Game-based Pursuit Evasion for Nonholonomic Wheeled Mobile Robots Subject to Wheel Slips

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Abstract

Pursuit Evasion (P-E) problem has been studied as a non-cooperative zero-sum game in Homicidal Chauffer problem [1] in 1960s and in the game of two identical cars [2] most recently. The capture conditions in the two games, which govern the capture behavior, can be determined by solving Hamilton-Jacobi-Isaacs (HJI) equations. However, the existing game theoretic solution does not consider wheel slip, and consequently, cannot answer the escape and capture conditions in the presence of wheel slip. In this paper we investigate how to predict capture and escape conditions when the pursuer has wheel slip. We study a dynamic P-E game problem with a nonholonomic Wheeled Mobile Robot (WMR) pursuer subject to wheel slip and propose an equivalent kinematic model to develop escape and capture conditions in the presence of wheel slip. To our knowledge, this is the first time the P-E game problems with WMR have been analyzed with wheel slip. The presented framework will allow future development of realistic P-E strategies that do not ignore wheel slip and thus will be able to model high speed P-E on different terrains.

Keywords: wheel slip, nonholonomic wheeled mobile robot, pursuit evasion

1. INTRODUCTION

Wheeled Mobile Robots (WMRs) have been extensively used in various applications such as in transportation, planetary exploration, intelligent surveillance, mining and military operations in recent years. Traditionally WMRs are modeled as nonholonomic systems that use regular wheels. Generally, no wheel slip is considered in modeling and control of nonholonomic WMRs, which is a reasonable assumption in many applications. In this work, however, we investigate how pursuit-evasion (henceforth P-E) problems, which is a family of problems in which one group of agents pursues to capture evading agents of another group, can be formulated with nonholonomic WMRs when no-slip assumption cannot be imposed. This is important since P-E problems require high speed chase with changing directions, and possibly on terrains that may have varied friction characteristics and therefore the likelihood of non-negligible wheel slip is real.

A typical example of P-E problem is that of a predator chasing a prey around until the prey is captured or it escapes beyond the reach of the predator. Such problems can be effectively analyzed using Game Theory [1]. A game-based P-E problem is a non-cooperative zero-sum game problem for two players, a pursuer and an evader, who have completely opposite interests. The pursuer tries to capture the evader while the evader tries to avoid being captured. Game-based P-E problem is the focus of this paper. The game arises in numerous situations. Typical examples would include battle field operations where one unmanned vehicle tries to chase another unmanned enemy vehicle, robotic soccer games, missile guidance to chase an aircraft, aircraft dogfight missions etc. In this paper, we focus on two game-based P-E problems where both the pursuer and the evader are WMRs.

There are several important works in autonomous P-E problems in the literature. A randomized pursuer strategy is applied to locate an unpredictable evader and to capture it in a visibility-based P-E problem in [3]. Dynamic programming is applied to find solution in a class of herding problem in [4], and in a multi-player P-E problem in [5] where cumulant-based control is used. In [6] nonlinear model predictive controller is applied to an evasive UAV in an aerial P-E problem to help evasion. In [7] a graph theoretic approach is proposed to a multi-player P-E problem. In [8] a time-optimal pursuit strategy is proposed in a P-E game and the pursuer takes the worst analysis to capture the evader in a time-efficient and robust fashion even when the evader is intelligent.

However, note that all these works assume kinematic constraints for the pursuer. For a WMR, this implies satisfaction of nonholonomic constraints that assume no-slipping condition, both longitudinally and laterally, at the wheels. However, for a realistic P-E problem with WMRs, wheel slip is inevitable, especially for motion at high speed in a dynamic and uncertain environment. In a P-E problem, if wheel slip is introduced, the behavior of the players may change and the existing P-E solution may no longer be valid. While it is necessary to study the effect of wheel slip in a P-E problem, wheel slip cannot be introduced without the dynamics of the WMR since the slip dynamics is governed by the traction forces.

In this paper, we explore how wheel slip can be introduced in game-based solutions of two P-E problems, namely the Homicidal Chauffer game [1] and the game of two identical cars [2], with the pursuer having dynamic model subject to wheel slip. We propose a mechanics-based approximation by introducing conceptually equivalent kinematic model for the pursuer to gain an understanding of how slip may affect the classical solutions of these two problems. *The contributions of this work are two-fold.* **First,** we show how slip may affect the solutions of the above-mentioned P-E problems that are derived without considering slip, and thus establish the need to model slip in P-E problems. **Second,** we present a new concept called "kinematically equivalent pursuer" that can be used to understand the impact of slip in P-E problem.

This paper is organized as follows. In Section II, two game-based kinematic P-E problems, namely the Homicidal Chauffeur game and the game of two identical cars, are introduced. In Section III, the importance of wheel slip consideration is discussed. In Section IV, the WMR model with wheel slip and the model of the traction force between the wheels and the surfaces are introduced. The model is then used in the game-based P-E problems with WMR pursuer subject to wheel slip. We then propose the equivalent kinematic model for the dynamic WMR subject to wheel slip in Section V. In Section VI, we present capture condition and backward reachable set in the P-E games subject to wheel slip where the equivalent kinematic model of the pursuer is used. In Section VII, we present simulation results to show the pursuit evasion behavior for dynamic WMR pursuer subject to wheel slip. We summarize our contributions in Section VIII.

2. GAME-BASED P-E PROBLEMS

P-E problems can be classified into one-sided optimization problems, which are non-game based, and two-sided optimization problems, which are game based. One-sided optimization problem is an optimal control problem where an objective function is optimized for one player, while in two-sided optimization problem an objective function needs to be maximized by one player and minimized by the other player simultaneously. Game-based P-E problem is the focus of the paper.

2.1 Homicidal Chauffeur game

The first P-E game is the Homicidal Chauffer game which was studied in 1960s [1]. In this game, the pursuer P moves at a fixed speed v_p, and its radius of curvature is bounded by a given quantity R. It steers by selecting the value of this curvature at each moment. The evader E moves at a fixed speed v_e ($v_e<v_p$) and it steers, at each moment, by choosing its direction of travel. Abrupt changes in this choice are allowed. Each player knows the other's relative location and orientation at each moment. Capture occurs when the distance between the pursuer and the evader, PE$\leq l$, where l is a given quantity. The solution of the game provides conditions that guarantee either capture (capture condition) or escape (escape condition) [1]. Guaranteed capture means that when the capture condition is satisfied, no matter what strategy the evader follows, there always exists a strategy for the pursuer to achieve capture. Guaranteed escape means that when the escape condition is satisfied, no matter what strategy the pursuer follows, there always exists a strategy for the evader to avoid being captured. It has been proved [1] that the inequality (1) is the capture condition and the inverse of it is the escape condition.

$$\frac{l}{R} > \sqrt{1-(\gamma)^2} + \gamma \sin^{-1}(\gamma) - 1 \tag{1}$$

where $\gamma = \frac{v_e}{v_p}$. The solution also provides optimal strategy for the pursuer and the evader to follow in order for each to achieve their conflicting goals [1]. While there is an analytic solution to this problem, the problem itself is of limited practical utility since the evader in the Homicidal Chauffer game does not have a turning constraint, which is not a realistic assumption. As a result, the game of two identical cars [2][9] has been studied with both players having turning constraints.

2.2 The game of two identical cars

In the game of two identical cars, the pursuer P and the evader E have fixed speed v_p and v_e ($v_p > v_e$), respectively, and their radii of curvature are bounded by given quantities R_p and R_e, respectively. They steer by selecting the value of their curvatures at each moment. Each player knows the other's relative location and orientation at each moment. Capture occurs if two cars come within a distance l of each other. However, it is shown in [9] that based solely on given initial conditions, the optimal play for the two players can not be derived analytically. Instead, a concept called the *backward reachable set* in the game space is computed as the game solution to describe the dependency of the game result on the initial conditions [2]. Capture is guaranteed to occur when the game starts from this set while escape is guaranteed when the game starts from the complement of this set. Generally computation of reachable set is used to verify and validate system design by catching every potential failure mode. Different from simulation, which only checks a single trajectory of a system each time, the reachable set is a way of checking the entire group of trajectories at once. Backward reachable set is a set of all states, starting from which trajectories can reach a given final set of states. In this game, the final set of states represents a set of all possible states at the moment of capture in the game space.

2.3 Game-based solution

The optimal play and the backward reachable set need to be solved for the Homicidal Chauffeur game and the game of two identical cars, respectively. The optimal inputs can be derived analytically in the Homicidal Chauffeur game [1]. In the game of two identical cars, the solution to the game can be characterized using Hamilton-Jacobi-Isaacs (HJI) theory. More precisely, the Hamiltonian of the system is the H function term in (2). The inputs that correspond to the optimal Hamiltonian are the optimal inputs. While it is not possible to solve for the optimal inputs, the backward reachable set can be solved instead. Let $\dot{x} = f(x, \omega_1, \omega_2)$ be the governing kinematics of the game, where x is the system state and ω_1, ω_2 are inputs for two players. Let $V(x,t)$ be the value function of the game. It has been shown in [2] that the solution of $V(x,t)$ to the HJI PDE

$$D_t V(x,t) + \min[0, H(x, D_x V(x,t))] = 0\,, \qquad (2)$$

where D_t is the derivative w.r.t time and D_x is the derivative w.r.t. x, $H(x,p) = \max_{\omega_1} \min_{\omega_2} p^T f(x, \omega_1, \omega_2)$ is the boundary of the backward reachable set when t=0, where $p = D_x V(x,t)$ in this case. It is not possible to determine the solution to (2) either analytically or numerically, however numeric approximation of the solution can be obtained by various techniques [2]. *Viscosity solution* to (2) has been proved in [2] to be the value function of the game. A family of algorithms called *level set methods* have been designed to compute approximations to the viscosity solution to (2). The Hamiltonian term, the time derivation term and the special derivation term in (2) are computed using Lax-Friedrichs approximation, a second order TVD RK approximation and a fifth order WENO spatial approximation, respectively. For the definition of above techniques, please refer to [2].

In this paper we want to find a solution to the game when dynamic model and wheel slip are introduced. If a game system has a full dynamic model, the model can be transformed into a higher dimensional system of first order ODEs and treated as a kinematic model such that the algorithms in [2] can be applied to solve the HJI equation. The highest dimension that has been solved in the literature is four as in the aircraft landing example [10], where the computation takes several days [11]. However, as the dimension becomes larger, the algorithms become computationally infeasible [11]. For example in the game of two identical cars, if the pursuer has a full dynamic model subject to wheel slip, the model can be transformed into a system of nine ODEs, which makes the computation extremely time consuming, if not impractical. Therefore, instead of pursuing the problem from a pure mathematical point of view, we approach the dynamic P-E problem from the viewpoint of mechanics-based approximation of the physical behavior of the WMR subject to wheel slip.

3. IMPORTANCE OF SLIP CONSIDERATION IN P-E GAMES

In this section, we demonstrate by examples, for each of the two games (i.e., the Homicidal Chauffer game and the game of two identical cars), that the introduction of wheel slip for the pursuer may invalidate the solution predicted by the Game Theory. An example of the original Homicidal Chauffeur game is shown in Fig. 1 where v_p=2m/s, v_e=1m/s, R=2m, l=0.5m, e_0=[0,0] and p_0=$[0,1,\pi/2]$. For these parameters, the Game Theory predicts a capture and the simulation confirms

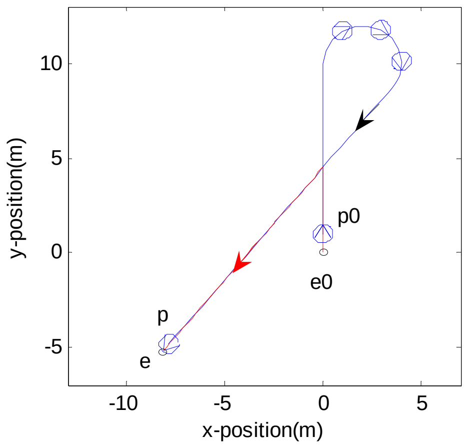

Fig. 1 Pursuit evasion paths in the original Homicidal Chauffeur game: red line is evader's path; blue curve is pursuer's path.

the prediction (Fig. 1). We then simulate the same problem with the same parameters but with wheel slip in Fig. 2 and Fig. 3 where the friction coefficients for the pursuer are 0.7 and 0.1, respectively. When the friction coefficient is 0.7, the slip is negligible so that the pursuer does not deviate much from its nominal path and the pursuer can still capture the evader (Fig. 2). However when the friction coefficient is 0.1, the pursuer deviates too much from its nominal path and therefore cannot capture the evader (Fig. 3). Thus it can be seen that introduction of wheel slip may invalidate the game theoretic prediction. Let us now consider an example of the game of two identical cars as shown in Fig. 4 where vp=ve=5m/s, R1=R2=5m, l=5m, p0=[0,0,0] and e0=[6,-11, $\pi/2$]. These parameters ensure that the game starts within the backward reachable set and thus the capture can be guaranteed. We simulate the problem with wheel slip in Fig. 5 and Fig. 6 where the friction coefficients for the

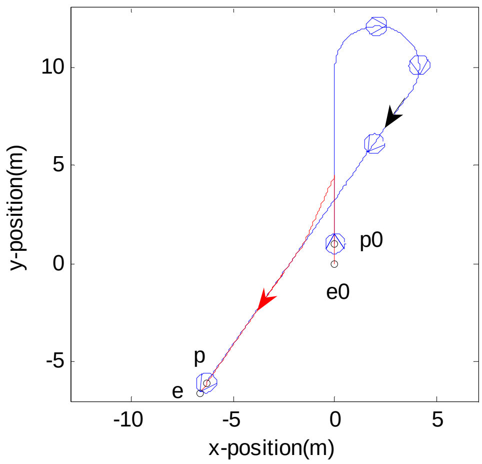

Fig. 2 Pursuit evasion paths in the Homicidal Chauffeur game subject to pursuer's wheel slip when friction coefficient is 0.7: red line is evader's path; blue curve is pursuer's path.

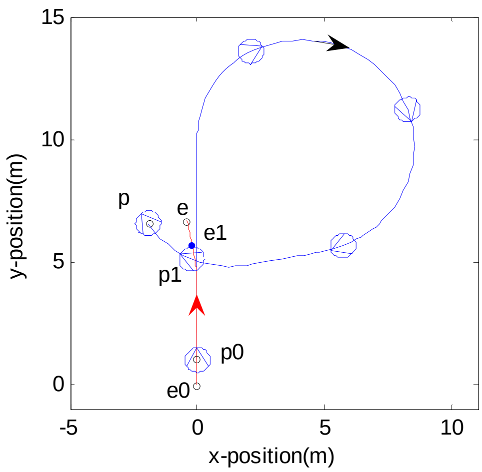

Fig. 3 Pursuit evasion paths in the Homicidal Chauffeur game subject to pursuer's wheel slip when friction coefficient is 0.1: red line is evader's path; blue curve is pursuer's path.

pursuer are 0.7 and 0.1, respectively. Since the game-based solution does not provide optimal play strategy, we let the pursuer take a pure pursuit [12] strategy while letting the evader maximize its distance from the pursuer. Fig. 4 confirms the capture as predicted by the backward reachable set consideration. Now we vary the friction coefficient to introduce slip. When the friction coefficient is 0.7, the slip is negligible so that the pursuer does not deviate much from its nominal path and the pursuer can still capture the evader (Fig. 5). However when the friction coefficient is 0.1, the pursuer deviates too much from its nominal path and therefore cannot capture the evader (Fig. 6). Note that in Fig. 3 and Fig. 6 when P is at p1 and E is at e1 they have the shortest distance from each other. The dynamic model of the pursuer used in this example is the one developed in next section. Note that even though we used a constant friction coefficient in these simulations to show the effect of slip, the

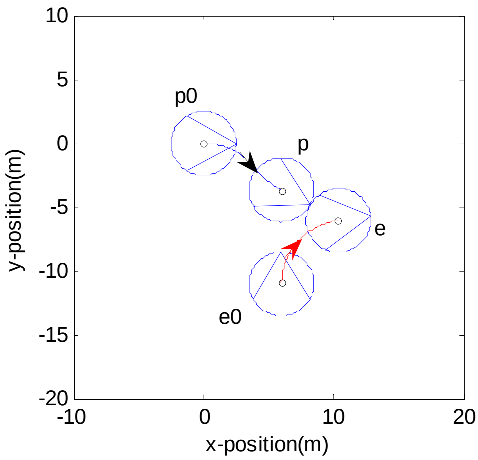

Fig. 4 Pursuit evasion paths in the original game of two identical cars: red line is evader's path; blue curve is pursuer's path.

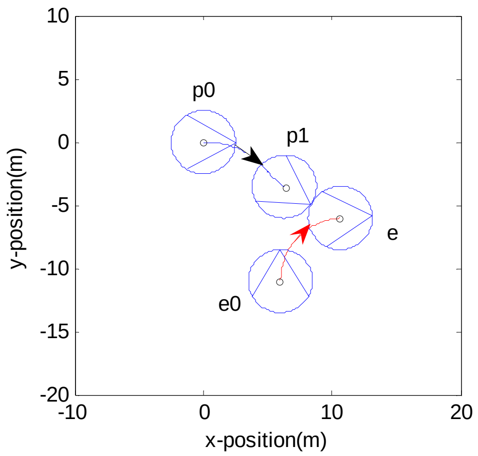

Fig. 5 Pursuit evasion paths in the game of two identical cars subject to pursuer's wheel slip when friction coefficient is 0.7: red line is evader's path; blue curve is pursuer's path.

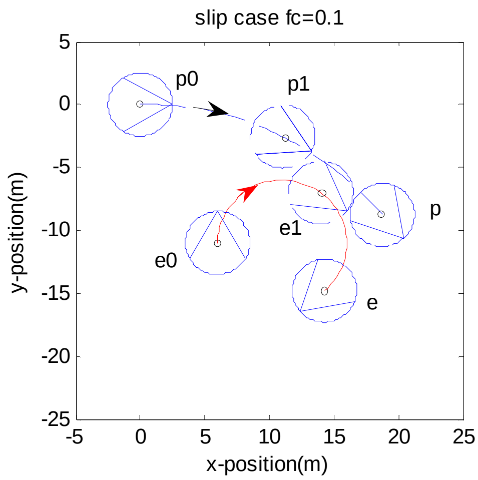

Fig. 6 Pursuit evasion paths in the game of two identical cars subject to pursuer's wheel slip when friction coefficient is 0.1: red line is evader's path; blue curve is pursuer's path.

formulation can account for variable friction coefficient. The above results demonstrate the need to consider wheel slip in P-E problems.

4. DYNAMIC WMR MODEL SUBJECT TO WHEEL SLIP

In recent literature there are some works that present approaches to model wheel slips. In [13] anti-slip factor was introduced to represent the amount of slip. However, traction force was considered as unmodeled dynamics. In [14-15] slip is introduced into kinematic WMR modeling and control. In [16][17] longitudinal slip dynamics was considered in WMR. However, the ideal WMR model was used in control design for simplicity. In [18][19][20] traction forces were introduced into WMR that were approximated to be linearly dependent on slips. In summary, either the slip has been ignored or assumed small for a WMR, or a nonlinear traction force model has not been considered so that the effect of traction forces to the WMR due to variation of slip can be investigated. In this paper, we want to model wheel slip in the overall nonholonomic WMR dynamics, investigate the effect of the nonlinear traction forces to the WMR due to variation of slip, and exploit the slip and traction force such that the maneuverability of the WMR can be improved.

The WMR subject to wheel slip is modeled as in Fig. 7, where P_c is the center of mass of the WMR, P_0 is the center of the wheel shaft, d is the distance from P_c to P_0, b is the distance from the center of each wheel to P_0. F_1 and F_2 are the longitudinal traction forces for $wheel_1$ and $wheel_2$, respectively. F_3 is the lateral traction force. Note that a dynamic model needs to be studied in order to take into account the effect of slip on the WMR. The equations for the dynamic WMR model are derived from Newton's Law as shown in (3).

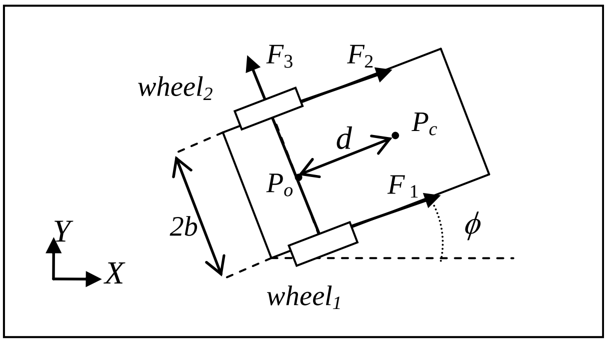

Fig. 7 WMR model subject to wheel slip.

$$\begin{cases} M\ddot{x}_c = (F_1 + F_2)\cos\phi - F_3 \sin\phi \\ M\ddot{y}_c = (F_1 + F_2)\sin\phi + F_3 \cos\phi \\ I\ddot{\phi} = (F_1 - F_2)b - F_3 d \end{cases} \tag{3.a}$$

$$\begin{cases} I_w\ddot{\theta}_1 = \tau_1 - F_1 r \\ I_w\ddot{\theta}_2 = \tau_2 - F_2 r \end{cases} \tag{3.b}$$

where M is the mass and I is its moment of inertia of the WMR, respectively, I_w is the moment of inertia of each wheel about the wheel axis, r is the wheel radius, ϕ is the orientation of the WMR, θ_i is the angular displacement of the i-th wheel, and τ_i is the wheel torque applied to the i-th wheel. Equation (3.a) represents the entire WMR dynamics in the plane while (3.b) represents the spinning dynamics of the wheels.

The lateral and longitudinal traction forces are functions of slip angle (sa) and slip ratio (sr), respectively, and are modeled following the so-called *Magic Formula* [21] in the literature. The slip angle and the slip ratio are defined as

$$sr_i = \frac{r\dot{\theta}_i - v_i}{v_i}, \quad sa = \tan^{-1}(\frac{\dot{\eta}}{v}) \tag{4}$$

where v_i is the longitudinal speed of the center of the i-th wheel, $v = (v_1 + v_2)/2$ is the forward speed, $\dot{\eta}$ is the lateral speed of the center of each wheel. They satisfy the following nonholonomic constraints [22]

$$v_1 = \dot{x}_c \cos\phi + \dot{y}_c \sin\phi + b\dot{\phi} \tag{5}$$

$$v_2 = \dot{x}_c \cos\phi + \dot{y}_c \sin\phi - b\dot{\phi} \tag{6}$$

$$\dot{\eta} = \dot{y}_c \cos\phi - \dot{x}_c \sin\phi - d\dot{\phi} \tag{7}$$

Note that, unlike classical nonholonomic constraints of WMR, the above constraints allow both longitudinal and lateral slips.

The traction force between a wheel and a surface is modeled as

$$F = K_1 \sin\left(K_2 \tan^{-1}\left(SK_3 + K_4\left(\tan^{-1}(SK_3) - SK_3\right)\right)\right) + S_v \tag{8}$$

where S is a function of slip angle for lateral traction force and slip ratio for longitudinal traction force. All other variables K_i, i=1,…,4 and S_v are constants and are determined from the curve fitting process of the empirical data. K_1 is proportional to friction coefficient. Fig. 8 shows an example of lateral traction forces with friction coefficient 0.7 and 0.3, respectively. The example of longitudinal traction force is omitted as its profile is similar to that of the lateral traction force.

Since F_i (i=1,2) is a functions of sr_i(i=1,2), sr_i(i=1,2) is a function of $\dot{\theta}_i$ (i=1,2) and $\ddot{\theta}_i$ (i=1,2) is a function of τ_i (i=1,2), $\dot{F}_i$ (i=1,2) becomes a function of τ_i (i=1,2), as shown in (9). Thus after taking the derivative of (3.a), it becomes a third order system with τ_i as the input. We observe that it is an underactuated system with a second order nonholonomic constraint.

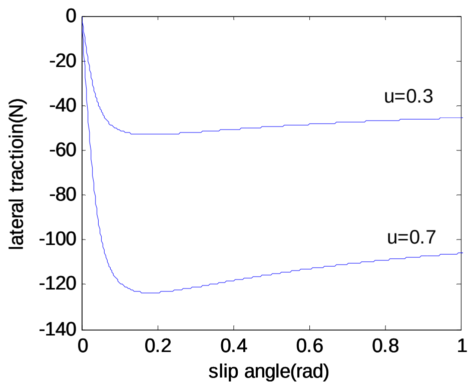

Fig. 8 Lateral traction for friction coefficients 0.7 and 0.3.

$$\dot{F}_i = \frac{K_1K_2\cos(K_2\tan^{-1}(K_3sr_i + K_4(\tan^{-1}(K_3sr_i) - K_3sr_i)))(K_3 + K_4(\frac{K_3}{1+K_3^2sr_i^2} - K_3))}{1+(K_3sr_i + K_4(\tan^{-1}(K_3sr_i) - K_3sr_i))^2}\frac{rv_i\tau_i - F_ir^2v_i - rI_w\dot{\theta}_i\dot{v}_i}{I_wv_i^2} \tag{9}$$

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5. EQUIVALENT KINEMATIC MODEL FOR THE DYNAMIC WMR SUBJECT TO WHEEL SLIP

5.1. Equivalent Kinematic Model

As discussed earlier, game theoretic solutions to the P-E problems do not consider wheel slip. However, we showed by simulation that wheel slip can make an ideally capture scenario to become an escape scenario (Figs. 3 and 6). Thus while considering wheel slip is important in a P-E problem, mathematically it becomes intractable if slip is included in the dynamic model. In order to avoid this problem, we propose a new concept called an *equivalent kinematic P-E model*, which is defined to be equivalent in terms of motion to a dynamic WMR subject to wheel slip if its lower bound of the radius of curvature (i.e., the minimum turning radius) is the same as the dynamic model's minimum allowed radius of curvature in a stable motion at a given speed. Let us explain the concept further. The game theoretic P-E problems (as discussed earlier) assume that the WMR has a given (maximum) forward speed and a given (minimum) turning radius. However, in reality when there is wheel slip, the forward speed and the turning radius will be determined by the slip-traction characteristics as well as the dynamics of the WMR and cannot be arbitrarily specified. Thus the idea of the equivalent kinematic model is to incorporate the allowable forward speed and turning radius from the dynamic WMR model and the slip-traction characteristics and then use these values as the given parameters in game theoretic solutions to predict the capture and escape conditions. If the slip-traction characteristics vary (e.g., change in terrain conditions) then a new forward speed and turning radius will be computed for each variation, which will then be used to predict capture and escape conditions at that instant. Note the difference of the solution obtained in this manner in contrast to traditional game theoretic solution. Instead of specifying arbitrary forward speed and turning radius in traditional P-E solutions that may or may not be achievable in reality, here the forward speed and the turning radius are computed from mechanics model of the dynamics of the WMR and the slip-traction characteristics of the terrain and thus provides more realistic capture and escape conditions.

When the WMR makes a turn at constant forward and angular velocity, the resultant tangential force is zero, the resultant normal force entirely contributes to the centripetal acceleration. Also, in this case, the resultant external moment is zero. In a kinematic model, it is assumed that the normal force can be as large as needed for turning. Thus the radius of curvature can be theoretically arbitrarily small when a kinematic model is used. However for the dynamic WMR subject to wheel slip, the dynamics is governed by (10) where the normal force is limited by traction forces.

$$\begin{aligned} \sum F_t &= (F_1 + F_2)\cos\phi - F_3 \sin\phi = 0 \\ \sum F_n &= (F_1 + F_2)\sin\phi + F_3 \cos\phi = M\,\frac{v^2 + \dot{\eta}^2}{R} \\ \sum M &= (F_1 - F_2)r - F_3 d = 0 \end{aligned} \tag{10}$$

When the lateral traction force F_3 reaches its maximum max(F_3), the lateral slip velocity at that moment is denoted as optimal slip $\dot{\eta}_{opt}$. When both max(F_3) and $\dot{\eta}_{opt}$ are known or estimated, for forward speed v, the minimum allowed radius of curvature is

$$R = \frac{M(v^2 + \dot{\eta}_{opt}{}^2)\cos\phi}{\max(F_3)} \quad \text{where} \quad \tan(\phi) = \frac{\dot{\eta}_{opt}}{v}. \tag{11}$$

In Table I we show the minimum turning radii of the dynamic WMR pursuer at various speeds on surfaces with various characteristics using the equivalent kinematic model developed in (11). Note that if the forward speed is variable during turning, then the minimum allowed turning radius derived from (11) will also vary at different parts of the path. We have presented a simulation in section VI where the pursuer accelerates and decelerates during turning.

TABLE I

EQUIVALENT KINEMATIC MODEL FOR DYNAMIC WMR

Velocity (m/s)	Friction Coefficient	Max (F_3) (N)	$\dot{\eta}_{opt}$ (m/s)	Radius of Curvature (m)
1	0.1	17.6	0.18	1
1	0.3	53	0.18	0.3
2	0.1	17.6	0.36	3.9
2	0.3	53	0.36	1.3
3	0.1	17.6	1.1	8.8
3	0.3	53	1.1	2.9

5.2. Control of a WMR in P-E Problems Subject to Slips

Capture condition for the Homicidal Chauffeur game with the equivalent kinematic pursuer

For the game with a WMR pursuer subject to slip, given instantaneous v_p and v_e, when (11) is substituted into (1), (12) is the instantaneous capture condition and its inverse is the escape condition.

$$\frac{l\max(F_3)}{M(v^2+\dot{\eta}_{opt}{}^2)\cos\theta} > \sqrt{1-(\gamma)^2}+\gamma\sin^{-1}(\gamma)-1 \tag{12}$$

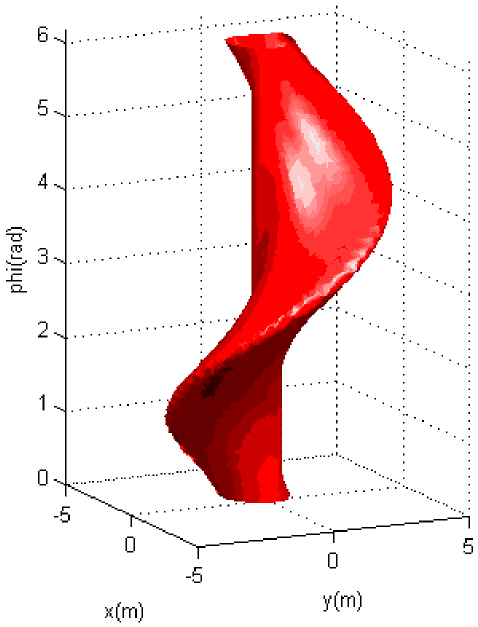

Fig. 9 Backward reachable set when $v_p=v_e=2$m/s, $R_p=0.48$m, $R_e=2$m, $l=0.48$m, $|\omega_2|\leq 1$, $|\omega_1|\leq 4.155$.

where $\tan(\theta) = \frac{\dot{\eta}_{opt}}{v}$.

The instantaneous capture condition presented by (12), unlike the classical solution (1), includes the effect of wheel slip and traction properties of the surface and thus allows us to understand the impact of slip in the Homicidal Chauffer game.

Backward reachable set for the game of two identical cars with the equivalent kinematic pursuer

Since there is no analytical capture or escape condition for this game, we simply demonstrate that it is possible to compute the backward reachable set in the presence of wheel slip using the equivalent kinematic model. In this game, we take an example with the pursuer having instantaneous speed v_p=2m/s and friction coefficient 0.3.The corresponding R_p=0.48m and we take v_e=2m/s, R_e=2m, $|\omega_2| \le \frac{v_e}{R_e} = 1$, l=0.48m. The instantaneous backward reachable set computed using the toolbox in [23] is shown in Fig. 9, where x, y, and ϕ represent coordinates and orientation of the evader relative to the pursuer.

6. SIMULATION RESULTS

In order to demonstrate the usefulness of the equivalent kinematic model in P-E problems, we needed to use a controller that would control the motion of the WMR pursuer in the presence of slip. We used a sliding mode controller that we had developed earlier [24] that controls the pursuer such that during turning it tracks the desired forward speed as well as maintain maximum lateral traction force. This controller does not assume the full knowledge of traction force. Instead traction forces are estimated using observers and we control the lateral traction force to reach its maximum by using a sliding mode-based extremum seeking control (ESC) technique [25][26]. We only assume traction forces are nonlinear and there is only one maximum both locally and globally. However, we assume that we can measure slip. While most current off-the-shelf WMRs are not equipped with slip sensors, this is not an unreasonable assumption. There are research groups in mobile robot community that have already developed various measurement and estimation techniques for detecting wheel slip in real-time for different surfaces [27-29]. We believe that such techniques will be available in many WMRs in the near future especially when there is a need to measure slip.

In the following simulations we simulate P-E games with a WMR pursuer subject to wheel slip to verify the capture condition and the backward reachable set as predicted by the equivalent kinematic model. We select initial conditions that satisfy the capture condition for the Homicidal Chauffeur game and select initial conditions from inside the backward reachable set for the game of two identical cars, using the equivalent kinematic model and show by simulation that capture truly occurs.

6.1 Homicidal Chauffer game with the WMR pursuer subject to wheel slip

We take a scenario where desired v_p=2m/s, friction coefficient is 0.3, equivalent R_p=1.3m, v_e=0.5m/s and l=0.24m. It can be shown by (12) that in this case the capture condition is satisfied. In Fig. 10, we select initial positions as $p_0=[0,1,\pi/2]$ and e_0=[0,0]. The simulations show the capture scenario for the WMR pursuer subject to wheel slip.

In another scenario, assuming everything is the same as above except that when the pursuer is turning it varies its speed, e.g., accelerates and decelerates as in Fig. 11, capture still occurs since (12) is satisfied for any instantaneous speed of the pursuer (see Fig. 12).

In the third scenario, assuming everything is the same except that there is 10% uncertainty in the measurements of both longitudinal and lateral slip velocities. Capture still occurs as shown in Fig. 13.

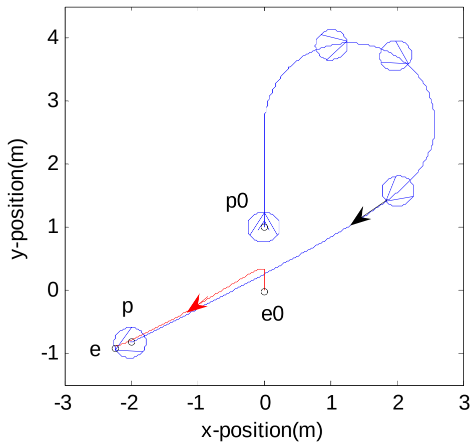

Fig. 10 Homicidal Chauffeur game for dynamic WMR pursuer subject to wheel slip governed by a sliding-mode based control.

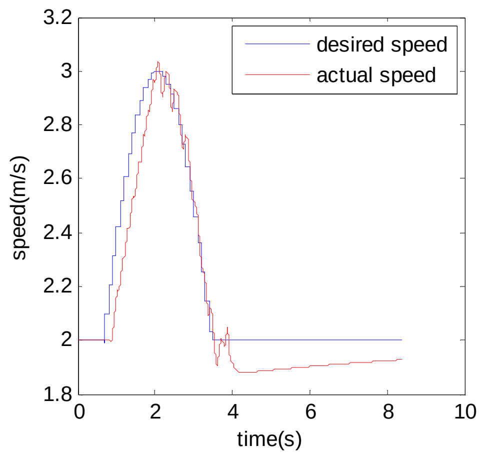

Fig. 11 Pursuer's varying speed in the Homicidal Chauffeur game for dynamic WMR pursuer subject to wheel slip governed by a sliding-mode based control

6.2 The game of two identical cars with the WMR pursuer subject to wheel slip

We take a scenario where v_p=2m/s, friction coefficient is 0.3, equivalent R_p=1.3m, v_e=2m/s, R_e=2m and l=0.48m. We select initial conditions from inside the backward reachable set in Fig. 9 as p_0=[0,0,0] and e_0=[3,0, π]. Since the game-based solution does not provide optimal play strategy, we let the pursuer take a pure pursuit [12] strategy while letting the evader maximize its distance from the pursuer. Fig. 14 shows the capture scenario for the WMR pursuer subject to wheel slip.

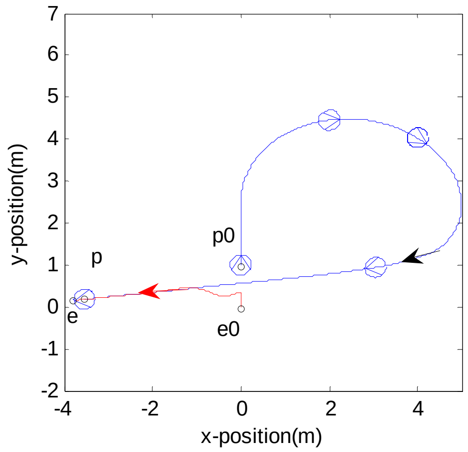

Fig. 12 Homicidal Chauffeur game for dynamic WMR pursuer subject to wheel slip governed by a sliding-mode based control. Note the change in turning radius in comparison to Fig. 10 due to varying speed during turning.

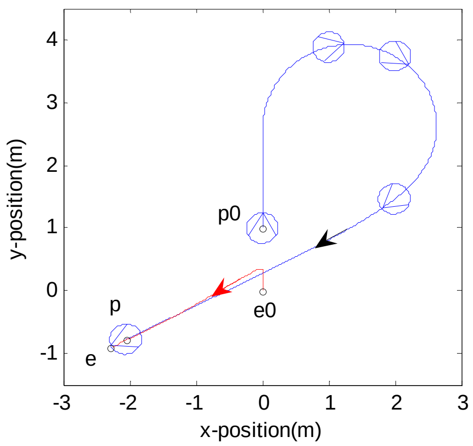

Fig. 13 Homicidal Chauffeur game for dynamic WMR pursuer subject to wheel slip governed by sliding-mode based control (10% uncertainty in slip velocity measurement).

6.3 Explanation of simulations in Section III

Back in Section II, in the Homicidal Chauffeur problem, in Fig. 1 capture condition (1) is satisfied in the original game and capture occurs under optimal play. When small amount of slip is introduced in Fig. 2, capture condition (12) is satisfied and capture still occurs. However when large amount of slip is introduced in Fig. 3, capture condition (12) is not satisfied and capture does not occur. In the game of two identical cars, in Fig. 4 the game starts from inside the backward reachable set so that capture occurs when the pursuer takes a pure pursuit [26] strategy. When a small amount of slip is introduced in Fig. 5, the game still starts from inside the backward reachable set and capture still occurs. However, when a large amount of slip is introduced in Fig. 6, the backward reachable set shrinks and the game starts from outside the set and capture does not occur.

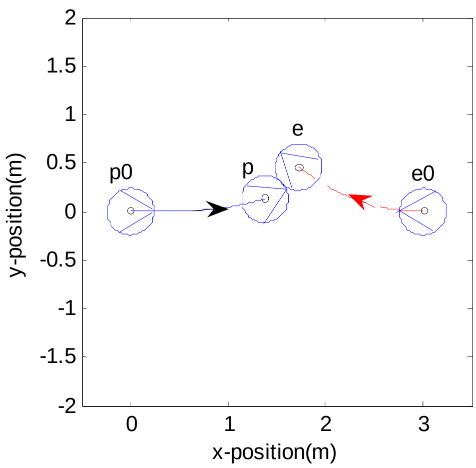

Fig. 14 Game of two identical cars for dynamic WMR pursuer subject to wheel slip governed by sliding-mode control.

7. CONCLUSION

In the P-E game problems with kinematic players, there exist analytic capture condition for the Homicidal Chauffer game and approximate backward reachable set for the game of two identical cars. In terms of wheeled mobile robots (WMR) as pursuers, the traditional solution implies kinematic model of the WMR with no wheel slip. However, we believe that for high speed P-E on uncertain terrains the wheel slip can be an important parameter that may impact the outcome of the P-E problem. Thus, in this paper, we wanted to extend our ability to predict whether a pursuer can capture an evader in the presence of slip. As discussed, the existing game theoretic solution does not consider wheel slip, and consequently, cannot answer the question that we ask in this paper. In fact, our first contribution is to show by simulation that prediction of capture based on game theory can be erroneous when there is wheel slip (Section II).

The second and more important contribution of our work is to provide a mechanism to study the P-E problem when there exists wheel slip. In order to be able to predict the outcome of a P-E problem in the presence of wheel slip, we formulated the problem in the dynamic level (as opposed to the kinematic formulation of game theory) and explicitly included a slip-traction characteristic within the WMR dynamics so that any pursuit is subject to the wheel slip dynamics. We then presented an equivalent kinematic model of this dynamic P-E game so that one can still use the classical game theoretic tools developed for kinematic game for the dynamic game presented here. To our knowledge, this is the first time the P-E game problems with WMR have been analyzed with wheel slip. The presented framework will allow future development of realistic pursuit evasion strategies that do not ignore wheel slip and thus will be able to model high speed P-E on different terrains.

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