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Gauge-Higgs Unification in Higher Dimensions

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Abstract

The electroweak Higgs doublets are identified as components of a vector multiplet in a higher dimensional supersymmetric field theory. We construct a minimal model in 6D where the electroweak $SU(2) \otimes U(1)$ gauge group is extended to $SU(3)$, and unified 6D models with the unified $SU(5)$ gauge symmetry extended to $SU(6)$. In these realistic theories the extended gauge group is broken by orbifold boundary conditions, leaving Higgs doublet zero modes which have Yukawa couplings to quarks and leptons on the orbifold fixed points. In one $SU(6)$ model the weak mixing angle receives power law corrections, while in another the fixed point structure forbids such corrections. A 5D model is also constructed in which the Higgs doublet contains the fifth component of the gauge field. In this case Yukawa couplings are introduced as non-local operators involving the Wilson line of this gauge field.

1 Introduction

Precision electroweak data suggest that the weak interactions are broken by the vacuum expectation value of a scalar field: the Higgs boson. Yet, the quadratic divergence in the Higgs mass tells us that the Higgs should become something other than just a scalar field at energies not far above the weak scale. What is the fundamental origin of the Higgs boson? A first step, which we adopt in this paper, is that the Higgs is a component of a 4D supersymmetric chiral multiplet. An economical possibility would be that the Higgs doublet is identified as the supersymmetric partner of the left-handed lepton doublet, but this has been hard to implement. This would have made significant progress in understanding the origin of the Higgs: instead of the three types of fields of the standard model (gauge, chiral matter and Higgs), the supersymmetric theory would have only two types of fields: vector multiplets and chiral multiplets which are chiral under the gauge group. Instead we are driven to the minimal supersymmetric theory where a third type of multiplet is added: a pair of Higgs doublets in chiral multiplets which are vector-like under the gauge group. Even though there are several ways to obtain such light vector-like Higgs multiplets, one cannot help but feel that the theory would look more elegant without them.

In this paper we study higher dimensional supersymmetric theories in which there are only two types of fields: a vector multiplet containing the gauge bosons, and matter multiplets which are chiral under the gauge group containing quarks and leptons. The gauge group is enlarged beyond that of the standard model and is broken by compactification on an orbifold, which nevertheless preserves a single supersymmetry. As expected, the resulting massless modes are found to include 4D vector multiplets and chiral multiplets which are chiral under the unbroken gauge group. We also find that there can be zero modes in chiral multiplets which are vector-like under the gauge group. These 4D Higgs multiplets originate from the higher dimensional vector multiplet. In this paper we identify the Higgs doublets as remnant zero modes of the higher dimensional vector multiplet. The vector multiplet transforms in the adjoint representation of the gauge group and, for the standard model gauge group, this does not contain weak doublets, hence the gauge group must be enlarged.

A simple implementation of this idea is for the Higgs doublets to be the higher dimensional components of the gauge fields [1]. This idea does not require supersymmetry — the quadratic divergence of the Higgs mass at low energies is regulated by local gauge invariance in the higher dimensions. However, in higher dimensional supersymmetric theories, the vector multiplet contains scalars of the higher dimensional Lorentz symmetry, allowing alternative identifications of the Higgs doublets.

An immediate objection to the Higgs boson originating from a vector multiplet is that in-

dependent Yukawa couplings of the Higgs to matter are forbidden by higher dimensional gauge and supersymmetries. Apparently the rich structure of Yukawa couplings of the standard model must somehow all arise from the gauge interaction, which would presumably have to be very complicated. This objection largely disappears in the case that the extra dimensional spacetime is an orbifold and that the orbifold boundary conditions break some of the gauge and supersymmetries. At some orbifold fixed points certain gauge transformation parameters are constrained to vanish, so that these fixed points feel only a restricted gauge symmetry [2]. While the Higgs is a component of the vector multiplet in the bulk, as far as these fixed points are concerned they are components of matter type multiplets, and the restricted symmetries may allow independent Yukawa interactions to be located on these fixed points.

In this paper we seek to implement this gauge origin for the Higgs doublets in simple, realistic effective field theories. Any such theory involves several choices: the number of extra dimensions, the number of supersymmetries, the orbifold spacetime and the gauge group. With a single extra dimension, the 5D vector multiplet contains a 4D vector multiplet and a 4D chiral adjoint field: (V, Φ) . The Higgs doublets would lie in Φ , and would therefore contain A_5 , the component of the gauge field in the fifth dimension. In this case, even though the broken gauge transformation parameters, $\xi(y)$, may vanish on an orbifold fixed point, the derivatives, $\partial_y \xi$, do not. Thus the Higgs will have an inhomogeneous transformation under the broken gauge generators, forbidding independent local Yukawa couplings from appearing at the fixed point — the objection of the previous paragraph remains. This situation is unchanged in 6D with $N = 1$ supersymmetry. Furthermore the 6D $N = 1$ vector multiplet has anomalies. We are therefore led to 6D $N = 2$ theories. The vector multiplet is anomaly free, and contains three chiral adjoints (V, Φ_i) . Since there are only two extra components of the gauge field, $A_{5,6}$, at least one of Φ_i does not contain any gauge fields and therefore gauge transforms homogeneously. Higgs doublets arising from such a Φ_i may have local Yukawa couplings at fixed points. The three models presented in sections 2, 3 and 4 all have such an origin for the Higgs doublets within $N = 2$ supersymmetry in 6D.

The simplest extension of the standard model gauge group, which gives weak doublets in the adjoint representation, is to embed $SU(2) \otimes U(1)$ into $SU(3)$, and this minimal case is explored in section 2. Indeed, the only addition to the adjoint is two weak doublets. This extension does not increase the rank of the gauge group, and hence the orbifold breaking to the standard model gauge group is particularly straightforward. In sections 3 and 4 we explore extending the $SU(5)$ grand unified theory to $SU(6)$. As well as weak doublets, the addition to the adjoint now contains color triplets. However, we find that orbifold gauge symmetry breaking can remove the unwanted colored triplets, by an extension of the $SU(5)$ case [3].

For a 1D bulk there is a unique orbifold of finite size: S^1/Z_2 , while for 2D there are many

possibilities. The Higgs can originate from a 6D vector multiplet by symmetry breaking on many 2D orbifolds. In sections 2 and 3 we make the simple choice of $T^2/(Z_2 \times Z'_2)$, which repeats the S^1/Z_2 structure in both of the extra dimensions. Nevertheless, the fixed point structure and therefore the nature of the Kaluza-Klein (KK) towers, is very sensitive to the orbifold choice. In the $SU(6)$ theory of section 3 we find that the KK towers lead to power law running of gauge couplings which is not $SU(5)$ invariant. This gives a power law correction to the weak mixing angle. In section 4 we construct an alternative $SU(6)$ theory, with symmetry breaking on T^2/Z_4 , where such power law corrections are absent.

In section 5 we return to the case that the Higgs doublets contain higher dimensional components of gauge fields. Although local Yukawa couplings are forbidden, the Higgs may couple to quarks and leptons via non-local interactions involving Wilson lines. We do not consider how such non-local interactions may be generated, but simply assume that all gauge invariant interactions occur in the effective field theory, local or not. In this case we are able to construct 5D theories with gauge symmetry broken on S^1/Z_2 .

Our discussions include further aspects of these models, including supersymmetry breaking and the location of quarks and leptons. We discuss the possibility that the third generation resides on an $SU(5)$ invariant 3 brane, yielding the successful b/τ mass relation, while the lighter two generations reside on a 4 brane and therefore have suppressed, non- $SU(5)$ invariant Yukawa couplings.

2 6D $SU(3)_C \otimes SU(3)_L$ Model on $T^2/(Z_2 \times Z'_2)$

In this section we present a minimal model which realizes the idea that the Higgs fields are components of the gauge supermultiplet in higher dimensions. Unlike previous works [1], the Higgs bosons here are not extra dimensional components of the gauge field, but rather scalar fields that are supersymmetric partners of the gauge field in higher dimensions.

2.1 Orbifold and Gauge Structure

We consider a 6D gauge theory with $N = 2$ supersymmetry. The extra dimensions are compactified on a $T^2/(Z_2 \times Z'_2)$ orbifold with radii $R_5 \sim R_6$. The $N = 2$ supersymmetry in 6D corresponds to $N = 4$ supersymmetry in 4D, so that only the gauge multiplet can be introduced in the bulk. We take an $SU(3)_C \otimes SU(3)_L$ gauge multiplet propagating in the bulk. This multiplet can be decomposed under a 4D $N = 1$ supersymmetry into a vector supermultiplet V and three chiral multiplets Σ_5 , Σ_6 and Φ in the adjoint representation. The fifth and sixth components of the

gauge field, A_5 and A_6 , are contained in the lowest component of Σ_5 and Σ_6 , respectively.

Using the 4D $N = 1$ language, the bulk action is written as [4]

$$\begin{aligned}
S = & \int d^6x \left\{ \text{Tr} \left[\int d^2\theta \left(\frac{1}{4kg^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{kg^2} \left(\Phi \partial_5 \Sigma_6 - \Phi \partial_6 \Sigma_5 - \frac{1}{\sqrt{2}} \Phi [\Sigma_5, \Sigma_6] \right) \right) + \text{h.c.} \right] \right. \\
& + \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[(\sqrt{2} \partial_5 + \Sigma_5^\dagger) e^{-V} (-\sqrt{2} \partial_5 + \Sigma_5) e^V + (\sqrt{2} \partial_6 + \Sigma_6^\dagger) e^{-V} (-\sqrt{2} \partial_6 + \Sigma_6) e^V \right. \\
& \left. \left. + \Phi^\dagger e^{-V} \Phi e^V + \partial_5 e^{-V} \partial_5 e^V + \partial_6 e^{-V} \partial_6 e^V \right] \right\}, \tag{1}
\end{aligned}$$

in the Wess-Zumino gauge. The $SU(3)_C \otimes SU(3)_L$ gauge transformation is given by

$$e^V \rightarrow e^\Lambda e^V e^{\Lambda^\dagger}, \tag{2}$$

$$\Sigma_5 \rightarrow e^\Lambda (\Sigma_5 - \sqrt{2} \partial_5) e^{-\Lambda}, \tag{3}$$

$$\Sigma_6 \rightarrow e^\Lambda (\Sigma_6 - \sqrt{2} \partial_6) e^{-\Lambda}, \tag{4}$$

$$\Phi \rightarrow e^\Lambda \Phi e^{-\Lambda}. \tag{5}$$

Here, we have used a short-handed notation for the $SU(3)_C \otimes SU(3)_L$ gauge structure as $\Lambda \equiv (\Lambda_C^a T_C^a \oplus \Lambda_L^a T_L^a)$, $V \equiv (V_C^a T_C^a \oplus V_L^a T_L^a)$, $\Sigma_5 \equiv (\Sigma_{5,C} \oplus \Sigma_{5,L})$, $\Sigma_6 \equiv (\Sigma_{6,C} \oplus \Sigma_{6,L})$, and $\Phi \equiv (\Phi_C \oplus \Phi_L)$, where T_C^a and T_L^a ($a = 1, \dots, 8$) are the generators of the $SU(3)_C$ and $SU(3)_L$ gauge groups, respectively. Similarly, the gauge coupling g should also be understood to contain two gauge couplings g_C and g_L for $SU(3)_C$ and $SU(3)_L$: $1/g^2 \equiv (1/g_C^2 \oplus 1/g_L^2)$.

Note that the $\Sigma_{5,6}$ fields transform non-linearly under the gauge transformation, since they contain $A_{5,6}$ as lowest components. This prevents us from writing down local operators which couple $\Sigma_{5,6}$ to matter fields on the orbifold fixed points, making it difficult to identify Σ_5 or Σ_6 as the Higgs field.¹ Fortunately, the 6D $N = 2$ gauge multiplet contains an additional adjoint chiral superfield Φ that does not contain components of the higher dimensional gauge bosons and that can thus couple to the matter fields localized on the fixed point. We will soon identify components of these superfields as the Higgs doublets of the minimal supersymmetric standard model (MSSM).

We now describe the model, following the notation of Ref. [5]. The orbifold $T^2/(Z_2 \times Z_2)$ is constructed by identifying points of the infinite plane R^2 under four operations, $\mathcal{Z}_5 : (x^5, x^6) \rightarrow (-x^5, x^6)$, $\mathcal{Z}_6 : (x^5, x^6) \rightarrow (x^5, -x^6)$, $\mathcal{T}_5 : (x^5, x^6) \rightarrow (x^5 + 2\pi R_5, x^6)$ and $\mathcal{T}_6 : (x^5, x^6) \rightarrow$

¹ Matter can couple to $A_{5,6}$ through non-local operators containing $\mathcal{P} \exp(\oint(dx^5 A_5 + dx^6 A_6))$, which may be generated by integrating out some physics at the compactification scale. We present a model of this kind in section 5.

$(x^5, x^6 + 2\pi R_6)$. Here, for simplicity, we have taken the two translations \mathcal{T}_5 and \mathcal{T}_6 to be in orthogonal directions. In general, various fields $\varphi(x^5, x^6)$ transform nontrivially under these operations as $\varphi(x^5, x^6) \rightarrow \mathcal{Z}_{5,6}[\varphi(x^5, x^6)]$ and $\varphi(x^5, x^6) \rightarrow \mathcal{T}_{5,6}[\varphi(x^5, x^6)]$. The consistency condition requires that these transformations must be symmetries of the bulk action. Then, the above identification is made by imposing the conditions $\mathcal{Z}_5[\varphi(x^5, x^6)] = \mathcal{Z}_6[\varphi(x^5, x^6)] = \mathcal{T}_5[\varphi(x^5, x^6)] = \mathcal{T}_6[\varphi(x^5, x^6)] = \varphi(x^5, x^6)$ for all the bulk fields present in the model.

Under the $\mathcal{Z}_{5,6}$ operations, various fields in a single irreducible gauge representation can transform differently. Thus, the gauge symmetry can be broken by identifications under $\mathcal{Z}_{5,6}$. We here require V to transform nontrivially under $\mathcal{Z}_{5,6}$ such that only $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ components have massless modes. This is accomplished by taking \mathcal{Z}_5 and \mathcal{Z}_6 identifications as

$$V(-x^5, x^6) = P_Z V(x^5, x^6) P_Z^{-1}, \quad (6)$$

$$\Sigma_5(-x^5, x^6) = -P_Z \Sigma_5(x^5, x^6) P_Z^{-1}, \quad (7)$$

$$\Sigma_6(-x^5, x^6) = P_Z \Sigma_6(x^5, x^6) P_Z^{-1}, \quad (8)$$

$$\Phi(-x^5, x^6) = -P_Z \Phi(x^5, x^6) P_Z^{-1}, \quad (9)$$

and

$$V(x^5, -x^6) = P_Z V(x^5, x^6) P_Z^{-1}, \quad (10)$$

$$\Sigma_5(x^5, -x^6) = P_Z \Sigma_5(x^5, x^6) P_Z^{-1}, \quad (11)$$

$$\Sigma_6(x^5, -x^6) = -P_Z \Sigma_6(x^5, x^6) P_Z^{-1}, \quad (12)$$

$$\Phi(x^5, -x^6) = -P_Z \Phi(x^5, x^6) P_Z^{-1}, \quad (13)$$

respectively. Here, P_Z is given by

$$P_Z = \text{diag}(1, 1, 1) \oplus \text{diag}(1, 1, -1). \quad (14)$$

Note that various signs appearing in Eqs. (6 – 13) are determined by invariance of the bulk action under the $\mathcal{Z}_{5,6}$ operations.

The \mathcal{Z}_5 identification breaks 4D $N = 4$ supersymmetry to 4D $N = 2$ supersymmetry (or equivalently, 6D $N = 2$ to 6D $N = 1$ supersymmetry), with (V, Σ_6) forming a vector multiplet and (Σ_5, Φ) forming a hypermultiplet. Similarly, the \mathcal{Z}_6 identification breaks 4D $N = 4$ supersymmetry to 4D $N = 2$ supersymmetry, with (V, Σ_5) forming a vector multiplet and (Σ_6, Φ) forming a hypermultiplet. This means that the two $N = 2$ supersymmetries remaining after the \mathcal{Z}_5 and \mathcal{Z}_6 operations are different subgroups of the original $N = 4$ supersymmetry. Thus, the combination of \mathcal{Z}_5 and \mathcal{Z}_6 identifications, *i.e.* the $T^2/(Z_2 \times Z'_2)$ compactification, breaks the original 6D $N = 2$ supersymmetry all the way down to 4D $N = 1$ supersymmetry.

Under the $\mathcal{T}_{5,6}$ operations also, various fields in a single irreducible gauge representation can transform differently, breaking the gauge symmetry. In the present $SU(3)_C \otimes SU(3)_L$ model, we do not introduce this non-trivial transformation in the gauge space for the $\mathcal{T}_{5,6}$ operations. However, for later use, we write \mathcal{T}_5 and \mathcal{T}_6 identifications as

$$V(x^5 + 2\pi R_5, x^6) = P_T V(x^5, x^6) P_T^{-1}, \quad (15)$$

$$\Sigma_5(x^5 + 2\pi R_5, x^6) = P_T \Sigma_5(x^5, x^6) P_T^{-1}, \quad (16)$$

$$\Sigma_6(x^5 + 2\pi R_5, x^6) = P_T \Sigma_6(x^5, x^6) P_T^{-1}, \quad (17)$$

$$\Phi(x^5 + 2\pi R_5, x^6) = P_T \Phi(x^5, x^6) P_T^{-1}, \quad (18)$$

and

$$V(x^5, x^6 + 2\pi R_6) = P_T V(x^5, x^6) P_T^{-1}, \quad (19)$$

$$\Sigma_5(x^5, x^6 + 2\pi R_6) = P_T \Sigma_5(x^5, x^6) P_T^{-1}, \quad (20)$$

$$\Sigma_6(x^5, x^6 + 2\pi R_6) = P_T \Sigma_6(x^5, x^6) P_T^{-1}, \quad (21)$$

$$\Phi(x^5, x^6 + 2\pi R_6) = P_T \Phi(x^5, x^6) P_T^{-1}, \quad (22)$$

respectively, with P_T given by

$$P_T = \text{diag}(1, 1, 1) \oplus \text{diag}(1, 1, 1). \quad (23)$$

Thus, Eqs. (15 – 22) just give periodic boundary conditions for all the fields. Again, no additional signs can be introduced in the above transformations due to the requirement of invariance of the bulk action under $\mathcal{T}_{5,6}$.

In general, we could use different P_Z matrices for \mathcal{Z}_5 and \mathcal{Z}_6 (Eqs. (6 – 9) and Eqs. (10 – 13)), and different P_T matrices for \mathcal{T}_5 and \mathcal{T}_6 (Eqs. (15 – 18) and Eqs. (19 – 22)). Here we have chosen the same P_Z, P_T matrices for the fifth and sixth directions. This can enhance the symmetry of the system: we have an extra symmetry described by $x^5 \leftrightarrow x^6$ and $\Sigma_5 \leftrightarrow \Sigma_6$ if $R_5 = R_6$. This implies that the choice is a natural one. It may also be important for fixing an unwanted moduli field, R_5/R_6 and θ_T (angle between \mathcal{T}_5 and \mathcal{T}_6), at the symmetry enhanced point $R_5/R_6 = 1$ and $\theta_T = \pi/2$. (We comment on the phenomenology of the case $R_5 \gg R_6$ later in this section.)

Having identified all the boundary conditions, let us consider the massless bulk fields in the model. To work this out, we consider the transformation properties for the fields under $\mathcal{Z}_{5,6}$. Since massless modes can arise only from fields that are even under both $x^5 \rightarrow -x^5$ and $x^6 \rightarrow -x^6$, we need only consider the components of V and Φ . Under the parities, the various

components transform as

$$V_C : \begin{pmatrix} (+, +) & (+, +) & (+, +) \\ (+, +) & (+, +) & (+, +) \\ (+, +) & (+, +) & (+, +) \end{pmatrix}, \quad V_L : \left(\begin{array}{cc|c} (+, +) & (+, +) & (-, -) \\ (+, +) & (+, +) & (-, -) \\ \hline (-, -) & (-, -) & (+, +) \end{array} \right), \quad (24)$$

$$\Phi_C : \begin{pmatrix} (-, -) & (-, -) & (-, -) \\ (-, -) & (-, -) & (-, -) \\ (-, -) & (-, -) & (-, -) \end{pmatrix}, \quad \Phi_L : \left(\begin{array}{cc|c} (-, -) & (-, -) & (+, +) \\ (-, -) & (-, -) & (+, +) \\ \hline (+, +) & (+, +) & (-, -) \end{array} \right), \quad (25)$$

where the first and second signs represent parities under $x^5 \rightarrow -x^5$ and $x^6 \rightarrow -x^6$, respectively.

From Eq. (24) we find that the low-energy gauge group is indeed $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, which we identify as the standard-model gauge group. In addition to these vector multiplets, however, extra massless modes arise from Φ fields. The quantum numbers of these extra massless states under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ are read off from Eqs. (25) as $(\mathbf{1}, \mathbf{2}, 1/2) \oplus (\mathbf{1}, \mathbf{2}, -1/2)$, which are exactly the correct quantum numbers for the two Higgs doublets, H_U and H_D , of the MSSM. (Here, we have normalized the $U(1)_Y$ charges to match convention.) In the next sub-section, we identify these extra massless states as the Higgs doublets and couple them to quarks and leptons on the orbifold fixed point.

2.2 Fixed Points and Quarks and Leptons

The $T^2/(Z_2 \times Z'_2)$ orbifold has four fixed points at $(x^5, x^6) = (0, 0), (\pi R_5, 0), (0, \pi R_6)$ and $(\pi R_5, \pi R_6)$. To understand what types of matter fields and interactions can be placed on a fixed point, we have to work out the symmetry structures of the fixed point [2, 5]. This can be done by investigating the profile of the symmetry transformation parameters in the extra dimension. We find that the gauge transformation parameters for $SU(3)_L/(SU(2)_L \otimes U(1)_Y)$ vanish on the four fixed points, so that the gauge symmetry on the fixed points is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. These four fixed points are connected by four fixed lines, on which the only non-trivial gauge transformations are again those of the standard model. As for supersymmetry, three of the four supersymmetry transformation parameters vanish on the four fixed points, so that the remaining supersymmetry on the fixed points is 4D $N = 1$ supersymmetry. (The supersymmetry on the four fixed lines is 4D $N = 2$ supersymmetry.) Therefore, we find that the original bulk symmetry is reduced to 4D $N = 1$ supersymmetry and $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry on each of the four fixed points. In fact, these four fixed points are completely equivalent due to the symmetry of the system. The matter fields and interactions located on the fixed points need (only) respect these symmetries.

	θ^α	V	Σ_5	Σ_6	Φ	Q	U	D	L	E	N
$Z_{4,R}$	1	0	0	0	2	-1	1	1	-1	1	1

Table 1: $Z_{4,R}$ charge assignment for the $SU(3)_C \otimes SU(3)_L$ model.

The 6D $N = 2$ supersymmetry prevents us from introducing quarks and leptons in the bulk, so that they must be localized on the orbifold fixed points or fixed lines. We here introduce quark and lepton chiral superfields, Q, U, D, L and E , on the $(x^5, x^6) = (0, 0)$ fixed point. The $N = 1$ supersymmetric Yukawa couplings are also introduced on this fixed point:

$$\mathcal{L}_6 \supset \delta(x^5)\delta(x^6) \int d^2\theta (\lambda_U QUH_U + \lambda_D QDH_D + \lambda_E LEH_D), \quad (26)$$

where the two Higgs doublets, H_U and H_D , are components of the Φ field in the higher dimensional $SU(3)_C \otimes SU(3)_L$ gauge multiplet. With these Yukawa couplings, the theory reduces to the MSSM below the compactification scale.

One can also induce small neutrino masses through the see-saw mechanism [6] by introducing right-handed neutrino superfields N on the fixed point.

2.3 R Symmetry

The bulk action Eq. (1) possesses a $U(1)_R$ symmetry. This $U(1)_R$ symmetry is extended to the full theory by assigning appropriate charges for the quark and lepton superfields. Here we impose the discrete Z_4 subgroup of this $U(1)_R$ symmetry on the model. The charge assignment of this $Z_{4,R}$, which allows the Yukawa couplings of Eq. (26) (and the Yukawa couplings and Majorana masses for N), is given in Table 1. Imposing the $Z_{4,R}$ symmetry on the theory forbids an unwanted large mass term for the Higgs doublets, $[H_U H_D]_{\theta^2}$, on the fixed point. (A mass term for the Higgs fields, $[H_U H_D]_{\theta^2}$, of the order of the electroweak scale is generated through the $Z_{4,R}$ breaking effect after supersymmetry is broken.) This symmetry contains the R -parity of the MSSM and thus forbids dangerous operators such as $[LH_U]_{\theta^2}$, $[QDL]_{\theta^2}$, $[UDD]_{\theta^2}$ and $[LLE]_{\theta^2}$; it also forbids the $d = 5$ proton decay operators $[QQQL]_{\theta^2}$ and $[UUDE]_{\theta^2}$.

2.4 Supersymmetry Breaking

In the present model the gauge and Higgs multiplets propagate in the bulk and the matter fields are localized on the fixed point at $(x^5, x^6) = (0, 0)$. This provides a natural setting [2] for

gaugino mediated supersymmetry breaking [7]. Supersymmetry is broken by the F -component expectation value, F_S , of a field S on either of the three fixed points $(x^5, x^6) = (\pi R_5, 0)$, $(0, \pi R_6)$ or $(\pi R_5, \pi R_6)$, and it is directly transmitted to the gauge and Higgs multiplets through the operators

$$\mathcal{L}_6 = \delta(x^5 - x_f^5)\delta(x^6 - x_f^6) \left[\int d^2\theta S \mathcal{W}_i^\alpha \mathcal{W}_{i\alpha} + \int d^4\theta (S^\dagger H_U H_D + S^\dagger S H_U H_D) + \text{h.c.} \right]. \quad (27)$$

Here, (x_f^5, x_f^6) is the coordinate of the fixed point where S field is localized, and we have omitted coefficients of order unity in units of the fundamental scale. Note that S has a vanishing $Z_{4,R}$ charge, so that F_S breaks the $Z_{4,R}$ symmetry. The interactions of Eq. (27) generate gaugino masses as well as the μ and μB parameters, while the squarks and sleptons obtain masses through radiative corrections so that the supersymmetric flavor problem is naturally solved. Since only the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry is preserved on the fixed point, the masses for the three gauginos can take different values.

2.5 Gauge Couplings and Compactification Scale

The $SU(3)_L$ unification in our model suggests that $g_1^{3\otimes 3} = g_2$ should be satisfied at the cutoff M_* . Here, $g_1^{3\otimes 3}$ is the correctly normalized hypercharge gauge coupling when the hypercharge operator is identified as the appropriate $SU(3)_C \otimes SU(3)_L$ generator T_Y , satisfying $\text{Tr}(T_Y)^2 = 1/2$. This boundary condition may instead be expressed in terms of g_Y , the conventionally normalized hypercharge coupling of the standard model. The $SU(3)_C \otimes SU(3)_L$ hypercharge generator is $T_Y = (1/2\sqrt{3})[\text{diag}(0, 0, 0) \oplus \text{diag}(1, 1, -2)]$, leading to hypercharge assignments for the Higgs doublets equal to $\pm\sqrt{3}/2$. These assignments are a factor $\sqrt{3}$ larger than the standard model ones, implying that $g_Y = \sqrt{3}g_1^{3\otimes 3}$, so that the correct boundary condition is $g_Y(M_*) = \sqrt{3}g_2(M_*)$.

If only massless zero modes contributed to the running of g_Y and g_2 , this boundary condition would require M_* to be well above the Planck scale for low energy data to be reproduced. However, at scales above the compactification scale M_c , g_Y and g_2 undergo power-law running [15], allowing M_* to be lowered.² By calculating the one-loop contributions to the running from the KK excitations of the 6D $N = 2$ vector multiplet we find that $\sqrt{3}g_2$ and g_1 unify beneath M_{Pl} provided that $M_*/M_c \gtrsim 40$ holds. Actually, this perturbative calculation is not trustworthy because the classical scaling of the gauge couplings makes the theory strongly coupled. At the scale $\mu > M_c$, the appropriate loop expansion parameter is $(\alpha/4\pi)(\mu/M_c)^2$. Taking $\mu = 40M_c$,

² As will be discussed in sub-section 3.6, the 4D $N = 4$ supersymmetry of the bulk is broken to $N = 2$ on 5D fixed lines of the orbifold, leading to a linear (rather than quadratic) evolution of the gauge couplings above M_c .

this parameter is larger than unity even for $\alpha = 0.01$. We are thus forced to conclude that the $SU(3)_L$ unification in this model is achieved in a non-perturbative regime.

2.6 Asymmetric Extra Dimension

So far, we have been considering $R_5 \sim R_6$. In this sub-section we comment on the phenomenology of the case where there is a (mild) hierarchy between R_5 and R_6 . We consider the case $R_5 \gg R_6$ without a loss of generality. In this case, between the two energy scales R_5^{-1} and R_6^{-1} , the theory appears as 5D $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge theory with the gauge multiplets and two Higgs doublet hypermultiplets in the bulk. Therefore, if R_5 is as low as TeV, there is the possibility that the theories discussed in Refs. [8, 9] are low energy effective theories of the present $SU(3)_C \otimes SU(3)_L$ model. Then, if quarks and leptons are localized on the fixed point $(x^5, x^6) = (0, 0)$, the lower bound on the scale R_5^{-1} comes from the production of single gauge KK modes with nonzero KK momentum in the fifth dimension and the generation of four zero-mode fermion operators [10], requiring R_5^{-1} to be larger than a few TeV.

However, instead of putting quarks and leptons on the fixed point, we could put them on the 5D fixed line $x^6 = 0$, since the gauge symmetry preserved on the fixed line is only that of the standard model. Although the fixed line preserves 4D $N = 2$ supersymmetry and quarks and leptons have to be introduced as hypermultiplets, the zero mode matter content is precisely that of the MSSM due to the orbifold operation \mathcal{Z}_5 . In this case, the bound on R_5^{-1} is significantly weaker, since the effects giving a strong bound are absent due to the conservation of the KK momentum in the fifth dimension [11, 12].

3 6D $SU(6)$ Unified Model on $T^2/(Z_2 \times Z'_2)$

In this section we construct a model that realizes the idea of Higgs fields as components of higher dimensional gauge supermultiplet, in which all the standard model gauge groups are unified into a single gauge group.

3.1 Orbifold and Gauge Structure

We consider a 6D $N = 2$ supersymmetric gauge theory as in the previous section. The extra dimensions are compactified on the $T^2/(Z_2 \times Z'_2)$ orbifold with radii $R_5 \sim R_6 \sim M_U^{-1}$, where $M_U \simeq 2 \times 10^{16}$ GeV is the conventional grand unification scale. We here set two radii equal, $R \equiv R_5 = R_6$, for simplicity. The gauge group is taken to be $SU(6)$, so that the only bulk field is

the $SU(6)$ gauge multiplet, $(V, \Sigma_5, \Sigma_6, \Phi)$. The orbifold boundary conditions are given by Eqs. (6 – 13) and Eqs. (15 – 22) with P_Z and P_T given by

$$P_Z = \text{diag}(1, 1, 1, 1, 1, -1), \quad (28)$$

$$P_T = \text{diag}(1, 1, 1, -1, -1, -1). \quad (29)$$

This breaks the $SU(6)$ gauge group to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ at low energies.

To identify the massless fields, we consider the transformation properties of V and Φ fields under $\mathcal{Z}_{5,6}$ and $\mathcal{T}_{5,6}$. They are written as

$$V : \left(\begin{array}{ccc|cc|cc} (+, +) & (+, +) & (+, +) & (+, -) & (+, -) & (-, -) \\ (+, +) & (+, +) & (+, +) & (+, -) & (+, -) & (-, -) \\ (+, +) & (+, +) & (+, +) & (+, -) & (+, -) & (-, -) \\ \hline (+, -) & (+, -) & (+, -) & (+, +) & (+, +) & (-, +) \\ (+, -) & (+, -) & (+, -) & (+, +) & (+, +) & (-, +) \\ \hline (-, -) & (-, -) & (-, -) & (-, +) & (-, +) & (+, +) \end{array} \right), \quad (30)$$

$$\Phi : \left(\begin{array}{ccc|cc|cc} (-, +) & (-, +) & (-, +) & (-, -) & (-, -) & (+, -) \\ (-, +) & (-, +) & (-, +) & (-, -) & (-, -) & (+, -) \\ (-, +) & (-, +) & (-, +) & (-, -) & (-, -) & (+, -) \\ \hline (-, -) & (-, -) & (-, -) & (-, +) & (-, +) & (+, +) \\ (-, -) & (-, -) & (-, -) & (-, +) & (-, +) & (+, +) \\ \hline (+, -) & (+, -) & (+, -) & (+, +) & (+, +) & (-, +) \end{array} \right), \quad (31)$$

where the first and second signs represent parities under $\mathcal{Z}_{5,6}$ and $\mathcal{T}_{5,6}$, respectively. We see from Eq. (30) that the massless vector multiplets are those of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. In addition, we have massless chiral superfields coming from Φ whose quantum numbers are given by $(\mathbf{1}, \mathbf{2}, 1/2, -2) \oplus (\mathbf{1}, \mathbf{2}, -1/2, 2)$. (We have normalized the $U(1)_{Y,X}$ charges to match the convention.) Since these quantum numbers are exactly those for the two Higgs doublets of the MSSM, we identify these massless states to be the Higgs fields. The Yukawa couplings to quarks and leptons are discussed in the next sub-section.

We here comment on the uniqueness of obtaining massless Higgs doublets. Note that we could have chosen $P_T = \text{diag}(1, 1, -1, -1, -1, -1)$ for the purpose of breaking the gauge group to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. In that case, however, the massless states coming from Φ are triplet Higgs fields, $(\mathbf{3}, \mathbf{1}, -1/3, -2) \oplus (\mathbf{3}^*, \mathbf{2}, 1/3, 2)$, instead of doublets.

(x^5, x^6)	4D supersymmetry	gauge symmetry
$(0, 0)$	$N = 1$	$SU(5) \otimes U(1)_X$
$(\pi R, 0)$	$N = 1$	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$
$(0, \pi R)$	$N = 1$	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$
$(\pi R, \pi R)$	$N = 1$	$SU(4)_{\bar{C}} \otimes SU(2)_L \otimes U(1)_{\bar{X}}$

Table 2: Supersymmetry and gauge symmetry on each of the four fixed points.

3.2 Fixed Points and Quarks and Leptons

The structure of the fixed points can be worked out by considering the profiles of symmetry transformation parameters in the extra dimensions. On each of the four fixed points of the $T^2/(Z_2 \times Z_2')$ orbifold, the remaining supersymmetry and gauge symmetry is given in Table 2 — matter multiplets and interactions placed on the fixed points must respect these symmetries. The fixed points are connected by fixed lines on which the gauge symmetries are $SU(5) \otimes U(1)_X$ for $x_5 = 0$ and $x_6 = 0$, and $SU(4)_{\bar{C}} \otimes SU(2)_L \otimes U(1)_{\bar{X}}$ for $x_5 = \pi R$ and $x_6 = \pi R$.

We first consider putting quark and lepton superfields on the $(x^5, x^6) = (0, 0)$ fixed point. Since the gauge symmetry preserved on this fixed point is $SU(5) \otimes U(1)_X$, we have to introduce matter and interactions respecting this symmetry. This $SU(5)$ contains unbroken $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as in the conventional way [13]. Thus, we introduce three generations of quarks and leptons, $3 \times [T(\mathbf{10}, 1), \bar{F}(\mathbf{5}^*, -3), N(\mathbf{1}, 5)]$, and couple them to the Higgs fields as

$$\mathcal{L}_6 = \delta(x^5)\delta(x^6) \int d^2\theta (y_T TTH + y_F T\bar{F}\bar{H} + y_N \bar{F}NH) + \text{h.c.} \quad (32)$$

Here, $H(\mathbf{5}, -2)$ and $\bar{H}(\mathbf{5}^*, 2)$ are components of the Φ field whose wavefunctions are nonvanishing at $(x^5, x^6) = (0, 0)$. Below the compactification scale, these interactions give the usual MSSM Yukawa couplings plus neutrino Yukawa couplings, since the only massless fields in H and \bar{H} are the doublet components. These couplings, however, precisely respect $SU(5)$ relations leading to unwanted predictions such as $m_s/m_d = m_\mu/m_e$. These relations can be avoided by mass mixing with heavy matter propagating on the 5D fixed lines [2]. Below we follow an alternative choice with quarks and leptons on another fixed point, even though this loses some understanding of the fermion quantum numbers. Further alternative models are discussed in sub-sections 3.7 and 3.8.

We locate quark and lepton chiral superfields on the fixed point where only $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ gauge symmetry is preserved. Since two fixed points at $(x^5, x^6) = (\pi R, 0)$ and

$(0, \pi R)$ are equivalent, we put them on the $(x^5, x^6) = (\pi R, 0)$ fixed point without a loss of generality. We introduce three generations of quarks and leptons, $3 \times [Q(\mathbf{3}, \mathbf{2}, 1/6, 1), U(\mathbf{3}^*, \mathbf{1}, -2/3, 1), D(\mathbf{3}^*, \mathbf{1}, 1/3, -3), L(\mathbf{1}, \mathbf{2}, -1/2, -3), E(\mathbf{1}, \mathbf{1}, 1, 1), N(\mathbf{1}, \mathbf{1}, 0, 5)]$, and couple them to the Higgs fields as

$$\mathcal{L}_6 = \delta(x^5)\delta(x^6 - \pi R) \int d^2\theta (y_U Q U H_U + y_D Q D H_D + y_E L E H_D + y_N L N H_U) + \text{h.c.} \quad (33)$$

Here, $H_U(\mathbf{1}, \mathbf{2}, 1/2, -2)$ and $H_D(\mathbf{1}, \mathbf{2}, -1/2, 2)$ are massless Higgs doublets coming from the Φ field, whose wavefunctions are nonvanishing at $(x^5, x^6) = (\pi R, 0)$. Since these Yukawa couplings need only respect $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ gauge symmetry, there are no unwanted $SU(5)$ fermion mass relations. Moreover, $d = 5$ proton decay due to triplet Higgs exchange [14] is absent, since there is no coupling of triplet Higgs fields to quarks and leptons.³ Similarly, $d = 6$ proton decay induced by the exchange of an X gauge boson is also absent, since the wavefunction of the X gauge boson vanishes on this fixed point [15].⁴ The $U(1)_X$ symmetry breaking and neutrino masses are discussed in the next sub-section.

3.3 $U(1)_X$ Symmetry Breaking

We have seen that, after the orbifolding, the $SU(6)$ bulk gauge multiplet provides massless modes of 4D $N = 1$ vector superfields of the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ gauge group and two Higgs chiral superfields H_U and H_D . To recover the MSSM at low energies, we have to break the $U(1)_X$ gauge symmetry. This $U(1)_X$ symmetry is the extra $U(1)$ symmetry in the usual $SO(10)$ grand unified theory, $U(1)_X = SO(10)/SU(5)$, as far as the quantum numbers for the quarks, leptons and Higgs fields are concerned. We here break it with the usual Higgs mechanism by introducing chiral superfields $X(\mathbf{1}, \mathbf{1}, 0, 10)$ and $\bar{X}(\mathbf{1}, \mathbf{1}, 0, -10)$ on the $(x^5, x^6) = (\pi R, 0)$ fixed point. We consider the following superpotential

$$\mathcal{L}_6 = \delta(x^5)\delta(x^6 - \pi R) \int d^2\theta \{Y(X\bar{X} - M_X^2) + \bar{X}N^2\} + \text{h.c.}, \quad (34)$$

where Y is a singlet superfield. This superpotential forces X and \bar{X} to have vacuum expectation values $\langle X \rangle = \langle \bar{X} \rangle = M_X$. It also gives Majorana masses for the right-handed neutrinos of order

³ We can write down couplings of triplet Higgs fields to quarks and leptons using derivatives of the extra dimensional coordinates. Even then, however, the mechanism of Ref. [2] ensures that the $d = 5$ proton decay is not caused by the exchange of the triplet Higgs fields.

⁴ There could be operators which couple to the X gauge boson to quarks and leptons through the derivative of the extra dimensional coordinates, but these operators are suppressed by the volume of the extra dimensions and thus expected to be small.

	θ^α	V	Σ_5	Σ_6	Φ	Q	U	D	L	E	N	X	\bar{X}	Y
$Z_{4,R}$	1	0	0	0	2	0	0	0	0	0	0	2	2	2

Table 3: $Z_{4,R}$ charge assignment for the $SU(6)$ model.

M_X , generating small neutrino masses through the see-saw mechanism [6]. Motivated by the observation of atmospheric neutrino oscillation [16], we take $M_X \sim 10^{14}$ GeV. An interesting point of breaking $U(1)_X$ by $\langle X \rangle = \langle \bar{X} \rangle \neq 0$ is that it leaves an unbroken Z_2 discrete gauge symmetry at low energies, which is precisely the matter parity in the MSSM. Therefore, unwanted operators such as $[LH_U]_{\theta^2}$, $[QDL]_{\theta^2}$, $[UDD]_{\theta^2}$ and $[LLE]_{\theta^2}$ are never generated even by quantum gravitational effects [17].

3.4 R symmetry

To make the model fully realistic, we have to forbid dangerous tree-level $d = 5$ proton decay operators, $[QQQL]_{\theta^2}$ and $[UUDE]_{\theta^2}$, as well as the tree-level Higgs mass term, $[H_U H_D]_{\theta^2}$. This can be done by imposing the discrete $Z_{4,R}$ symmetry on the model, whose charge assignment is given in Table 3. This $Z_{4,R}$ could be gauged if we employ the Green-Schwarz mechanism [18] to cancel anomalies [19]. The expectation values $\langle X \rangle = \langle \bar{X} \rangle \neq 0$ break both $Z_{4,R}$ and $U(1)_X$ symmetries, but it leaves another unbroken discrete $Z'_{4,R}$ symmetry that is a linear combination of $Z_{4,R}$ and $U(1)_X$: $Z'_{4,R} = Z_{4,R} + (1/5)U(1)_X$. (To make all charges integer, we have to take a linear combination, $Z_{4,R} + (1/5)U(1)_X + (24/5)U(1)_Y$.) This $Z'_{4,R}$ symmetry is sufficient to forbid the above unwanted operators, and thus no large μ term is generated by the $Z_{4,R}$ - $U(1)_X$ breaking. A μ term of the order of the weak scale is generated through the $Z'_{4,R}$ breaking effect after supersymmetry is broken.

3.5 Supersymmetry Breaking

Since we have put matter on the $(x^5, x^6) = (\pi R, 0)$ fixed point, supersymmetry breaking must happen either on the $(x^5, x^6) = (0, 0)$, $(0, \pi R)$ or $(\pi R, \pi R)$ brane for gaugino mediation to work. Supersymmetry is broken by the F -component expectation value, F_S , of a field S localized on the fixed point. Supersymmetry breaking effects are transmitted to the gauge and Higgs multiplets through the operators given in Eq. (27). Since the $Z'_{4,R}$ charge of S is zero, F_S breaks the $Z'_{4,R}$ symmetry, generating gaugino masses and the μ and μB parameters.

On the three fixed points, $(x^5, x^6) = (0, 0), (0, \pi R)$ and $(\pi R, \pi R)$, the “unbroken” gauge groups are different as was shown in Table 2. Therefore, depending on where supersymmetry breaking occurs, we obtain different relations for the gaugino masses. In the case that supersymmetry is broken at $(x^5, x^6) = (0, 0)$ (S is located on $(x^5, x^6) = (0, 0)$), we obtain the gaugino mass relations $m_{SU(3)_C} = m_{SU(2)_L} = m_{U(1)_Y}$ at the compactification scale, since the interactions in Eq. (27) must respect the $SU(5) \otimes U(1)_X$ gauge symmetry remaining on this fixed point. On the other hand, if S is located on the $(x^5, x^6) = (0, \pi R)$ fixed point, the four gaugino masses, $m_{SU(3)_C}, m_{SU(2)_L}, m_{U(1)_Y}$ and $m_{U(1)_X}$, can take arbitrary values, since the interactions in Eq. (27) need only respect $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ gauge symmetry. Finally, if S is on $(x^5, x^6) = (\pi R, \pi R)$, we obtain a relation among $m_{SU(3)_C}, m_{U(1)_Y}$ and $m_{U(1)_X}$, but it is irrelevant for low energy phenomenology.

3.6 Gauge Couplings and Compactification Scale

In the present $SU(6)$ model, the $SU(3)_C, SU(2)_L$ and $U(1)_Y$ gauge groups are unified into $SU(5)$ as in the conventional way. Therefore, the compactification scale is given by $1/R = M_U$ in the zero-th order approximation. There are, however, two types of corrections to this naive identification [2, 20].

First, we can write down tree-level gauge kinetic terms that do not respect full $SU(6)$ symmetry on subspaces of the 6D spacetime. Specifically, we can write 5D gauge kinetic terms respecting only $SU(5) \otimes U(1)_X$ gauge symmetry on the $(4 + 1)$ -dimensional spaces $x^5 = 0$ and $x^6 = 0$. Similarly, 5D gauge kinetic terms respecting only $SU(4)_{\tilde{C}} \otimes SU(2)_L \otimes U(1)_{\tilde{X}}$ can be written on the $(4 + 1)$ -dimensional spaces $x^5 = \pi R$ and $x^6 = \pi R$. Finally, 4D gauge kinetic terms are also introduced on the four fixed points, which need only respect gauge symmetries specified in Table 2. However, the corrections from these operators are generically suppressed by the volume of the extra dimension(s), so that we will neglect these contributions in the following analysis.

The second correction originates from the running of the gauge couplings above the compactification scale due to KK modes. Since the present model is a 6D theory, the zero-mode gauge couplings g_{0i} at the compactification scale $M_c (\equiv 1/R)$ receive power-law corrections as [15]

$$\frac{1}{g_{0i}^2(M_c)} \simeq \frac{1}{g_0^2(M_*)} - \frac{b}{8\pi^2}((M_*R)^2 - 1) - \frac{b'_i}{8\pi^2}(M_*R - 1) + \frac{b''_i}{8\pi^2} \ln(M_*R), \quad (35)$$

where b, b'_i and b''_i are constants of $O(1)$ and M_* the cutoff scale of the theory. In 6D picture, the last three terms correspond to 6D, 5D and 4D gauge kinetic terms generated by loop effects in the 6D bulk, on the 5D subspaces and the 4D fixed points, respectively. An interesting point

is that the present model possesses 6D $N = 2$ supersymmetry in the bulk, so that the term quadratically sensitive to the cutoff does not appear, $b = 0$. The other two terms give the correction to the relation between M_c and M_U and to the standard 4D supersymmetric $SU(5)$ prediction of $\sin^2 \theta_w$.

To estimate the threshold correction coming from this second source, we consider the one-loop renormalization group equations for the three gauge couplings. Assuming that the couplings take a unified value g_* at M_* , they take the following form:

$$\alpha_i^{-1}(m_Z) = \alpha_*^{-1}(M_*) + \frac{1}{2\pi} \left\{ \alpha_i \ln \frac{m_{\text{SUSY}}}{m_Z} + \beta_i \ln \frac{M_*}{m_Z} + \gamma_i \sum_{n=0}^{N_l} \ln \frac{M_*}{(n+1)M_c} \right\}, \quad (36)$$

where $(\alpha_1, \alpha_2, \alpha_3) = (-5/2, -25/6, -4)$, $(\beta_1, \beta_2, \beta_3) = (33/5, 1, -3)$ and $(\gamma_1, \gamma_2, \gamma_3) = (12/5, -4, -12)$. Here, we have assumed a common mass m_{SUSY} for the superparticles for simplicity, and the sum on n includes all KK modes below M_* , so that $(N_l + 1)M_c \leq M_*$. As expected, the term which involves the double sum of the KK towers does not appear, but there is still non-universal linear running of the gauge couplings above the compactification scale, as indicated by the presence of the single sum. This power-law contribution yields a large threshold correction that could spoil the successful prediction of $\sin^2 \theta_w$, if N_l is taken too large. For $N_l = 3$, we estimate the threshold correction to the value of $\sin^2 \theta_w$ to be $\sim (2 - 3)\%$, while for $N_l = 10$, the correction is $\sim 10\%$. Consistency with low-energy data requires some degree of cancellation between threshold corrections coming from unknown cutoff-scale physics and this correction arising from the vector multiplet KK modes. In section 4, we describe a slightly different $SU(6)$ model in which the vector multiplet KK excitations give a vanishing contribution to the gauge coupling running so that the successful prediction of $\sin^2 \theta_w$ is preserved.

3.7 Quarks and Leptons in 5D subspace

So far, we have been considering a model with quarks and leptons localized on a 4D fixed point. In this sub-section, we present an alternative model in which they propagate in a 5D subspace of the original 6D spacetime.

First, we note that the $T^2/(Z_2 \times Z'_2)$ orbifold is equivalent to an $(S^1/Z_2)^2$ orbifold where each of the fifth and sixth dimensions is compactified on S^1/Z_2 . Therefore, we can imagine the situation where matter is localized on the fixed point of the S^1/Z_2 in the x^5 direction but is freely propagating in the x^6 direction. This means that $T^2/(Z_2 \times Z'_2)$ has four $(4+1)$ -dimensional subspaces where matter and/or interactions can be placed. On each subspace, which we call a fixed line, the remaining supersymmetry and gauge symmetry are given in Table 4.

fixed lines	4D supersymmetry	gauge symmetry
$x^5 = 0$	$N = 2$	$SU(5) \otimes U(1)_X$
$x^6 = 0$	$N = 2$	$SU(5) \otimes U(1)_X$
$x^5 = \pi R$	$N = 2$	$SU(4)_{\tilde{C}} \otimes SU(2)_L \otimes U(1)_{\tilde{X}}$
$x^6 = \pi R$	$N = 2$	$SU(4)_{\tilde{C}} \otimes SU(2)_L \otimes U(1)_{\tilde{X}}$

Table 4: Supersymmetry and gauge symmetry on each of the four fixed lines.

We here locate quarks and leptons on the fixed line where $SU(5) \otimes U(1)_X$ gauge symmetry is preserved. Since the two fixed lines $x^5 = 0$ and $x^6 = 0$ are equivalent, we choose to put quarks and leptons on the $x^6 = 0$ fixed line without a loss of generality. Then, we have to introduce quarks and leptons in the form of hypermultiplets, since 4D $N = 2$ supersymmetry (5D $N = 1$ supersymmetry) is preserved on the fixed line. We find that introducing only three hypermultiplets $\mathcal{T}(\mathbf{10}, 1)$, $\mathcal{F}(\mathbf{5}^*, -3)$ and $\mathcal{N}(\mathbf{1}, 5)$ for each generation does not work due to an automatic “double-triplet splitting” mechanism caused by non-trivial matrix P_T . Rather, we have to introduce at least five hypermultiplets, $\mathcal{T}(\mathbf{10}, 1)$, $\mathcal{T}'(\mathbf{10}, 1)$, $\mathcal{F}(\mathbf{5}^*, -3)$, $\mathcal{F}'(\mathbf{5}^*, -3)$ and $\mathcal{N}(\mathbf{1}, 5)$, to obtain the correct low energy matter content [2, 21, 22].

We explicitly show the boundary conditions for the matter fields located on the fixed line. Each hypermultiplet \mathcal{M} ($\mathcal{M} = \mathcal{T}, \mathcal{T}', \mathcal{F}, \mathcal{F}', \mathcal{N}$) is decomposed into two chiral superfields $M^{(+)}$ and $M^{(-)}$ under 4D $N = 1$ supersymmetry. Then, the boundary conditions are written as

$$M^{(\pm)}(-x^5, x^6) = \pm M^{(\pm)}(x^5, x^6), \quad (37)$$

and

$$T^{(\pm)}(x^5 + 2\pi R_5, x^6) = \hat{P}_T T^{(\pm)}(x^5, x^6) \hat{P}_T, \quad (38)$$

$$T'^{(\pm)}(x^5 + 2\pi R_5, x^6) = -\hat{P}_T T'^{(\pm)}(x^5, x^6) \hat{P}_T, \quad (39)$$

$$F^{(\pm)}(x^5 + 2\pi R_5, x^6) = F^{(\pm)}(x^5, x^6) \hat{P}_T^{-1}, \quad (40)$$

$$F'^{(\pm)}(x^5 + 2\pi R_5, x^6) = -F'^{(\pm)}(x^5, x^6) \hat{P}_T^{-1}, \quad (41)$$

$$N^{(\pm)}(x^5 + 2\pi R_5, x^6) = N^{(\pm)}(x^5, x^6), \quad (42)$$

where we have used matrix notation; \hat{P}_T is given by $\hat{P}_T = \text{diag}(1, 1, 1, -1, -1)$, which is obtained by projecting the matrix P_T on the $SU(5)$ subspace. With these boundary conditions, the correct low energy matter content follows. Specifically, we find that $\{U, E\}, Q, D, L$ and N come from $T^{(+)}, T'^{(+)}, F^{(+)}, F'^{(+)}$ and $N^{(+)}$, respectively.

The Yukawa couplings and the $U(1)_X$ breaking can be located either on the $(x^5, x^6) = (0, 0)$ or $(\pi R, 0)$ fixed point. We here put them on the $(x^5, x^6) = (0, 0)$ fixed point as

$$\begin{aligned} \mathcal{L}_6 = & \delta(x^5)\delta(x^6) \int d^2\theta \left\{ (y_{T,1} T T H + y_{T,2} T T' H + y_{T,3} T' T' H) \right. \\ & + (y_{F,1} T F \bar{H} + y_{F,2} T F' \bar{H} + y_{F,3} T' F \bar{H} + y_{F,4} T' F' \bar{H}) + (y_{N,1} F N H + y_{N,2} F' N H) \\ & \left. + Y(X\bar{X} - M_X^2) + \bar{X} N^2 \right\} + \text{h.c.}, \end{aligned} \quad (43)$$

where we have omitted the superscript (+) from each superfield $T^{(+)}, T'^{(+)}, F^{(+)}, F'^{+}$ and $N^{(+)}$; $H(\mathbf{5}, -2)$ and $\bar{H}(\mathbf{5}^*, 2)$ are components of the Φ field whose wavefunctions are nonvanishing at $(x^5, x^6) = (0, 0)$, and $X(\mathbf{1}, 10), \bar{X}(\mathbf{1}, -10)$ and $Y(\mathbf{1}, 0)$ are chiral superfields localized on the $(x^5, x^6) = (0, 0)$ fixed point.

How about R symmetry and supersymmetry breaking? We can impose $Z_{4,R}$ symmetry on the model as in sub-section 3.4. The $Z_{4,R}$ charge for the $M^{(+)}$ and $M^{(-)}$ chiral superfields are 0 and 2, respectively. Then, all the couplings in Eq. (43) are allowed, while dangerous operators such as tree-level $d = 5$ proton decay operators and a Higgs mass term are not. As for the supersymmetry breaking, it can be either on the $(x^5, x^6) = (0, \pi R)$ or $(\pi R, \pi R)$ fixed point. In either case, there is no specific relation for the three gaugino masses, $m_{SU(3)_C}, m_{SU(2)_L}$ and $m_{U(1)_Y}$, at low energies.

We finally comment on the phenomenology of the model with matter on the fixed line. In this case, the quarks and leptons which would be unified into a single multiplet in the usual 4D grand unified theories come from different $SU(5)$ multiplets. Specifically, D and L (Q and U, E) come from different (hyper)multiplets. Therefore, proton decay from broken gauge boson exchange is absent in this case [2]. Furthermore, there is no unwanted $SU(5)$ relation among the low energy Yukawa couplings arising from the interactions given in Eq. (43) [2]. This is reminiscent of the situation in certain string motivated theories [23]. Nevertheless, the theory still keeps the desired features of the usual 4D grand unified theory: the quantization of hypercharge and the unification of the three gauge couplings [23, 2]. Therefore, this type of theory, with matter in the bulk, preserves (experimentally) desired features of 4D grand unified theories, while not necessarily having the problematic features, such as proton decay and fermion mass relations.

3.8 A Theory of Flavor

In this sub-section, we present a model where some matter lives on a fixed point and some on a fixed line. An important point of this model is that it partially explains the mass hierarchies

among the three generations of quarks and leptons, and simultaneously explains why the $SU(5)$ mass relation holds for the heavier generation but fails for lighter generations. The mechanism presented here also applies for the 5D $SU(5)$ model discussed in Refs. [3, 2].

We put one generation of quarks and leptons on the $(x^5, x^6) = (0, 0)$ fixed point. Since this fixed point preserves $SU(5) \otimes U(1)_X$ symmetry, we introduce $T_3(\mathbf{10}, 1)$, $\bar{F}_3(\mathbf{5}^*, -3)$ and $N_3(\mathbf{1}, 5)$ chiral superfields. The meaning of the suffix 3 becomes apparent later when we identify these fields as the third generation quarks and leptons. The other two generations are located on the fixed line $x^6 = 0$. Thus, we introduce hypermultiplets $\mathcal{T}_i(\mathbf{10}, 1)$, $\mathcal{T}'_i(\mathbf{10}, 1)$, $\mathcal{F}_i(\mathbf{5}^*, -3)$, $\mathcal{F}'_i(\mathbf{5}^*, -3)$ and $\mathcal{N}_i(\mathbf{1}, 5)$ on this line, where $i = 1, 2$ represents the generation index. The $Z_{4,R}$ symmetry and supersymmetry breaking are the same as before. Since the first two generations are located on the fixed line, proton decay is suppressed. The supersymmetry breaking occurs either at $(x^5, x^6) = (0, \pi R)$ or $(\pi R, \pi R)$ fixed point and is mediated by gaugino interaction.

A distinctive feature of the present setup comes from the structure of the Yukawa couplings. They are located on the $(x^5, x^6) = (0, 0)$ fixed point. We allow all the couplings of the forms TTH , $TF\bar{H}$ and FNH , where T collectively represents $T_i^{(+)}$, $T'_i^{(+)}$ and T_3 and similarly for F and N . Specifically, we introduce

$$\begin{aligned}
\mathcal{L}_6 = & \delta(x^5)\delta(x^6) \int d^2\theta \left\{ \left((y_T)_{33}TTH + (y_F)_{33}TF\bar{H} + (y_N)_{33}FNH \right) \right. \\
& + \sum_{i=1}^2 \left((y_{T,1})_{3i}TT_iH + (y_{T,1})_{i3}T_iTH + (y_{T,2})_{3i}TT'_iH + (y_{T,2})_{i3}T'_iTH \right. \\
& \quad + (y_{F,1})_{3i}TF_i\bar{H} + (y_{F,1})_{i3}T_iF\bar{H} + (y_{F,2})_{3i}TF'_i\bar{H} + (y_{F,2})_{i3}T'_iF\bar{H} \\
& \quad \left. + (y_{N,1})_{3i}FN_iH + (y_{N,1})_{i3}F_iNH + (y_{N,2})_{i3}F'_iNH \right) \\
& + \sum_{i,j=1}^2 \left((y_{T,1})_{ij}T_iT_jH + (y_{T,2})_{ij}T_iT'_jH + (y_{T,3})_{ij}T'_iT'_jH \right. \\
& \quad + (y_{F,1})_{ij}T_iF_j\bar{H} + (y_{F,2})_{ij}T_iF'_j\bar{H} + (y_{F,3})_{ij}T'_iF_j\bar{H} + (y_{F,4})_{ij}T'_iF'_j\bar{H} \\
& \quad \left. + (y_{N,1})_{ij}F_iN_jH + (y_{N,2})_{ij}F'_iN_jH \right) \\
& \left. + Y(X\bar{X} - M_X^2) + \bar{X}N^2 \right\} + \text{h.c.}, \tag{44}
\end{aligned}$$

where we have dropped the superscript (+) from the fields which come from bulk hypermultiplets, and omitted order one coefficients in units of fundamental scale M_* of the theory. Yukawa couplings between members of the lighter two generations are also located at $(x^5, x^6) = (\pi R, 0)$.

An important point is that since the couplings of bulk fields are suppressed by the volume of the extra dimension, we obtain the Yukawa structure that the couplings for the first two

generations are suppressed compared with the third generation ones. Specifically, we obtain the following structure for the Yukawa couplings

$$y_u \sim y_d \sim y_e \sim y_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad (45)$$

where $\epsilon = 1/(\pi R M_*)$ is a small parameter representing the volume suppression factor. This provides a partial understanding of the mass hierarchy among the generations from a geometrical viewpoint. In particular, it is very natural to identify the fields on the fixed line with the first two generations of matter and those on the fixed point with the third generation, since the former receive wavefunction suppressions while the latter does not.

Another important point is that since the $(x^5, x^6) = (0, 0)$ fixed point respects $SU(5) \otimes U(1)_X$ gauge symmetry, the Yukawa couplings among the fields localized on this fixed point must respect $SU(5)$ relations. Since the fields on this point are identified with the third generation matter, we obtain the relation $(y_d)_{33} = (y_e)_{33}$, which means that the b/τ unification is preserved in the present model. On the other hand, the first two generations of matter come from the hypermultiplets located on the fixed line $x^6 = 0$ so that they do not respect the $SU(5)$ mass relations. This means that we do not get unwanted fermion mass relations such as $m_s/m_d = m_\mu/m_e$. Together with the argument of volume suppression leading to Eq. (45), the present setup provides an understanding for why the heaviest generation respect $SU(5)$ mass relation while the lighter ones do not.

4 6D $SU(6)$ Unified Model on T^2/Z_4

One less than ideal feature of the $T^2/(Z_2 \times Z'_2)$ orbifold model discussed in the previous section is the non-universal power-law running of the gauge couplings above the compactification scale. As discussed in sub-section 3.6, this running leads to corrections to the standard supersymmetric $SU(5)$ prediction of $\sin^2 \theta_w$. These corrections can be suppressed by taking M_* to be not far above M_c . Here we instead consider an alternative $SU(6)$ model in which the running of the gauge couplings is just as in the MSSM, even above the compactification scale.

The orbifold for this model is T^2/Z_4 . As before the only bulk fields are those of a 6D $N = 2$ vector multiplet. Defining $z \equiv x_5 + ix_6$ and $\partial \equiv \partial_5 - i\partial_6$, the bulk action may be written using 4D $N = 1$ language as [4]

$$S = \int d^6x \left\{ \text{Tr} \left[\int d^2\theta \left(\frac{1}{4kg^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{kg^2} \left(\Phi^c \partial \Phi - \frac{1}{\sqrt{2}} \Sigma[\Phi, \Phi^c] \right) \right) + \text{h.c.} \right] \right\}$$

$$+ \int d^4\theta \frac{1}{kg^2} \text{Tr} \left[(\sqrt{2}\partial^\dagger + \Sigma^\dagger) e^{-V} (-\sqrt{2}\partial + \Sigma) e^V + \Phi^\dagger e^{-V} \Phi e^V + \Phi^{c\dagger} e^{-V} \Phi^c e^V \right] \Big\}, \quad (46)$$

in the Wess-Zumino gauge. Here the gauge field components A_5 and A_6 are both contained in Σ , so that both Φ and Φ^c transform linearly under gauge transformations. When expressed in terms of components, this action and that of Eq. (1) have identical forms. The orbifold of the present model will preserve a different 4D $N=1$ supersymmetry than the orbifold of the previous model (namely, one in which A_5 and A_6 are paired in the same superfield), and we have chosen to make this different 4D $N=1$ supersymmetry manifest.

The orbifold T^2/Z_4 is constructed by identifying points of the infinite plane R^2 under three operations, $\mathcal{Z} : z \rightarrow iz$, $\mathcal{T}_5 : z \rightarrow z + 2\pi R$ and $\mathcal{T}_6 : z \rightarrow z + 2\pi iR$. The identifications for the fields under \mathcal{Z} are taken to be

$$V(iz) = P_Z V(z) P_Z^{-1}, \quad (47)$$

$$\Sigma(iz) = -i P_Z \Sigma(z) P_Z^{-1}, \quad (48)$$

$$\Phi(iz) = -P_Z \Phi(z) P_Z^{-1}, \quad (49)$$

$$\Phi^c(iz) = -i P_Z \Phi^c(z) P_Z^{-1}, \quad (50)$$

and the identifications under \mathcal{T}_5 and \mathcal{T}_6 are

$$V(z + 2\pi R) = P_T V(z) P_T^{-1}, \quad (51)$$

$$\Sigma(z + 2\pi R) = P_T \Sigma(z) P_T^{-1}, \quad (52)$$

$$\Phi(z + 2\pi R) = P_T \Phi(z) P_T^{-1}, \quad (53)$$

$$\Phi^c(z + 2\pi R) = P_T \Phi^c(z) P_T^{-1}, \quad (54)$$

and

$$V(z + 2\pi iR) = P_T V(z) P_T^{-1}, \quad (55)$$

$$\Sigma(z + 2\pi iR) = P_T \Sigma(z) P_T^{-1}, \quad (56)$$

$$\Phi(z + 2\pi iR) = P_T \Phi(z) P_T^{-1}, \quad (57)$$

$$\Phi^c(z + 2\pi iR) = P_T \Phi^c(z) P_T^{-1}, \quad (58)$$

respectively. As before, we take

$$P_Z = \text{diag}(1, 1, 1, 1, 1, -1), \quad (59)$$

$$P_T = \text{diag}(1, 1, 1, -1, -1, -1), \quad (60)$$

which breaks the $SU(6)$ gauge group to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ at low energies. It is not difficult to see that the only massless zero modes besides those of V come from Φ and have the quantum numbers of the two MSSM Higgs doublets.

This orbifold has two fixed points located at $(x_5, x_5) = (0, 0)$ and $(\pi R, \pi R)$. The remaining supersymmetry on both of these fixed points is 4D $N=1$, and the gauge symmetries are $SU(5) \otimes U(1)_X$ and $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$, respectively. The important distinction between this orbifold and the $T^2/(Z_2 \times Z'_2)$ orbifold considered previously is that there are no longer 5D fixed lines on which only 4D $N=2$ supersymmetry remains. Thus, the bulk 4D $N=4$ supersymmetry ensures that the terms quadratically *and* linearly sensitive to the cutoff vanish in Eq. (35). In fact, explicit calculation of the KK modes for V , Σ , Φ , and Φ^c reveals that at *each* massive level the states are arranged in $N=4$ multiplets, so that the only states that contribute to the running of gauge couplings are the massless zero modes. This means that Eq. (36) holds with each of the γ coefficients set to zero. The fundamental scale M_* should then be chosen to be close to the usual 4D unification scale $M_U \simeq 2 \times 10^{16}$ GeV. The compactification scale should not be more than a factor ~ 10 lower, so that the theory remains perturbative up to M_* , but the prediction for $\sin^2 \theta_w$ is insensitive to the precise value of M_c/M_* .

If the quarks and leptons are placed on the $SU(5) \otimes U(1)_X$ preserving fixed point, $SU(5)$ mass relations follow, and the setups of sub-sections 3.7 and 3.8 cannot be used to alter these relations because of the absence of fixed lines with reduced supersymmetry. Thus we choose instead to introduce matter on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ preserving fixed point. The discussions of $U(1)_X$ breaking, R symmetry, and supersymmetry breaking from sub-sections 3.3 – 3.5 then carry over in the obvious way.

5 5D $SU(6)$ Unified Model on S^1/Z_2

In this section, we give a model in which Higgs fields arise from the extra dimensional components of the gauge fields. The model is based on a 5D $N = 1$ supersymmetric gauge theory with the extra dimension y compactified on S^1/Z_2 . We take the gauge group to be $SU(6)$. In terms of 4D $N = 1$ supersymmetry language, we have a vector superfield V and a chiral superfield Φ both in the adjoint representation of the $SU(6)$.

The orbifold is defined by specifying boundary conditions for the bulk fields under two operations $\mathcal{Z} : y \rightarrow -y$ and $\mathcal{T} : y \rightarrow y + 2\pi R$ as

$$V(-y) = P_Z V(y) P_Z^{-1}, \tag{61}$$

$$\Phi(-y) = -P_Z \Phi(y) P_Z^{-1}, \tag{62}$$

and

$$V(y + 2\pi R) = P_T V(y) P_T^{-1}, \quad (63)$$

$$\Phi(y + 2\pi R) = P_T \Phi(y) P_T^{-1}, \quad (64)$$

respectively. Here, $V = V^A T^A$, $\Phi = \Phi^A T^A$, P_Z and P_T are 6×6 matrices.

We take $P_Z = \text{diag}(1, 1, 1, 1, 1, -1)$ and $P_T = \text{diag}(1, 1, 1, -1, -1, -1)$, so that $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ remains unbroken at low energies. Then, the transformation properties for the bulk fields are explicitly given in Eqs. (30, 31) with the first and second signs representing quantum numbers under \mathcal{Z} and \mathcal{T} respectively. We find that two zero-mode chiral superfields from Φ , which have $(+, +)$ transformation properties, have precisely the quantum numbers of the two Higgs doublets under the standard model gauge group. Therefore, we identify these massless fields as the MSSM Higgs fields. In contrast to the model presented in previous sections, here some components of the Higgs doublets are extra dimensional components of the heavy unified gauge bosons.

The quarks and leptons are introduced either on $y = 0$ or $y = \pi R$ fixed point. Here, for an illustrative purpose, we introduce them on the $y = 0$ fixed point where $SU(5) \otimes U(1)_X$ gauge invariance is manifest. The $y = \pi R$ fixed point case can be worked out quite similarly. Since the $y = 0$ fixed point preserves only $SU(5) \otimes U(1)_X$ gauge symmetry, we can introduce three generations of quarks and leptons, $3 \times [T(\mathbf{10}, 1), F(\mathbf{5}^*, -3), N(\mathbf{1}, 5)]$. Here, we have normalized the $U(1)_X$ charges to match the convention, $U(1)_X = SO(10)/SU(5)$.

An immediate difficulty in the present model compared with the previous models is that, since the Higgs fields are the extra dimensional components of the gauge fields, 5D gauge invariance prevents us from introducing Yukawa couplings between the Higgs field Φ and the quarks and leptons. Specifically, the Yukawa couplings are forbidden by non-linear transformation of the Φ field, $\Phi \rightarrow e^\Lambda (\Phi - \sqrt{2} \partial_y) e^{-\Lambda}$, under the 5D gauge transformation. To circumvent this problem, we consider the Wilson line operator, $\mathcal{P} \exp(\int_0^{2\pi R} \Phi dy)$, where \mathcal{P} represents the path ordered product. This object transforms linearly under the gauge transformation, so that we can couple it to the quark and lepton fields. Since the $y = 0$ fixed point preserves only $SU(5) \otimes U(1)_X$ gauge symmetry, we consider the subsets of the 6×6 Wilson line matrix which contain linear terms in the zero modes. They are given by the upper-right five-dimensional column vector H and the lower-left five-dimensional row vector \bar{H} :

$$H(x^\mu) = \mathcal{P} \exp \left(\int_0^{2\pi R} \Phi dy \right) \Big|_{(\mathbf{5}, -2)_{y=2\pi R}, (\mathbf{1}, -10)_{y=0}}, \quad (65)$$

$$\bar{H}(x^\mu) = \mathcal{P} \exp \left(\int_0^{2\pi R} \Phi dy \right) \Big|_{(\mathbf{1}, 10)_{y=2\pi R}, (\mathbf{5}^*, 2)_{y=0}}. \quad (66)$$

Here, H transforms as $(\mathbf{5}, -2)$ under the gauge transformation $SU(5) \otimes U(1)_X$ at $y = 2\pi R$ but as $(\mathbf{1}, -10)$ under that at $y = 0$. Similarly, \bar{H} transforms as $(\mathbf{1}, 10)$ at $y = 2\pi R$ and as $(\mathbf{5}^*, 2)$ at $y = 0$.

Introducing the brane fields $X(\mathbf{1}, 10)$ and $\bar{X}(\mathbf{1}, -10)$, we can write down gauge-invariant non-local interactions among the Higgs field and quarks and leptons,

$$S = \int d^4x \int d^2\theta \left\{ y_T(TT)|_{y=2\pi R} X|_{y=0} H + y_F(TF)|_{y=0} \bar{X}|_{y=2\pi R} \bar{H} + y_N(FN)|_{y=2\pi R} X|_{y=0} H \right\}, \quad (67)$$

with each term suppressed by appropriate powers of the fundamental scale M_* . Here we will not consider the physics which may generate these non-local operators, but simply take a viewpoint that they can be written in the effective field theory since they are gauge invariant. We also introduce the brane-localized superpotential

$$S = \int d^4x dy \delta(y) \int d^2\theta \left\{ Y(X\bar{X} - M_X^2) + \bar{X}N^2 \right\}, \quad (68)$$

where Y is the singlet superfield. This superpotential forces X and \bar{X} to have vacuum expectation values $\langle X \rangle = \langle \bar{X} \rangle = M_X$, breaking $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ down to the standard model gauge group. Then, assuming $M_X \sim M_*$, Eq. (67) gives the usual Yukawa couplings at low energies. The Majorana masses for the right-handed neutrinos of order M_X also arise from Eq. (68), so that small neutrino masses are generated through the see-saw mechanism.

We finally comment on the phenomenological issues. The present model does not suffer from the power-law correction for $\sin^2 \theta_w$; above the compactification scale $1/R \approx M_U$, differential running between the three standard-model gauge couplings is logarithmic. Therefore, the situation is similar to the $SU(5)$ case discussed in Ref. [2], and retains an exciting possibility that the proton decay caused by dimension 6 operators may be seen in the near future. (This proton decay is absent if we put quarks and leptons on the $y = \pi R$ fixed point, since the wavefunctions for the X, Y gauge bosons vanish there.)

Proton decay from dimension 5 operators is forbidden in a similar way to the $SU(5)$ case in Ref. [2], by imposing a $U(1)_R$ symmetry. The higher KK modes of Φ form mass terms together with V , becoming a part of the massive vector multiplets. The $U(1)_R$ charge assignment is given as $\Phi(0), T(1), F(1), N(1), X(0), \bar{X}(0), Y(2)$. A good thing here compared with the $SU(5)$ case is that the $U(1)_R$ charges for the Higgses, Φ , are automatically fixed to the desired value: the bulk $U(1)_R$ is just a subgroup of the $SU(2)_R$ automorphism group of $N = 2$ supersymmetry algebra. This $U(1)_R$ also forbids dimension 4 proton decay, since it contains R -parity as a subgroup.

As for the μ term and the supersymmetry breaking, the situation is the same with the $SU(5)$ case. If the supersymmetry breaking occurs on the $y = \pi R$ fixed point, gaugino mediation could

naturally occur and there is no supersymmetric flavor problem. Realistic fermion masses could result from mixing of brane and bulk matter [2]. (In the case of matter on the $y = \pi R$ fixed point, realistic fermion masses are more easily obtained, since the Yukawa couplings need only preserve $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ on that fixed point.)

6 Conclusions

In this paper we have explored the idea that the Higgs doublets of the minimal supersymmetric standard model originate from the same higher dimensional supermultiplet that contains the standard model gauge fields. This requires an extension of the standard model gauge group, and is particularly suited to the situation where gauge symmetry breaking is induced by orbifold boundary conditions. The Higgs doublets emerge naturally as zero modes, and despite the bulk gauge symmetry, there are orbifold fixed points where independent Yukawa couplings of the Higgs to matter are allowed. This idea demonstrates that it is natural for a supersymmetric gauge theory in higher dimensions to give states below the fundamental scale which are vector-like with respect to the gauge group.

We have written several explicit models with such a unification of the Higgs doublets with the standard model gauge bosons. In the minimal case, where the electroweak gauge group is extended to $SU(3)$, the compactification scale could be as low as a few TeV. Alternatively the compactification scale could be at the unification scale, and we have shown that the appearance of Higgs doublets from compactification of the gauge multiplet allows a preservation of the successful weak mixing angle prediction.

In the supersymmetric standard model some mechanism is needed to solve the μ problem — that is to explain why the vector-like Higgs doublets only pick up a mass at the scale of supersymmetry breaking. Similarly, a mechanism is needed to break supersymmetry in a way which does not introduce too large flavor violations. Such mechanisms are also required in the present case, and, given the higher dimensional orbifold context, a particularly natural possibility emerges. An R symmetry protects the μ parameter, and this is broken on the same brane where supersymmetry is broken, yielding a TeV mass for the Higgs. Furthermore, as long as this brane is not the one containing matter, the supersymmetry breaking preserves flavor by the gaugino mediation mechanism. The higher dimensional framework not only provides a reason for the existence of the Higgs, but allows a simple explanation for why it is light.

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