Gamma-ray Burst Luminosity Relations: Two-dimensional versus Three-dimensional Correlations

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ABSTRACT

The large scatters of luminosity relations of gamma-ray bursts (GRBs) have been one of the most important reasons that prevent the extensive applications of GRBs in cosmology. In this paper, we extend the two-dimensional (2D) luminosity relations with τ_{lag}, V, E_{peak}, and τ_{RT} as the luminosity indicators to three dimensions (3D) using the same set of luminosity indicators to explore the possibility of decreasing the intrinsic scatters. We find that, for the 3D luminosity relations between the luminosity and an energy scale (E_{peak}) and a time scale (τ_{lag} or τ_{RT}), their intrinsic scatters are considerably smaller than those of corresponding 2D luminosity relations. Enlightened by the result and the definition of the luminosity (energy released in units of time), we discussed possible reasons behind, which may give us helpful suggestions on seeking more precise luminosity relations for GRBs in the future.

Subject headings: Gamma rays: bursts

1. Introduction

Gamma-ray bursts (GRBs) have recently attracted much attention in their cosmological applications as the most luminous astrophysical events observed today. Based on the correlations between the luminosity/energy and the measurable parameters of light curves and/or spectra, GRBs can be used as standard candles after calibration (see, for example, Dai et al. 2004; Ghirlanda et al. 2004b; Firmani et al. 2005; Ghirlanda et al. 2006; Schaefer 2007; Amati et al. 2008; Basilakos & Perivolaropoulos 2008, etc.). The advantage of the GRBs is their high redshifts due to their high luminosities. The 69 GRBs compiled in Schaefer (2007) extend the redshift to $z > 6$. Recently observed GRB 090423 has a redshift of $z = 8.3$ (Tanvir et al. 2009; Salvaterra et al. 2009). However, GRBs are not as ideal standard candles as type Ia supernovae (SNe Ia). Compared with SNe Ia, GRBs suffer from the circularity problem due to the lack of low-redshift samples and the scatters of known luminosity relations of GRBs are still very large. There are a few ways proposed in literatures to avoid the circularity problem. One can simply fit the calibration parameters and cosmological parameters simultaneously (Li et al. 2008; Qi et al. 2008); and in Wang (2008), GRB data are summarized by a set of model-independent distance measurements; calibrating GRBs using SNe Ia in their overlapping redshift range was also proposed (Kodama et al. 2008; Liang et al. 2008) and adopted (Wei & Zhang 2009; Cardone et al. 2009) in cosmological studies using GRBs.

Till now in most works of cosmological studies using GRBs, two-dimensional (2D) luminosity relations are used, which have least calibration parameters. For example τ_{lag}–L (Norris et al. 2000), V–L (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001, there exist several definitions of V, mainly depending on the smoothing time intervals the reference curve is built upon, and on the normalization as well), E_{peak}–$E_{\gamma,\mathrm{iso}}$ (Amati et al. 2002), E_{peak}–E_{γ} (Ghirlanda et al. 2004a), E_{peak}–L (Schaefer 2003), and τ_{RT}–L (Schaefer

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2007) relations. However, the intrinsic scatters of 2D luminosity relations are usually very large, which may imply hidden parameters considering the complication of GRBs. There are already works which explored the possibility of a three-dimensional (3D) correlation with negligible scatter. For example, Firmani et al. (2006) claimed that a temporal parameter of the prompt emission, the $T_{0.45}$, could reduce the scatter of the correlation of $L_{\rm iso}$–$E_{\rm peak}$ to a negligible value. But it was later found that the new proposed relation does not appear to be as tight as it seemed to be (Rossi et al. 2008; Collazzi & Schaefer 2008). In this paper, we extend the luminosity relations used in Schaefer (2007) from 2D to 3D and explore the possibility of decreasing the scatters in the correlations.

2. Methodology

In Schaefer (2007), five luminosity relations of GRBs as follows are used:

$$\log \frac{L}{1\ \rm erg\ s^{-1}} = a_1 + b_1 \log \left[\frac{\tau_{\rm lag}(1+z)^{-1}}{0.1\ \rm s} \right] \quad (1)$$

$$\log \frac{L}{1\ \rm erg\ s^{-1}} = a_2 + b_2 \log \left[\frac{V(1+z)}{0.02} \right], \quad (2)$$

$$\log \frac{L}{1\ \rm erg\ s^{-1}} = a_3 + b_3 \log \left[\frac{E_{\rm peak}(1+z)}{300\ \rm keV} \right] \quad (3)$$

$$\log \frac{E_\gamma}{1\ \rm erg} = a_4 + b_4 \log \left[\frac{E_{\rm peak}(1+z)}{300\ \rm keV} \right] \quad (4)$$

$$\log \frac{L}{1\ \rm erg\ s^{-1}} = a_5 + b_5 \log \left[\frac{\tau_{\rm RT}(1+z)^{-1}}{0.1\ \rm s} \right] \quad (5)$$

where the luminosity L and the total collimation-corrected energy E_γ of GRBs are derived respectively from the bolometric peak flux $P_{\rm bolo}$ and the bolometric fluence $S_{\rm bolo}$ of GRBs through

$$L = 4\pi d_L^2 P_{\rm bolo}, \quad (6)$$
$$E_\gamma = E_{\gamma,\rm iso} F_{\rm beam} = 4\pi d_L^2 S_{\rm bolo}(1+z)^{-1} F_{\rm beam} \quad (7)$$

Here d_L is the luminosity distance, which depends on the cosmological model and is inversely proportional to the value of Hubble parameter of today. In this paper, we have adopted the flat ΛCDM model with $\Omega_m = 0.27$ and in the calculation, we actually replace d_L with $\bar{d}_L = \frac{H_0}{c} d_L \times 1\,{\rm cm}$ in Eq. (6) and Eq. (7), so that the dependence on Hubble constant is absorbed into the intercepts of the linear luminosity relations. For later convenience, we denote these luminosity relations by

$$y^{(i)} = c_0^{(i,\,i)} + c_1^{(i,\,i)} x^{(i)}, \quad (8)$$

where

$$x^{(1)} = \log \left[\frac{\tau_{\rm lag}(1+z)^{-1}}{0.1\ \rm s} \right], \quad (9)$$

$$x^{(2)} = \log \left[\frac{V(1+z)}{0.02} \right], \quad (10)$$

$$x^{(3)} = x^{(4)} = \log \left[\frac{E_{\rm peak}(1+z)}{300\ \rm keV} \right], \quad (11)$$

$$x^{(5)} = \log \left[\frac{\tau_{\rm RT}(1+z)^{-1}}{0.1\ \rm s} \right], \quad (12)$$

$$y^{(1)} = y^{(2)} = y^{(3)} = y^{(5)} = \log \frac{L}{1\ \rm erg\ s^{-1}} \quad (13)$$

$$y^{(4)} = \log \frac{E_\gamma}{1\ \rm erg}, \quad (14)$$

and

$$c_0^{(i,\,i)} = a_i, \quad c_1^{(i,\,i)} = b_i. \quad (15)$$

The coefficient c is given two superscripts to incorporate 3D correlations introduced just below. Hereafter, we denote a luminosity relation by the superscript pair of the corresponding c.

The above 2D luminosity relations connect measurable parameters of light curves and/or spectra with GRB luminosity/energy. They are empirical though there exist some explanations (see Schaefer (2007) and the references therein). Since the physical processes of GRBs are very complicated, the real correlations between GRB luminosity/energy and the parameters of light curves and/or spectra, generally speaking, should be more complicated than the above simple relations. The large scatters in the above luminosity relations also imply that there may be hidden parameters not included. Motivated by this, we extend the 2D luminosity relations to 3D and investigate if there are any improvements. See Eq. (11), Eq. (13), and Eq. (14), since $x^{(3)} = x^{(4)}$ and $y^{(4)}$ is different from $y^{(i)}$ ($i = 1, 2, 3, 5$), we introduce 3D correlations as follows. For (i, j) with both i and j in (1, 2, 3, 5) and $i < j$, we introduce 3D correlations as

$$y^{(i)} = c_0^{(i,\,j)} + c_1^{(i,\,j)} x^{(i)} + c_2^{(i,\,j)} x^{(j)}, \quad (16)$$

for the case of $(i, j) = (3, 4)$, the 3D correlation is defined as

$$y^{(3)} = c_0^{(3,\,4)} + c_1^{(3,\,4)} x^{(3)} + c_2^{(3,\,4)} y^{(4)}, \quad (17)$$

and for $(i, j) = (1, 4)$, $(2, 4)$, or $(4, 5)$

$$y^{(4)} = c_0^{(i, j)} + c_1^{(i, j)} x^{(i)} + c_2^{(i, j)} x^{(j)}. \qquad (18)$$

The luminosity relation $(3, 4)$ is different from other 3D luminosity relations in that there is only one luminosity indicator (i.e. E_{peak}) in it, while there are two in other 3D luminosity relations. The luminosity relations in Eq. (16) and those in Eq. (18) are different from each other in that the left-hand side of Eq. (16) is $\log \frac{L}{1 \text{ erg s}^{-1}}$, while the left-hand side of Eq. (18) is $\log \frac{E_\gamma}{1 \text{ erg}}$.

We investigate the quality of the luminosity relations mainly by comparing their intrinsic scatters. Since we are free to multiply the luminosity relations by a constant when introducing 3D correlations and such a factor would increase the intrinsic scatter by the same multiple as the factor, we need to normalize the relations in order to compare the intrinsic scatters. Bearing in mind that one of the most important purposes for exploring the luminosity relations is to use them in distance measurements, what we do here is just dividing Eq. (17) by a factor of $1 - c_2^{(3, 4)}$ so that $\log(d_L)$ have the same coefficient in all luminosity relations above.

Comparing the 3D luminosity relations with the 2Ds, as an example, comparing Eq. (16) with Eq. (8), one can see that, by introducing the 3D correlations in this way, we actually treat $x^{(j)}$ as the hidden parameter of the 2D correlation of Eq. (8) and write it explicitly in the 3D correlation of Eq. (16). So, in addition to examine the quality of the luminosity relations by comparing their intrinsic scatters, we also calculated the correlation coefficients between the residual of the fit of 2D correlations and the possible hidden parameters we introduced to extend correlations from 2D to 3D.

In the fit of the luminosity relations, we used the techniques presented in D'Agostini (2005), following which the likelihood function for the coefficients c and the intrinsic scatter is (for the cases of Eq. (16). The other cases of 3D luminosity relations are similar and the likelihood function for the 2D luminosity relations can be obtained just

by setting $c_2 = 0$)

$$\mathcal{L}(c, \sigma_{\text{int}}) \propto \prod_k \frac{1}{\sqrt{\sigma_{\text{int}}^2 + \sigma_{y_k^{(i)}}^2 + c_1^2 \sigma_{x_k^{(i)}}^2 + c_2^2 \sigma_{x_k^{(j)}}^2}}$$
$$\times \exp \left[-\frac{\left(y_k^{(i)} - c_0 - c_1 x_k^{(i)} - c_2 x_k^{(j)} \right)^2}{2 \left(\sigma_{\text{int}}^2 + \sigma_{y_k^{(i)}}^2 + c_1^2 \sigma_{x_k^{(i)}}^2 + c_2^2 \sigma_{x_k^{(j)}}^2 \right)} \right],$$
$$(19)$$

where k runs over GRBs with corresponding quantities available. In the calculation, Markov chain Monte Carlo techniques are used. For each luminosity relations, A Markov chain with samples of order 10^6 is generated according to the likelihood function and then properly burned in and thinned to derive statistics of interested parameters.

For the GRB data, we used the compilation in Schaefer (2007), which includes 69 GRBs. When considering error propagation from a quantity, say ξ with error σ_ξ, to its logarithm, we set $\frac{\log(\xi + \sigma_\xi^+) + \log(\xi - \sigma_\xi^-)}{2}$ and $\frac{\log(\xi + \sigma_\xi^+) - \log(\xi - \sigma_\xi^-)}{2}$ as the center value and the error of the logarithm correspondingly. This requires $\xi > \sigma_\xi^-$ (the quantities we are interested in here are all positive). Due to the limitation of the data, for a given luminosity relation (i, j), not all the GRBs have all of the needed observational quantities available and satisfy $\xi > \sigma_\xi^-$ at the same time. By set (i, j) we denote the maximum GRB set that can be used in the luminosity relation (i, j). The numbers of GRBs of different sets are presented in Table 1.

3. Results and discussion

We summarize our results in Tables 1 and 2. In Table 1, by comparing the intrinsic scatters of the 3D luminosity relations with those of the corresponding 2D luminosity relations, we find that only for the cases of $(1, 3)$ and $(3, 5)$ the intrinsic scatters of the 3D luminosity relations are considerably smaller than those of their corresponding 2D luminosity relations, and for all other cases, the intrinsic scatters of the 3D luminosity relations are either very close to the smaller one of the intrinsic scatters of the corresponding 2D luminosity relations (for correlations in Eq. (18), close to the intrinsic scatter of luminosity relation $(4, 4)$) or even greater than those of the corresponding 2D

(i, j)	1	2	3	4	5
1	32 $(-3.994^{+0.078}_{-0.077}, -0.79^{+0.11}_{-0.11})$ $0.404^{+0.067}_{-0.055}$	22 $(-3.96^{+0.10}_{-0.10}, -0.68^{+0.19}_{-0.18}, 0.51^{+0.38}_{-0.36})$ $0.414^{+0.087}_{-0.067}$ $[-0.01^{+0.09}_{-0.10}, 0.09^{+0.09}_{-0.10}]$	30 $(-4.013^{+0.059}_{-0.060}, -0.618^{+0.091}_{-0.091}, 0.80^{+0.16}_{-0.16})$ $0.279^{+0.052}_{-0.042}$ $[0.124^{+0.079}_{-0.076}, 0.142^{+0.064}_{-0.066}]$	13 $(-5.575^{+0.074}_{-0.083}, 0.08^{+0.14}_{-0.14}, 1.44^{+0.17}_{-0.18})$ $0.15^{+0.11}_{-0.08}$ $[0.01^{+0.10}_{-0.11}]$	31 $(-3.952^{+0.081}_{-0.081}, -0.60^{+0.14}_{-0.14}, -0.31^{+0.17}_{-0.17})$ $0.344^{+0.063}_{-0.051}$ $[0.059^{+0.084}_{-0.083}, 0.111^{+0.072}_{-0.076}]$
2	-	44 $(-3.712^{+0.080}_{-0.081}, 1.02^{+0.23}_{-0.22})$ $0.508^{+0.066}_{-0.055}$	42 $(-3.875^{+0.067}_{-0.067}, 0.65^{+0.19}_{-0.19}, 1.09^{+0.19}_{-0.19})$ $0.366^{+0.051}_{-0.043}$ $[0.141^{+0.078}_{-0.075}, 0.055^{+0.064}_{-0.065}]$	22 $(-5.652^{+0.052}_{-0.057}, 0.15^{+0.23}_{-0.22}, 1.56^{+0.15}_{-0.15})$ $0.178^{+0.064}_{-0.051}$ $[-0.020^{+0.075}_{-0.078}]$	41 $(-3.702^{+0.071}_{-0.071}, 0.46^{+0.22}_{-0.23}, -0.65^{+0.16}_{-0.16})$ $0.432^{+0.060}_{-0.050}$ $[0.075^{+0.082}_{-0.081}, 0.023^{+0.071}_{-0.074}]$
3	-	-	63 $(-3.999^{+0.058}_{-0.058}, 1.37^{+0.12}_{-0.12})$ $0.422^{+0.048}_{-0.041}$	27 $(-3.3^{+1.6}_{-1.6}, 0.91^{+0.48}_{-0.43}, 0.12^{+0.27}_{-0.29})$ $0.50^{+0.26}_{-0.14}$ $[-0.08^{+0.14}_{-0.26}, -0.34^{+0.15}_{-0.26}]$	56 $(-3.852^{+0.048}_{-0.048}, 0.90^{+0.11}_{-0.11}, -0.624^{+0.089}_{-0.090})$ $0.293^{+0.037}_{-0.032}$ $[0.128^{+0.057}_{-0.055}, 0.162^{+0.060}_{-0.057}]$
4	-	-	-	27 $(-5.626^{+0.044}_{-0.047}, 1.51^{+0.11}_{-0.11})$ $0.159^{+0.054}_{-0.046}$	23 $(-5.661^{+0.055}_{-0.059}, 1.56^{+0.15}_{-0.14}, 0.00^{+0.11}_{-0.12})$ $0.176^{+0.064}_{-0.052}$ $[-0.017^{+0.075}_{-0.078}]$
5	-	-	-	-	61 $(-3.766^{+0.065}_{-0.065}, -0.88^{+0.11}_{-0.11})$ $0.456^{+0.051}_{-0.044}$

Table 1: Fit of 2D and 3D luminosity relations. In every grid, the first row is the number of GRBs of set (i, j), the vector below enclosed by parentheses is the vector of c for the luminosity relation (i, j), and what follows next is the intrinsic scatter. For 3D luminosity relations, their reduction in the intrinsic scatters compared to corresponding 2D luminosity relations are presented in the brackets: for 3D luminosity relations in Eq. (16) and Eq. (17), the reduction corresponds to the 2D luminosity relation (i, i) and (j, j) in turn and for those in Eq. (18), corresponds to $(4, 4)$. The statistics in the table are for the median values and the errors of 1σ (68.3%) confidence level.

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(5)}$	$y^{(3)}$	$y^{(4)}$
(1, 1)	-	$0.31^{+0.12}_{-0.14}$	$0.712^{+0.053}_{-0.077}$	$-0.18^{+0.15}_{-0.14}$	-	-
(2, 2)	$-0.647^{+0.093}_{-0.069}$	-	$0.689^{+0.042}_{-0.060}$	$-0.474^{+0.097}_{-0.081}$	-	-
(3, 3)	$-0.732^{+0.046}_{-0.040}$	$0.472^{+0.038}_{-0.044}$	-	$-0.634^{+0.050}_{-0.044}$	-	$-0.21^{+0.10}_{-0.09}$
(4, 4)	$0.223^{+0.008}_{-0.012}$	$0.380^{+0.014}_{-0.020}$		$-0.007^{+0.037}_{-0.037}$	$-0.09^{+0.11}_{-0.11}$	-
(5, 5)	$-0.36^{+0.10}_{-0.10}$	$0.185^{+0.087}_{-0.089}$	$0.740^{+0.034}_{-0.045}$	-	-	-

Table 2: Correlation coefficients between the residuals of the fit of 2D correlations and possible hidden parameters. The statistics in the table are for the median values and the errors of 1σ (68.3%) confidence level.

luminosity relations (for the case (3, 4)). The correlations presented in Table 2 also give the same implication. Only the correlation coefficients between the residual of fit of (1, 1) and $x^{(3)}$, (3, 3) and $x^{(1)}$, (5, 5) and $x^{(3)}$ are approximately greater than 0.7. We presented the plots of the luminosity relation (1, 3) and (3, 5) in Figure 1.
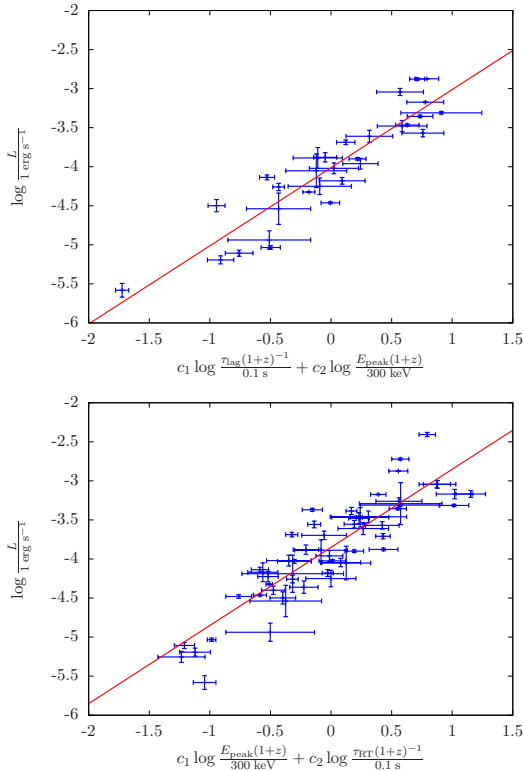

Fig. 1.— Plots of the 3D luminosity relation (1, 3) and (3, 5).

It is interesting to note that the 3D luminosity relations that have considerable improvement in the intrinsic scatter are the luminosity relations in Eq. (16), with one of the luminosity indicators being an energy scale (E_{peak}) and the other a time scale (τ_{lag} or τ_{RT}). Enlightened by this, one may naturally guess that probably the luminosities of GRBs are mainly determined by a characteristic energy scale and a characteristic time scale and E_{peak} is correlated to the characteristic energy scale, τ_{lag} and τ_{RT} to the characteristic time scale, so that 3D luminosity relations with E_{peak} and τ_{lag} or τ_{RT} as luminosity indicators could significantly reduce the intrinsic scatters compared to corre-

sponding 2D luminosity relations. This is easy to understand, since the definition of the luminosity is the amount of the energy released in units of time. If one knows the released energy and the time duration of a GRB, the (averaged) luminosity can be calculated immediately. Obviously, the 2D correlation between the luminosity and the energy or the time duration would not be complete unless the time duration or the energy is constant for all samples or there is a strong correlation between the energy and the time duration. This, if proven true, also explains why the 2D luminosity relations (3, 3) and (4, 4) have little correlation with each other despite their similarity at first sight (see discussion in Section 4.7 of Schaefer (2007); the correlation coefficient between the residual of fit of (3, 3) and $y^{(4)}$ and the correlation coefficient between the residual of fit of (4, 4) and $y^{(3)}$ also show their weak correlation). This is because of the difference between E_γ and L, where a new independent variable—the characteristic time scale—enters. In fact, we could check the guess partially with current data by examining the correlation between $x^{(i)}$s. In Table 3, one can find that $x^{(1)}$, $x^{(2)}$, and $x^{(5)}$ are strongly correlated with each other, while all of them have only weak correlation with $x^{(3)}$ (see also Schaefer et al. (2001) and Section 4.5 of Schaefer (2007) for discussions on the correlations between the luminosity indicators). This is consistent with the guess that the luminosity indicators τ_{lag} and τ_{RT} are correlated with a characteristic time scale and E_{peak} is correlated with a characteristic energy scale which is independent of the characteristic time scale. In addition, the correlations presented in Table 3 seems to imply that V is also correlated with the characteristic time scale. Accordingly, there is indeed some reduction in the intrinsic scatter for the 3D luminosity relation (2, 3) compared with corresponding 2D luminosity relations, but such reduction is relatively smaller than that of (1, 3) and (3, 5). Maybe the correlation of V with the characteristic time scale is not so strong as that of τ_{lag} or τ_{RT}.

If our guess about the GRB luminosity relations is correct, it would be very enlightening. It suggests us to include an energy scale and a corresponding time scale for seeking more precise luminosity relations for GRBs in the future. However, it should be emphasized that, even if it is true, only appropriate energy and time

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(5)}$
$x^{(1)}$	1.0	-0.73	-0.35	0.72
$x^{(2)}$	-0.73	1.0	0.41	-0.63
$x^{(3)}$	-0.35	0.41	1.0	-0.30
$x^{(5)}$	0.72	-0.63	-0.30	1.0

Table 3: Correlation coefficients between $x^{(1)}$, $x^{(2)}$, $x^{(3)}$, and $x^{(5)}$.

scales might significantly reduce the intrinsic scatter bearing in mind the situation of some 3D luminosity relations (see, for example, Firmani et al. 2006; Rossi et al. 2008; Collazzi & Schaefer 2008).

4. Summary

In this paper, we extend the widely used 2D luminosity relations to 3D by using the same set of luminosity indicators, i.e. τ_{lag}, V, E_{peak}, and τ_{RT}, and check the improvement in the quality of the luminosity relations. We find that, for the 3D luminosity relations between the luminosity and an energy scale (E_{peak}) and a time scale (τ_{lag} or τ_{RT}), their intrinsic scatters are considerably smaller than those of corresponding 2D luminosity relations. The correlations between the residuals of fit of the 2D luminosity relations and the luminosity indicators also give the same implication. Enlightened by the result and the definition of the luminosity (energy released in units of time), we discussed possible reasons behind, which may give us helpful suggestions on seeking more precise luminosity relations for GRBs in the future.

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