Gantry-Tau – A New Three Degrees of Freedom Parallel Kinematic Robot

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Abstract

In the last decades, an increasing attention has been paid to the study of different parallel structure mechanisms and their applications, mainly triggered by Stewart that presented an aircraft simulator system. Parallel structure features provide big advantages in potential applications. For example, parallel robots may give higher speed and acceleration, higher static and dynamic accuracy and higher stiffness than what is possible with the industrial robots used today.

A typical limitation with many of the parallel structures is that their workspace is small compared to the serial structures. This paper presents a new parallel structure, the Gantry-Tau, which provides 3 degrees of freedom (DOF) translational motion with a large workspace. The structure of the robot is patented by ABB. The Gantry-Tau robot is a six link parallel kinematic structure with the links configured according to 3-2-1. The 3-2-1 notation refers to how many links form each resulting kinematic cluster of the robot. Orientational DOF of the robot could be provided by a decoupled system.

For a conventional 3 DOF serial gantry robot two of the actuators contribute to the moving mass. The Gantry-Tau can be constructed with exceptionally low moving mass since the actuators are stationary and the structure has inherently high stiffness. The structure is thus ideal for many applications with demands on high accelerations, for instance for the pick and place operations.

The nominal inverse and direct kinematics of the structure are developed and optimization is used to find a construction of Gantry-Tau with maximum workspace volume.

1 Introduction

Most of the robots used in the industry are serial manipulators. A serial manipulator has an open kinematic chain structure. This type of robots offers high generality and can be used for various applications. However the serial manipulators suffer from a low ratio between load capacity and robot mass. The main reasons for this are that the robot's actuators contribute to the moving mass and that each link is subjected to the weight of the following links. Thus the links have to be dimensioned with respect to large flexure torques, which means that the structure has to be stiffened, and thus become heavier. Accuracy is limited by the fact that the links magnify errors throughout the chain. For instance a small angular error in a revolute joint early in the chain will induce a large error for the tool center point (*TCP*).

A parallel manipulator is a closed kinematic chain mechanism. There exist a variety of architectures designed for different applications. The parallel manipulator can be characterized with comparison to it serial counterpart as a system with [4]:

- higher ratio between load capacity and robot mass,
- higher stiffness,
- higher absolute accuracy,
- simpler inverse kinematics,
- more difficult direct kinematics,
- smaller workspace.

The higher ratio between load capacity and robot mass is due to that the actuators often are located on a fixed platform and for many of the structures the links are only subjected to axial forces and that the load is distributed over the chains. Higher stiffness is due to that the external force is distributed over the chains. Higher absolute accuracy is due to non cumulative joint error and the higher stiffness. The inverse kinematics problem is often solved easily since the chains can be studied separately and that different configurations are generally fixed in the design process. The solution of the direct kinematics problem is often difficult since in the general case there is no unique solution. The constant orientation workspace is also often limited for most 6 DOF fully parallel manipulators. One approach to get better workspace properties is to develop manipulators where the translational degrees of freedom are separated from the rotational degrees of freedom or to design manipulators that are not fully parallel.

This paper presents a new parallel structure, the Gantry-Tau, which provides 3 DOF translational motions with a large workspace. The structure of the robot is patented by ABB [2] and early results indicate that the structure could outperform the serial gantry structure for many applications.

2 Kinematic description

The Gantry-Tau is a six link parallel kinematic structure with the links configured according to 3-2-1. The 3-2-1 notation refers to how many links form each resulting kinematic arm. Gantry-Tau belongs to the PRRS family of parallel manipulators with the HexaGlide as one of its closest relative [4]. The PRRS notation describes the joints in the kinematic chains from actuation to the *TCP*. Thus each chain is formed by a prismatic joint with actuation (P), a universal joint (RR), and finally a spherical joint which connects to the moving plate. These 6 chains form three kinematic clusters where the chains are organised as a double parallelogram, a single parallelogram and a single link which all connect to the moving plate. The prismatic joints are three parallel linear tracks.

Figure 1 shows a schematic for the Gantry-Tau structure. By moving *A*, *B* and *C* along the tracks from $S_{A,B,C,0}$ to $S_{A,B,C,1}$ the translational motion is controlled for the *TCP* while the orientation of the moving plate is maintained.

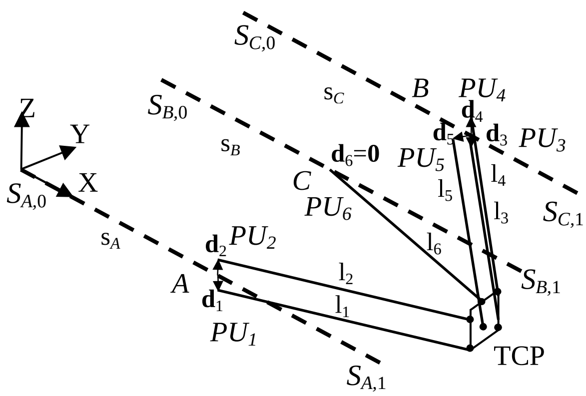

Fig: 1. Schematic Gantry-Tau. Global coordinate system is defined with the X-axis along the direction from $S_{A,0}$ to $S_{A,1}$. The black dots represent spherical joints.

The vectors $\mathbf{d_i}$ i=1-6 define the locations for the universal joints PU_i i=1-6 from points *A*, *B* and *C* (figure 1). The vectors $\mathbf{n_i}$ i=1-6 define the relative locations for the *TCP* with respect to the spherical joints PS_i i=1-6 (figure 2).

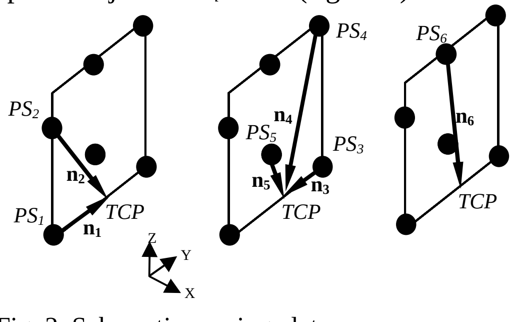

Fig: 2. Schematic moving plate.

The length of the links l_i must be the same for links belonging to the same cluster. The vectors $\mathbf{d_i}$ i=1-5 and $\mathbf{n_i}$ i=1-5 are prerequisite to fulfill the condition that the vector between PS_1 and PS_2 must be parallel to the vector between PU_1 and PU_2, and that the vector between PS_3 and PS_4 must be parallel to the vector between PU_3 and PU_4, and that the vector between PS_4 and PS_5 must be parallel to the vector between PU_4 and PU_5. Another perhaps obvious prerequisite is that PS_5 must be located outside the plane $PS_3PU_3PS_4$.

Spherical joints, allowing the links to spin around their principal axis, can of course replace the universal joints. For some applications it might be favourable to use only universal joints. This can be achieved by adding a revolute joint on each link that prevents the structure from being over constrained. A 4 DOF endless tool orientation arrangement can be achieved by adding a double cardan axis as shown in figure 3. This decoupled arrangement is used in the Delta robot design [3]. Another variant of the Gantry-Tau is shown in figure 4. This arrangement offers 5 DOF limited tool tilt and could be used for water jet cutting, plasma cutting and laser cutting.

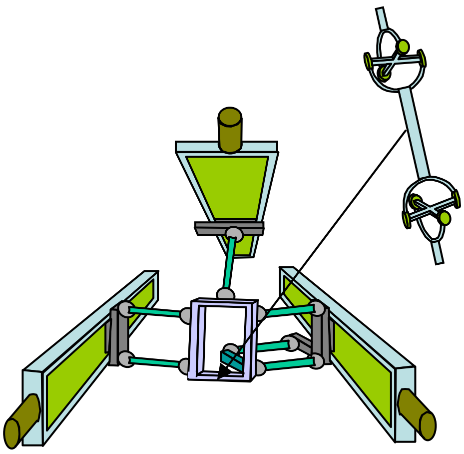

Fig: 3. 4 DOF Endless tool rotation.

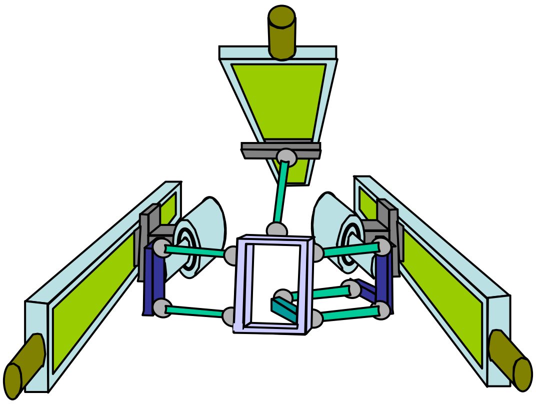

Fig:4. 5 DOF Tool tilt.

2.1 Inverse kinematics

For the considered parallel robot the inverse kinematics problem is formulated as follow. Calculate the location of points A, B and C along the linear tracks for a given TCP location. Let

$$A = (s_a \quad 0 \quad 0)^T + S_{A,0},$$
$$B = (s_b \quad 0 \quad 0)^T + S_{B,0},$$
$$C = (s_c \quad 0 \quad 0)^T + S_{C,0},$$
$$TCP = (x \quad y \quad z)^T.$$

Here the parameters s_a, s_b and s_c are to be determined and can be found as the intersection between spheres with midpoints at $TCP - \mathbf{d_1} - \mathbf{n_1}$, $TCP - \mathbf{d}_3 - \mathbf{n}_3$ and $TCP - \mathbf{d}_6 - \mathbf{n}_6$ and the respective linear track. The spherical equations can be written as follows:

$$(S_{A,0,x} + s_a - x + d_{1,x} + n_{1,x})^2 + (S_{A,0,y} - y + d_{1,y} + n_{1,y})^2$$
$$+ (S_{A,0,z} - z + d_{1,z} + n_{1,z})^2 = l_1^2$$

$$(S_{B,0,x} + s_b - x + d_{3,x} + n_{3,x})^2 + (S_{B,0,y} - y + d_{3,y} + n_{3,y})^2$$
$$+ (S_{C,0,z} - z + d_{3,z} + n_{3,z})^2 = l_3^2$$

$$(S_{C,0,x} + s_c - x + d_{6,x} + n_{6,x})^2 + (S_{C,0,y} - y + d_{6,y} + n_{6,y})^2$$
$$+ (S_{C,0,z} - z + d_{6,z} + n_{6,z})^2 = l_6^2$$

Then we can determine the parameters

$$s_a = x - d_{1,x} - n_{1,x} - S_{A,0,x} \pm \sqrt{l_1^2 - \left(S_{A,0,y} - y + d_{1,y} + n_{1,y}\right)^2 - \left(S_{A,0,z} - z + d_{1,z} + n_{1,z}\right)^2}$$

$$s_b = x - d_{3,x} - n_{3,x} - S_{B,0,x} \pm \sqrt{l_3^2 - \left(S_{B,0,y} - y + d_{3,y} + n_{3,y}\right)^2 - \left(S_{B,0,z} - z + d_{3,z} + n_{3,z}\right)^2}$$

$$s_c = x - d_{6,x} - n_{6,x} - S_{C,0,x} \pm \sqrt{l_6^2 - \left(S_{C,0,y} - y + d_{6,y} + n_{6,y}\right)^2 - \left(S_{C,0,z} - z + d_{6,z} + n_{6,z}\right)^2}$$

The sign before the root expression decides the configuration of the robot.

2.2 Direct kinematics

For the considered parallel robot the direct kinematics problem can be formulated as follows. Calculate the location of the *TCP* for given *A, B* and *C*.

Three spheres with radius l_1, l_3 and l_6 describe all possible location for the *TCP* for fixed *A*, *B* and *C*. The intersection points between the spheres describe the location of the *TCP*.

The midpoints of the spheres and the spherical equations are:

$$A_1 = [x_{a1} \quad y_{a1} \quad z_{a1}] = S_{A,0} + s_a [1 \quad 0 \quad 0]^T + d_1 + n_1$$

$$B_1 = [x_{b1} \quad y_{b1} \quad z_{b1}] = S_{B,0} + s_b [1 \quad 0 \quad 0]^T + d_3 + n_3$$

$$C_1 = [x_{c1} \quad y_{c1} \quad z_{c1}] = S_{C,0} + s_c [1 \quad 0 \quad 0]^T + d_6 + n_6$$

$$(x_{a1} - x)^2 + (y_{a1} - y)^2 + (z_{a1} - z)^2 = l_1^2$$

$$(x_{b1} - x)^2 + (y_{b1} - y)^2 + (z_{b1} - z)^2 = l_3^2$$

$$(x_{c1} - x)^2 + (y_{c1} - y)^2 + (z_{c1} - z)^2 = l_6^2$$

Mathematical symbolic software can solve the spherical equations, but produces a rather extensive solution. Proficient use of simplification rules is needed in order to simplify the solution. This problem is avoided by solving the equations in two steps. First find the intersection between two of the spheres. The intersection is either a circle or a point. Ignore the point case for now. The intersection between the third sphere and one of the other forms of course also a circle. Derive the plane where this circle is located. Secondly the intersections of this plane and the first circle describe the possible location for the *TCP*.

In the solution below the intersection circle between spheres with midpoints at A_1 and C_1 is calculated. All calculations are then done in a coordinate system with the z-axis pointing from A_1 to C_1.

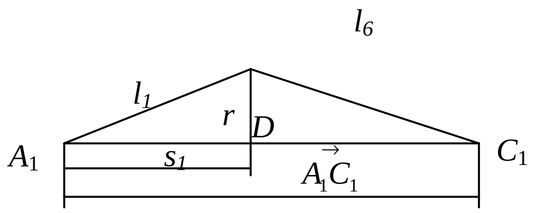

Fig:5. Intersection between two spheres.

$$s_1 = \frac{l_1^2 + \left|\overrightarrow{A_1C_1}\right|^2 - l_6^2}{2\left|\overrightarrow{A_1C_1}\right|} \quad , \qquad r = \sqrt{l_1^2 - s_1^2}$$

Midpoint for the circle:

$$D = A_1 + s_1 \frac{\overrightarrow{A_1C_1}}{\left|\overrightarrow{A_1C_1}\right|}, \quad s_2 = \frac{l_1^2 + \left|\overrightarrow{B_1C_1}\right|^2 - l_6^2}{2\left|\overrightarrow{B_1C_1}\right|}$$

A point on the plane:

$$E = B_1 + s_2 \frac{\overrightarrow{B_1C_1}}{\left|\overrightarrow{B_1C_1}\right|}$$

The normal vector for the plane:

$$N = \overrightarrow{B_1C_1}$$

Deriving the rotation matrix:

$$\theta = \tan^{-1}\left(\frac{x_a - x_c}{y_c - y_a}\right), \quad \beta = \sin^{-1}\left(\frac{z_c - z_a}{\overrightarrow{A_1C_1}}\right)$$

$$Rot_z = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Rot_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta-\pi/2) & \sin(\beta-\pi/2) \\ 0 & -\sin(\beta-\pi/2) & \cos(\beta-\pi/2) \end{pmatrix}$$

$$Rot_{xz} = Rot_x Rot_z$$

The normal vector for the plane and points D and E are transformed into a coordinate system with the z-axis pointing from A_1 to C_1.

$$(N_x \quad N_y \quad N_z)^T = Rot_{xz} N$$

$$(x_d \quad y_d \quad z_d)^T = Rot_{xz} D$$

$$(x_e \quad y_e \quad z_e)^T = Rot_{xz} E$$

The spherical equations can now be written in the new coordinate system as the intersection between a circle and a sphere.

$$(x_r - x_d)^2 + (y_r - y_d)^2 = r^2$$

$$N_x(x_r - x_e) + N_y(y_r - y_e) + N_z(z_r - z_e) = 0$$

$$z_r = z_d ,$$

where

$$(x_r \quad y_r \quad z_r)^T = Rot_{xz} TCP$$

$$x_{r1} = \frac{N_y^2 x_d + N_x^2 x_e - N_{xy} P_2 - N_{xz} P_3 - N_y S}{N_q}$$

$$y_{r1} = \frac{N_y^2 y_e + N_x^2 y_d - N_{xy} P_1 - N_{yz} P_3 + N_x S}{N_q}$$

$$x_{r2} = \frac{N_y^2 x_d + N_x^2 x_e - N_{xy} P_2 - N_{xz} P_3 + N_y S}{N_q}$$

$$y_{r2} = \frac{N_y^2 y_e + N_x^2 y_d - N_{xy} P_1 - N_{yz} P_3 - N_x S}{N_q}$$

$$P_1 = x_d - x_e \ , \quad P_2 = y_d - y_e \ , \quad P_3 = z_d - z_e$$

$$N_{xy} = N_x N_y \ , \quad N_{yz} = N_y N_z \ , \quad N_{xz} = N_x N_z$$

$$N_q = N_x^2 + N_y^2 \ , \quad Q = N_y r \ , \quad R = N_y P_2 + N_z P_3$$

$$S = \sqrt{N_x^2 (r + P_1)(r - P_1) - 2 N_x P_1 R + Q^2 - R^2}$$

$$TCP = Rot_{xz}^{-1} \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix}$$

The configuration of the robot decides which solution is valid.

3 Workspace optimization

In order to characterize the workspace of the manipulator the following optimization problem is formulated. Find the distances between the tracks that give the largest cross-section workspace for a manipulator with links of equal length *l*. Only symmetrical placements of the tracks are considered. The workspace is further restricted in three directions with the requirement that the workspace must be a part of the open rectangular area formed by the linear tracks. Two types of joints are considered both shown in figure 6. The cardan joint puts no restrictions on the cross-section workspace while the ball and socket joint limits the workspace along one direction.

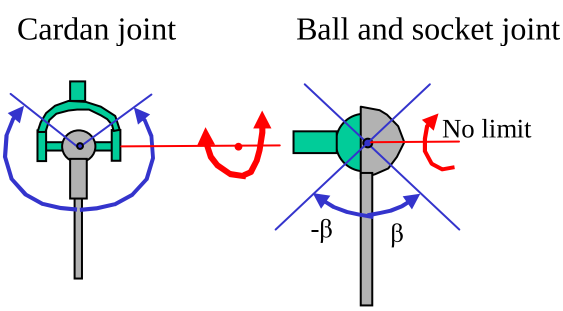

Fig: 6. Joints

The problem is solved independently from the vectors $\mathbf{n_i}$ and $\mathbf{d_i}$ by imposing the parameterization on the midpoints of the three spheres that intersect at the *TCP*. The optimization parameters are q_1 and q_2 as shown in figure 7. When the ball and socket joint is used, the optimal orientation of the joint must also be considered.

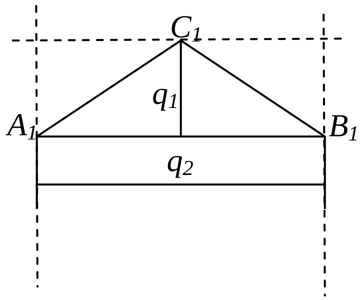

Fig:7. Optimization parameters. The dashed lines form the open rectangular area which restricts the workspace.

The optimization problem is solved by using a non linear programming routine. The obtained optimal cross section area is shown in figure 8.

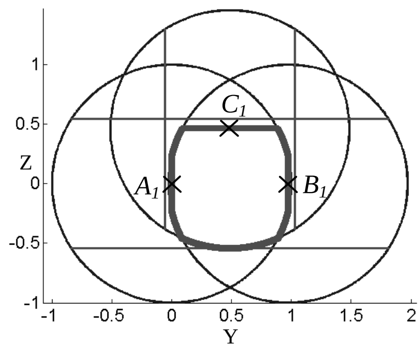

Fig:8. Optimal cross section workspace. The straight lines show the limits for ball and socket joints with $\beta = 32.86^{o}$. The circles show the maximum reachability for each cluster without limitation imposed by the joints.

As long as $\beta \geq \sin(|q_2 - 1|)$ and the ball and socket joints are orientated as in figure 8 the optimal relations between link length l and the optimization parameters are the same for both joint types, namely:

$$q_1/l \approx 0.97345833971302$$

$$q_2/l \approx 0.45745029339798$$

$$\beta \geq 32.86$$

The relation between optimal area and link length is as follows:

$$Area \approx 0.90310315654235 * l^2$$

4 Conclusions

In the paper the solutions of the inverse kinematics and direct kinematics problems for the novel parallel structure robot have been obtained. The initial study of the Gantry-Tau structure has demonstrated good workspace properties of the robot. Further studies are needed in order to examine how competitive the considered structure is.

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