Gang rivalry dynamics via coupled point process networks

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Abstract

We introduce a point process model for inter-gang violence driven by retaliatory behavior. For this purpose we use a coupled system of state-dependent jump stochastic differential equations to model the conditional intensities of the directed network of gang rivalries. The system admits an exact simulation strategy based upon Poisson thinning. The model produces a wide variety of transient or stationary weighted network configurations and in certain parameter regimes rivalry interdependency is shown to have surprising consequences for policing strategies. We investigate the stability of gang rivalries in Los Angeles by fitting the model to gang violence data provided by the Hollenbeck district of the Los Angeles Police Department.

Key words: point process, network, jump stochastic differential equation

1 Introduction

The behavioral basis of clustering in crime data has recently been investigated using reaction-diffusion PDEs [6,10,12–14], agent based models [2,6,13], and point processes [2,7,15]. In the case of burglary, an initial crime has been shown to increase the likelihood of more crime at the same location, as well as neighboring houses within a few hundred meters [3,5,7,15]. In [13] this phenomenon is modeled using the coupled PDE system,

$$\frac{\partial A}{\partial t} = D\nabla^2 A - A + A_0 + \rho A, \tag{1}$$

$$\frac{\partial \rho}{\partial t} = \vec{\nabla} \cdot \left[\vec{\nabla}\rho - \frac{2\rho}{A}\vec{\nabla}A\right] - \rho A + \overline{A} - A_0\ , \tag{2}$$

where A denotes target attractiveness and ρ denotes criminal density. Here criminals diffuse in their urban environment, biased towards areas of high attractiveness. As they commit crimes the local attractiveness is increased and macroscopic crime "hotspots" can form, where a majority of the total crime is localized in a few small areas. The stability of crime hotspots implies important consequences for policing strategies [6,10,14], as some hotspots can be suppressed while others can only be relocated.

In the context of gang violence, "hotspots" can form within the inter-gang rivalry network, localized within a small subset of the rivalry links between gangs. In [2], temporal clustering in inter-gang violence data from the Hollenbeck district in Los Angeles is modeled using self-exciting point processes. In particular, a Hawkes Process with conditional intensity,

$$\lambda(t) = \mu + \theta \sum_{t>t_i} \omega e^{-\omega(t-t_i)}, \tag{3}$$

is used to model the increase in the rate of crime between gangs due to retaliation. Here crimes occur according to a Poisson process with intensity μ, representing attacks that occur by random chance. The overall rate λ is increased as each event t_i occurs, reflecting the propensity of gangs to respond to previous attacks. This leads to clusters of gang violence, with intermittent periods of inactivity due to the exponential decay of the second term in (3). Similar models are used to explain clustering patterns of earthquake aftershocks [8].

It is our goal here to capture excitation and inhibition in gang rivalry networks within the point process modeling framework. For example, if two gangs are focused on retaliating against each other, they may be significantly less likely to attack a third gang in the short term. When the ability of a particular gang to attack is suppressed through police intervention, new rivalries may form between the former rivals of the disrupted gang. To capture these history dependent relationships in gang rivalry networks we propose a coupled system of state-dependent jump stochastic differential equations in Section 2. The solution to the system defines the conditional intensity of a marked point process, characterized by events between two gangs exciting that particular rivalry, while inhibiting others. In Section 3, we outline an exact simulation strategy for the model. We then present numerical examples of the variety of transient and stationary rivalry networks that can form in Section 4. We show that in certain parameter regimes rivalry interdependency can lead to surprising consequences for policing strategies. We also investigate the stability of gang rivalries in Los Angeles by fitting the model to gang violence data provided by the Hollenbeck district of the Los Angeles Police Department.

2 Description of the Model

We consider a network of M gangs labeled by $i = 1, 2, ..., M$ and model the distribution of marked event times when gang i attacks gang j with a counting measure $N_{ij}(t)$. Letting t_{ij}^k denote the time of the kth event where i attacks j, the counting measure can be interpreted through delta spikes located at the marked event times,

$$dN_{ij}(t) = \sum_{k=1}^{\infty} \delta(t - t_{ij}^k)dt \tag{4}$$

or through its right continuous conditional intensity [1] $\lambda_{ij}(t)$, the rate of events when i attacks j (analogous to the rate of a Poisson process). The dependence on time reflects the fact that the intensity of the rivalry will be changing as events occur. We assume that the rivalry intensity can be written as a sum,

$$\lambda_{ij}(t) = \mu + g_{ij}(t). \tag{5}$$

Here μ (independent of time) represents the baseline rate of attacks from gang i towards gang j, as some gang crimes occur by random encounters between gangs. The next term, $g_{ij}(t)$, represents the change in rivalry intensity due to past crimes where gang j has attacked gang i (excitation) or gang k has attacked gang i (inhibition). The dynamic portion of the overall intensity evolves according to the following nonlinear stochastic differential equation,

$$dg_{ij}(t) = -\omega g_{ij}(t)dt - \chi g_{ij}(t_-) \sum_{k \neq j,i} dN_{ki}(t) + f(\lambda_{ij}(t_-))dN_{ji}(t), \tag{6}$$

where $g_{ij}(t_-) = \lim_{s \uparrow t} g_{ij}(s)$. In this equation the first term represents the decay in rivalry strength back to the baseline rate in the absence of violence, as gangs eventually forget past attacks. The second term represents the decrease in the likelihood of gang i attacking gang j when gang k attacks i. The last term represents the increase in the rate of i attacking j after j attacks i (in other words i wants to retaliate when attacked by j). For the state-dependent response to attacks we consider the function $f(x) = \theta e^{-\nu x}$, where for low values of the intensity the response is greater than for high values, as we assume gangs have finite resources and thus a bound on the rate, λ_{ij}, at which they can attack.

3 Simulation Methodology

In this section we describe how the solution to Equation (6) can be sampled exactly, up to the error due to round off and uniform random number generation. Because the intensity in (6) is non-increasing in between events, Poisson thinning can be used to compute the event times [9]. Since the intensity decays exponentially in between events, no numerical discretization is necessary. The overall simulation strategy is as follows. Given the solution $\lambda_{ij}(t)$ to (6) and event times $t^1, ..., t^n < s$ have been computed up to time s:

(1) Set $\beta = \sum_{i \neq j} \lambda_{ij}(s)$.

(2) Sample candidate event time $\tilde{t}$ according to a Poisson process with parameter β: $\tilde{t} = s - \log(u)/\beta$ where $u \sim U[0, 1]$.

(3) Compute $\lambda_{ij}(\tilde{t}_-) = \mu + g_{ij}(s) \exp(-\omega(\tilde{t} - s))$. Accept event with probability $\sum_{i \neq j} \lambda_{ij}(\tilde{t}_-)/\beta$ and go to (4) otherwise set $s = \tilde{t}$, $\lambda_{ij}(s) = \lambda_{ij}(\tilde{t}_-)$, and go to (1).

(4) Set $s, t^{n+1} = \tilde{t}$. Choose directed rivalry $l_1 \rightarrow l_2$ as the mark for event time t^{n+1} with probability $\lambda_{l_1 l_2}(s_-)/\sum_{i \neq j} \lambda_{ij}(s_-)$.

(5) Update intensities:
$\lambda_{ij}(s) = \lambda_{ij}(s_-) - \chi g_{ij}(s_-) \sum_{k \neq j,i} 1_{\{k=l_1, i=l_2\}} + f(\lambda_{ij}(s_-)) 1_{\{j=l_1, i=l_2\}}$
where 1 denotes the indicator function. Go to (1).

We note that for models where the differential equation is not analytically integrable, or when other sources of stochasticity are included through a Brownian force, thinning can still be used to simulate the process. In this case a standard numerical method for the approximation of deterministic or stochastic differential equations is used to evolve the state variable between events [4].

4 Results

4.1 Network Classification

The solutions to Equation (6) exhibit either transient or stationary networks depending on the parameter regime. Transient rivalry networks are characterized by either short bursts of event clusters between gang pairs that quickly subside and are replaced by new rivalries (Figure 1, top left) or by event clusters that are longer in duration and appear stable over intermediate timescales

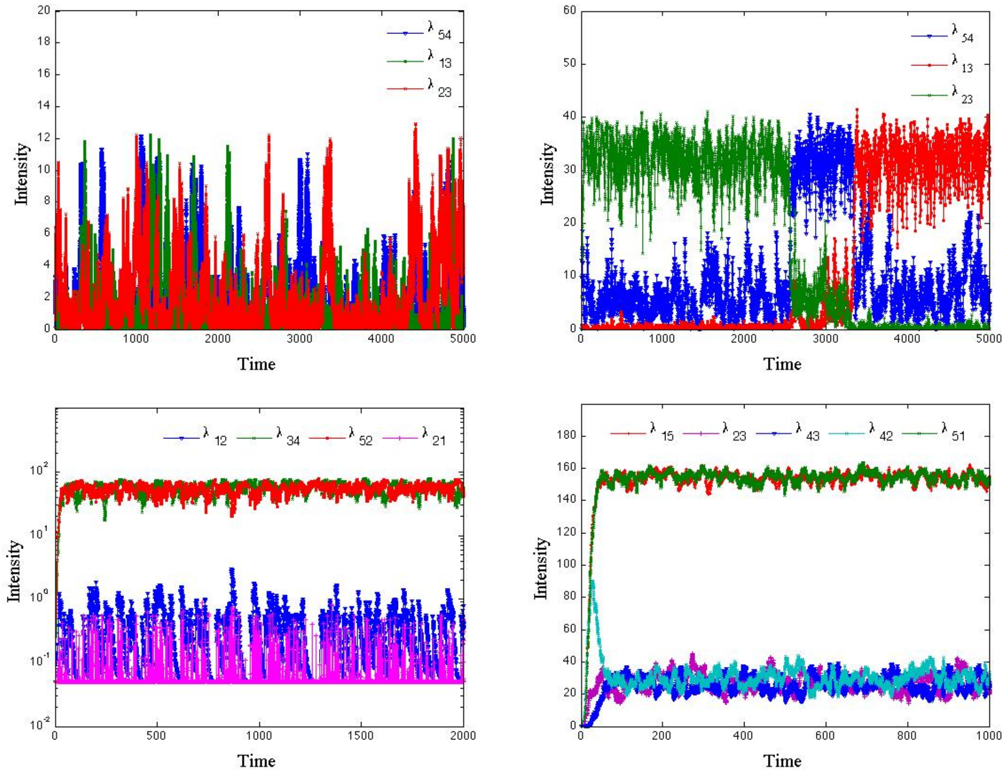

Fig. 1. Network configurations for 5 gangs. **Top Left**: Transient gang rivalries with $\mu = .1$, $\omega = .5$, $\chi = .1$, $\theta = .8$, $\nu = .05$. **Top Right**: Transient gang rivalries with $\mu = .1$, $\omega = .1$, $\chi = .1$, $\theta = .8$, $\nu = .05$. **Bottom Left**: Stationary gang rivalries with $\mu = .05$, $\omega = .1$, $\chi = .15$, $\theta = .5$, $\nu = .02$ characterized by two sets of two gangs rivalries corresponding to the highest intensity and an intermediate intensity corresponding to attacks between the first gang and each of the other four gangs. The first gang attacks several times more than is attacked. **Bottom Right**: Stationary gang rivalries with $\mu = .1$, $\omega = .1$, $\chi = .01$, $\theta = .5$, $\nu = .01$ characterized by one two gang rivalry (highest intensity) and one three gang rivalry (intermediate intensity).

(Figure 1, top right). As χ is increased and ω, μ are decreased relative to θ, a phase transition occurs and stationary rivalries form between subsets of two or more gangs. For example, plotted in Figure 1 are two possible stationary network configurations for 5 rival gangs. The first network (Figure 1, bottom left) is characterized by four rivalry intensity levels, the highest corresponding to two sets of two gang rivalries (between gangs 2 and 5, 3 and 4), the next highest corresponding to gang 1 attacking each of gangs 2 through 5, the third corresponding to gangs 2 through 5 attacking gang 1, and the lowest intensity level, slightly above the baseline rate μ, corresponds to all other rivalries. The second network (Figure 1, bottom right) is characterized by one set of two gangs (1 and 5) with a high intensity level and one set of three gangs (2 through 4) with an intermediate intensity level, where all other intensities are close to the baseline rate. For certain parameter values more than one equilibrium state is possible, for example two sets of 3-gang rivalries or three sets of 2-gang rivalries are possible network topologies in the case of a 6 gang network. The variety of possible equilibrium configurations is due to the high degree of nonlinearity in Equation (6).

4.2 Policing Strategies

Reducing gang violence is a high priority in many police departments in large cities and proactive methods include directed patrols and gang injunctions. The point process model introduced in Section 2 offers a platform on which policing strategies can be tested and optimized for reducing overall violence. As an example, we consider a 7 gang network of rivalries initially characterized by one set of 3-gang rivalries and two sets of 2-gang rivalries. We first investigate a policing strategy characterized by focusing all attention on gang 7, the gang with the highest intensity of crime. We assume that through directed patrols or injunctions all crime can be suppressed associated with this gang, i.e. $\lambda_{7k} = 0$ for all k. In the top of Figure 2 we plot the results of such a strategy. Around time 325 the directed patrols begin and initially the overall rate of crime is reduced (top right). However, gang 6 disrupts the 3-gang set leading to the formation of 3 stable 2-gang rivalries, which actually has a higher rate of crime compared to the original network of 7 gangs. Thus this type of policing strategy actually leads to *more* crime. A more dynamic strategy is displayed in the bottom of Figure 2, where every seven time units the directed patrols switch their focus to whichever gang currently is associated with the highest intensity. The result of such a strategy is that all rivalries become transient and the overall rate of crime is reduced (bottom right).

4.3 Hollenbeck Gang Rivalries

In the Hollenbeck district of Los Angeles, approximately 33 gangs reside in a 5km by 3km region. Some of the rivalries date back decades, while others are more transient in nature. The complex rivalry network has been examined in [2,11] and the degree of each node of the network ranges from a few to over 10 rivalry connections, where links can be defined either through surveys of gang members and police officers or through violent crime data where the suspect and victim gang are known (we use the latter in the analysis that follows).

Here we use Equation (6) to investigate the stability of gang rivalries in Hollenbeck over the time period 1999 to 2002. The data set we consider consists of 349 Part one violent crimes committed over a 1044 day span between 1999 and 2002 by one of 33 gangs in Hollenbeck against another one of the 33 gangs. Each event includes the time (in days past the start of the time window),

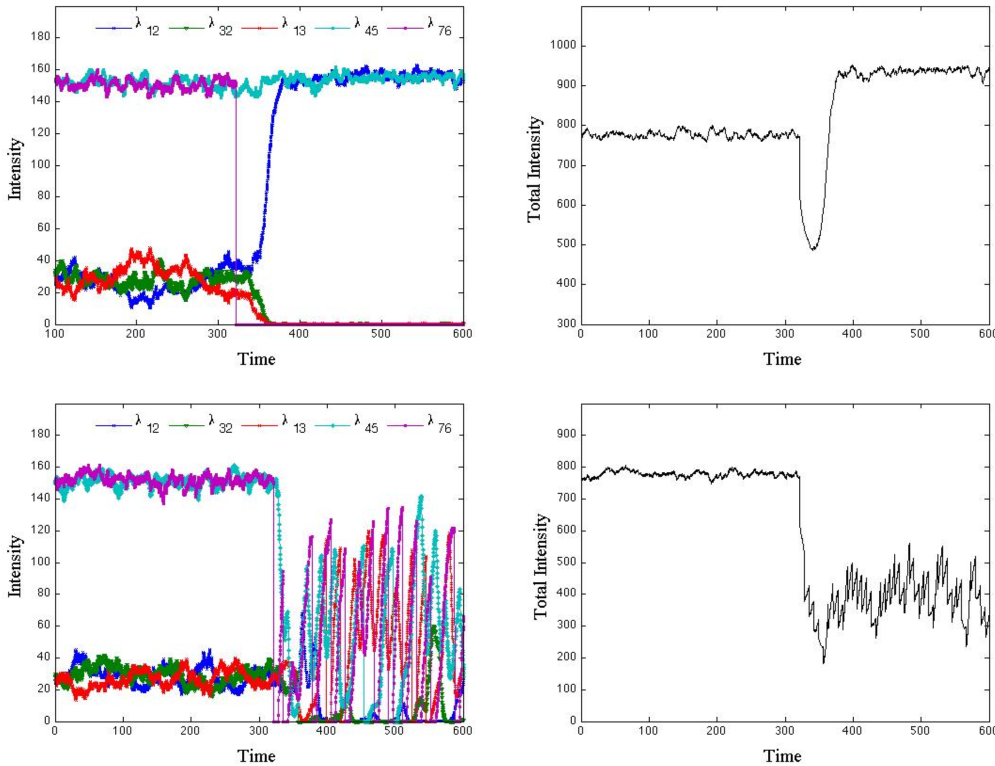

Fig. 2. Top: Static policing strategy. 7 gang rivalry network intensity (left) and total intensity (right) before and after suppression. Bottom: Dynamic policing strategy for a different realization of the 7 gang network. Parameter values are $\mu = .1$, $\omega = .1$, $\chi = .01$, $\theta = .5$, $\nu = .01$

geocoded spatial location, suspected gang, and victim gang (in this study we ignore the spatial information).

We fit Equation (6) using Maximum Likelihood Estimation, where the log-likelihood function [1] is given by,

$$(\hat{\mu}, \hat{\omega}, \hat{\chi}, \hat{\theta}, \hat{\nu}) = \underset{(\mu,\omega,\chi,\theta,\nu)}{\operatorname{argmax}} \left\{ \sum_{i \neq j} \left(\sum_{k=1}^{K_{ij}} \log\left(\lambda_{ij}(t_{ij}^k)\right) - \int_0^T \lambda_{ij}(t)dt \right) \right\}, \tag{7}$$

and K_{ij} is the total number of events in the data set where i attacked j. The first term in the log-likelihood function leads to the selection of parameter values for which more probability mass is placed at the event times and the second term forces the intensity to (approximately) integrate to the total number of events in the data set. The negative log-likelihood function is minimized using the built in Matlab routine 'fminsearch' and we use the homogeneous initial condition $\lambda_{ij}(0) = 349/(33 \cdot 32 \cdot 1044)$ for the starting value of each rivalry intensity.

Maximum Likelihood Estimation yields the parameter estimates $\hat{\mu} = .000124$,

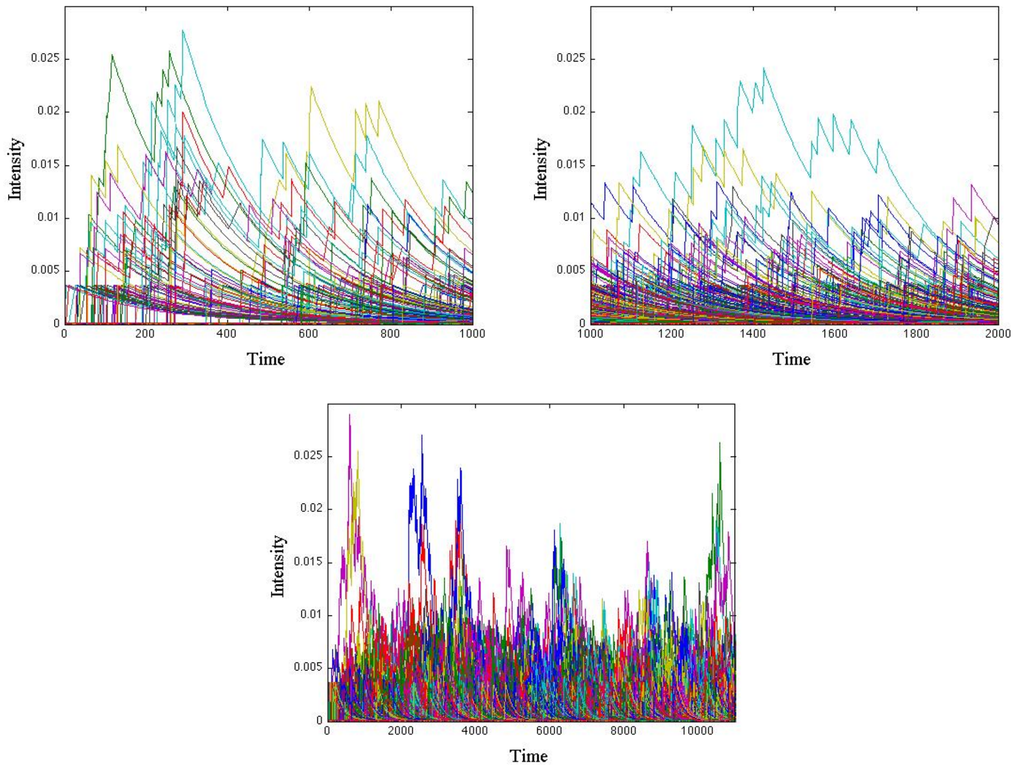

Fig. 3. Top: Observed (left) and simulated (right) fitted conditional intensities for the 33×32 directed gang rivalries in Hollenbeck over 1,000 days. Bottom: Simulated conditional intensities of directed gang rivalries over 11,000 days.

$\hat{\omega} = .00491$, $\hat{\chi} = .0164$, $\hat{\theta} = .00356$, and $\hat{\nu} = .0193$. The baseline rate μ corresponds to an approximate rate of 1 event per 10,000 days, thus an unprovoked attack between any rivalry pair is a relatively rare event. However, the intensity increases by a factor of (approx.) 30 after an initial attack, leading to the formation of either transient or stationary hotspots in the rivalry network. There is also a significant inhibitory effect through the rivalry network due to occurrence of an attack, reflected in the estimate of χ.

In the top of Figure 3 we plot the fitted conditional intensity over 1,000 days using the observed 349 events (top left) and simulated events (top right). The point process is characterized by the majority of the 33×32 directed rivalries having intensities close to the baseline rate, though the strongest rivalries reach a rate as high as .025 events per day. A simulation for a longer time period, 11,000 days, is plotted in the bottom of Figure 3. Here we observe that the estimated parameter values lead to transient rivalries similar to those plotted in the top left of Figure 1. It is also interesting to note that maximum of the intensity fluctuates between .01 and .03, and thus, according to the model, the observed Hollenbeck rivalries were at a high point during 1999-2002. These results indicate that crime hotspots in Hollenbeck are produced by stochastic fluctuations, rather than by static rivalry relations.

5 Conclusion

We developed a point process model for the simulation of gang rivalry networks, paying close attention to the network dynamics that emerge due to excitation and inhibition. The model is capable of reproducing hotspots similar to those observed in gang violence data and is amenable to fast simulation and parameter estimation. We illustrated how the model can be used to test policing strategies before they are implemented in the field. This could be important since the complex, nonlinear network of gang rivalries may respond in counter-intuitive ways to a given strategy. By fitting the model to gang violence data, we explored the stability of gang rivalries in Hollenbeck, Los Angeles through numerical simulation. Our results indicate that rivalries between the 33 gangs in that district are transient as opposed to stationary.

Acknowledgements

This work was supported in part by ARO grant 58344-MA and NSF grant DMS-0968309. The authors would also like to thank Tawny Lim and Anthony DeCino for helpful discussions and George Tita and the LAPD for providing the data used in this study.

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