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Gas-Liquid Slip Ratio and MHD Pressure Drop in Two-Phase Liquid Metal Flow in Strong Magnetic Field

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The gas-liquid slip ratio and the MHD pressure drop in the two-phase liquid metal flow under a strong magnetic field are treated in relation to the distributions of the gaseous phase and the fluid velocity. From NaK-N_2 two-phase experiments, it is found that:

(1) The ratio of the two-phase MHD pressure drop to that of the homogeneous non-slip two-phase flow model is independent of the applied magnetic field strength and the channel geometry (ratio of wall thickness to channel width) and is presented as a function of the ratio of the gas-liquid volumetric flow rate.

(2) The void distribution is affected by the applied magnetic field. The gas bubbles may be pushed away towards the both side walls due to the pinch effect, resulting in the increase of the ratio of the mean void fraction to the one at the channel center with the increasing applied magnetic field.

(3) The gas-liquid slip ratio is not directly dependent on the applied magnetic field strength and the channel geometry and is presented as a function of the ratio of the gas-liquid volumetric flow rate. As a result, the slip ratio decreases with the increasing system pressure and/or the decreasing mixture quality.

***KEYWORDS**: two-phase flow, liquid metals, magnetic fields, MHD pressure drop, gas-liquid slip ratio, flow pattern, void distribution, velocity distribution, electrical conductivity, flow rate, pinch effect*

I. INTRODUCTION

MHD power generation system with use of liquid metal as a working fluid is considered to be more hopeful as a new energy conversion system as well as the potassium turbine system for the comparatively high temperature heat source of about 1,000°C.

Recent cycle studies(1)(2) on liquid metal MHD power generation have indicated the possibility of the overall cycle efficiency in excess of 50% in advanced power cycles utilizing two-phase liquid metal MHD power generation. The two-phase liquid metal MHD generator, however, which is one of the most important components in the system, may be perhaps required at least 70% generating efficiency. But it has not been developed yet and its performance will be much different from that of a liquid metal single phase MHD generator because the thermal energy is converted first into the kinetic energy and subsequently into the electrical energy in the generator itself.

In the previous papers(3)~(5) the theoretical analysis and the experimental studies have been presented on the performance characteristics of the two-phase liquid metal MHD induction converter, in which the void fraction and consequently the equivalent electrical conductivity and the fluid velocity of the two-phase flow varied along the generator channel.

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However, to obtain essentially the high generating efficiency, it is the first priority to convert the thermal energy to the kinetic energy efficiently and for its sake it is desirable that a gas-liquid slip ratio is slightly above unity in consideration of liquid acceleration due to the expansion of the gas in the generator.

On the other hand, MHD pressure drop due to the interaction of the two-phase liquid metal flow with the applied magnetic field is one of the most important problems in the design of the energy extraction system from the blanket of the thermonuclear fusion reactor using two-phase liquid metal flow, which has been proposed by one of the authors(6).

These problems, the gas-liquid slip ratio and the two-phase MHD pressure drop, are closely related to the flow pattern, which is affected strongly by the electromagnetic force, that is, a body force. Therefore it may be much different from that of the two-phase flow without a magnetic field, which is dominated mainly by the shear stress due to the channel wall. Therefore it is difficult to predict them theoretically because of the inherent complication of the two-phase fluid dynamics.

In the present paper, the gas-liquid slip ratio and the MHD pressure drop in the two-phase liquid metal flow under the strong magnetic field are treated theoretically and experimentally in relation to the flow pattern by introducing the parameter K_1 which presents the distributions of the gaseous phase and the fluid velocity in the cross section perpendicular to the flow. The parameter K_1 is equivalent to the inverse of Bankoff's "flow parameter"(7) in the mass conservation law because of the assumption of the local non-slip flow. However, while he discusses the two-phase fluid dynamics dominated by the shear stress, in the present paper the ones in the MHD flow having a large Hartmann number are discussed from the viewpoint of the electromagnetic force balance.

Still more, the experiments with use of NaK-N_2 two-phase flow are performed to investigate the effects of the applied magnetic field and other system parameters, the mass flow rates of gas and liquid, the system pressure and the channel geometry *etc.*, on the parameter K_1. Then the effects of these system parameters on the gas-liquid slip ratio and the two-phase MHD pressure drop under the strong magnetic field are also discussed.

II. THEORETICAL TREATMENTS

1. Distribution Flow Model

The flow treated in the paper is steady and flowing in the z-direction with a small drift of the fluid in the direction of the applied magnetic field and has the distributions of the void fraction and the fluid velocity in the flow direction as well as in the y-direction (the direction of the applied magnetic field). However, the effects of these variations in the flow direction on Maxwell's equations can be negligible because the gradients of these variations in the flow direction are much smaller than ones in the y-direction and still more the effects brought by two variations of the void fraction, that is, electrical conductivity and the fluid velocity in the flow direction generally tend to act in opposite directions and thus to cancel each other(3)(4).

On the other hand, the effects of the drift of the fluid in the direction of the applied magnetic field on the solutions of the Maxwell's equations will be also negligible because this small drift has no direct interaction with the applied magnetic field and interact only with the magnetic field induced in the flow direction which is much smaller than the applied magnetic field, which is discussed in more detail in the latter chapter. Consequently in the theoretical analysis it is assumed that the fluid flows in the z-direction only and has the distributions of the void fraction and the fluid velocity in the y-direction only as shown in **Fig. 1.** Further assumptions are:

(1) The gas and liquid have the same velocity in the flow direction, the relative velocity of the gaseous phase with respect to the surrounding liquid phase being considered to be negligible compared to the bulk gas velocity in the flow direction $u_l=u_g=u_t(y)$.

(2) The applied magnetic field B_0 is uniform and in the y-direction only.

(3) The electrodes in the both sides are perfect electrical conductors.

(4) The local two-phase electrical conductivity is a function of the local void fraction only, *i.e.*, $\sigma_t = \sigma_s f(\alpha)$.

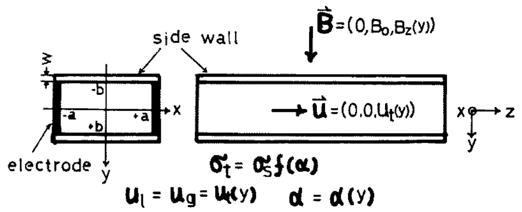


Fig. 1 Schematic diagram of MHD channel

From the above assumptions, all current loops are assumed to lie in the x-y plane, with result that the induced magnetic field is in the z-direction $\boldsymbol{B}=(0, B_0, B_z(y))$. Therefore, the current density has only x component and the electric field has also x component only, which is constant in space from $\nabla\cdot\boldsymbol{E}=0$ and $\nabla\times\boldsymbol{E}=0$.

The total current in the fluid is obtained by integrating Ohm's law as follows:

$$I_f=\int_0^L\int_{-b}^{b}\sigma_t(y)(E_x-u_t(y)B_0)dydz$$

$$=ZbL(\langle\sigma_t\rangle E_x-\langle\sigma_t u_t\rangle B_0), \tag{1}$$

where $\langle X\rangle\equiv\frac{1}{2b}\int_{-b}^{b}X(y)dy$,

and the current flowing in the external circuit is described as

$$I_0=2aE_x/R_0, \tag{2}$$

where R_0 is the external electrical resistance.

Since

$$I_f+I_0=0, \tag{3}$$

substitution of Eqs. (1) and (2) into Eq. (3) yields

$$E_x=\frac{B_0}{1+C_t}\cdot\frac{\langle\sigma_t u_t\rangle}{\langle\sigma_t\rangle}, \tag{4}$$

where $$C_t=\frac{C_s\sigma_s}{\langle\sigma_t\rangle}, \tag{5a}$$

$$C_s=R_s/R_0, \tag{5b}$$

$R_s=\frac{a}{\sigma_s bL}$: Internal electrical resistance for single phase flow.

The voltage between the two electrodes is obtained by integrating Eq. (4) as follows:

$$V_t=\int_{-a}^{a}E_x dx$$

$$=\frac{2aB_0\langle u_t\rangle K_2}{1+C_t} \tag{6a}$$

$$=2aB_0\langle u_t\rangle\varepsilon_t, \tag{6b}$$

where K_2 is defined as

$$K_2\equiv\langle\sigma_t u_t\rangle/\langle\sigma_t\rangle\langle u_t\rangle, \tag{7}$$

and ε_t is the effective load factor in the two-phase MHD flow defined as

$$\varepsilon_t=\frac{\text{Terminal voltage}}{\text{Mean induced e.m.f.}}=\frac{V_t}{2a\langle u_t\rangle B_0} \tag{8a}$$

$$=\varepsilon_0 K_2, \tag{8b}$$

where $\varepsilon_0=1/(1+C_t)$: Load factor in homogeneous two-phase flow.

When the gravitational force and the viscous force are neglected compared with the electromagnetic force, the pressure is given as follows:

$$p+\frac{B_z^2(y)}{2\mu}=p_0, \tag{9}$$

where P_0 is the pressure at $y=0$ and $B_z^2/2\mu$ the pressure caused by the induced magnetic field B_z.

The MHD pressure drop in the flow direction ($J_x\times B_0$) is constant in the cross section perpendicular to the flow in this case. Therefore it is equal to the value averaged over the y-direction and obtained as follows:

$$\left(\frac{\Delta p}{\Delta z}\right)_t=-\frac{C_t}{1+C_t}\langle\sigma_t\rangle\langle u_t\rangle B_0^2K_2 \tag{10a}$$

$$=-(1-\varepsilon_0)\langle\sigma_t\rangle\langle u_t\rangle B_0^2K_0. \tag{10b}$$

2. Parameter K_2 and Distribution of Gaseous Phase

To express the deviations of the MHD characteristics here concerned from those of the homogeneous flow due to the distributions of the electrical conductivity and the fluid velocity, the parameter K_2 is defined in the previous section. The parameter K_2 is also able to be presented in connection with the distribution of the gaseous phase in the two-phase flow through the correlation between the two-phase electrical conductivity and the void fraction.

The two-phase electrical conductivity decreases monotonously with increase of the void fraction(5)(6)(8). To clarify the effects of the distribution of the gaseous phase in the two-phase flow on MHD characteristics, the assumption is made that the two-phase electrical conductivity is a function of the void fraction only and expressed as follows:

$$\frac{\sigma_t}{\sigma_s} \equiv f(\alpha) = a_1 + a_2\alpha\,. \qquad (11)$$

Then the parameter K_2 is written as follows:

$$K_2 = \frac{a_1 + a_2 K_1\langle\alpha\rangle}{a_1 + a_2\langle\alpha\rangle}\,, \qquad (12)$$

where $K_1 \equiv \langle\alpha u_t\rangle / \langle\alpha\rangle\langle u_t\rangle$.

In **Fig. 2** the parameter K_2 is shown as a function of the mean void fraction $\langle\alpha\rangle$ with K_1 as parameter for $a_1=1$ and $a_2=-1.1$ which are obtained from the results in Ref. (6) as fitting. If the gas bubbles concentrate toward the higher velocity region, the parameter K_1 becomes larger than unity from its definition, resulting in decrease of K_2 and then decreases of the terminal voltage and the MHD pressure drop and *vice versa*.

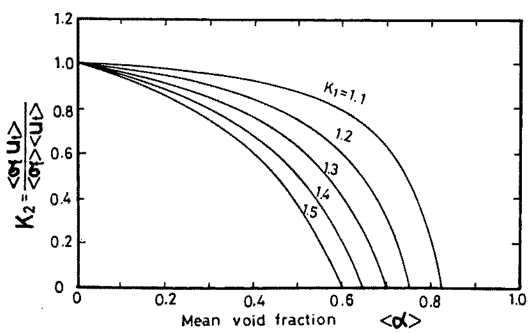


Fig. 2 Parameter K_2 as function of mean void fraction

3. Parameter K_1 and Gas-liquid Slip Ratio

The theoretical treatments of the two-phase flow dynamics without a magnetic field have been performed by Bankoff assuming the distributions of the void fraction and the fluid velocity as the power law and the local non-slip flow(7). He showed that the mean velocity of the gaseous phase is greater than that of liquid phase when gas is concentrated in the higher velocity region even if the local relative velocity between two phases is neglected. Similar treatments are made in the present paper. It is, however, difficult to assume the power law distributions in the two-phase liquid metal flow under the strong magnetic field, where the dominant restrictive force is a body force due to the electromagnetic interactions, which is almost constant in all positions in the cross section perpendicular to the flow. In addition, the pinch effect due to the large current density in the fluid acts on the fluid towards the channel center, that is mentioned in detail in the latter chapter. Therefore in the present paper the gas-liquid slip ratio is discussed by assuming only the local non-slip flow.

The mass conservation laws of the both phases in the two-component two-phase flow are presented with use of the parameter K_1 as follows:

$$M_l = 2a\int_{-b}^{b} \rho_l(1-\alpha)u_t\,dy = 4ab\,\rho_l(1-K_1\langle\alpha\rangle)\langle u_t\rangle\,, \qquad (13)$$

$$M_g = 2a\int_{-b}^{b} \rho_g\alpha u_t\,dy = 4ab\,\rho_g K_1\langle\alpha\rangle\langle u_t\rangle\,. \qquad (14)$$

Consequently, the gas-liquid slip ratio is expressed as

$$S_f \equiv \frac{\chi}{1-\chi}\cdot\frac{\rho_l}{\rho_g}\cdot\frac{1-\langle\alpha\rangle}{\langle\alpha\rangle} = \frac{1-\langle\alpha\rangle}{1/K_1-\langle\alpha\rangle} \qquad (15a)$$

$$= (K_1-1)\beta + K_1\,, \qquad (15b)$$

where $\beta = \{\chi/(1-\chi)\}(\rho_l/\rho_g)$: Ratio of gas-liquid volumetric flow rate

and the mean void fraction is described as

$$\langle\alpha\rangle=\frac{1}{K_1}\cdot\frac{\beta}{\beta+1} \qquad (16a)$$

$$=\alpha_0/K_1\,, \qquad (16b)$$

where $\alpha_0=\beta/(1+\beta)$: Void fraction in homogeneous non-slip flow.

In **Fig. 3**, the mean void fraction is shown as a function of the ratio of the gas-liquid volumetric flow rate β with K_1 as parameter and also is shown by dotted lines with the gas-liquid slip ratio as parameter. It is seen that the gas-liquid slip ratio increases with increase of β for a constant K_1.

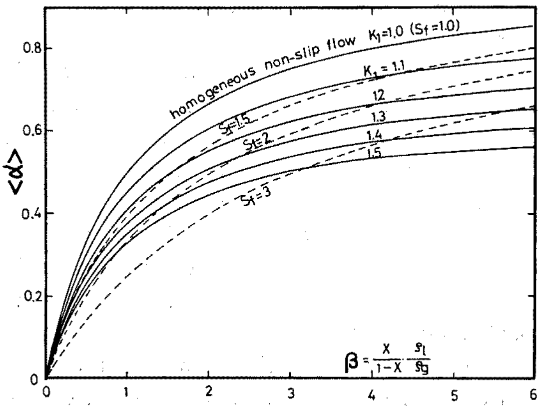


Fig. 3 Relation between mean void fraction and ratio of gas-liquid volumetric flow rate

On the other hand, the parameter K_2 can be rewritten with the parameter K_1 and β using Eq. (16a). This means that the two-phase MHD performance characteristics, for example two-phase MHD pressure drop and terminal voltage, are able to be obtained directly for the given mass flow rates of the gas and liquid phases and the system pressure if the value of K_1 is known under the strong magnetic field.

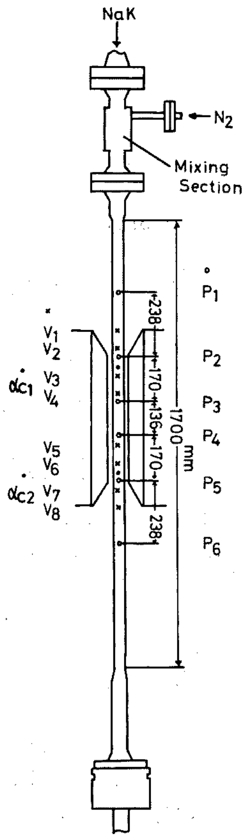
From the above discussions, the gas-liquid slip ratio and the two-phase MHD pressure drop are able to be discussed concerning with the distribution of theg aseous phase with use of the parameter K_1. For its sake, however, it is necessary to know how the parameter K_1 is affected by some system parameters, the applied magnetic field, the gas and liquid mass flow rates, the system pressure and the channel geometry *etc.*

III. EXPERIMENTAL

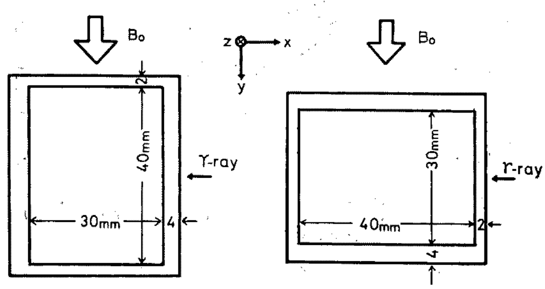
The experiments were performed with NaK-blow down facility, which is presented in detail in the previous paper(9).

The test section and the mixing section are illustrated schematically in **Fig. 4**. The test section consists of a constantarea, rectangular channel built up from 2 mm or 4 mm-thick stainless steel plate with the inside dimensions of 40×30 mm² as shown in **Fig. 5**. The mixing section consists of a perforated pipe surrounded by a stainless steel tube. In this mixing section, N_2 gas is introduced into the vertically-flowing NaK stream through the holes of 0.5 mm diameter spaced uniformly in the inner channel wall.

Pressures were measured with semiconductor pressure transducers at the six positions of the test channel as shown in Fig. 4. Pressure taps of P_2~P_6 are installed in the region of the uniform magnetic field



Experimental channel
Fig. 4



Case A (C_s=0.067) Case B (C_s=0.18)

Fig. 5 Geometry of test section

and pressure tap 1 is located upstream of the uniform magnetic field and pressure tap 6, downstream of the magnetic field to estimate the end effects.

The terminal voltages were measured with the potential taps of $V_3 \sim V_6$ in the uniform magnetic field region and V_1, V_2, V_7 and V_8 in the fringing region.

The flow rates of NaK and N_2 gas were measured by means of an electromagnetic flow meter installed upstream of the mixing section and by an orifice flow meter in the gas injection line, respectively.

The void fractions at entrance and exit of the test region with the uniform magnetic field were measured by the γ-ray attenuation technique and the measurements were made by the "one-shot" technique with 5 mm diameter γ beam at the center of the channel to investigate the effects caused by the applied magnetic field on the void fraction profile. The γ-ray source was ^{241}Am of 30 mCi. The mean void fraction was estimated by the terminal voltage measured by the potential tap, using Eqs. (6), (12)~(14).

In order to examine the effects of the channel geometry, two series of the experiments were performed for Case A ($C_s=0.067$) and Case B ($C_s=0.18$) as shown in Fig. 5.

IV. Experimental Results and Discussions

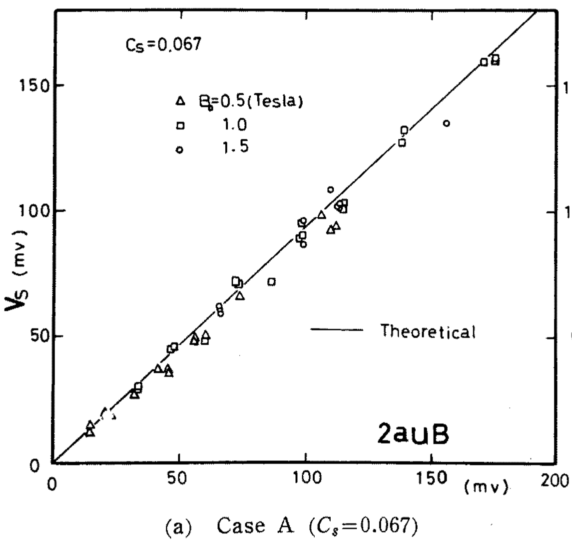
Data were taken over the following parameter ranges:

Magnetic field B_0: 0.5, 1.0, 1.5 T
NaK mass flow rate M_l: 7 kg/sec (max.)
Mixture quality χ: 0~0.015
Mean void fraction $\langle\alpha\rangle$: 0~50%
Pressure P: 4~10 kg/cm^2
Hartmann number H_a: 500~1,500 (for single phase flow)

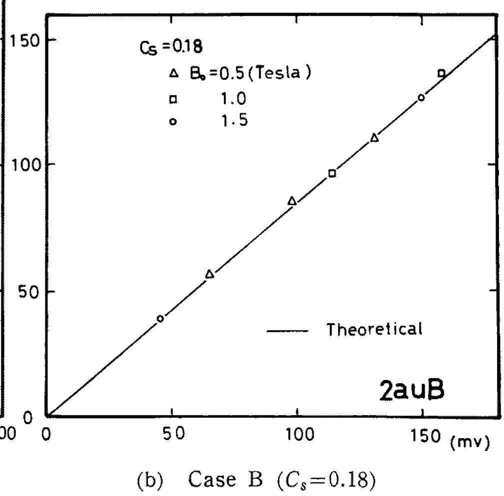
In the present experiments, the external load was substituted by the resistance of the wall. The values of C_s defined in Eq. (5b) as the ratio between the internal resistance in NaK single phase flow and that of the external circuit were 0.067 for Case A and 0.18 for Case B.

1. Terminal Voltage and Load Factor

The experiments using NaK single phase flow were performed to check the propriety of the assumptions in the theoretical treatments. In **Fig. 6**(a) and (b), the terminal voltage in the NaK single phase flow are plotted as a function of the mean internal electrical motive force for Cases A and B. The theoretical curve obtained from Eq. (6) is also shown in the figure. It is seen that it gives a fairly good correlation for the experimental data. In the case of the two-phase



(a) Case A ($C_s=0.067$)

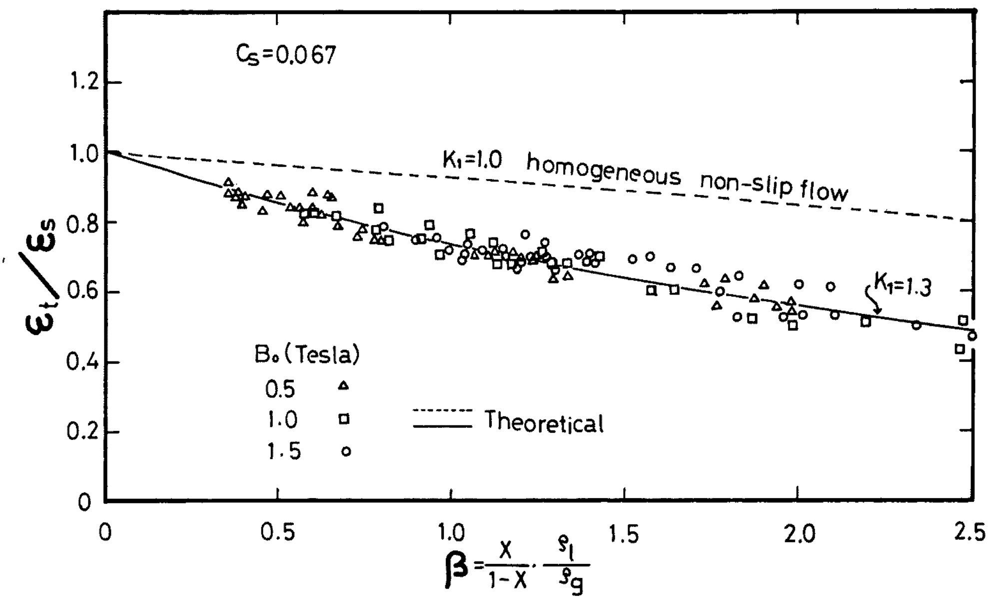


(b) Case B ($C_s=0.18$)

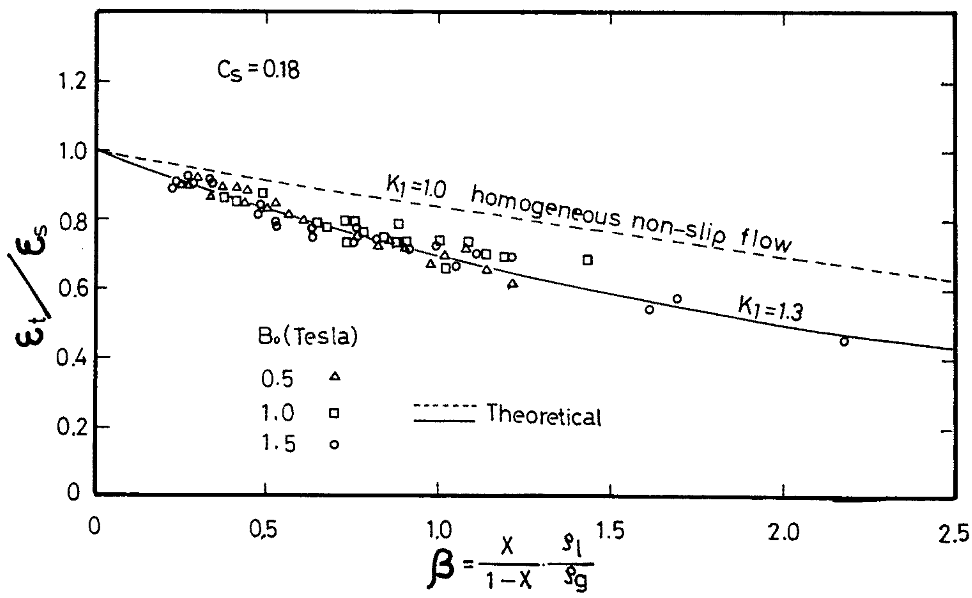
Fig. 6 Terminal voltage in NaK single phase flow

flow the approximation of the perfect conductor for the electrodes can be more reasonable because of the decreasing electrical conductivity of the fluid.

The terminal voltage in the two-phase flow is affected by, at first, the increase of the internal electrical resistance caused by containing the gas bubbles. In addition, it is also affected by the distributions of the void fraction and the fluid velocity as mentioned in the previous section. In **Fig.** 7(a) and (b) the ratio of the load factor in NaK-N_2 two-phase flow to the one in NaK single phase flow obtained from the terminal voltages at



(a) Case A (C_s=0.067)



(b) Case B (C_s=0.18)

Fig. 7 Ratio of load factor in NaK-N_2 two-phase flow to the one in NaK single phase flow the positions of $V_3 \sim V_6$ in the uniform magnetic field are plotted as a function of the ratio of the gas-liquid volumetric flow rate with the applied magnetic field as parameter for Cases A and B, respectively. The experimental data show that the terminal voltages are much lower than those of the homogeneous non-slip flow and these deviations are little dependent on the applied magnetic field strength and the channel geometry but only on β at any position in the uniform magnetic field region. It is also seen that the theoretical curve corresponding to $K_1=1.3$ gives a fairly good correlation for all the experimental data.

2. MHD Pressure Drop

In **Fig.** 8(a) and (b) the ratio of the pressure drop in NaK-N_2 two-phase flow in the uniform magnetic field to the one in NaK single phase flow with the same NaK mass flow rate are shown as a function of β with the applied magnetic field strength as parameter and also compared with the theoretical values. Being compared in the same NaK mass flow rate, the ratio of the MHD pressure drop is larger than unity because the velocity of the two-phase flow is greater than that of the single phase flow. The theoretical curve of $K_1=1.0$ shows the value of the homogeneous non-slip flow. The experimental data are lower than the values of the homogeneous non-slip flow and the curve corresponding to $K_1=1.3$ shows the best correlation for the both Cases A and B. The same tendencies of the deviations as seen in Fig. 7 are also found clearly. It is clear that these deviations are caused due to the non-uniformities of the void fraction and the fluid velocity in the cross section as discussed in the previous section.

3. Void Distribution

There are two large differences between the two-phase liquid metal flow under the strong magnetic field and the one without a magnetic field. First, it is the electromagnetic force, that is a body force, to dominate the flow pattern in the two-phase liquid metal flow under the strong magnetic field, while in the two-phase flow without a magnetic field it is the shear stress. Second, in addition, there is the pinch effect acting on the fluid towards the center of the channel caused in the consequence of the interaction between the current in the fluid and the magnetic field induced by the current itself. The flow pattern in the two-phase liquid metal flow under the strong magnetic field may be different from that in the two-phase flow without a magnetic field.

In **Fig. 9** the ratio of the mean void fraction to the one at the center of the channel is shown as a function of the effective magnetic interaction number written as follows:

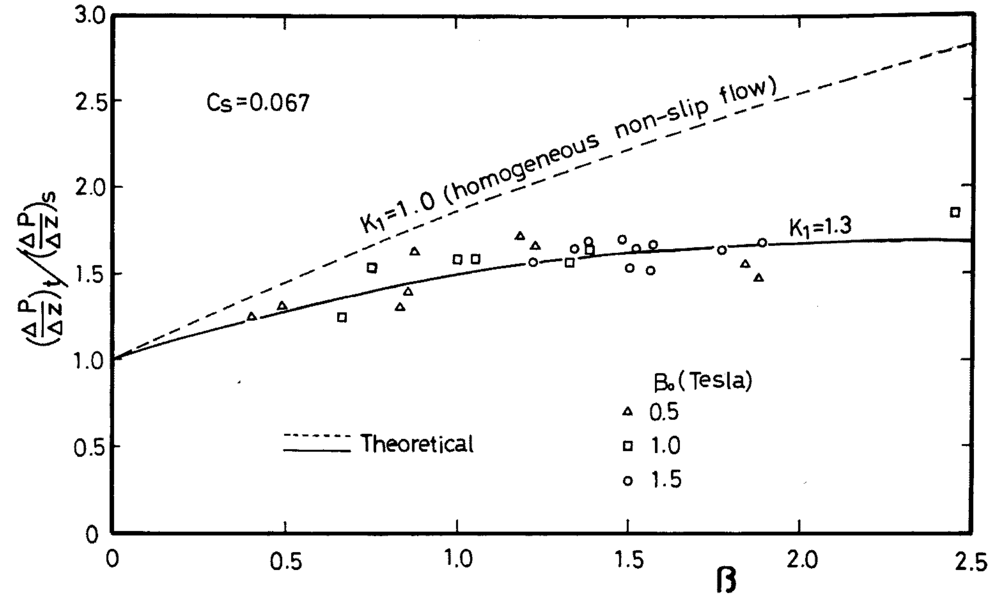
$$N=\frac{\text{Electromagnetic force}}{\text{Inertia}}$$

$$=\frac{\langle\sigma_t\rangle B_0^2 b}{\langle\rho_t\rangle\langle u_t\rangle}\,. \qquad (17)$$

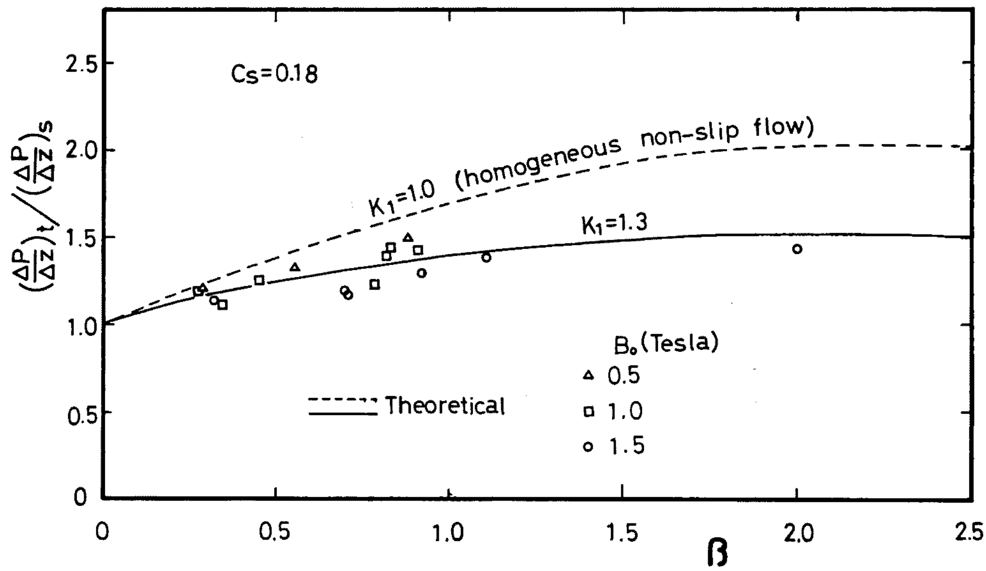
It is seen clearly in Fig. 9 that the void distribution is much affected by the applied magnetic field. The gas bubbles may be pushed away toward the channel walls in the both sides, resulting the ratio of the mean void fraction to the one at the center of the channel from less than unity to more than unity with the increasing applied magnetic field strength.

These phenomena can be explained by the pinch effect caused as the consequence of the interaction between the current in the fluid and the magnetic field induced by the current itself as follows.

The momentum equation in the flow direction can be presented only by the electromagnetic force in the down stream region beyond the point of the equivalent length b from the entrance of the magnetic field because of large magnetic interaction numbers and large Hartmann numbers, resulting in a constant current density J_x to keep the constant pressure drop in the flow direction in the cross section perpendicular to the flow. However, if some force acts on the fluid in the direction of the applied magnetic field, it is possible that the fluid drifts in the direction under the condition of the constant



(a) Case A ($C_s=0.067$)



(b) Case B ($C_s=0.18$)

Fig. 8 Ratio of MHD pressure drop in NaK-N_2 two-phase flow to the one in NaK single phase flow with the same NaK mass flow rate

current density because the drift of the fluid in this direction can not be restrained directly by the applied magnetic field.

The induced magnetic field in the fluid is obtained as

$$B_z(y) = -\frac{C_t}{1+C_t} R_m B_0 K_2 \frac{y}{b}, \tag{18}$$

where $R_m = \mu\langle\sigma_t\rangle\langle u_t\rangle b$: Effective magnetic Reynolds number.

This induced magnetic field interacting with the current in the fluid increases the pressure in the y-direction as shown in Eq. (9). The pressure drop averaged over the y-direction is presented as:

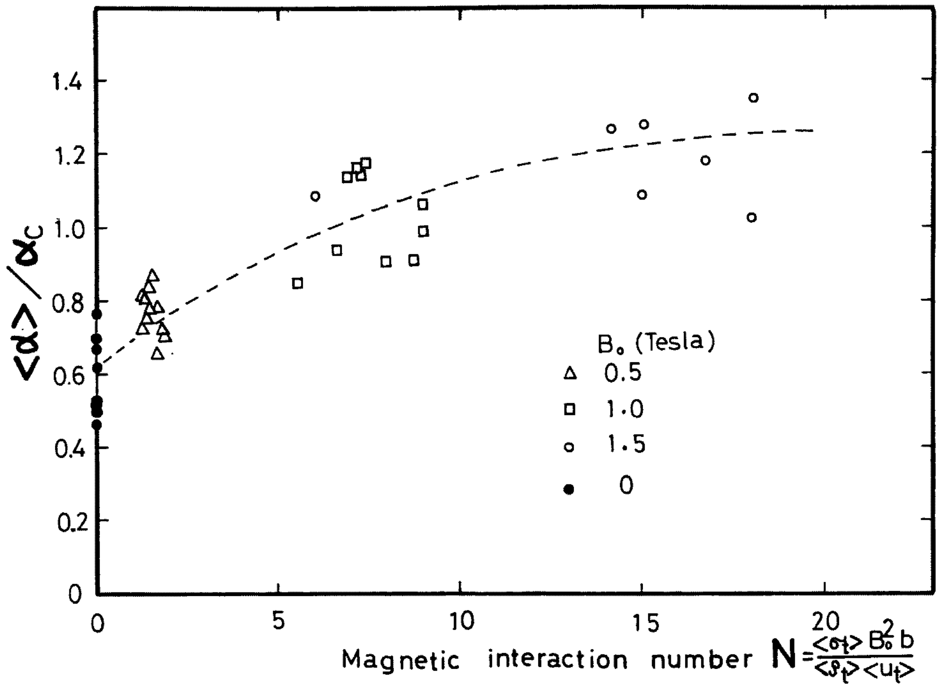


Fig. 9 Ratio of mean void fraction to the one at center of channel

$$\left\langle \frac{\Delta p}{\Delta y} \right\rangle = -\left(\frac{C_t}{1+C_t} R_m K_2\right)^2 \frac{B_0^2}{2\mu} \cdot \frac{1}{b} \tag{19a}$$

$$= \frac{1}{2} \cdot \frac{C_t}{1+C_t} R_m K_2 \left(\frac{\Delta p}{\Delta z}\right)_t, \tag{19b}$$

which is proportional to the MHD pressure drop in the flow direction. For example, in the case of the applied magnetic field of 1 T, the mean fluid velocity of 10 m/sec and the mean void fraction of 50%, the induced current density in the fluid J_x is approximately 10^6 A/m^2 and then the induced magnetic field B_z becomes about 10^{-2}T resulting in the averaged pinching force of about 10^4 N/m^3.

In **Fig. 10** the averaged drift velocity of the gas bubble, which is obtained assuming that the gas bubble is spherical and receives Stokes' restrictive force only when it drifts towards the channel wall due to the pinch effect, is shown as a function of the average pinching force with the diameter of the gas bubble as parameter. The time required to pass through the uniform magnetic field region is about 0.1 sec for the mean fluid velocity of about 5 m/sec which is an example of the present experiments for the applied magnetic field of 1.5 T. The time required to reach the side wall is about 0.04 sec even if the average drift velocity is about 0.5 m/sec considering the pinching force is roughly 0.1 atm/m and assuming that a diameter of the gas bubble, which will be perhaps greater than 0.5 mm which is a diameter of the

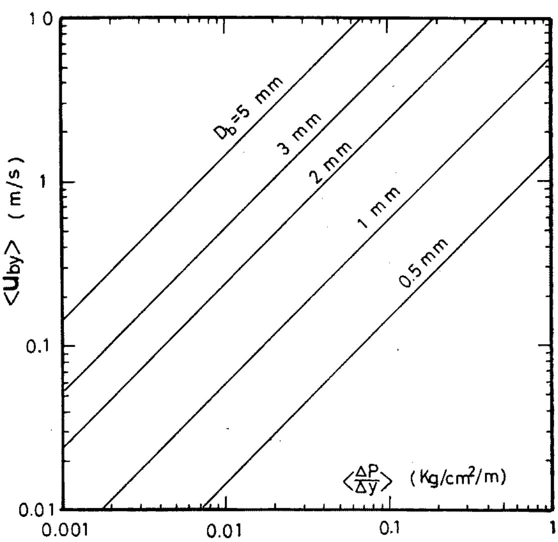


Fig. 10 Drift velocity of gas bubble due to pinch effect

orifices for the gas injection at the mixing section, is also roughly 1 mm. This means that the gas bubble at the center region of the channel can arrive at the side wall at 2/5 of the test section from the entrance. In the case of the applied magnetic field of 0.5 T, when the fluid velocity in the flow direction is about 10 m/sec which is an example for the case of 0.5 T in the present experiments, the averaged pinching force is about 4/9 times compared with that in the case of 1.5 T assuming the same diameter of the gas bubble. This means that the gas bubble at the center can not arrive at the side wall while the fluid passes through the applied magnetic field region in the case of 0.5 T, resulting in the ratio of the mean void fraction to the one at the center less than unity while it is more than unity in the case of 1.5 T.

The discussions above may be too rough and it will be required to consider the transformation and the coalesence of the bubbles and the interactions between the bubbles to discuss the more fine void profile. However, the drift of the gas bubbles due to the pinch effect is possible. Concerning the more fine void profile in the two-phase liquid metal flow under the strong magnetic field, further study is required in future.

On the other hand, the effects of the drift of the fluid in this direction on the solutions of Maxwell's equations will be negligible as mentioned in the previous section.

4. Gas-liquid Slip Ratio

In **Fig. 11** the gas-liquid slip ratios are shown as a function of β for Cases A and B with the applied magnetic field strength as parameter. From this figure it is seen that the effects of the applied magnetic field strength and the channel geometry on the gas-liquid slip ratio are scarcely observed and they depend only on the ratio of the gas-liquid volumetric flow rate.

The theoretical curve of $K_1=1.3$ represented by the solid line makes a good agreement with the experimental data, which lie almost in the range between the curves corresponding to 10% above and under of K_1 shown by two dotted lines. This means that the gas-liquid slip ratio is a function of the mixture quality and the ratio of the densities between the liquid and gaseous phases and

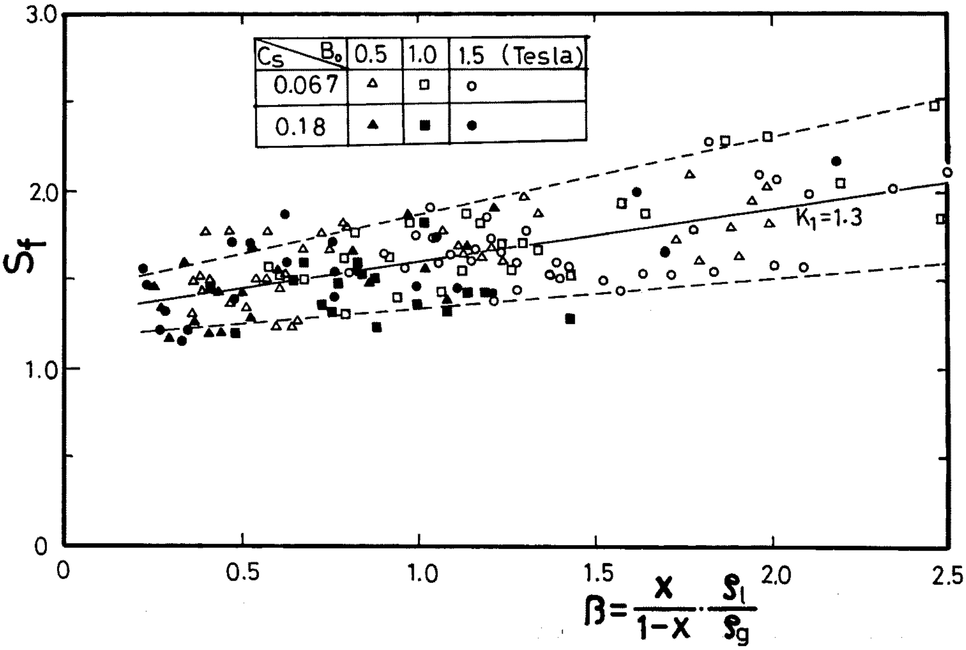


Fig. 11 Correlation between gas-liquid slip ratio and ratio of gas-liquid volumetric flow rate

that it decreases as the mixture quality decreases and/or as the density of the gaseous phase increases, that is, the system pressure increases. Consequently, it is also concluded that the gas-liquid slip ratio increases along the flow direction in the magnetic field because of the increase of the ratio of the gas-liquid volumetric flow rate due to the MHD pressure drop along the flow direction.

In **Fig. 12** the gas-liquid slip ratios in the experiments performed by Thome at ANL using NaK-N_2 two-phase flow[10] are rearranged with the present parameters. He concluded in his paper that the gas-liquid slip ratio increases with the increasing applied magnetic field strength in all cases in his experiments. Rearranging the same data as a function of β, although his data are slightly larger, the similar tendency as the present results is seen clearly. It will be concluded from these results that even if the applied magnetic field strength increases, the gas-liquid slip ratio is not affected directly by the applied magnetic field in spite of the disturbance of the void distribution by the applied magnetic field and is affected always only through the variations of the mixture quality and/or the system pressure caused by the electromagnetic interaction between the two-phase flow and the applied magnetic field.

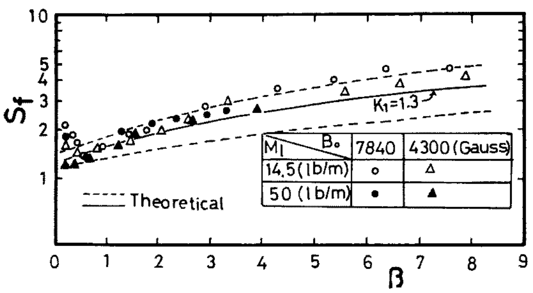


Fig. 12 Correlation between gas-liquid slip ratio and ratio of gas-liquid volumetric flow rate obtained from data of Thome[10]

The reasons why the gas-liquid slip ratio is little dependent directly on the applied magnetic field strength are discussed below. The distribution of the fluid velocity is affected consequently by the disturbance of the void distribution due to the applied magnetic field to balance the MHD pressure drop in the flow direction through the two-phase electrical conductivity. The velocity profile is obtained approximately for the two-phase flow of a large Hartmann number from the momentum equation $((\Delta P/\Delta z)_t = J_x \times B_0)$ as follows:

$$u(y) = \frac{E_x}{B_0} + \frac{-(\Delta p/\Delta z)_t}{\sigma_t(y) B_0^2} . \tag{20}$$

Consequently, the parameter K_2 is given as

$$K_2 = \frac{E_x}{\langle u_t \rangle B_0} + \frac{-(\Delta p/\Delta z)_t}{\langle \sigma_t \rangle \langle u_t \rangle B_0^2} , \tag{21}$$

where the second term is approximately equal to $C_t/(1+C_t)$ and negligible in the case of $C_t \ll 1$ being compared with the first term, which is the load factor and approximately equal to $1/(1+C_t)$. Therefore, the parameter K_2 is presented approximately as

$$K_2 \cong \frac{1}{1+C_t} . \tag{22}$$

From Eqs. (22) and (13) the parameter K_1 is obtained as a function of the mean void fraction as follows:

$$K_1 \cong \frac{a_1 + a_2\langle \alpha \rangle - a_1 C_s / a_2 \langle \alpha \rangle}{a_1 + C_s + a_2 \langle \alpha \rangle} . \tag{23}$$

Using Eq. (19) it is also possible to obtain K_1 as a function of the ratio of the gas-liquid volumetric flow rate β, which is shown in **Fig. 13** where experimental data are also plotted. Reasonable agreement between the theoretical predictions and the experimental data and as expected, little dependence of K_1 on the applied magnetic field strength and the ratio of the gas-liquid volumetric flow rate are seen in Fig. 13.

As a conclusion, it can be considered that even if the distribution of the void fraction is disturbed by the applied magnetic field, the distribution of the fluid velocity varies simultaneously to compensate the effect of the applied magnetic field on K_1, resulting in little direct influence of the applied magnetic field on the gas-liquid slip ratio.

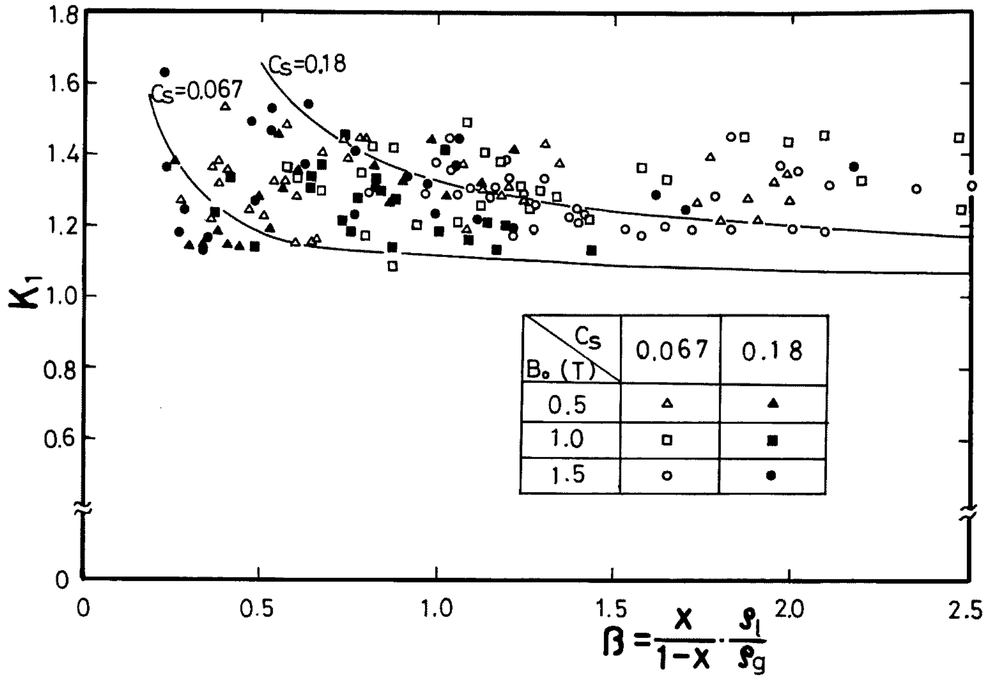


Fig. 13 Dependence of parameter K_1 on system parameters

V. Conclusions

The gas-liquid slip ratio and the MHD pressure drop in the two-phase liquid metal flow under a strong magnetic field are treated in relation to the flow pattern by introducing the parameter K_1 which presents the distributions of the gaseous phase and the fluid velocity.

From the experiments with use of NaK-N_2 two-phase flow, it is found that the parameter K_1 is little dependent on the system parameters such as the applied magnetic field strength, mass flow rates of gas and liquid phases, the system pressure and the channel geometry and almost constant as $K_1=1.3$ within the range of $\pm 10\%$.

This leads to the following conclusions:

(1) The ratio of the two-phase MHD pressure drop to that of the homogeneous non-slip two-phase flow model is independent of the applied magnetic field strength and the channel geometry and is presented as a function of the ratio of the gas-liquid volumetric flow rate.

(2) The distribution of the void fraction is affected by the applied magnetic field and the gas bubble may be pushed away towards the both side walls due to the pinch effects, resulting in the ratio of the mean void fraction to the one at the center of the channel from less than unity to more than unity with the increasing applied magnetic field strength.

(3) The gas-liquid slip ratio is also little dependent directly on the applied magnetic field strength in spite of the disturbance of the void distribution due to the applied magnetic field and independent of the channel geometry. It is presented as a function of only the ratio of the gas-liquid volumetric flow rate β as follows:

$$S_f=0.3\beta+1.3\,,$$

with which the slip ratio is possible to be predicted for the given mass flow rates of the gas and liquid phases and the given system pressure. As a result, the slip ratio decreases with the increasing system pressure and/or the decreasing mixture quality.

[NOMENCLATURE]

a: Half width of channel
b: Half height of channel
B_0: Applied magnetic field
C: Ratio of electrical resistance between that of fluid and that of external circuit
D_b: Diameter of gas bubble
$K_1=\langle\alpha u_t\rangle/\langle\alpha\rangle\langle u_t\rangle$, $K_2=\langle\sigma_t u_t\rangle/\langle\sigma_t\rangle\langle u_t\rangle$
L: Length of channel, M: Mass flow rate
N: Effective magnetic interaction number $=\langle\sigma_t\rangle B_0^2 b/\langle\rho_t\rangle\langle u_t\rangle$
R_m: Effective magnetic Reynolds number $=\mu\langle\sigma_t\rangle\langle u_t\rangle b$
S_f: Gas-liquid slip ratio
u: Velocity of fluid
u_{by}: Drift velocity of gas bubble
α: Void fraction
β: Ratio of gas-liquid volumetric flow rate
ε: Load factor, μ: Magnetic permeability
σ: Electrical conductivity
ρ: Density of fluid
χ: Mixture quality$=M_g/(M_g+M_l)$

(Subscripts)

x, y, z: Quantities for x-, y- and z-direction, respectively
g: Gaseous phase, l: Liquid phase
s: Single phase, t: Two-phase

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