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Gauge Groups of Z_N Orbifold Models

Yasuhiko KATSUKI, Yoshiharu KAWAMURA,
Tatsuo KOBAYASHI, Noriyasu OHTSUBO*
and Kazutaka TANIOKA

Department of Physics, Kanazawa University, Kanazawa 920
**Kanazawa Institute of Technology, Ishikawa 921*

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Z_N orbifold models which have $N=1$ space-time supersymmetry are examined. All gauge groups of Z_N orbifold models are classified completely.

§ 1. Introduction

The $E_8 \times E_8$ heterotic string theory[1)] has drawn much attention as the unified theory of all known interactions. But it is ten-dimensional theory and has unrealistic gauge group $E_8 \times E_8$ with no matter field. We need several compactification schemes to lead to a four-dimensional theory. The toroidal compactification, which is the simplest way to reduce space-time dimensions, however, leads to four-dimensional theory with $N=4$ space-time supersymmetry.

To obtain four-dimensional theory with $N=1$ space-time supersymmetry, more realistic gauge group and matters, one of the most interesting ideas is the Z_N orbifold compactification.[2)] The Z_N orbifold is the quotient of an extra six-dimensional torus T^6 divided by a discrete rotation. It has been known that orders N of the discrete rotations to preserve only $N=1$ space-time supersymmetry should be 3, 4, 6, 7, 8 and 12.[3)]

The simplest orbifold among them is the Z_3 orbifold,[2)] whose models have been studied in detail and classified into five types (including a model with the unbroken $E_8 \times E_8$ gauge group). But these five models are far from realistic. That requires some mechanism to lead to *real world* theory, e.g., the Wilson line mechanism.

It has been shown in recent papers[4),5)] that the Z_4, Z_6 and Z_7 orbifold models are obtained through the same construction as one of Z_3 orbifold models and that the Z_4, Z_6 and Z_7 orbifold models have more diverse and smaller gauge groups than ones of Z_3 orbifold. The other Z_N orbifold models can be obtained in the same way as the above one and it can be expected that they also have more variant, smaller and more realistic gauge groups. Here, we study all gauge groups that can be got from Z_N orbifolds.

In § 2 we review the construction of Z_N orbifold models and the division of the six-dimensional space which preserve $N=1$ space-time supersymmetry. In § 3, we discuss breakings of an E_8 group and classifications of Z_N orbifold models. Conclusions and discussion are given in the last section.

§ 2. Z_N orbifold

Let us start from the ten-dimensional $E_8 \times E_8$ heterotic string in the bosonized form. That is a combination of ten-dimensional supersymmetric right movers and twenty-six-dimensional bosonic left movers. Momenta P^I ($I=1, \cdots, 16$) of the gauge left movers are on an $E_8 \times E_8$ root lattice $\Gamma_{E_8 \times E_8}$ and momenta p^t ($t=1, \cdots, 4$) of the bosonized fermionic right movers are on an SO_8 weight lattice Γ_{SO_8}. Physical massless states of the heterotic string correspond to a ten-dimensional supergravity multiplet coupled to $E_8 \times E_8$ super Yang-Mills fields. That means momenta P^I of massless states span an $E_8 \times E_8$ root system $\Lambda_{E_8} \oplus \Lambda_{E_8}$.

Next, let us discuss a construction of Z_N orbifold models to get realistic four-dimensional theory from the ten-dimensional theory. The Z_N orbifold is the quotient of a six-dimensional torus devided by one Z_N rotation, or the quotient of a six-dimensional Euclidean space devided by a space group which consists of the discrete rotation θ and discrete translations (shifts) e, represented by (θ, e). The rotation should be automorphisms of the lattice spanned by shifts. Of course, the orbifold differs from manifolds because the former has singular points (fixed points) while the latter does not. We shall not discuss fixed points in detail.

When the six-dimensional space is divided, we suppose the SO_8 weight lattice Γ_{SO_8} and the $E_8 \times E_8$ root lattice $\Gamma_{E_8 \times E_8}$ are divided simultaneously, i.e., in terms of elements,

$$
\begin{aligned}
&(\theta, 0) \text{ in } R^6 \longrightarrow (1, v^t) && \text{in } \Gamma_{SO_8}, \\
&\qquad\qquad (\Theta, 0) \text{ or } (1, V^I) && \text{in } \Gamma_{E_8 \times E_8}, \\
&(1, e) \text{ in } R^6 \longrightarrow (1, a) && \text{in } \Gamma_{E_8 \times E_8},
\end{aligned}
$$

where the v^t's, V^I's are shifts in the Γ_{SO_8} and $\Gamma_{E_8 \times E_8}$, respectively and the Θ represents some automorphism of the $\Gamma_{E_8 \times E_8}$, so that we could get $N=1$ space-time supersymmetry and more realistic and smaller gauge group. Here, the a's correspond to background gauge field, called "Wilson lines". We consider the case where the Wilson lines vanish. Note that we have two types of the embeddings of the space group into the $\Gamma_{E_8 \times E_8}$. One is an "automorphism embedding" type and the other is a "shift embedding" one. Remark that $\theta^N=1$ implies $\Theta^N=1$ and the NV^I is on the $\Gamma_{E_8 \times E_8}$, i.e.,

$$
N \sum_{I=1}^{8} V^I = N \sum_{I=9}^{16} V^I = 0 . \qquad (\text{mod } 2) \tag{2·1}
$$

Of closed strings on the orbifold, some are closed even in the torus. These are called untwisted strings and the others are called twisted strings, whose momenta and mass formulae differ from ones of the untwisted strings. It is remarkable that models should be constrained from the modular invariance, which is important in the string theory.

Now, our first problem is how many kinds of Z_N orbifolds are allowed to leave one unbroken space-time supersymmetry. So let us consider the six-dimensional torus formed through a division of six-dimensional Euclidean space by some root lattice of semisimple Lie algebra of rank 6. We describe a Weyl reflection corresponding to each simple root e_i as s_i. The Coxeter element c is defined as the product of all Weyl reflections, i.e.,

$$c = s_1 s_2 \cdots s_6 \,. \tag{2·2}$$

We get the Z_N orbifold by dividing a torus by a discrete rotation, i.e., the Coxeter element of the lattice or the generalized Coxeter element including an outer automorphism of the Lie algebra. This rotation is diagonalizable under a suitable complex basis,

$$\theta = \mathrm{diag}[\exp 2\pi i(\eta_1, \eta_2, \eta_3)] \,. \tag{2·3}$$

Further, v^t is put equal to η_α ($\alpha = 1, 2, 3$).

For massless states, momenta p^t of NS and R right movers belong to the weights of the SO_8 vector and spinor representations, respectively. Therefore supersymmetric charges correspond to the weights of the SO_8 conjugate spinor representation, which is represented by the u^t's. When the SO_8 weight lattice Γ_{SO_8} is divided in terms of shifts v^t, the number of unbroken supercharges is a half of the number of the u^t's satisfying the condition,

$$\sum_t v^t u^t = \text{integer} \,. \tag{2·4}$$

So, to leave one unbroken space-time supersymmetry, the above equation must have only two solutions u^t. Up to the SO_8 rotation, the above condition is equivalent to

$$v^1 + v^2 + v^3 = 1 \,, \tag{2·5}$$

where v^t ($t = 1, 2, 3,$) are non-zero.

Table I. Numbers of gauge groups in Z_N orbifold.

Point Group	Exponent η	6-dim. Lattice	No. of Gauge Groups Automor.	Shift
Z_3	$(1, 1, -2)/3$	SU_3^3	4+(1)	4+(1)
Z_4	$(1, 1, -2)/4$	SU_4^2	12	12
Z_6-I	$(1, 1, -2)/6$	$SU_3 \times G_2^2$	26	48
Z_6-II	$(1, 2, -3)/6$	$SU_6 \times SU_2$ $SU_3 \times SO_8$	28	54
Z_7	$(1, 2, -3)/7$	SU_7	2+(1)	39+(1)
Z_8-I	$(1, 2, -3)/8$	$SO_5 \times SO_8$ $SO_5 \times SO_9$	25	119
Z_8-II	$(1, 3, -4)/8$	$SO_4 \times SO_8$	24	120
Z_{12}-I	$(1, 4, -5)/12$	E_6 $SU_3 \times F_4$ $SU_3 \times SO_8$	92	581
Z_{12}-II	$(1, 5, -6)/12$	$SO_4 \times F_4$	110	603

Unbroken gauge group ($E_8 \times E_8$) are denoted by (1) in the fourth and fifth columns.

All lattices and discrete rotations corresponding to each lattice that satisfy the above condition have been known. A complete list of them is given in Table I. Exponents η and lattices are found in the second and third columns, respectively. Note that Z_6, Z_8 and Z_{12} have two types of rotations (type I and type II). Of Z_N orbifold models in Table I, Z_3, Z_4, Z_6 and Z_7 models have been classified.[2),4),5)] In this paper we shall complete classifications of gauge groups from Z_N orbifold models.

§ 3. Breaking of gauge group

In the previous section, we have mainly considered the division of the six-dimensional space. In this section, we discuss the division of the sixteen-dimensional space, i.e., breaking the gauge group. Momenta P^I of massless gauge left movers span the $E_8 \times E_8$ root system $\Lambda_{E_8} \oplus \Lambda_{E_8}$. First of all, let us investigate breaking of an E_8 group. As said in the previous section, there are two types of breakings of the gauge group, the automorphism type and the shift type. However any breaking through an automorphism can be equivalently realized through a shift. It has been shown there are 112 possible breakings of the E_8 through each shift corresponding to all automorphism.[6)] In the following, when we consider some automorphism, we shall consider the corresponding shift instead of automorphism.

When the E_8 root system Λ_{E_8} divided with respect to several shift V^I, unbroken gauge bosons are states whose momenta P^I (in Λ_{E_8}) satisfy the condition $\Sigma P^I V^I$ =integer. Then all we do is to look for group root system which consists of the P^I's satisfying the above condition for all the possible shifts V^I constrainted from the algebraic requirement Eq. (2·1), e.g., by computers.

There is an alternative intuitive and diagrammatical approach,[6)] which is reviewed in the following. We review that approach. (See Ref. 4) or 5) for the former.) The whole E_8 root system Λ_{E_8} is described by the extended Dynkin diagram of the E_8 group in Fig. 1, where the α_0 is the lowest root and the other α_i^I's ($i=1, \cdots, 8$) are simple roots of the E_8 group. Let k_i be an expansion coefficient of the highest root in terms of the simple root α_i, i.e.,

$$(k_1, k_2, \cdots, k_8)=(2, 4, 6, 5, 4, 3, 2, 3)\,. \tag{3·1}$$

Next, let k_0 be equal to one. It is convenient to expand shifts V^I in terms of the fundamental weights W_i^I of the E_8 group,

$$V^I=\frac{1}{N}\sum_i s_i W_i^I\,, \tag{3·2}$$

where the order N is obtained by

$$N=\sum_{i=0}^{8} s_i k_i\,. \tag{3·3}$$

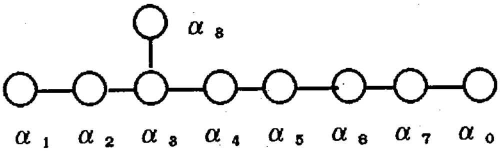

Fig. 1.

A product of this shift and some simple root α_i^I is

$$\sum_I \alpha_i^I V^I=\frac{1}{N}s_i\,, \tag{3·4}$$

and a product of this shift and the lowest root $\alpha_0{}^I$ is

$$\sum_I \alpha_0{}^I V^I = -\frac{1}{N}\sum_{i=1}^{8} k_i s_i = \frac{s_0}{N} - 1\,. \tag{3·5}$$

Therefore, if the s_i does not vanish, the i-th spot in the extended Dynkin diagram is broken except a trivial case $N=s_0$, so that the remaining Dynkin diagram represents a new group, which is a subgroup of the E_8 group. But the states whose momenta linearly depend on the α_i are still candidates for physical massless states.

Let us demonstrate the above breaking, e.g., through a shift $V^I=(1/3)W_6{}^I$. In that case, part of the Λ_{E_8} linearly independent of $\alpha_6{}^I$ corresponds to an adjoint representation of $E_6\times SU_3$ and the other part corresponds to a $(27, 3)$ representation under the group. In fact, it corresponds to the Z_3 orbifold models with "standard embedding" which has $E_6\times SU_3$ group and the $(27, 3)$ physical massless multiplet in the untwisted sector.

After all, we only have to classify combinations of coefficients s_i with the order N fixed to be 3, 4, 6, 7, 8 or 12. For example, in a case, $N=3$, there are five types, i.e.,

Table II. Gauge groups in E_8.

No.	Gauge Group	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}	No.	Gauge Group	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}
0	E_8	*	*	*	*	*	*	26	$SU_6\times SU_3\times U_1$			AS			AS
1	$E_7\times SU_2$		AS	AS		AS	AS	27	$SU_6\times SU_2^2\times U_1$			AS		S	AS
2	$E_7\times U_1$	AS	AS	AS	S	AS	AS	28	$SU_6\times SU_2\times U_1^2$				S	AS	AS
3	$E_6\times SU_3$	AS		AS			AS	29	$SU_6\times U_1^3$					AS	AS
4	$E_6\times SU_2\times U_1$		AS	S	S	AS	AS	30	$SU_5\times SU_4\times U_1$			AS	S	S	AS
5	$E_6\times U_1^2$			AS	S	S	AS	31	$SU_5\times SU_3\times SU_2\times U_1$				S	S	S
6	SO_{16}		AS	AS		AS	AS	32	$SU_5\times SU_3\times U_1^2$					AS	S
7	$SO_{14}\times U_1$	AS	AS	AS	S	AS	AS	33	$SU_5\times SU_2^2\times U_1^2$					AS	S
8	$SO_{12}\times SU_2\times U_1$		AS	AS		AS	AS	34	$SU_5\times SU_2\times U_1^3$						S
9	$SO_{12}\times U_1^2$			S	S	AS	AS	35	$SU_5\times U_1^4$						S
10	$SO_{10}\times SU_4$		AS			AS	AS	36	$SU_4^2\times SU_2\times U_1$					AS	AS
11	$SO_{10}\times SU_3\times U_1$			S	S	S	S	37	$SU_4^2\times U_1^2$					AS	AS
12	$SO_{10}\times SU_2^2\times U_1$			AS		S	AS	38	$SU_4\times SU_3\times SU_2^2\times U_1$					AS	S
13	$SO_{10}\times SU_2\times U_1^2$			AS	S	S	AS	39	$SU_4\times SU_3\times SU_2\times U_1^2$						S
14	$SO_{10}\times U_1^3$					AS	S	40	$SU_4\times SU_3\times U_1^3$						S
15	$SO_8\times SU_4\times U_1$			AS		AS	AS	41	$SU_4\times SU_2^3\times U_1^2$						AS
16	$SO_8\times SU_3\times U_1^2$				AS	AS	S	42	$SU_4\times SU_2^2\times U_1^3$						AS
17	$SO_8\times SU_2^2\times U_1^2$					AS	AS	43	$SU_4\times SU_2\times U_1^4$						AS
18	$SO_8\times SU_2\times U_1^3$						AS	44	$SU_3^3\times SU_2\times U_1$						AS
19	$SO_8\times U_1^4$						AS	45	$SU_3^3\times U_1^2$						AS
20	SU_9	AS		AS			AS	46	$SU_3^2\times SU_2^2\times U_1^2$						AS
21	$SU_8\times SU_2$		AS			AS	AS	47	$SU_3^2\times SU_2\times U_1^3$						AS
22	$SU_8\times U_1$		AS	AS	S	AS	AS	48	$SU_3\times SU_2^4\times U_1^2$						AS
23	$SU_7\times SU_2\times U_1$			S	S	S	S	49	$SU_3\times SU_2^3\times U_1^3$						AS
24	$SU_7\times U_1^2$			AS	S	S	AS		Total # of A	4	9	17	1	21	37
25	$SU_6\times SU_3\times SU_2$			AS			AS		Total # of S	4	9	21	14	30	49

Gauge groups realized by the shift (automorphism) of E_8 lattice are denoted by $S(A)$.

Table III. Gauge groups and shifts in Z_3 orbifold models.

No.	Gauge Group	Shift ($3V^J$)
0	E_8	*
1	$E_7\times U_1$	$(11000000)^A$
2	$E_6\times SU_3$	$(21100000)^A$
3	$SO_{14}\times U_1$	(20000000)
4	SU_9	$(21111000)^A$
Total # of Shifts (Auto.)		4(4)+ *

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

Table IV. Gauge groups and shifts in Z_4 orbifold models.

No.	Gauge Group	Shift ($4V^J$)
0	E_8	*
1	$E_7\times SU_2$	$(22000000)^A$
2	$E_7\times U_1$	$(11000000)^A$
3	$E_6\times SU_2\times U_1$	$(21100000)^A$
4	SO_{16}	$(40000000)^A$
5	$SO_{14}\times U_1$	$(20000000)^A$
6	$SO_{12}\times SU_2\times U_1$	$(31000000)^A$
7	$SO_{10}\times SU_4$	$(22200000)^A$
8	$SU_8\times SU_2$	$(31111100)^A$
9	$SU_8\times U_1$	$(1111111\text{-}1)^A$
Total # of Shifts (Auto.)		9(9)+ *

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

Table V. Gauge groups and shifts in Z_6 orbifold models.

No.	Gauge Group	Shift ($6V^J$)	
0	E_8	*	
1	$E_7\times SU_2$	$(33000000)^A$	
2	$E_7\times U_1$	(11000000)	$(22000000)^A$
3	$E_6\times SU_3$	$(42200000)^A$	
4	$E_6\times SU_2\times U_1$	(21100000)	
5	$E_6\times U_1^2$	$(32100000)^A$	
6	SO_{16}	$(60000000)^A$	
7	$SO_{14}\times U_1$	(20000000)	$(40000000)^A$
8	$SO_{12}\times SU_2\times U_1$	(42000000)	$(51000000)^A$
9	$SO_{12}\times U_1^2$	(31000000)	
10	$SO_{10}\times SU_3\times U_1$	(22200000)	
11	$SO_{10}\times SU_2^2\times U_1$	$(33200000)^A$	
12	$SO_{10}\times SU_2\times U_1^2$	$(41100000)^A$	
13	$SO_8\times SU_4\times U_1$	$(51110000)^A$	
14	SU_9	$(51111111)^A$	
15	$SU_8\times U_1$	(1111111-1)	$(5111111\text{-}1)^A$
16	$SU_7\times SU_2\times U_1$	(7111111-1)/2	
17	$SU_7\times U_1^2$	(31111111)	$(91111111)/2^A$
18	$SU_6\times SU_3\times SU_2$	$(51111100)^A$	
19	$SU_6\times SU_3\times U_1$	$(93311111)/2^A$	
20	$SU_6\times SU_2^2\times U_1$	$(3311111\text{-}1)^A$	
21	$SU_5\times SU_4\times U_1$	$(22222000)^A$	
Total # of Shifts (Auto.)		26 (17) + *	

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

Table VI. Gauge groups and shifts in Z_7 orbifold models.

No.	Gauge Group	Shift ($7V^J$)		
0	E_8	*		
1	$E_7\times U_1$	(11000000)	(11111111)	(33000000)
2	$E_6\times SU_2\times U_1$	(11111100)	(22222200)	(33333300)
3	$E_6\times U_1^2$	(22111111)		
4	$SO_{14}\times U_1$	(11110000)	(22220000)	(33330000)
5	$SO_{12}\times U_1^2$	(21111110)	(22221111)	(33220000)
6	$SO_{10}\times SU_3\times U_1$	(22111100)	(33222200)	(33331100)
7	$SO_{10}\times SU_2\times U_1^2$	(22221100)	(22222110)	(32222210)
8	$SO_8\times SU_3\times U_1^2$	(3222211-1)^A		
9	$SU_8\times U_1$	(1111111-1)	(2222222-2)	(4222222-2)
10	$SU_7\times SU_2\times U_1$	(2211111-1)	(22222220)	(3322222-2)
11	$SU_7\times U_1^2$	(22211110)	(3221111-1)	(33321000)
12	$SU_6\times SU_2\times U_1^2$	(32221110)	(2222221-1)	(3222222-1)
13	$SU_5\times SU_4\times U_1$	(2222111-1)	(3332211-1)	(3333111-1)
14	$SU_5\times SU_3\times SU_2\times U_1$	(3322111-1)	(33222110)	(3322221-1)
Total # of Shifts (Auto.)		38 (1) +*		

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

Table VII. Gauge groups and shifts in Z_8 orbifold models.

No.	Gauge Group	Shift ($8V^J$)				
0	E_8	*				
1	$E_7\times SU_2$	(22222222)^A				
2	$E_7\times U_1$	(11000000)	(11111111)^A	(33000000)		
3	$E_6\times SU_2\times U_1$	(11111100)	(22222200)^A	(33333300)		
4	$E_6\times U_1^2$	(22111111)	(22222211)			
5	SO_{16}	(80000000)^A				
6	$SO_{14}\times U_1$	(11110000)	(22220000)^A	(33330000)		
7	$SO_{12}\times SU_2\times U_1$	(32222221)	(33331111)^A	(44330000)		
8	$SO_{12}\times U_1^2$	(21111110)	(22221111)^A	(33220000)		
9	$SO_{10}\times SU_4$	(3333311-1)^A				
10	$SO_{10}\times SU_3\times U_1$	(22111100)	(33332200)			
11	$SO_{10}\times SU_2^2\times U_1$	(33222211)^A				
12	$SO_{10}\times SU_2\times U_1^2$	(22221100)	(22222110)	(33222200)	(33331100)	
13	$SO_{10}\times U_1^3$	(32222210)^A				
14	$SO_8\times SU_4\times U_1$	(44331100)^A				
15	$SO_8\times SU_3\times U_1^2$	(3222211-1)^A				
16	$SO_8\times SU_2^2\times U_1^2$	(33332110)^A				
17	$SU_8\times SU_2$	(4422222-2)^A				
18	$SU_8\times U_1$	(1111111-1)	(2222222-2)^A	(4442111-1)	(44431000)	
19	$SU_7\times SU_2\times U_1$	(2211111-1)	(3333332-2)			
20	$SU_7\times U_1^2$	(22211110)	(22222220)	(3221111-1)	(33321000)	(4222222-2)

(continued)

Table VII.

No.	Gauge Group	Shift $(8V^J)$				
21	$SU_6\times SU_2^2\times U_1$	(33222220)	(4332222-2)			
22	$SU_6\times SU_2\times U_1^2$	(2222221-1)	(32221110)	(3322222-2)	(33322210)	(4322222-1)^A
23	$SU_6\times U_1^3$	(3222222-1)^A				
24	$SU_5\times SU_4\times U_1$	(2222111-1)	(3333222-2)			
25	$SU_5\times SU_3\times SU_2\times U_1$	(3322111-1)	(3333221-1)			
26	$SU_5\times SU_3\times U_1^2$	(33222110)	(3332211-1)^A	(4332221-1)		
27	$SU_5\times SU_2^2\times U_1$	(3322221-1)^A				
28	$SU_4^2\times SU_2\times U_1$	(4333211-1)^A				
29	$SU_4^2\times U_1^2$	(3333111-1)^A				
30	$SU_4\times SU_3$	(3332222-1)^A				
Total # of Shifts (Auto.)		62 (22) + *				

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

Table VIII. Gauge groups and shifts in Z_{12} orbifold models.

No.	Gauge Group	Shift $(12V^J)$				
0	E_8	*				
1	$E_7\times SU_2$	(33333333)^A				
2	$E_7\times U_1$	(11000000)	(11111111)	(22222222)^A	(33000000)^A	(55000000)
3	$E_6\times SU_3$	(44444400)^A				
4	$E_6\times SU_2\times U_1$	(11111100)	(22222200)	(33333300)^A	(66111111)	
5	$E_6\times U_1^2$	(22111111) (44111111)	(22222211) (44444411)	(33222222)	(33333311)^A	(33333322)
6	SO_{16}	(12,0000000)^A				
7	$SO_{14}\times U_1$	(11110000)	(22220000)	(33330000)^A	(44440000)^A	(55550000)
8	$SO_{12}\times SU_2\times U_1$	(43333332)	(44442222)	(55551111)^A	(63333330)^A	(66550000)
9	$SO_{12}\times U_1^2$	(21111110) (33332222)	(22221111) (44330000)^A	(32222221) (44441111)	(33220000) (55220000)	(33331111) (55440000)
10	$SO_{10}\times SU_4$	(66333300)^A				
11	$SO_{10}\times SU_3\times U_1$	(22111100)	(3333311-1)	(55552200)		
12	$SO_{10}\times SU_2^2\times U_1$	(44333322)	(44444220)^A			
13	$SO_{10}\times SU_2\times U_1^2$	(22221100) (33332200) (44443300)^A (55551100)	(22222110) (33333221) (44443311) (55553300)	(33222200) (44333300)^A (4444421-1)	(33222211) (44441100) (44444310)	(33331100) (44442200)^A (55222200)
14	$SO_{10}\times U_1^3$	(32222210) (44331111)	(33332211) (44333311)	(33333210) (44442211)	(43333310) (55441111)	(43333321)
15	$SO_8\times SU_4\times U_1$	(5555311-1)^A	(66551100)			
16	$SO_8\times SU_3\times U_1^2$	(3222211-1)	(44331100)	(4444311-1)	(5444431-1)	(55441100)
17	$SO_8\times SU_2^2\times U_1^2$	(44443221)	(5444422-1)	(55442211)^A	(55552110)	
18	$SO_8\times SU_2\times U_1^3$	(33332110)	(4333321-1)	(43333220)	(44442110)^A	(55442200)

(continued)

Table VIII.

No.	Gauge Group	Shift $(12V^J)$				
19	$SO_8\times U_1^4$	$(44443210)^A$				
20	SU_9	$(4444444\text{-}4)^A$				
21	$SU_8\times SU_2$	$(6633333\text{-}3)^A$				
22	$SU_8\times U_1$	(1111111-1) (66651000)	(2222222-2)	$(3333333\text{-}3)^A$	$(6664111\text{-}1)^A$	(66642000)
23	$SU_7\times SU_2\times U_1$	(2211111-1)	(4422222-2)	(6631111-1)		
24	$SU_7\times U_1^2$	(22211110) (3333333-1) (44431000) (55532000)	(22222220) (33333331) (4444444-2) (55541000)	(3221111-1) (4222222-2) $(5333333\text{-}3)^A$ (65551000)	(33321000) (4431111-1) (54441000) (6663211-1)	(33322221) (4442111-1) (5444444-3)
25	$SU_6\times SU_3\times SU_2$	$(5553333\text{-}3)^A$				
26	$SU_6\times SU_3\times U_1$	(4444441-1)	$(4444442\text{-}2)^A$	$(4444443\text{-}3)^A$	(6653222-2)	
27	$SU_6\times SU_2^2\times U_1$	(44333331)	$(5533333\text{-}1)^A$	(6655111-1)		
28	$SU_6\times SU_2\times U_1^2$	(2222221-1) (3333332-2) (5522222-2) (6533333-2)	(32221110) (43332221) (5533333-3) (6543333-3)	(3322222-2) (4433333-3) (55532220) (65551110)	(33222220) (44433321) (55542111)	(3333331-1) $(54333330)^A$ (6444443-3)
29	$SU_6\times U_1^3$	(3222222-1) $(4333333\text{-}2)^A$ (4444443-1) (6433333-3)	(33322210) (43333330) (5333333-1)	(33333320) (44431110) (5432222-2)	(4322222-1) (44432111) (54441110)	(43322220) (44433310) (55541110)
30	$SU_5\times SU_4\times U_1$	(2222111-1)	$(4444222\text{-}2)^A$	(5555222-2)		
31	$SU_5\times SU_3\times SU_2\times U_1$	(3322111-1)	(5555332-2)			
32	$SU_5\times SU_3\times U_1^2$	(33222110) (4442222-2) (6444333-3)	(3332211-1) (4444422-2) (65552100)	(33332220) (5542221-1)	(4433311-1) (5544421-1)	(44333221) (55543200)
33	$SU_5\times SU_2^2\times U_1^2$	(3322221-1) (44443320) (5544111-1)	(3333221-1) (4444433-2) (5544333-3)	(4332222-2) (54443310) (5544443-3)	(4433331-1) (5533222-2)	(4444331-1) (5533332-2)
34	$SU_5\times SU_2\times U_1^3$	(3333322-1) (44333320) (4444322-1) (5443333-3)	(4332221-1) (4433333-1) (4444432-1) (5444443-2)	(43332210) (44432100) (5333332-2) (55542100)	(4432222-1) (4443321-1) (5443331-1) (55542210)	(44333210) (44433220) (54433320)
35	$SU_5\times U_1^4$	(4333332-1)	(5433332-1)	(5433333-2)		
36	$SU_4^2\times SU_2\times U_1$	$(5544422\text{-}2)^A$	(6633332-2)			
37	$SU_4^2\times U_1^2$	(3333111-1) (5554322-2)	(3333222-2) (5555111-1)	(4444111-1) (6555211-1)	$(4444333\text{-}3)^A$	(5444433-3)
38	$SU_4\times SU_3\times SU_2^2\times U_1$	(44433330)				
39	$SU_4\times SU_3\times SU_2\times U_1^2$	(3332222-1) (5544222-2)	(4333211-1) (5555221-1)	(4433222-2)	(44332220)	(4444333-1)

(continued)

Table VIII.

No.	Gauge Group	Shift $(12V^I)$				
40	$SU_4\times SU_3\times U_1^3$	(4333222-1) (5433322-2) (5554211-1)	(4333322-2) (5443222-2) (55543000)	(4443222-1) (5444322-2) (6543332-2)	(4443333-2) (5444432-2)	(44442220) (55442110)
41	$SU_4\times SU_2^3\times U_1^2$	$(5443333\text{-}1)^A$	(54443220)	$(5543333\text{-}2)^A$	(6443333-2)	(6544322-2)
42	$SU_4\times SU_2^2\times U_1^3$	(4433322-1) $(5444321\text{-}1)^A$	$(4433332\text{-}2)^A$ (5444332-1)	(4444221-1) (5543332-1)	$(4444332\text{-}2)^A$ $(55543110)^A$	(5433222-1)
43	$SU_4\times SU_2\times U_1^4$	$(5443322\text{-}1)^A$				
44	$SU_3^3\times SU_2\times U_1$	$(5544331\text{-}1)^A$				
45	$SU_3^3\times U_1^2$	$(5553332\text{-}2)^A$				
46	$SU_3^2\times SU_2^2\times U_1^2$	(4433221-1)	$(4443322\text{-}2)^A$	(5544332-2)		
47	$SU_3^2\times SU_2\times U_1^3$	(4443332-1)	$(5444222\text{-}1)^A$	(5444333-2)	$(5543322\text{-}2)^A$	
48	$SU_3\times SU_2^4\times U_1^2$	$(5544322\text{-}1)^A$				
49	$SU_3\times SU_2^3\times U_1^3$	$(5443332\text{-}2)^A$				
Total # of Shifts (Auto.)		269 (49) + *				

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism.

$$(s_1, s_2, \cdots, s_8, s_0)=\begin{cases}(1,0,0,0,0,0,0,0,1)\,,\\(0,0,0,0,0,1,0,0,0)\,,\\(0,0,0,0,0,0,1,0,1)\,,\\(0,0,0,0,0,0,0,1,0)\,,\\(0,0,0,0,0,0,0,0,3)\,,\end{cases}$$

where the last are trivial, i.e., unbroken. Under a suitable basis, explicit examples of shifts leading to each group are found in Table III. (In the following tables gauge groups of the other Z_N orbifolds and examples of corresponding shifts are listed.) All actions of these shifts are also realized through some automorphism of order 3, hence a breaking by any automorphism of order 3 is realized through a shift among the above five. The same phenomenon also appears in order 4, where are ten types of shifts (automorphism). The groups broken from the E_8 through these shifts are in the second column of Table II, where the symbols $\boldsymbol{S}$ mean that given groups are broken by shifts and the symbols $\boldsymbol{A}$ mean that given groups are broken by shifts corresponding to automorphisms.

The following columns are groups broken from the E_8 in the same way as the above with the other orders. The table shows that the shift breaking is not always equivalent to the automorphism breaking. The models from shift breakings always include gauge groups from automorphism breaking.

Up to now, we have discussed division of six-dimensional space and breaking of one E_8 group, separately. To consider the whole gauge group, i.e., combinations of groups broken from each E_8, it is neccessary to consider closed strings on the orbifold as the whole. Namely, we combine the six-dimensional compact space (orbifold) with the sixteen-dimensional torus of gauge group. Not all of combinations are allowed, because the models should be modular invariant. The condition of the modular invariance (the level matching condition) is

$$N\sum_{t=1}^{4}(v^t)^2=N\sum_{I=1}^{16}(V^I)^2 . \qquad (\text{mod } 2) \tag{3·6}$$

Let us investigate whole possible combinations of V^I ($I=1, \cdots, 8$) and V^J ($J=9, \cdots, 16$) with N and v^t fixed, using Tables III~VIII, so that we get all the possible gauge groups of Z_N orbifold models.

In the result, the numbers of models through automorphism and shift breaking are given in the fourth and the last columns, respectively. It is remarkable that the unbroken gauge group $E_8\times E_8$ is allowed in the cases of the Z_3 and Z_7 orbifolds, but not in the other cases.

§ 4. Conclusion and remark

We have classified Z_N orbifold models with vanishing Wilson lines in terms of gauge groups, completely. We have got various groups. It is remarkable that models through automorphism and shift breakings are equivalent for the Z_3 and Z_4 orbifolds but not for the others. Among the given models, *a la* flipped $SU(5)\times U(1)\times G_{11}$ gauge groups (G_{11} is several groups of rank 11), i.e., models including Nos. 30 ~35 in Table II are found in the Z_6, Z_7, Z_8 and Z_{12} orbifold models. Several standard-like gauge groups, e.g., $SU_3\times SU_2\times U_1\times G_{12}$ (Nos. 31, 38, 39, 44 and 46~49 in Table II) also appear among the Z_8 and Z_{12} orbifold models. If G_{11} or G_{12} is completely hidden, the model might be promising.

For the above models to be the standard model (or the flipped $SU_5\times U_1$ models), the matters of models should completely decouple into an observable part and a hidden one, and they represent the suitable multiplets corresponding to the matters of the standard model under the group. Therefore, we have to study matter contents of the given models. They have been studied for the Z_3, Z_4, Z_6 and Z_7 orbifolds. They are under investigation at present for the others.

When we study matter contents we might find that several U_1 might be anomalous. The anomalous U_1 suggests that vacuum of the model is not stable and leads to the breaking of supersymmetry. Such a mechanism requires the anomalous U_1 symmetry should be broken, and it might lead to further gauge symmetry breaking followed by rank reduction of the gauge group.[7),8)]

Furthermore, there is an alternative mechanism to break the gauge group, that is the Wilson line mechanism. In that case, the rank of the gauge group is reduced by non-commutability between two independent divisions corresponding to rotation and shifts of the space group. This mechanism gives us *thousand and one* models. Through this mechanism, we may have gauge groups including new U_1's. These U_1 symmetry might be anomalous and the above U_1 breaking could be applied to this case.

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