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Game Theory and the Flat-Fading Gaussian Interference Channel

Analyzing resource conflicts in wireless networks

Most human activity is limited in one way or the other by the finite availability of resources. In communications engineering, the fundamental resources are bandwidth (spectrum) and power. Power is needed to overcome noise at the receiver, and it is basically limited because of regulations and due to limitations on storage (i.e., batteries). Spectrum is fundamentally in shortage, because the part of the electromagnetic (EM) spectrum where radio wave propagation behaves well is rather limited. As a result, licensed (dedicated) spectrum has become very expensive, as evidenced by the spectrum license auctions for third generation (3G) telephony, for example. At the same time, unlicensed spectrum has become overcrowded and at many locations so cluttered with interference that systems provide a very poor grade of service or even cease to work. One example of this can be seen in places where many wireless local area network (LAN) base stations are placed too closely.

Game theory is a branch of mathematics and provides a toolset for analyzing resource conflicts, or more generally, optimization problems with multiple conflicting objective functions. In the context of this article, the finiteness of spectrum and power, and especially the situation when multiple operators are allowed to use the same spectrum, creates a resource conflict. The goal of this article is to explain some basic terminology and to convey why game theory is a useful tool for analyzing this resource conflict. The specific focus will be on the physical layer of wireless links, including in particular beamforming and array signal processing aspects for multiantenna systems. The article will build on material from the literature [1]–[6] and on our own work on the topic (e.g., [7]–[9]).

Before we begin our discussion, we should note that there is relatively rich literature on applications of game theory to the study of various other aspects of resource allocations in

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communications. A comprehensive summary of this literature is presented in "A Brief Survey of Related Tutorial Articles," which also puts the current tutorial article in context.

GAME THEORY IS A BRANCH OF MATHEMATICS AND PROVIDES A TOOLSET FOR ANALYZING RESOURCE CONFLICTS, OR MORE GENERALLY, OPTIMIZATION PROBLEMS WITH MULTIPLE CONFLICTING OBJECTIVE FUNCTIONS.

BASIC GAME THEORY–DEFINITIONS AND NOTIONS

We first review some basic terminology. In general, a game $\mathcal{G}$ consists of three elements and can be represented as follows:

$$\mathcal{G} = (\{1, \ldots, K\}, S, \{R_1, \ldots, R_K\}).$$

First, there are the parties involved in the resource conflict. These will be called players, consistent with mainstream game theory literature [10]–[15]. In communication games, the players, $\{1, \ldots, K\}$, are usually the transmitters and receivers that share the wireless channel. Second, the actions or moves that can be taken by the players are called strategies. They belong to the strategy space S. In the wireless communications application, a set of strategies may refer to which spectral band a user is transmitting in, or how much power she spends. The third element is the payoffs or utilities $\{R_1, \ldots, R_K\}$ obtained by the players. These will depend on their selected strategies. Utility is a measure of how much something is worth to someone. The players are assumed to be rational and selfish that characterizes their objective: to maximize their utility in the game. In our work, the utilities are the rates at the receivers. In a more general context, utility may represent real or monetary values and it may be measured in arbitrary units. Utility is not necessarily linear in the amount owned. For

A BRIEF SURVEY OF RELATED TUTORIAL ARTICLES

We provide here a brief survey over related tutorial articles. MacKenzie et al. [25] discuss the application of game theory to code division multiple access (CDMA) power control problems and to medium access control in an ALOHA system. Srivastava et al. [26] illustrate the utility of game theory for ad hoc networks. In [19], Papadimitriou analyzes the Internet and its protocols from a game-theoretic point of view. Moreover, an overview of game theoretic approaches and a set of noncooperative games for energy-efficient resource allocation are presented in [27]. A survey on game theory influenced by transportation engineering, but with application of the concepts to telecommunications, was given by Altman in [28]. A recent book on game theory and its application in wireless communications is [29]. This book first addresses fundamental aspects in game theory and then goes on to discussing different examples and applications in wireless communications and networking.

There is a fairly rich literature on applications of game theory, specifically to spectrum conflicts in wireless systems, even though this topic is relatively new. In [4], the spectrum-sharing problem is studied from a game-theoretic viewpoint in search of fair, effective, and self-enforcing protocols. The players should be compelled to use proportional fair and Pareto efficient operating points. The strategy enforcement idea is backed by the use of a repeated game where users can punish one another if deviating from a desired strategy. Specifically, if a player defects from the proposed fair and globally efficient power strategy, the other players would also defect by punishing that player with a lower utility corresponding to the Nash equilibrium strategy. Consequently, no player has any incentive to defect. In [30], the authors characterize the conditions under which the Nash equilibrium is inefficient for a two-player spectrum-sharing game, and they introduce a distributed coordination algorithm to improve the performance of the system through optimizing the frequency allocation between the users.

A game-theoretic analysis of cognitive radio is presented in [31]. The authors formulate the spectrum-sharing problem between a primary user and several secondary users as a static and dynamic (repeated) Cournot game. Here, the setup is described as an oligopoly market, and the objective is to maximize the payoffs of the secondary users. In [32], a repeated game is analyzed for a spectrum sharing situation in cognitive radio that can be described as a "prisoner's dilemma" game. As a strategy to achieve higher outcomes, the iterated prisoner's dilemma is used applying different decision rules. These rules decide the moves of a player in response to previous moves. Using these rules, a comparison of the different algorithms is performed. A further work is the power control game presented in [33]. There, an approach based on pricing is used to obtain efficient operating points. It is found that when a cost function is inserted into the defined utility function, the players reduce their powers simultaneously, achieving higher payoffs at the Nash equilibrium operating point.

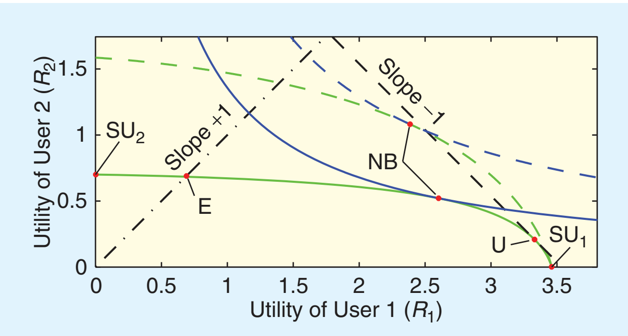
In terms of cooperative game theory, [6] and [34] study coalitions, coordination, and Nash bargaining theory for interference channels. Cooperative and noncooperative schemes are studied in [35] for power control optimization in interference networks. In [36], cooperation is made to agree on fair allocation of the spectrum.

More closely related to the topic discussed in this article, we note the following work. A minimax approach is used in [1] to show optimality of equal power allocation in single-user multiple antenna channels without channel state information at the transmitter. The work in [3] addresses transmission strategies in noncooperative systems. In the first part of [3], Nash equilibria are explored for different optimization problems. The second part deals with algorithmic aspects to obtain the Nash equilibria.

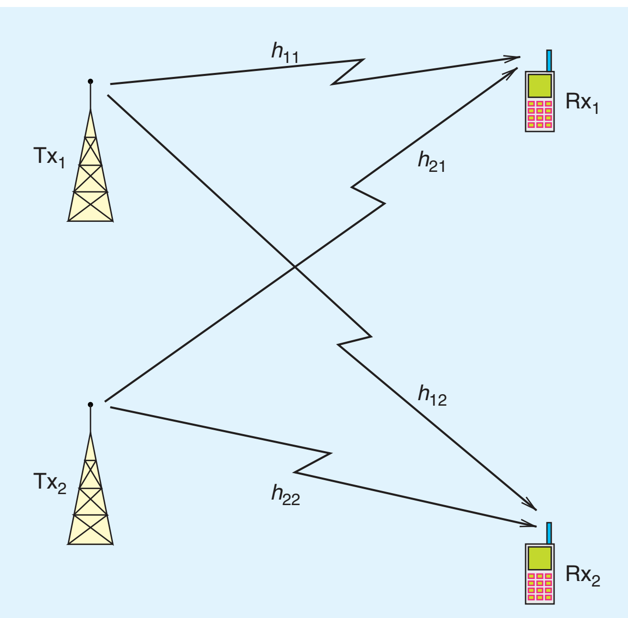
example, utility of money is more often argued to be logarithmic in the amount $\$x$: utility $= \log(\$x)$. That is, one dollar has much more worth for someone who has nothing than for someone who already owns a fortune.

When increasing someone else's utility means decreasing your own, we say that we have a conflict. All resource allocation problems are conflicts in this sense. The set of all possible outcomes of a conflict is called the utility region. The northeast boundary of the utility region is called the Pareto boundary, because it consists of Pareto optimal operating points. These are points at which increasing the utility for one of the players necessarily must decrease the utility for the other. Figure 1 illustrates the utility region for a fictitious two-player game. Figure 1 also shows the following special operating points:

- The utilitarian point (U) is the point where the sum of the utilities $(R_1 + R_2)$ is maximized. (In communications this point is also called the "sum-rate" point.) This is the point where a straight line with slope -1 touches the Pareto boundary.
- The egalitarian point (E) is the point where $\min(R_1, R_2)$ is maximized. This point is the intersection between the Pareto boundary and a straight line with slope $+1$, which passes through the origin.
- The single-user (SU_1, SU_2) are the two points where $R_2 = 0$ and where $R_1 = 0$, respectively.



[FIG1] Example of a utility region (inside the solid green curve) and some interesting points: the egalitarian (E) point, utilitarian (U) point, single-user (SU_i) points, and the Nash bargaining (NB) solution.



[FIG2] Interference channel encountered in the power game.

RESOURCE CONFLICTS ON THE GAUSSIAN INTERFERENCE CHANNEL

We will next introduce three running examples that will be used to illustrate how game theory can be used to analyze resource conflicts in communications, and specifically situations that are well modeled by an interference channel [16]. For simplicity of representation, we restrict ourselves to two-player games, $K = 2$. The first example is the so-called single-input, single-output (SISO) interference channel, and the corresponding resource allocation problem is formulated as a power game. The second example is the multiple-input, single-output (MISO) interference channel. The resource allocation problem is interpreted as a beamforming game. Finally, in the third example, we consider the multiple-input, multiple-output (MIMO) interference channel. Here, the strategy space is the space of positive semidefinite matrices that satisfy a trace constraint.

EXAMPLE 1: POWER (SISO) GAME

In this running example we consider a "power game," which is simple but provides some insights into what game theory can offer. Consider two transmitter-receiver pairs, $TX_1 \to RX_1$ and $TX_2 \to RX_2$, that operate in the same spectral band and create mutual interference to each other. Suppose that the first system transmits the signal $x_1[n]$ using the power P_1, and so does the second system transmits $x_2[n]$ with power P_2. The signals at the two receivers can then be modeled as

$$\begin{aligned} y_1[n] &= h_{11}x_1[n] + h_{21}x_2[n] + e_1[n] \\ y_2[n] &= h_{22}x_2[n] + h_{12}x_1[n] + e_2[n], \end{aligned} \quad (1)$$

where the channels between the transmitters and the receivers of both systems are defined in Figure 2 and denoted by h_{ij}, $i, j \in \{1, 2\}$. Also, $e_1[n]$ and $e_2[n]$ are samples of a zero-mean circularly symmetric complex Gaussian noise process with variance σ^2. The discrete-time signals in (1) are obtained either by filtering and sampling, or by projection of continuous-time waveforms onto an appropriate set of basis functions. The assumption made on the powers implies that $\mathrm{E}[|x_i[n]|^2] = P_i$.

The systems compete with each other for resources, because if one of the transmitters increases its transmit power in an attempt to improve performance (SINR at its receiver), then it will simultaneously increase the amount of interference generated to the other system. The "strategy" space for the two systems consists of how much power to spend (P_1, P_2) and during what fraction of the available time, to transmit. Clearly, the problem is only well formulated if we impose constraints on the power that can be spent, say

$$P_1 \leq \bar{P}_1 \text{ for system 1, and } P_2 \leq \bar{P}_2 \text{ for system 2.} \quad (2)$$

Throughout this example we will assume that both systems operate under the same constraint, i.e., that $\bar{P}_1 = \bar{P}_2 = \bar{P}$ for some $\bar{P}$.

The model (1) is recognized as an interference channel (IFC) in the literature. The IFC is a complex topic and the capacity of this channel is unknown [16]. However, it is known that if the interference at any of the receivers is strong enough to be decoded by treating the desired signal as noise when doing this decoding, then the following scheme is optimal: The receiver first decodes the interference, and then subtracts the decoded interference from the received signal to obtain interference-free data. Conversely, if the interference at any receiver is very weak, then it is optimal to just treat it as additional additive noise. Throughout this article, we will assume that the receivers treat the interference as noise. The main reason for this is that a decode-and-subtract-interference strategy would require all systems to know the coding and modulation formats of all other systems. This is a questionable assumption. In addition, interference subtraction brings difficult synchronization problems.

Throughout, we will also assume that the transmitters use capacity-achieving coding, so that Shannon's $\log(1 + \text{SNR})$ formula is applicable. While this formula only provides an (achievable) upper bound, with some modification it provides a reasonable model for practical systems. More precisely, the achievable rate, at some given error probability, of most adaptive coding and modulation schemes behaves as $\log(1 + \gamma\text{SNR})$ for some penalty factor γ that measures how far from the Shannon limit the system is operating [17]. (For simplicity of the exposition we will omit γ in what follows). The $\log(1 + \gamma\text{SNR})$ scaling law holds also to some extent for fast (ergodic) fading channels.

To understand the possible operating points for the system we first assume that both systems transmit continuously with powers P_1, P_2, and therefore produce interference to each other continuously as well, see Figure 3(a). In this case, we can write the rates at the receivers as

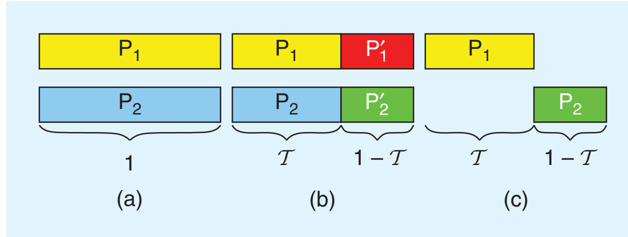
$$R_1 = \log_2\left(1 + \frac{P_1|h_{11}|^2}{P_2|h_{21}|^2 + \sigma^2}\right)$$
$$R_2 = \log_2\left(1 + \frac{P_2|h_{22}|^2}{P_1|h_{12}|^2 + \sigma^2}\right). \quad (3)$$

The rate (utility) region is

$$\mathcal{R} = \bigcup_{P_1 \leq \bar{P},\, P_2 \leq \bar{P}} (R_1, R_2).$$

Clearly, to achieve points on the boundary at least one of the transmitters must use maximum power.

To achieve points outside the region $\mathcal{R}$ one can use a technique called time-sharing. This amounts to splitting the available time into two subslots of relative lengths τ and $1 - \tau$, where $0 \leq \tau \leq 1$ and use two different pairs of transmit powers (P_1, P_2) and (P_1', P_2') that yield two different rate pairs (R_1, R_2) and (R_1', R_2'), during the two subslots [see Figure 3(b)]. Equivalently, the systems can split the available bandwidth, or code space (in orthogonal CDMA) into two parts with a relative number of degrees of freedom equal to τ and $1 - \tau$. For a given τ, the achievable rate pair becomes $(\tau R_1 + (1-\tau)R_1', \tau R_2 + (1-\tau)R_2')$ were R_1 and R_2 are defined in (3) and where



[FIG3] Different time-sharing schemes are shown in (a)–(c).

$$R_1' = \log_2\left(1 + \frac{P_1'|h_{11}|^2}{P_2'|h_{21}|^2 + \sigma^2}\right)$$
$$R_2' = \log_2\left(1 + \frac{P_2'|h_{22}|^2}{P_1'|h_{12}|^2 + \sigma^2}\right). \quad (4)$$

The power constraint in (2) can be interpreted either as a peak constraint, or as a limit on the average transmit power (in case the transmission is intermittent). Depending on how the power constraint is interpreted, two rate regions will emerge with time-sharing. We will assume that the peak power is constrained. The resulting rate region with time-sharing is

$$\bar{\mathcal{R}} = \bigcup_{\substack{\tau: 0 \leq \tau \leq 1 \\ 0 \leq P_1, P_1' \leq \bar{P},\, 0 \leq P_2, P_2' \leq \bar{P}}} (\tau R_1 + (1-\tau)R_1', \tau R_2 + (1-\tau)R_2').$$

We recognize $\bar{\mathcal{R}}$ as the convex hull of $\mathcal{R}$. Therefore, time-sharing with a peak power constraint corresponds to convexification of the rate region $\mathcal{R}$.

A special case of time-sharing is when the strategies are chosen such that the systems do not create any interference to each other. This is illustrated in Figure 3(c). In practice, this can be simply accomplished by separating the systems in time or frequency. In accordance with most literature, we call this "orthogonal transmission." Then we get the rate region (assuming again a peak power constraint)

$$\bar{\mathcal{R}}_{\text{orth}} = \bigcup_{\substack{\tau: 0 \leq \tau \leq 1 \\ 0 \leq P_1 \leq \bar{P},\ 0 \leq P_2 \leq \bar{P}}} (\tau R_1, (1-\tau)R_2).$$

Generally, $\bar{\mathcal{R}}_{\text{orth}} \subseteq \bar{\mathcal{R}}$.

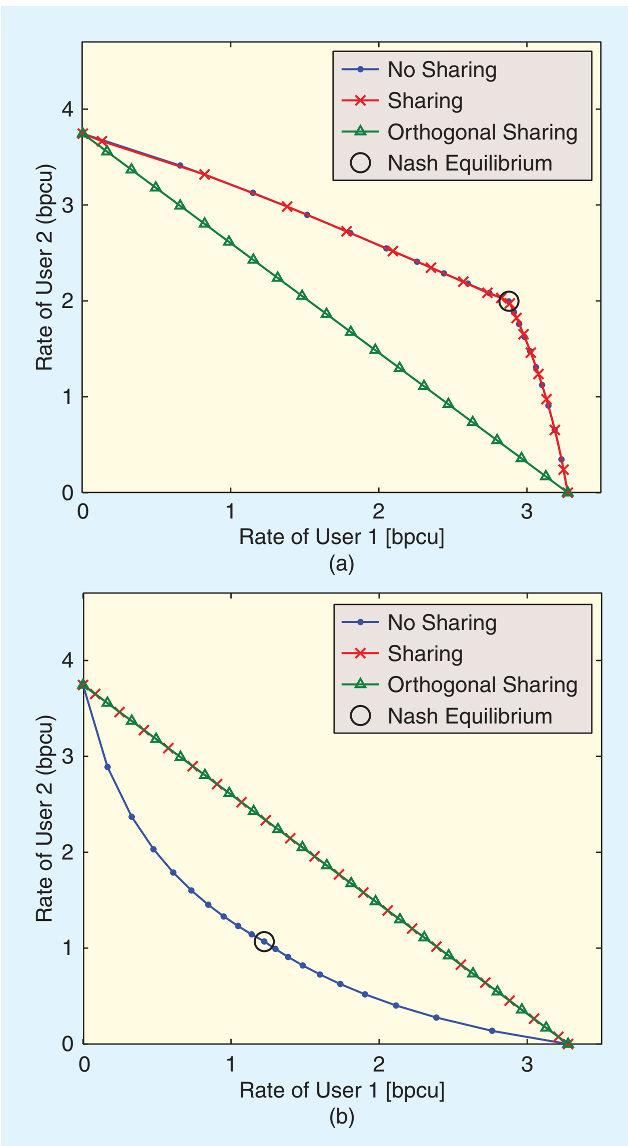
Figure 4 illustrates the regions $\mathcal{R}$, $\bar{\mathcal{R}}$ and $\bar{\mathcal{R}}_{\text{orth}}$ for two randomly chosen channel realizations: one corresponding to a situation with weak interference between the systems [Figure 4(a)], and the other corresponding to strong interference [Figure 4(b)]. For the case of strong interference, $\mathcal{R}$ is nonconvex so time-sharing enlarges the rate region. In this case, there is no loss induced by forcing the time-sharing to be orthogonal. For the example with weak interference, $\mathcal{R}$ is convex so time-sharing cannot make it larger. However, with weak interference, orthogonal time-sharing shrinks the rate region; in fact, here $\overline{\mathcal{R}}_{\text{orth}}$ is much smaller than $\mathcal{R}$. The channel realizations in this example were chosen such that both $|h_{11}|^2$ and $|h_{22}|^2$ have the same values for the weak and the strong interference case. This is the reason for why the regions $\overline{\mathcal{R}}_{\text{orth}}$ coincide for the weak and the strong interference case.

The basic problem with the power game is that if the systems act unilaterally (not cooperating) then no system has any incentive to do time-sharing (stop transmitting for a period) nor to transmit with less than the maximal possible power $\overline{P}$. In the next section, we make this precise and see what tools game theory offers to understand this problem. We end by noting that the power game is formally defined as

[LICENSED (DEDICATED) SPECTRUM HAS BECOME VERY EXPENSIVE, AS EVIDENCED BY THE SPECTRUM LICENSE AUCTIONS FOR 3G TELEPHONY.]

$$\mathcal{G}_{\text{SISO}} = (\{1, 2\}, [0, \overline{P}]^2 \times [0, 1], \{R_1, R_2\}).$$

■



[FIG4] Examples (a) and (b) of rate region boundaries for two randomly chosen channel realizations. Results are shown for all the three sharing schemes defined in Figure 3. Also, the two Nash equilibria are pointed out (to be explained later).

EXAMPLE 2: BEAMFORMING (MISO) GAME

We will consider two multiple-antenna setups, one in this example and one in Example 3. In this example, we assume that the transmitters TX_1 and TX_2 have n transmit antennas each, that can be used with full phase coherency. Receivers RX_1 and RX_2, however, have only a single receive antenna each. Hence our problem setup constitutes a MISO IFC [18].

This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at RX_1 and RX_2

$$y_1 = \mathbf{h}_{11}^T \boldsymbol{w}_1 s_1 + \mathbf{h}_{21}^T \boldsymbol{w}_2 s_2 + e_1$$
$$y_2 = \mathbf{h}_{22}^T \boldsymbol{w}_2 s_2 + \mathbf{h}_{12}^T \boldsymbol{w}_1 s_1 + e_2,$$

where s_1 and s_2 are transmitted symbols, $\mathbf{h}_{ij}$ is the (complex-valued) $n \times 1$ channel-vector between TX_i and RX_j, and $\boldsymbol{w}_i$ is the beamforming vector used by TX_i. The variables e_1, e_2 are noise terms that we model as i.i.d. complex Gaussian with zero mean and variance σ^2. We assume that each base station can use the transmit power $\overline{P}$, but that power cannot be traded between the base stations. Without loss of generality, we shall take $\overline{P} = 1$. This gives the power constraint $\|\boldsymbol{w}_i\|^2 \leq 1, \;\; i = 1, 2$. Various schemes that we will discuss require that the transmitters have different forms of channel state information (CSI). However, at no point we will require phase coherency between TX_1 and TX_2.

The following beamformers are well known in literature, and their operational meaning in a game-theoretic framework is studied in [7]. The maximum-ratio transmission (MRT) beamforming vectors maximize the power of the received desired signal component and are given by

$$\boldsymbol{w}_1^{\text{MRT}} = \frac{\boldsymbol{h}_{11}^*}{\|\boldsymbol{h}_{11}\|} \quad \text{and} \quad \boldsymbol{w}_2^{\text{MRT}} = \frac{\boldsymbol{h}_{22}^*}{\|\boldsymbol{h}_{22}\|}.$$

The zero-forcing (ZF) beamformers assure that the transmitter generates no interference to the other system, and they are given by

$$\boldsymbol{w}_1^{\text{ZF}} = \left(\frac{\Pi_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}}{\|\Pi_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}\|}\right)^* \quad \text{and} \quad \boldsymbol{w}_2^{\text{ZF}} = \left(\frac{\Pi_{\boldsymbol{h}_{21}}^{\perp} \boldsymbol{h}_{22}}{\|\Pi_{\boldsymbol{h}_{21}}^{\perp} \boldsymbol{h}_{22}\|}\right)^* \qquad (5)$$

for TX_1 and TX_2, respectively, where $\Pi_X^{\perp} \triangleq I - X(X^H X)^{-1} X^H$ denotes orthogonal projection onto the orthogonal complement of the column space of X.

[THE PRICE OF ANARCHY MEASURES THE COST THAT A SYSTEM PAYS FOR OPERATING WITHOUT COOPERATION.]

In [8], we showed that all efficient, i.e., Pareto optimal, beamforming vectors can be parameterized by

$$w_1(\lambda_1) = \frac{\lambda_1 w_1^{\text{MRT}} + (1-\lambda_1) w_1^{\text{ZF}}}{\|\lambda_1 w_1^{\text{MRT}} + (1-\lambda_1) w_1^{\text{ZF}}\|} \quad \text{and}$$

$$w_2(\lambda_2) = \frac{\lambda_2 w_2^{\text{MRT}} + (1-\lambda_2) w_2^{\text{ZF}}}{\|\lambda_2 w_2^{\text{MRT}} + (1-\lambda_2) w_2^{\text{ZF}}\|} \tag{6}$$

for some $0 \le \lambda_1, \lambda_2 \le 1$. The parameterization in (6) says that each transmitter needs to know only its MRT and ZF beamformers to achieve the points on the Pareto boundary. To compute these beamformers, knowledge of the transmitters' own channels to all other users is sufficient. The parameter λ_k, $0 \le \lambda_k \le 1$ can be interpreted as the "selfishness" of user k. For $\lambda_k = 1$ the transmitter falls back to the selfish MRT solution (which turns out to be a so-called Nash equilibrium). For $\lambda_k = 0$ the transmitter acts in a completely altruistic way and uses the ZF beamforming vectors, which spread no interference to the other system.

The main problem is to find beamforming vectors that yield a rational, efficient, and fair performance tuple (R_1, R_2). Noncooperative as well as cooperative game theory provide a systematic way of approaching this problem. Let us define the game as a tuple

$$\mathcal{G}_{\text{MISO}} = (\{1,2\}, [0,1]^2, \{R_1, R_2\})$$

with players one and two strategy space $0 \le \lambda_1, \lambda_2 \le 1$ and payoff functions

$$R_1(\lambda_1, \lambda_2) = \log\left(1 + \frac{|w_1^T(\lambda_1) h_{11}|^2}{\sigma^2 + |w_2^T(\lambda_2) h_{21}|^2}\right)$$

$$R_2(\lambda_1, \lambda_2) = \log\left(1 + \frac{|w_2^T(\lambda_2) h_{22}|^2}{\sigma^2 + |w_1^T(\lambda_1) h_{12}|^2}\right).$$

EXAMPLE 3: MIMO GAME

We consider the setup of Figure 2, but where both the transmitters and the receivers are equipped with multiple antennas. The received signal at receiver i can be described via the following baseband signal model:

$$y_i = H_{ii} x_i + \sum_{j \ne i} H_{ji} x_j + e_i, \quad \text{for } 1 \le i \ne j \le 2,$$

where x_i is the vector transmitted by TX_i, H_{ji} is the flat-fading MIMO channel matrix between TX_j and RX_i, and y_i is the received vector at RX_i. Also, e_i is a vector of received noise at RX_i, and we model its elements as i.i.d. circularly symmetric complex Gaussian with zero mean and variance σ^2. The transmit covariance matrix associated with TX_i is given by $Q_i = \mathrm{E}[x_i x_i^H]$ and the individual power constraint is $\bar{P}_i$; hence, $\mathrm{E}[\|x_i\|^2] = \mathrm{tr}(Q_i) \le \bar{P}_i$.

We continue with the assumption of low-complexity terminals and do not allow for interference cancellation at the receivers. Hence, the receivers simply treat the multiuser interference as additive spatially colored noise. We assume that the receivers have perfect CSI. The maximum achievable information rate on link i is a function of the transmit covariance matrices Q_1, Q_2

$$R_1(Q_1, Q_2) = \log\det(I + H_{11}^H \Psi_1^{-1}(Q_2) H_{11} Q_1),$$

where the noise-plus-interference covariance matrix is

$$\Psi_1(Q_2) \triangleq \sigma^2 I + H_{21} Q_2 H_{21}^H.$$

The achievable rate R_2 for the link $TX_2 \rightarrow RX_2$ has a similar form. The achievable rate region is a function of Q_1, Q_2 and is given by

$$\mathcal{R} = \bigcup_{Q_i \in \mathcal{D}_i} \{R_1(Q_1, Q_2), R_2(Q_1, Q_2)\},$$

where the constraint set $\mathcal{D}_i$ contains all feasible strategies for user i

$$\mathcal{D}_i \triangleq \{Q \succcurlyeq 0 : \mathrm{tr}(Q) \le \bar{P}_i\}.$$

The resource allocation problem amounts to finding reasonable operating points in $\mathcal{R}$. In the next sections, we will approach this problem using noncooperative and cooperative game theoretic tools. The game is formally defined by

$$\mathcal{G}_{\text{MIMO}} = (\{1,2\}, \{\mathcal{D}_1, \mathcal{D}_2\}, \{R_1, R_2\}).$$

NONCOOPERATIVE GAME THEORY

Resource conflicts in wireless systems can be understood using game theory. Game theory is a branch of mathematics. The theory splits into noncooperative and cooperative game theory. In noncooperative games, the players strictly compete and cannot strike deals. In cooperative games, the players can negotiate with one another and form joint strategies. This section will explain static noncooperative game theory, and the next section deals with cooperative theory.

If two players do not cooperate, then the only reasonable operating point will be at a so-called Nash equilibrium. A (Nash) equilibrium is an operating point where no player can improve her situation by changing strategy unilaterally, assuming everyone else continues their current strategy.

EXAMPLE 1 CONTINUED

In the power game, the strategy space consists of the powers P_1, P_2 and the time-sharing factor τ. Considering first only the parameters P_1, P_2, the Nash equilibrium is the set of P_1, P_2 for which

$$R_1^{\text{NE}} = \log_2\left(1 + \frac{P_1^{\text{NE}}|h_{11}|^2}{P_2^{\text{NE}}|h_{21}|^2 + \sigma^2}\right) \geq \log_2\left(1 + \frac{P_1|h_{11}|^2}{P_2^{\text{NE}}|h_{21}|^2 + \sigma^2}\right)$$

for all P_1 with $P_1 \leq \overline{P}$ and

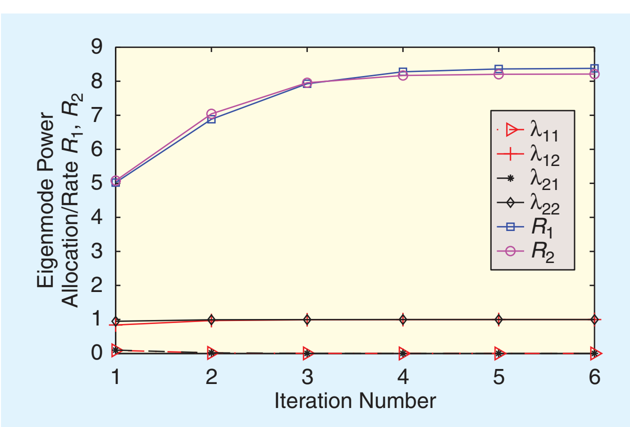
$$R_2^{\text{NE}} = \log_2\left(1 + \frac{P_2^{\text{NE}}|h_{22}|^2}{P_1^{\text{NE}}|h_{12}|^2 + \sigma^2}\right) \geq \log_2\left(1 + \frac{P_2|h_{22}|^2}{P_1^{\text{NE}}|h_{12}|^2 + \sigma^2}\right)$$

for all P_2 with $P_2 \leq \overline{P}$. In practice, we must also consider the time-sharing factor τ. In the power game, it turns out that there is a trivial Nash equilibrium that consists of transmitting with maximum power ($P_1 = P_2 = \overline{P}$) and not doing time-sharing [4]. To see that one should transmit continuously at the equilibrium, one can waterfill the available power over the noise and interference. If user two transmits with constant power over all time slots, then the waterfilling power allocation will give a constant power allocation for user one too. The Nash equilibria are pointed out in Figure 4. ■

Very often, the Nash equilibrium is a bad outcome in the sense that the selfishness of the players does not pay off. For example, in the power game with strong interference, [Figure 4(b)], any point on the single-user time-sharing line beats any point inside the region (especially the Nash equilibrium). However for weak interference, the Nash equilibrium is a good outcome in this example. Indeed, for the weak-interference case in Figure 4(a), the Nash equilibrium is sum-rate optimal.

EXAMPLE 2 (BEAMFORMING GAME) CONTINUED

For the MISO IFC game $\mathcal{G}_{\text{MISO}}$, there is a unique and pure Nash equilibrium [7]. At this equilibrium point, both systems use their maximum-ratio transmission beamforming vectors $\boldsymbol{w}_1^{\text{NE}} = \boldsymbol{w}_1^{\text{MRT}} = \boldsymbol{h}_{11}^*/\|\boldsymbol{h}_{11}\|$ and $\boldsymbol{w}_2^{\text{NE}} = \boldsymbol{w}_2^{\text{MRT}} = \boldsymbol{h}_{22}^*/\|\boldsymbol{h}_{22}\|$. The selfish strategies $\lambda_1 = \lambda_2 = 1$ achieve the equilibrium. Unfortunately, the corresponding rate tuple is not Pareto optimal in general.



[FIG5] Convergence of IWF for a 2 × 2 MIMO IFC with two users. The SNR was 15 dB and the channels were randomly chosen.

EXAMPLE 3 (MIMO GAME) CONTINUED

The Nash equilibrium for the MIMO IFC game $\mathcal{G}_{\text{MIMO}}$ always admits a NE, for any set of channel matrices and transmit powers. In contrast to the SISO and MISO IFCs, the NE for the MIMO IFC is not necessarily unique. It is shown in [5] that the NE is unique if the following condition is satisfied

$$\rho(S) < 1 \quad \text{with } [S]_{ji} = \begin{cases} \rho(\boldsymbol{H}_{ji}^H \boldsymbol{H}_{ii}^{-H} \boldsymbol{H}_{ii}^{-1} \boldsymbol{H}_{ji}), & \text{if } j \neq i \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

where $\rho(\boldsymbol{A})$ denotes the largest eigenvalue of $\boldsymbol{A}$. If the NE exists, it can be approached by the so-called synchronous or asynchronous iterative waterfilling (IWF). Each user performs single-user waterfilling with respect to the effective channel and by treating signals from the other users as noise. More precisely, the first user computes the transmit covariance matrix

$$\boldsymbol{Q}_1^{\text{opt}}(\boldsymbol{Q}_2) = \boldsymbol{U}_1(\mu_1 \boldsymbol{I} - \boldsymbol{D}_1^{-1})^+ \boldsymbol{U}_1^H, \tag{8}$$

where $\boldsymbol{U}_1$ and $\boldsymbol{D}_1$ are obtained from the eigenvalue decomposition of the effective channel $\boldsymbol{H}_{11}^H \Psi_1^{-1}(\boldsymbol{Q}_2)\boldsymbol{H}_{11} = \boldsymbol{U}_1\boldsymbol{D}_1\boldsymbol{U}_1^H$ and μ_1 is chosen to satisfy $\text{tr}[(\mu_1\boldsymbol{I} - \boldsymbol{D}_1^{-1})^+] = P_1$ with $(x)^+ \triangleq \max(0, x)$. The second user computes an expression similar to (8). The computations of $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are then iterated. One can show that the IWF algorithm converges if (7) holds.

In Figure 5, we illustrate the convergence of the IWF for a two-user 2 × 2 MIMO IFC operating at an SNR of 15 dB. The channel realization was chosen at random, but such that (7) is fulfilled. The horizontal axis shows the iteration number. On the vertical axis, the eigenvalues of the two transmit covariance matrices $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are shown (denoted as $(\lambda_{11}, \lambda_{12})$ for the first user and $(\lambda_{21}, \lambda_{22})$ for the second user). It holds that $\lambda_{\ell 1} + \lambda_{\ell 2} = 1$. Furthermore, the rates R_1 and R_2 of the two users are shown. It can be observed that the algorithm converges very fast.

PRICE OF ANARCHY

One characterization of the efficiency of the Nash equilibrium is the so-called price of anarchy (PoA) [19], [20]. The PoA measures the cost that a system pays for operating without cooperation. It is defined as the ratio of the profit obtained at the optimal operating point, over the profit when functioning at the worst-case Nash equilibrium. The question arises here to the distinction of optimal operating points and what would be the "social good." For this purpose, several global objective functions have been proposed, two of which are the utilitarian and the egalitarian solutions (cf. the discussion of Figure 1). The utilitarian solution could be the one of most interest to network operators, while the egalitarian solution may be perceived as "fairer." Here, we use the utilitarian social welfare function to express the PoA

$$\text{PoA} = \frac{\max_{\|\boldsymbol{w}_1\|^2 = \|\boldsymbol{w}_2\|^2 = 1} R_1(\boldsymbol{w}_1, \boldsymbol{w}_2) + R_2(\boldsymbol{w}_1, \boldsymbol{w}_2)}{\min_{\text{NE}}(R_1^{\text{NE}} + R_2^{\text{NE}})}. \tag{9}$$

The PoA is always greater than or equal to one. If PoA = 1, the NE achieves the utilitarian optimal solution. The PoA can be interpreted as follows: If e.g., PoA = 2, the optimal solution is twice as good as the selfish NE solution.

> **RESOURCE CONFLICTS IN WIRELESS SYSTEMS CAN BE UNDERSTOOD USING GAME THEORY.**

In Figure 6, we show the PoA for the MISO IFC game for different SNRs. Three transmit antennas and four representative channel realizations are compared. The channels $\boldsymbol{h}_{11}$ and $\boldsymbol{h}_{22}$ are chosen randomly. The channels $\boldsymbol{h}_{12}$ and $\boldsymbol{h}_{21}$, corresponding to the cross-talk, are chosen in four different ways: i) orthogonal ($\boldsymbol{h}_{11}^H\boldsymbol{h}_{12} = \boldsymbol{h}_{22}^H\boldsymbol{h}_{21} = 0$); ii) parallel ($\boldsymbol{h}_{11} = \gamma_1\boldsymbol{h}_{12}$ and $\boldsymbol{h}_{22} = \gamma_2\boldsymbol{h}_{21}$ for real-valued γ_1, γ_2); iii) regular ($\boldsymbol{h}_{11}, \boldsymbol{h}_{22}, \boldsymbol{h}_{12}, \boldsymbol{h}_{21}$ are all chosen randomly and independently); and iv) conjugate ($\boldsymbol{h}_{11} = \boldsymbol{h}_{12}^*$ and $\boldsymbol{h}_{22} = \boldsymbol{h}_{21}^*$).

For the extreme case of orthogonal channels, the NE solution is always Pareto optimal. For a regular (i.e., fixed but random) channel realization, the NE solution is close to the Pareto boundary for low SNR whereas for high SNR, the PoA increases without bound. For the case when the channels are parallel, the PoA is bounded by a constant. Finally, for the case when the channels are conjugate, the PoA increases without bound but the increase is slower than in the regular case. Note that ZF is always sum-rate optimal at high SNR for the MISO IFC [9].

COOPERATIVE GAME THEORY

In cooperative games, players (here: systems) are allowed to bargain and strike deals with one another. The theory for cooperative games splits into the cases of transferable utility and nontransferable utility. In the case of transferable utility, the players can pay one another side payments; with nontransferable utility, this is not allowed. We deal only with nontransferable utility games here. A fundamental point we must understand is that a player can be cooperative and rational at the same time. That is, being cooperative does not mean the same thing as being altruistic. The point is that even if players are eventually interested in maximizing their own outcome, they may be willing to accept a bargaining solution that is found to be good enough for both. One way of modeling this behavior mathematically is by using Nobel laureate (economics) John Nash bargaining theory [21].

EXAMPLE 4:
THE $100 QUESTION

This classic example (from [15]) is meant to illustrate the basic issues involved in modeling bargaining situations. Two men, one rich and one poor, meet a genie on the street. The genie offers them $100 to share, provided that they can agree on how to split the money. What will be the outcome of this event? The question, while somewhat imaginary, captures the same fundamental behavioral issues as the games in Examples 1–3 do. Thus, if we can understand how to deal with this question, we will also have gained some insight into the power and beamforming games. ■

The Nash bargaining theory answers the $100 question by formulating a set of axioms and proving the existence of a unique "bargaining solution." The Nash bargaining theory predicts to some extent what is likely to happen in practice if all parties act strictly rationally. An important point is that Nash bargaining has nothing to do with the Nash equilibrium. (The latter applies only to noncooperative games and there bargaining makes no sense.) The Nash bargaining outcome is not necessarily "fair" (as defined by most), as the Nash solution to the $100 question below will show.

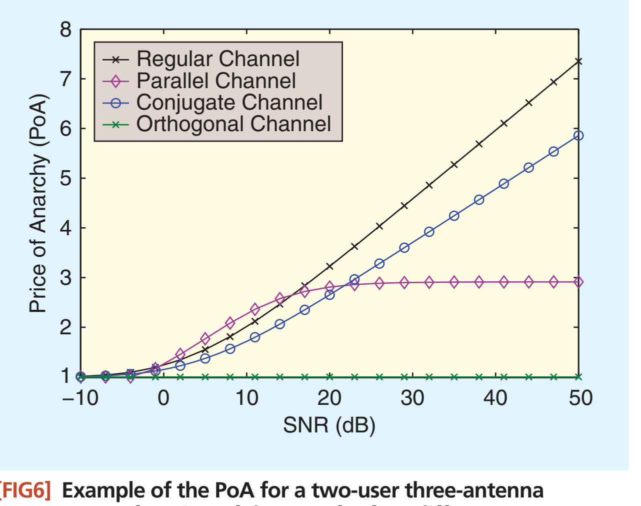
The main result by Nash is the following theorem. Let $\mathcal{S}$ be a utility region. Suppose $\mathcal{S}$ is compact and convex and let u_1^*, u_2^* be a so-called threat point. This threat point is the outcome that is achieved if the players cannot agree on any bargaining outcome. It may be taken, for example, to be the Nash equilibrium of the game, i.e., the likely outcome without any cooperation. Obviously, any meaningful threat point u_1^*, u_2^* must lie inside $\mathcal{S}$. Next, consider a function that maps the set of possible utility regions and the set of possible threat points onto a bargaining solution $(\bar{u}, \bar{v})$:

$$(\bar{u}, \bar{v}) = f(\mathcal{S}, u^*, v^*) \in \mathcal{S}.$$

Nash' bargaining theorem states that the function $f(\cdot)$, and therefore the bargaining outcome, is uniquely defined under relatively general circumstances. Moreover, the bargaining outcome can be easily computed.

More precisely, the conditions under which $f(\cdot)$ exists are given by a set of axioms (to be presented shortly). Under these axioms, Nash showed that $f(\cdot)$ is unique and given by

$$(\bar{u}, \bar{v}) = \max_{(u,v)\in\mathcal{S}} (u - u^*)(v - v^*). \tag{10}$$



[FIG6] Example of the PoA for a two-user three-antenna MISO IFC as a function of the SNR, for four different channel realizations.

The axioms are the following:

1) *Feasibility:* $(\bar{u}, \bar{v}) \geq (u^*, v^*)$. This condition is nearly trivial and just says that the outcome of a bargain cannot be worse for any of the players than the outcome if no bargaining occurs (that is, the treat point).

2) *Pareto optimal:* $(u, v) \in \mathcal{S}$ and $(u, v) \geq (\bar{u}, \bar{v}) \Rightarrow (u, v) = (\bar{u}, \bar{v})$. This condition is also nearly trivial and states that the outcome (u, v) of a bargaining must lie on the Pareto boundary of the utility region. Clearly, if this were not the case then there would be another outcome that is better for both players, and no reasonable bargaining scheme would choose (u, v).

3) *Independence of irrelevant alternatives (IIA):* If $(\bar{u}, \bar{v}) \in \mathcal{T} \subset \mathcal{S}$ and $(\bar{u}, \bar{v}) = f(\mathcal{S}, u^*, v^*)$, then $(\bar{u}, \bar{v}) = f(\mathcal{T}, u^*, v^*)$. This condition says that if bargaining in the utility region $\mathcal{S}$ results in a solution $(\bar{u}, \bar{v})$ that lies in a subset of $\mathcal{T}$ of $\mathcal{S}$, then a hypothetical bargaining in the region $\mathcal{T}$ would have resulted in the same outcome.

This axiom appears highly natural: The outcome of a bargaining should not be affected by the presence of alternative possible solutions that both players consider irrelevant. However, the axiom is controversial, because one can easily find examples where expanding the utility region will increase the bargaining outcome for one of the players but decrease it for the other. More precisely, suppose $\mathcal{T} \subset \mathcal{S}$ and let $(\bar{u}_1^{\mathcal{S}}, \bar{u}_2^{\mathcal{S}}) = f(\mathcal{S}, u^*, v^*)$ and $(\bar{u}_1^{\mathcal{T}}, \bar{u}_2^{\mathcal{T}}) = f(\mathcal{T}, u^*, v^*)$. Then we can construct cases where $\bar{u}_1^{\mathcal{S}} \geq \bar{u}_1^{\mathcal{T}}$ but $\bar{u}_2^{\mathcal{S}} < \bar{u}_2^{\mathcal{T}}$. Figure 1 exemplifies this point. The solid green curve is the Pareto boundary of the original region $\mathcal{S}$. The dashed green curve is the Pareto boundary of the expanded region $\mathcal{T}$. The Nash bargaining solutions occur when the Pareto boundaries intersect the corresponding hyperbola defined by (10). Clearly, expanding the region improves one of the utilities but not both. This illustrates that any requirement that $\mathcal{T} \subset \mathcal{S} \Rightarrow \bar{u}_1^{\mathcal{S}} \geq \bar{u}_1^{\mathcal{T}}, \bar{u}_2^{\mathcal{S}} \geq \bar{u}_2^{\mathcal{T}}$ would be incompatible with the IIA axiom.

4) *Symmetry:* If $\mathcal{S}$ is symmetric around $u = v$ then $u^* = v^* \Rightarrow \bar{u} = \bar{v}$. This just means that if the utility region is symmetric around a line with slope 45° then the bargaining outcome will lie on the line of symmetry.

5) *Invariance to linear transformation:* Let $a_1, a_2, b_1, b_2 \in \mathbb{R}$, $a_1 > 0, a_2 > 0$ be arbitrary. Then this axiom says that if $(a_1 u^* + b_1, a_2 v^* + b_2) \in S$ then $f(\mathcal{S}, a_1 u^* + b_1, a_2 v^* + b_2) = [a_1, a_2] f(\mathcal{S}, u^*, v^*) + [b_1, b_2]$.

A FUNDAMENTAL POINT WE MUST UNDERSTAND IS THAT A PLAYER CAN BE COOPERATIVE AND RATIONAL AT THE SAME TIME.

EXAMPLE 4 ($100 QUESTION) CONTINUED

Now let us solve the $100 question by using the Nash bargaining theorem. To formulate the problem more precisely, let us assume that the utility of money is logarithmic in the amount owned. Also, we assume that the rich man (R) is near infinitely rich ($x_r = 10^{10}$) but that the poor man (P) owns only $x_p = 10$ in total.

Let x be the amount R gets in the bargain. After bargaining the utility for R is

$$u_r^{\text{bargain}} = \log(10^{10} + x)$$

and the utility for P is

$$u_p^{\text{bargain}} = \log(10 + (100 - x)).$$

We take the threat point to be $(u_r^*, u_p^*) = (\log(x_r), \log(x_p))$ since if no bargain occurs, both R and P will leave with exactly the initial amount they owned. We now look for

$$\max_{u_r, u_p, x \in [0,100]} (u_r - u_r^*)(u_p - u_p^*).$$

The solution can be easily found graphically. It is the point where the Pareto boundary has a unique intersection with a hyperbola parametrized by $(u_r - u_r^*)(u_p - u_p^*) = \text{constant}$ (cf. the Nash bargaining points in Figure 1). The Nash bargaining solution of the $100-question with $x_r = 10^{10}$ and $x_p = 10$ is $x \approx 66$. Evidently, the bargaining outcome favors the rich man, who gets the most part of the money. For comparison, if instead P had initially owned only $x_p = 0.1 = 10$ cents (R has still $x_r = 10^{10}$), then the Nash bargaining solution would be $x \approx 84$ and the outcome would be even more unbalanced. The reason is that R has much more bargaining power. In fact, he can dictate a "my way or no way" outcome by threatening to walk away without a deal if he does not get a larger share of the money. Especially he knows that not being able to reach a deal will hurt P more than R so that P will be more willing to accept a bad deal than no deal at all. In the special cases when $x_p \to 0$ the bargaining solution $x \to 100$ and when $x_p \to x_r$ the solution approaches $x \to 50$.

Is the outcome of Example 4 fair? This depends on how one defines fairness, which is by necessity a highly subjective notion. Bargaining theory does not aspire to model fairness in the sense that most humans interpret the term. Rather, it should be seen as a mathematical model for the fact that a stronger part in a (resource) conflict always has a larger power of negotiation and therefore will achieve a better outcome [22].

The cooperative game theory framework outlined here, and the Nash bargaining theory in particular, can be used to analyze both the SISO (power), MISO and MIMO games in Examples 1–3. In essence, the approach consists of computing the Pareto boundary with time-sharing (to ensure convexity) and then solve (10) to find the Nash bargaining point. This solution must be found numerically, but the computation can be aided by the efficient parameterization of the Pareto boundary presented in [8]. A detailed treatment of the results here would lead too far and we instead refer the reader to the relevant literature [6] and [7].

A different cooperative approach for MIMO interference networks is studied in [23], where axiomatic bargaining theory is used to find the Kalai-Smorodinsky operating point. This approach can also be applied to nonconvex utility regions.

CONCLUDING REMARKS

In this article, we described some basic concepts from noncooperative and cooperative game theory and illustrated them by three examples using the interference channel model, namely, the power allocation game for SISO IFC, the beamforming game for MISO IFC, and the transmit covariance game for MIMO IFC. In noncooperative game theory, we restricted ourselves to discuss the NE and PoA and their interpretations in the context of our application. Extensions to other noncooperative approaches include Stackelberg equilibria and the corresponding question "Who will go first?" We also correlated equilibria where a certain type of common randomness can be exploited to increase the utility region. We leave the large area of coalitional game theory [24] open.

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