v_{\rm rot}{}^2 = 4\pi G\mu(1-\Omega_{\rm B}) + GM_{\rm b}(r)/r\,, \tag{3·17}$$

$M_{\rm B}(r)$ and $\Omega_{\rm B}$ being the baryonic mass distribution and density parameter of $\Omega_{\rm B} = 0.1 \sim 0.03$. In any cases, the estimation of μ from the observed $v_{\rm rot}$ would be modified from Eq. (3·14). From other arguments, the slightly larger values of $G\mu$ like $10^{-6} \sim 10^{-5}$ has been estimated.[7),12)]

§ 4. Cold matter and hot matter

The density distribution (3·4) is determined solely by the fundamental quantity μ and it is independent of the loop mass or size. However, we will see in this section that a validity of these relations is in fact dependent on the initial loop size and a type of the collisionless matter, i.e., the cold matter or the hot matter such as light neutrinos for which the velocity dispersion is not negligible in our problem. In the latter discussion, we omit numerical coefficients of order unity for simplicity.

For both the cold and hot matter cases, a necessary condition for the growth of density fluctuation is to be in the matter dominant phase. Therefore, the smaller loops which had disappeared in $t < t_{\rm eq}$ have had no chance to aggregate the matter. Putting $t_{\rm GW} > t_{\rm eq}$, we get

$$m_{\rm s} > m_{\rm sc3} = \gamma(G\mu)^2 z_{\rm eq}{}^{-1.5} M_{\rm H}\,, \tag{4·1}$$

where $z_{\rm eq} = (\rho_{\rm m}/\rho_{\rm r})_0$. The decay of the loop mass will not cause a disruption of the virialized system if it occurs after the loop has aggregated the larger amount of mass than $m_{\rm s}$. The system is kept in a bound state during the decay by a tiny ejection of the weakly bound particles under the condition of (the dynamical time)$\ll$(the decay time of the loop mass). Thus, the loop works as a catalyzer to initiate the galaxy formation in the otherwise uniform matter.

Furthermore, the fluctuation growth had been suppressed by $t_{\rm eq}$ even if the mass shell gets out from the string shell and $M_{\rm vir}$ cannot be larger than $z_{\rm eq} m_{\rm s}$, because $(m_{\rm s}/M)a(t_0)/a(t_{\rm eq}) > 1$ from Eq. (3·6) and $a(t_0)/a(t_{\rm eq}) = z_{\rm eq}$. This argument introduces a critical loop mass such as

$$m_{\rm sc1} = z_{\rm eq}{}^{-1} M_{\rm vir}\,. \tag{4·2}$$

For $m_{\rm s} < m_{\rm sc1}$, the larger M mass shells had passed the string shell before $t_{\rm eq}$ and the relation (3·6) is modified as

$$(m_{\rm s}/M)(r_{M\rm max}/r_M(t_{\rm eq})) = 1\,, \tag{4·3}$$

where

$$\rho(t_{\rm eq}) r_M(t_{\rm eq})^3 = M\,. \tag{4·4}$$

From Eqs. (4·3) and (4·4), $M(r) \propto r^{3/4}$ or $\rho(r) \propto r^{-9/4}$

$$\text{in} \quad r_{\rm vir}(m_{\rm s}/m_{\rm sc1})^3 < r < r_{\rm vir}(m_{\rm s}/m_{\rm sc1})^{1/3}\,. \tag{4·5}$$

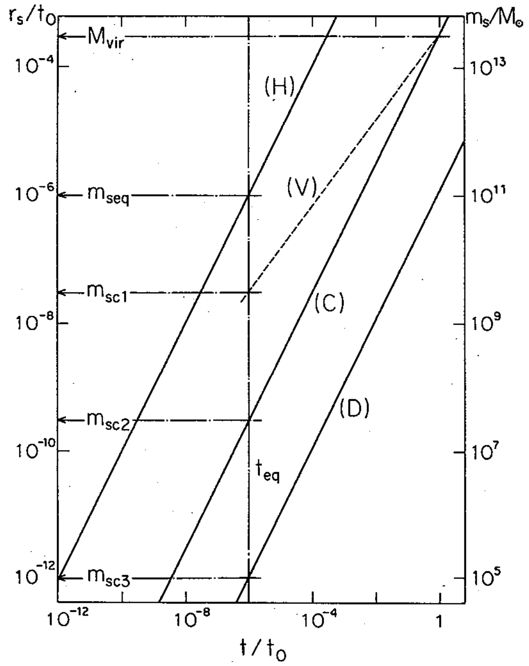

Fig. 1. The loop of size r_s is formed at the horizon $r_s=t$ denoted by (H), the aggregation of the expanding matter starts at t_{eq} and the virialized core of m_s is formed when $m_s=\rho(t_{vir})r_s^3$ or $t_{vir}=(G\mu)^{-0.5}r_s$ denoted by (C). After this epock, the virialization proceeds from the mass shell of m_s to those with the larger masses. The loops decay by the gravitational wave emission at $t=t_{GW}$ denoted by (D). The evolution of the mass shell r_M for $M=M_{vir}$ is denoted by (V).

The characteristic masses such as M_{vir}, m_{seq}, m_{sc1}, m_{sc2} and m_{sc3} are given taking the parameters as $G\mu=10^{-7}$, $\gamma=10$, $z_{eq}=10^4(2h)^2$, $M_H=10^{24}(2h)^{-1}M_\odot$ and $(2h)=1$.

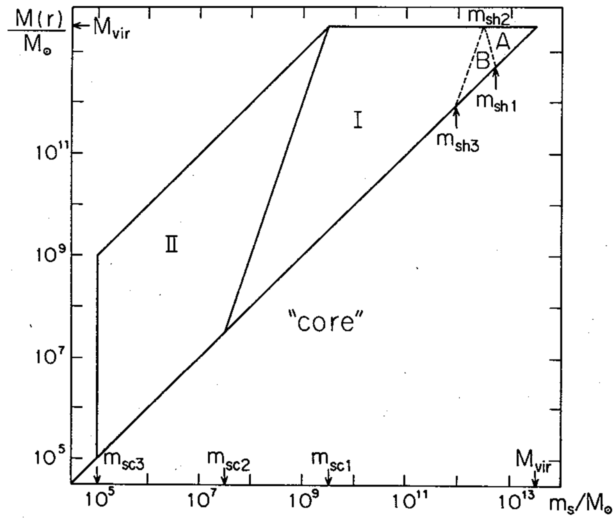

Fig. 2. The virialized mass of matter M versus the loop mass m_s. For the cold matter, the total virialized mass is M_{vir} for $m_{sc1}<m_s<M_{vir}$. The local density $\rho(r)$ falls as r^{-2} in the region I (the regions A and B are included in I) and as $r^{-9/4}$ in the region II. For the hot matter, the matter is aggregated only for $m_{sh3}<m_s<M_{vir}$, and the local density falls as r^{-2} in the region A and $r^{-3/2}$ in the region B. The parameters are same with those in Fig. 1 and $g=1$.

As the lower limit of r takes smaller value than r_s for

$$m_s<m_{sc1}=z_{eq}^{-0.5}m_{sc1}\,, \tag{4·6}$$

the density falls as $r^{-9/4}$ in $r_s<r<r_{vir}(m_s/m_{sc1})^{1/3}$ for $m_{sc3}<m_s<m_{sc2}$.

The above results are summarized in Figs. 1 and 2. The M_{vir} is also the maximum loop mass for which the virialization of the collapsing shell has just started. For the larger m_s, the aggregated matter has not yet virialized and still in a state of linear perturbation. From $\delta(r)=(m_s/M(r))(t_0/t_{cross})^{2/3}$ and $M(r)=\rho(t_{cross})r_s^3$, we get

$$\delta(r)=(M_{vir}/M(r))^{2/3}=(r_{vir}/r)^2 \quad \text{for } M(r)>m_s>M_{vir}\,. \tag{4·7}$$

In case of the hot matter, the fluctuation growth is possible for the scales larger than the Jeans length, which is given as[17)]

$$\lambda_J=g^{-1/3}t_{nr}(t/t_{nr})^{1/3} \quad \text{for} \quad t>t_{nr}\,, \tag{4·8}$$

the nonrelativistic time t_{nr} being defined as $T(t_{nr})=m$. If we write $\rho_m=gmn_r$, $t_{nr}=g^2 t_{eq}$. The motion of the mass shell begins to be decelerated only when $r_M(t)>\max\{r_s, \lambda_J(t)\}$.

The crossing time of the Jeans length over the string shell is given as

$$t_{Jcross}=gr_s^3/t_{nr}^2 \tag{4·9}$$

from $\lambda_J(t_{Jcross})=r_s$. The range of the mass shells which have passed the string shell by t_{Jcross} is estimated as $M>m_{sh1}$, where

$$m_{sh1}=g^{-2.5}M_{Ch}(t_{nr}/r_{sc1})^3=(G\mu)^{3/4}g^{1.5}z_{eq}^{-1.5}M_H \tag{4·10}$$

and $M_{Ch}=m_{pl}^3/m^2$ is the Chandrasekhar mass for m. Then, if $m_s>m_{sh1}$, the suppression of the growth does not occur and the same result holds with the cold matter case.

For the smaller m_s, $\rho(r)$ is modified in the inner region of $r_s<r<r_1$. The critical radius r_1 is estimated from $m_{sh1}=\rho(r_1)r_1^3=\mu r_1$ as

$$r_1=g^{-2}(G\mu)^{-1}t_{nr}^4/r_s^3=r_{vir}(m_{sh2}/m_s)^3\,, \tag{4·11}$$

where $m_{sh2}=g^2(G\mu)^{0.5}z_{eq}^{-2}M_H$. For the mass shells of $M<m_{sh1}$, the growth starts when $r_M(t)>\lambda_J(t)$ or $t>t_{Jcross}=t_{nr}^2/gGM$, and the perturbation reaches nonlinear at

$$r_{M\max}=(M/m_s)\lambda_J(t_{Jcross})\,. \tag{4·12}$$

Then, from Eqs. (4·11) and (4·12), we get

$$M(r)=(m_s r_1/r_s)(r/r_1)^{1.5} \quad \text{for } r_s<r<r_1\,. \tag{4·13}$$

If $m_s<m_{sh2}$, this distribution is given by Eq. (4·13) in all the region of $r_s<r<r_2$, where

$$r_2=g^{-2}(G\mu)z_{eq}^2 r_s \tag{4·14}$$

is derived from $\rho(r_2)=M(r_2)/r_2^3=\rho(t_0)$. Then, the total virialized mass is now

$$M_{vir}^h=M_{vir}(r_s/r_{sh2})^3 \tag{4·15}$$

and $M_{vir}^h<m_s$ for $m_s<m_{sh3}$, where

$$m_{sh3}=g^3 z_{eq}^{-3}M_H\,. \tag{4·16}$$

This implies that the loops with mass $m_s<m_{sh3}$ do not aggregate an appreciable mass.

The above results for the hot matter are summarized also in Fig. 2. We can see that the hot matter is aggregated only by the large loops compared with the cold matter. This tendency introduces a natural cutoff for the virialized objects. Then, in the hot matter scenario, the subgalactic objects with masses less than m_{sh3} are expected not to have a dark matter halo.

§ 5. Discussion

(a) *Peculiar velocity*

The loops are formed generally with a large peculiar velocity v_i and it decreases as $v\propto a^{-1}$ after the formation at $t=r_s$. For the loops of $r_s>t_{eq}$ or $m_s>m_{seq}$, where $m_{seq}=(G\mu)z_{eq}^{-1.5}M_H=m_{sc1}(z_{eq}G\mu)^{-0.5}$, the mass contained in the drifted size is $M_{drift}=v_i^3 r_s/G$. If this size is much smaller than the mass shell with M, the effect due to the peculiar velocity will be negligible; the condition $M>M_{drift}$ is rewritten as $r_s<(G\mu)v_i^{-3}(M$

$/M_{\rm vir})r_{\rm vir}$ and, since $M<M_{\rm vir}$ for the virialized shells, a necessary condition is

$$v_i<(z_{\rm eq}G\mu)^{0.5}(t_{\rm eq}/r_{\rm s})^{1/3}\,. \tag{5·1}$$

For the loops of $r_{\rm s}<t_{\rm eq}$, the peculiar velocity has been reduced to $v_i(r_{\rm s}/t_{\rm eq})^{0.5}$ by $t_{\rm eq}$ and the condition $M>M_{\rm drift}$ is now

$$v_i<(z_{\rm eq}G\mu)^{0.5}(t_{\rm eq}/r_{\rm s})^{1/6} \tag{5·2}$$

for $m_{\rm s}<m_{\rm scl}$, since $M/m_{\rm s}<z_{\rm eq}$ for the virialized shells.

The computer simulation of the loop formation has suggested such a large velocity as $v_i=0.1$. Therefore, the above criteria are just marginal to be satisfied. However, the condition $M>M_{\rm drift}$ is a qualitative one and we can require a different condition such that the peculiar velocity is smaller than the outflowing Hubble velocity at $r_{\rm s}$, $r_{\rm s}/t>v(t)$ for $t>(G\mu)^{-0.5}r_{\rm s}$, the epock after which the virialization starts. Once the virialized system of the matter is formed, the source of central gravity has been replaced by the growing virialized system and the drifting of the loop will not affect the aggregation of matter anymore. The above criterion derives

$$v_i<(G\mu)^{1/6}\,. \tag{5·3}$$

(b) *Baryonic components*

The virialized object consists of an almost uniform core within $r_{\rm s}$ and the extended halo, whose density structure has been discussed in §§ 3 and 4. The baryonic component is heated once upto $T=(G\mu)\,m_{\rm N}=10^6(G\mu/10^{-7})$K, $m_{\rm N}$ being proton mass, and it is in a fully ionized state. This core will be opaque if $(\mu\Omega_{\rm B}/m_{\rm N}r_{\rm s}^2)\sigma_{\rm TH}r_{\rm s}>1$ or

$$m_{\rm s}<\mu^2\Omega_{\rm B}\sigma_{\rm TH}/m_{\rm N}=10^{8.8}(\Omega_{\rm B}/0.1)(G\mu/10^{-7})^2{\rm M}_\odot\,. \tag{5·4}$$

The baryonic matter in such state might evolve into supermassive stars and finally into massive black holes, those would correspond to quasars and galactic nuclei.

The baryonic matter in the halo will contract toward the core if the cooling is efficient enough as $t_{\rm dyn}>t_{\rm cool}$, although the virialized collisionless matter keeps its equilibrium structure. Since the bound objects are in isothermal state, the cooling is more efficient in the inner dense part. The critical density $\rho_{\rm cool}$ above which $t_{\rm cool}<t_{\rm dyn}$ has been computed for the non-metalic baryonic matter.[18] The baryon density in the halo takes $\rho_{\rm cool}$ at $r_{\rm cool}=(\mu\Omega_{\rm B}/\rho_{\rm cool})^{1/2}$. The total mass within $r_{\rm cool}$ is

$$\begin{aligned} M_{\rm B}&=\Omega_{\rm B}\mu r_{\rm cool}=M_{\rm vir}\Omega_{\rm B}(\Omega_{\rm B}\rho_0/\rho_{\rm cool})^{0.5}\\ &=10^{10}(G\mu\Omega_{\rm B}/10^{-8})^{1.5}(10^{-25}{\rm g\,cm^{-3}}/\rho_{\rm cool})^{0.5}{\rm M}_\odot \end{aligned} \tag{5·5}$$

for sufficiently large $m_{\rm s}$. This amount of matter may have contracted toward the core. The baryonic matter in $r>r_{\rm cool}$ will remain uncontracted like collisionless matter.

(c) *Bound mass and unbound mass*

The uniform matter is aggregated by the loops and, if all the loops have aggregated the possible maximum mass, the averaged density of the bound mass is estimated as

$$\begin{aligned} \bar\rho_{\rm V}(M_{\rm vir})&=M_{\rm vir}\int_{r_{\rm scl}}^{r_{\rm vir}}n(r_s,\,t_0)\,dr_s\\ &=8\pi^2\nu(z_{\rm eq}G\mu)^{3/4}\rho(t_0) \end{aligned} \tag{5·6}$$

for the larger loops like $r_s > r_{sc1}$ and

$$\bar{\rho}_V(M < M_{vir}) = z_{eq}\int_{r_{sc3}}^{r_{sc1}} r_s n(r_s, t_0)\, dr_s$$

$$= 12\pi\nu z_{eq}\gamma^{-0.5}(G\mu)^{0.5}\rho(t_0) \qquad (5\cdot7)$$

for the smaller loops of $r_s > r_{sc3} = (\gamma G\mu)t_{eq}$. The condition that the virialized bound matter is less than the total matter, i.e., $\bar{\rho}_V < \rho(t_0)$, is written as

$$G\mu < 10^{-6}(0.01/\nu)^2(\gamma/10)(10^4/z_{eq})^2 . \qquad (5\cdot8)$$

Although it is almost marginal, the previous estimation (3·14) does satisfy this condition. If this is not satisfied, the larger loops formed later would not have enough accreting matter and might remain as a naked string loop. Anyhow, the larger loops can aggregate the virialized objects formed from the smaller loops, those would correspond to a cluster of galaxies.[7),11)]

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