Regular Article - Theoretical Physics

Gauge invariance and the CPT and Lorentz violating induced Chern–Simons-like term in extended QED

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Abstract The radiative induction of the CPT and Lorentz violating Chern–Simons (CS) term is reassessed. The massless and massive models are studied. Special attention is given to the preservation of gauge symmetry at higher orders in the background vector b_μ when radiative corrections are considered. Both the study of the odd and even parity sectors of the complete vacuum polarization tensor at one-loop order and a non-perturbative analysis show that this symmetry must be preserved by quantum corrections. As a complement we obtain the result that transversality of the polarization tensor does not fix the value of the coefficient of the induced CS term.

PACS 11.30.Cp · 11.30.Er · 11.30.Qc · 11.15.Bt

1 Introduction

Symmetries are the cornerstones of the systematical study of any theory. Lorentz and CPT invariances have supreme importance in the elaboration of modern quantum field theory models. However, in the previous decade the possibility of violation of these symmetries has been investigated on a vast scale [1–46]. A standard model description, where possible violations of such invariances are considered, was developed by Colladay and Kostelecký [4, 6] and by Coleman and Glashow [5, 8]. One of the most discussed terms that incorporates these features has the Chern–Simons (CS) form

$$\Sigma_{CS} = -\frac{1}{4} \int d^4x \, c_\mu A_\nu F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}, \qquad (1)$$

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in which c_μ is a constant four-vector that selects a space-time direction. Therefore, it gives rise to optical activity of the vacuum. Although astrophysical results put very stringent limits on the magnitude of c_μ [1–3], many controversies have emerged from the discussion whether such term could be generated by means of radiative corrections from the fermionic sector with the inclusion of the CPT and Lorentz violating axial term, $b_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi$, b_μ being a constant background vector. The main point concerns the regularization dependence of this term. As discussed in many papers, the finite radiative correction Δc_μ comes from the cancellation of divergences and, therefore, is in principle regularization dependent. Symmetries are invoked to argue against or in favor of the generation of such a term.

Other discussions focus on the possibility of considering a source for the b_μ field and in stating gauge invariance in a weak way, which means that we have gauge invariance of the action and not necessarily of the Lagrangian density.

In this paper, we reassess the discussion on the radiative generation of the CS-like term. We give particular attention to the possibility of gauge symmetry violation coming from quantum corrections. We also analyze if gauge invariance can impose some constraint on the coefficient of the CS-like term. The paper is divided as follows. In Sect. 2, the massless model is considered. It is shown at one-loop order that the even parity piece of the polarization tensor has the structure of simple spinorial QED, by arguments of assigning a constant background field to arbitrary rooting of the internal momentum dependence. The implicit regularization (IR) technique allows one to pin down the exact form of surface terms and its a priori arbitrary parameterized values. A weak constraint, i.e. a relation among two of the parameters, instead of fixing both values independently, is seen to be sufficient to ensure gauge invariance. The odd parity term is shown not to deliver further constraints on this relation, which leads finally to the indeterminacy of the coefficient of

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the CS term. This result is compared to the chirally rotated theory and the Jacobian of the transformation. In Sect. 3, we carry out the analysis of the massive model, expanding the exact propagator to second order in the constant background field. Again, a careful study of the surface terms with the IR method leaves the CS-like term coefficient undetermined. We summarize our results in Sect. 4.

2 The massless model

Concerning the modified QED which includes the axial term in the fermionic sector, some papers were devoted to discussion of the gauge invariance of the model [17, 18, 20, 21]. In Ref. [17], Altschul has analyzed the massless model and argued that gauge symmetry is violated at b^2 order of the vacuum polarization tensor. In [18], the author has shown that an adequate Pauli–Villars regulator is compatible with the gauge invariance of the model.

We would like to show here that the gauge invariance of the action enforces a certain relation among the a priori arbitrary coefficients which parameterize surface terms. This can be achieved by considering the whole amplitude. Nevertheless, this is not sufficient to fix unambiguously the coefficient of the radiatively generated CS term.

We begin our reasoning by analyzing the massless case, for which the fermion action is given by

$$\Sigma_\psi = \int d^4x \, \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - b\!\!\!/\gamma_5)\psi. \tag{2}$$

It is instructive to discuss first a non-perturbative calculation (in b and in the coupling constant) of the induced CS-type term. This calculation has been performed by Chung in [22]. By making the chiral transformation,

$$\psi \to e^{-i\gamma_5 b\cdot x}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\gamma_5 b\cdot x}, \tag{3}$$

we can eliminate the b_μ vector from the classical action. Nevertheless, at the quantum level, the measure of the generating functional acquires a factor given by the Jacobian [61],

$$J[b_\mu, A_\mu] = \exp\left\{-i\int d^4x \, (b\cdot x)\mathcal{A}[A_\mu](x)\right\}, \tag{4}$$

with

$$\mathcal{A}[A_\mu](x) = \frac{1}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{5}$$

We can write

$$J[b_\mu, A_\mu] = \exp\left\{-i\int d^4x \, \frac{1}{4\pi^2}(b\cdot x)\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu\partial_\alpha A_\beta\right\}, \tag{6}$$

which after an integration by parts turns out to be

$$J[b_\mu, A_\mu] = \exp\left\{i\int d^4x \, \frac{1}{4\pi^2}b_\mu\epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta\right\}. \tag{7}$$

We see that after the chiral transformation the axial term disappears from the fermionic sector. As a result the QED Lagrangian is obtained, together with a Jacobian which is taken into account when quantum corrections are calculated. Therefore, the non-massive model with Lorentz and CPT violation in the fermionic sector is equivalent to the non-massive QED, if a CS-type term coming from the Jacobian is added to the radiative correction of the photon self-energy. As a consequence, since the latter model is gauge invariant, so must the original one be. We will comment at the end of this section on the coefficient of the CS term.

We now carry out an one-loop analysis for the massless case, following Ref. [17]. If the fermion is non-massive, its propagator can be decomposed as

$$\frac{i}{k\!\!\!/ - b\!\!\!/\gamma_5} = \frac{i}{k\!\!\!/ - b\!\!\!/}P_L + \frac{i}{k\!\!\!/ + b\!\!\!/}P_R, \tag{8}$$

where we use the chiral projectors

$$P_{R,L} = \frac{1 \pm \gamma_5}{2}. \tag{9}$$

It is now simple to analyze the full vacuum polarization tensor and achieve some conclusions with respect to the gauge invariance of the theory and the generation of the CS term. The amplitude is easily written as

$$\Pi^{\mu\nu} = \frac{1}{2}\{\Pi^{\mu\nu}_+ + \Pi^{\mu\nu}_- + \Pi^{\mu\nu}_{5+} + \Pi^{\mu\nu}_{5-}\}, \tag{10}$$

with

$$\Pi^{\mu\nu}_\pm = \int_k^\Lambda \text{tr}\left\{\frac{\gamma^\mu(k\!\!\!/ \pm b\!\!\!/)\gamma^\nu(k\!\!\!/ + p\!\!\!/ \pm b\!\!\!/)}{(k \pm b)^2(k + p \pm b)^2}\right\} \tag{11}$$

and

$$\Pi^{\mu\nu}_{5\pm} = \pm\int_k^\Lambda \text{tr}\left\{\frac{\gamma^\mu(k\!\!\!/ \pm b\!\!\!/)\gamma^\nu(k\!\!\!/ + p\!\!\!/ \pm b\!\!\!/)\gamma_5}{(k \pm b)^2(k + p \pm b)^2}\right\}, \tag{12}$$

where $\int_k \equiv \int \frac{d^4k}{(2\pi)^4}$ and the superscript Λ is used to indicate that some regularization has been applied. Note that $\Pi^{\mu\nu}_\pm$ is simply the vacuum polarization tensor of the simple spinorial QED evaluated with an arbitrary momentum distribution in the internal lines, $k + k_1$ and $k + k_2$, with $p = k_2 - k_1$, for which $k_1 = \pm b$ and $k_2 = p \pm b$. Any regularization scheme that respects gauge invariance will return an answer which is transverse and depends only on $p = k_2 - k_1$. In Ref. [24], this amplitude has been calculated by means of implicit regularization [47] with arbitrary k_1 and k_2. In the massless

limit, it gives

$$
\begin{aligned}
\Pi_\pm^{\mu\nu} = {}& \Pi(p^2)(p^\mu p^\nu - p^2 g^{\mu\nu}) \\
& - 4\big(\alpha_1 g^{\mu\nu} - (k_1^2 + k_2^2)\alpha_2 g^{\mu\nu} \\
& + \tfrac{1}{3}(k_{1\alpha}k_{1\beta} + k_{2\alpha}k_{2\beta} + k_{1\alpha}k_{2\beta})\alpha_3 g^{\{\mu\nu\alpha\beta\}} \\
& - (k_1 + k_2)^\mu (k_1 + k_2)^\nu \alpha_2\big).
\end{aligned}
\tag{13}
$$

In the equation above, $\Pi(p^2)$ includes the divergent part. The quantity $g^{\{\mu\nu\alpha\beta\}}$ denotes the symmetrized product of two $g^{\mu\nu}$. Now, the momentum routing dependent terms, which cause violation of gauge symmetry, are proportional to the α_i, namely

$$
\begin{aligned}
\alpha_1 g_{\mu\nu} & \equiv \int_k^\Lambda \frac{g_{\mu\nu}}{k^2 - m^2} - 2\int_k^\Lambda \frac{k_\mu k_\nu}{(k^2 - m^2)^2} \\
& = \int_k^\Lambda \frac{\partial}{\partial k^\mu}\left(\frac{k_\nu}{(k^2 - m^2)}\right),
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\alpha_2 g_{\mu\nu} & \equiv \int_k^\Lambda \frac{g_{\mu\nu}}{(k^2 - m^2)^2} - 4\int_k^\Lambda \frac{k_\mu k_\nu}{(k^2 - m^2)^3} \\
& = \int_k^\Lambda \frac{\partial}{\partial k^\mu}\left(\frac{k_\nu}{(k^2 - m^2)^2}\right)
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
\alpha_3 g_{\{\mu\nu\alpha\beta\}} & \equiv g_{\{\mu\nu\alpha\beta\}}\int_k^\Lambda \frac{1}{(k^2 - m^2)^2} \\
& - 24\int_k^\Lambda \frac{k_\mu k_\nu k_\alpha k_\beta}{(k^2 - m^2)^4},
\end{aligned}
\tag{16}
$$

which means

$$
g_{\{\mu\nu\alpha\beta\}}(\alpha_3 - \alpha_2) = \int_k^\Lambda \frac{\partial}{\partial k^\beta}\left[\frac{4 k_\mu k_\nu k_\alpha}{(k^2 - m^2)^3}\right].
\tag{17}
$$

In the equation above the limit $m^2 \to 0$ is supposed to be taken. For the traditional spinorial QED, gauge invariance is obtained by setting these surface terms to zero. Pauli–Villars regularization, for example, is constructed to warrant gauge invariance, and in this case the surface terms cancel out. The diagrammatic proof of gauge invariance [62] is based on the assumption that there exists a regularization which allows for shifts in the integration momenta. Dimensional regularization has been developed exactly with this characteristic. So, if some technique works in the proper dimension of the theory, the preservation of the Ward identities depends on the elimination of the surface terms by means of symmetry restoring counterterms. The exception takes place when anomalies are involved. For situations like that, it is not possible to eliminate all the surface terms.

Now let us return to equation (13) and see how the result of [17] can be obtained. We turn our attention to the second

order contribution in the external vector b_μ. We have

$$
\begin{aligned}
& \tfrac{1}{2}\big(\Pi_{bb-}^{\mu\nu} + \Pi_{bb+}^{\mu\nu}\big) \\
& = -4\big\{(b^2 g^{\mu\nu} + 2 b^\mu b^\nu)(\alpha_3 - 2\alpha_2)\big\}.
\end{aligned}
\tag{18}
$$

If one uses symmetric integration when calculating α_2 and α_3, such that $k^\mu k^\nu \to g^{\mu\nu} k^2 / 4$ and $k^\mu k^\nu k^\alpha k^\beta \to g^{\{\mu\nu\alpha\beta\}} k^4 / 24$, one obtains

$$
\alpha_2 = \frac{\mathrm{i}}{32\pi^2} \quad \text{and} \quad \alpha_3 = \frac{5\mathrm{i}}{96\pi^2},
\tag{19}
$$

so that

$$
\tfrac{1}{2}\big(\Pi_{bb-}^{\mu\nu} + \Pi_{bb+}^{\mu\nu}\big) = \frac{\mathrm{i}}{24\pi^2}\big(b^2 g^{\mu\nu} + 2 b^\mu b^\nu\big).
\tag{20}
$$

It is the same result as in [17], apart from some factors of i and e in the definition of the Feynman rules. It is also interesting to see how the b_μ independent part of the photon self-energy (the simple QED vacuum polarization tensor) depends on the surface terms. From (13), we have

$$
\begin{aligned}
\Pi_0^{\mu\nu} = {}& \Pi(p^2)(p^\mu p^\nu - p^2 g^{\mu\nu}) - 4\alpha_1 g^{\mu\nu} \\
& - \tfrac{4}{3}\big\{\alpha_2(p^\mu p^\nu - p^2 g^{\mu\nu}) \\
& + (2 p^\mu p^\nu + p^2 g^{\mu\nu})(\alpha_3 - 2\alpha_2)\big\}.
\end{aligned}
\tag{21}
$$

By examining the expression above, we conclude that if gauge invariance is broken by the $\Pi_{bb\pm}^{\mu\nu}$ term, so it is also broken in the zeroth order, which is the QED vacuum polarization tensor. A simple calculation shows that any of the two possibilities for gauge invariance in the zeroth and second order in b_μ for the photon's self-energy, the stronger one, $\alpha's = 0$, or the weaker, $\alpha_1 = 0$ and $\alpha_3 = 2\alpha_2$, set to zero the first order dependence of $\Pi_\pm^{\mu\nu}$ on b_μ.

Hence, gauge invariance must be preserved in the full amplitude. Note that this implies the complete disappearance of b_μ from $\Pi_\pm^{\mu\nu}$. This means that the shifts $k \to k \pm b$ must be allowed independently in the two contributions, not necessarily being a global shift.

However, even by enforcing gauge invariance, we still have the freedom of choice of the parameter α_2. Now, we have not yet considered the $\Pi_{5\pm}^{\mu\nu}$ parts. The two contributions are equal and we have

$$
\Pi_5^{\mu\nu} = \tfrac{1}{2}\big(\Pi_{5+}^{\mu\nu} + \Pi_{5-}^{\mu\nu}\big) = \Pi_{5+}^{\mu\nu},
\tag{22}
$$

which, after Dirac algebra, can be written as

$$
\begin{aligned}
\Pi_5^{\mu\nu} & = -4\mathrm{i} b_\beta \epsilon^{\mu\alpha\nu\beta}\int_k^\Lambda \frac{(p+k)_\alpha}{(k+b)^2(k+p+b)^2} \\
& = -4\mathrm{i} b_\beta \epsilon^{\mu\alpha\nu\beta}(p_\alpha I + I_\alpha),
\end{aligned}
\tag{23}
$$

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with

$$I, I_\alpha = \int_k^\Lambda \frac{1, k_\alpha}{(k+b)^2(k+p+b)^2}. \tag{24}$$

The results of these integrals by means of implicit regularization [47] are given by

$$I = I_{\log}(\lambda^2) - \frac{i}{16\pi^2}\left[\ln\left(-\frac{p^2}{\lambda^2}\right) - 2\right] \tag{25}$$

and

$$I_\alpha = -\frac{(p+2b)_\alpha}{2}\left\{I_{\log}(\lambda^2)\right.$$
$$\left. - \frac{i}{16\pi^2}\left[\ln\left(-\frac{p^2}{\lambda^2}\right) - 2\right] - \alpha_2\right\}, \tag{26}$$

where

$$I_{\log}(\lambda^2) = \int_k^\Lambda \frac{1}{(k^2 - \lambda^2)^2} \tag{27}$$

is the remaining regularization dependent part and λ^2 is a mass parameter characteristic of the procedure. Substituting these results in equation (23), we get

$$\Pi_5^{\mu\nu} = -4i\alpha_2 p_\alpha b_\beta \epsilon^{\mu\nu\alpha\beta} \quad \Rightarrow \quad \Delta c_\mu = 2i\alpha_2 b_\mu. \tag{28}$$

We see that the coefficient of the CS-type generated term is proportional to the surface term α_2. It is the same parameter that could not be fixed on gauge invariance grounds. Of course, symmetric integration will give us the traditional result

$$\Delta c_\mu = -\frac{1}{16\pi^2}b_\mu. \tag{29}$$

Nevertheless, we would like to argue that this result is ambiguous and regularization dependent. If we decide to calculate all the surface terms by using symmetric integration, gauge invariance is violated even in the zeroth order in b (from (13), and transversality is violated in all orders in b if symmetric integration is applied to calculate the surface terms). This means that we have violation of gauge symmetry in simple QED. Moreover, the non-perturbative functional calculus has shown that the model must be gauge invariant. Concerning the coefficient which was obtained in the functional calculation, as discussed in [13, 22], there is an unavoidable ambiguity coming from the definition of the current operator.

One comment on the meaning of implicit regularization is in order. Although the regulator needs not to be specified, it serves to obtain the following crucial features of IR: all infinities and the differences of infinite integrals of the same degree of divergency (the surface terms) occurring in

a certain amplitude do not involve loop propagators that depend on external momenta. Thus, they decouple from the physical content and dynamics of the amplitude. The latter is contained in strictly finite integrals, which are integrated without restriction. As a consequence the regulator does not need to be a gauge invariant one (an example is the simple cutoff), since the symmetry is restored by fixing the surface terms (it corresponds to making use of symmetry restoring counterterms). Nevertheless, the regulator should not modify the dimension of the integrals, or one would face the problems related to the dimension specific theories. Keeping in mind that a regularization is implicit, all the integrals of the same amplitude are treated on the same footing. In other words, the same unregularized integrals will give us the same result when regularized. The α_i (see (13)) coming from these integrals are the same and are adjusted according to the symmetry. On the other hand, since the adjustment of the α_i corresponds to using symmetry restoring counterterms, for different amplitudes they can be fixed at different values. It is in this sense that we can say that the same unregularized integrals can lead to different results, i.e. when they stem from different amplitudes. This is not the case here. We treat only one amplitude, the vacuum polarization tensor of the model.

3 The massive model

In this section, we complement our analysis with the massive model. It is instructive to see that the same conditions for preserving gauge symmetry are obtained. First, we consider the complete photon's self-energy, $\Pi_{\mu\nu}(p)$, which on gauge invariance grounds must be transverse:

$$p^\mu \Pi_{\mu\nu} = 0. \tag{30}$$

The modified fermionic propagator is given by

$$S(k) = \frac{i}{\slashed{k} - m - \slashed{b}\gamma_5}, \tag{31}$$

and so we have

$$\Pi^{\mu\nu} = -\int_k^\Lambda \text{tr}\left\{\gamma^\nu \frac{1}{\slashed{k} + \slashed{p} - m - \slashed{b}\gamma_5}\gamma^\mu \frac{1}{\slashed{k} - m - \slashed{b}\gamma_5}\right\}. \tag{32}$$

Now we perform the contraction with the external momentum p and use the identity $\slashed{p} = (\slashed{k} + \slashed{p} - m - \slashed{b}\gamma_5) - (\slashed{k} - m - \slashed{b}\gamma_5)$ and trace properties to obtain

$$p_\mu \Pi^{\mu\nu} = \int_k^\Lambda \text{tr}\left\{\frac{1}{\slashed{k} - m - \slashed{b}\gamma_5}\gamma^\nu\right\}$$
$$- \int_k^\Lambda \text{tr}\left\{\frac{1}{\slashed{k} + \slashed{p} - m - \slashed{b}\gamma_5}\gamma^\nu\right\}. \tag{33}$$

It is clear from the equation above that gauge invariance is obeyed under the condition that the shift $k \to k + p$ in the first integral is allowed (or $k \to k - p$ in the second one). Note that the shift is not carried out in the amplitude as a whole, but only in one of the terms. In other words, it is not a global shift. The same occurs if we are dealing with traditional QED ($b = 0$). Nevertheless, the shift is carried out in the contracted amplitude $p_\mu \Pi^{\mu\nu}$, and this will be important for the conclusions on the generation of the CS term.

Now we consider the possibilities of violation of gauge symmetry, specifically the contribution quadratic in b to the vacuum polarization tensor. For the modified fermion propagator, it is possible to write an expansion, which reads

$$\frac{i}{\slashed{k} - m - \slashed{b}\gamma_5} = \sum_{n=0}^{\infty} \frac{i}{\slashed{k} - m} \left\{ -i\slashed{b}\gamma_5 \frac{i}{\slashed{k} - m} \right\}^n$$

$$= \sum_{n=0}^{\infty} S_n(k). \tag{34}$$

For the b^2 order, we have

$$\Pi_{bb}^{\mu\nu} = -\int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_1(p+k)\gamma^\mu S_1(k)\}$$

$$- \int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_2(p+k)\gamma^\mu S_0(k)\}$$

$$- \int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_0(p+k)\gamma^\mu S_2(k)\}. \tag{35}$$

When the contraction with p_μ is performed and the identity $\slashed{p} = (\slashed{k} + \slashed{p} - m) - (\slashed{k} - m)$ is used, there remain only two terms:

$$p_\mu \Pi_{bb}^{\mu\nu} = -\int_k^\Lambda \mathrm{tr}\left\{\gamma^\nu \frac{1}{\slashed{k} + \slashed{p} - m} \slashed{b}\gamma_5 \frac{1}{\slashed{k} + \slashed{p} - m} \right.$$

$$\times \slashed{b}\gamma_5 \frac{1}{\slashed{k} + \slashed{p} - m}\Bigg\}$$

$$+ \int_k^\Lambda \mathrm{tr}\left\{\gamma^\nu \frac{1}{\slashed{k} - m}\slashed{b}\gamma_5 \frac{1}{\slashed{k} - m}\slashed{b}\gamma_5 \frac{1}{\slashed{k} - m}\right\}. \tag{36}$$

The second term is zero, and the first differs from zero by a surface term (they differ by a shift). So, all we have to do is to identify the surface terms. They are easily identified by using, after the Dirac algebra has been performed, the identity

$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{(k^2 - m^2)} - \frac{p^2 + 2p \cdot k}{(k^2 - m^2)[(p+k)^2 - m^2]} \tag{37}$$

to separate the divergent (regularization dependent) terms. The surface terms come from differences between integrals

of the same degree of divergence. Using the definitions of (15) and (16), we get

$$p_\mu \Pi_{bb}^{\mu\nu} = -4(\alpha_3 - 2\alpha_2)[b^2 p^\nu + 2(b \cdot p)b^\nu], \tag{38}$$

and the same condition for transversality is achieved.

To complete our analysis, we look at the linear contribution in the external vector b_μ. We have two contributions:

$$\Pi_b^{\mu\nu} = \int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_0(p+k)\gamma^\mu S_1(k)\}$$

$$+ \int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_1(p+k)\gamma^\mu S_0(k)\}. \tag{39}$$

Using

$$S_1(k) = -iS_0(k)\slashed{b}\gamma_5 S_0(k), \tag{40}$$

we obtain

$$\Pi_b^{\mu\nu} = -i\int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_0(p+k)\gamma^\mu S_0(k)\slashed{b}\gamma_5 S_0(k)\}$$

$$- i\int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_0(k+p)\slashed{b}\gamma_5 S_0(k+p)\gamma^\mu S_0(k)\}. \tag{41}$$

We have already seen on gauge invariance grounds that a shift $k \to k \pm p$ must be allowed in $p_\mu \Pi^{\mu\nu}$, not necessarily as a global shift. However, in this contracted amplitude the term linear in b has already disappeared. The reason is the presence of the antisymmetric Levi-Cìvita tensor. So it does not say anything about the surface term that emerges from a shift $k \to k - p$ in the first integral. By performing this shift and using the fact that the integral is odd in p and its antisymmetry under the exchange $\mu \leftrightarrow \nu$, we see that we have two equal terms plus the surface term. This surface term is easily identified. Then we have

$$\Pi_b^{\mu\nu} = -2ib_\alpha \int_k^\Lambda \mathrm{tr}\{\gamma^\nu S_0(k+p)\gamma^\alpha \gamma_5 S_0(k+p)\gamma^\mu S_0(k)\}$$

$$+ 4i\alpha_2 b_\alpha p_\beta \epsilon^{\mu\nu\alpha\beta}$$

$$\equiv -2ib_\alpha T^{\mu\nu\alpha} + 4i\alpha_2 b_\alpha p_\beta \epsilon^{\mu\nu\alpha\beta}. \tag{42}$$

We now follow the calculations of Refs. [24, 25]. Carrying out the Dirac algebra, we get

$$T^{\mu\nu\alpha} = \int_k^\Lambda \frac{\mathcal{N}^{\mu\nu\alpha}}{D}, \tag{43}$$

with

$$\mathcal{N}^{\mu\nu\alpha} = -4\{\{[(p+k)^2 - m^2]k_\beta - 2m^2 p_\beta\}\epsilon^{\mu\nu\alpha\beta}$$

$$- 2p_\sigma k_\beta k^\alpha \epsilon^{\mu\nu\sigma\beta}\} \tag{44}$$

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and

$$D = (k^2 - m^2)\big[(p+k)^2 - m^2\big]^2. \tag{45}$$

We can write

$$T^{\mu\nu\alpha} = -4\big\{(I_\beta - 2m^2 p_\beta J)\epsilon^{\mu\nu\alpha\beta}$$
$$- 2p_\sigma g^{\alpha\lambda} J_{\beta\lambda}\epsilon^{\mu\nu\sigma\beta}\big\}, \tag{46}$$

where

$$I_\beta = \int_k^\Lambda \frac{k_\beta}{(k^2 - m^2)[(p+k)^2 - m^2]} \tag{47}$$

and

$$J, J_{\beta\lambda} = \int_k^\Lambda \frac{1, k_\beta k_\lambda}{(k^2 - m^2)[(p+k)^2 - m^2]^2}. \tag{48}$$

The only finite integral is J. For the others a regularization method is needed. Since we are going to analyze the regularization dependence of the generated term, we opt to keep the regularization implicit. So, we adopt here the procedure of implicit regularization (IR) [24, 25, 47–60], since it permits one to leave the evaluation of divergent integrals to the end of the calculation. The results of the divergent integrals are given by [24]

$$I_\beta = -\frac{p_\beta}{2}\left\{I_{\log}(m^2) - \frac{i}{16\pi^2}Z_0(p^2, m^2) - \alpha_2\right\} \tag{49}$$

and

$$J_{\beta\lambda} = \frac{g_{\beta\lambda}}{4}\left\{I_{\log}(m^2) - \frac{i}{16\pi^2}Z_0(p^2, m^2) - \alpha_2\right\}$$
$$+ \text{terms in } p_\beta p_\lambda, \tag{50}$$

where

$$Z_0(p^2, m^2) = \int_0^1 dx \ln\left(\frac{p^2 x(1-x) - m^2}{(-m^2)}\right). \tag{51}$$

For the finite one, we obtain

$$J = \frac{i}{16\pi^2}\int_0^1 dx \frac{(1-x)}{(p^2 x(1-x) - m^2)}. \tag{52}$$

When these integrals are substituted into equation (46), only the term in J survives:

$$T^{\mu\nu\alpha} = 8m^2 J p_\beta \epsilon^{\mu\nu\alpha\beta}. \tag{53}$$

In the limit $p^2 \to 0$, we have

$$T^{\mu\nu\alpha} \to \frac{i}{4\pi^2}p_\beta\epsilon^{\mu\nu\alpha\beta} \quad \Rightarrow$$

$$\Pi_b^{\mu\nu} \to \frac{1}{2\pi^2}b_\alpha p_\beta \epsilon^{\mu\nu\alpha\beta}\big(1 + 8i\pi^2\alpha_2\big). \tag{54}$$

This will give

$$\Delta c_\mu = \frac{1}{4\pi^2}b_\mu\big(1 + 8i\pi^2\alpha_2\big). \tag{55}$$

Some comments are in order. We can make explicit the regularization required to manipulate the divergent integrals. Then renormalization allows one to remove the regulator in all the finite integrals, and one is left with the integrals which depend on the regularization, specifically the basic divergence $I_{\log}(m^2)$ and the surface term parameterized as $\alpha_2 g_{\mu\beta}$. We see that I_β and $J_{\mu\beta}$ do depend on the regularization to be used. Nevertheless, these ambiguous terms cancel out exactly when the integrals are inserted into the amplitude. Actually, the real ambiguity in all this calculation comes from the shift carried out in one of the terms of (41) in order to obtain the expression (42).

The result of the massless model can be recovered if the limit $m^2 \to 0$ is taken before the other one, $p^2 \to 0$. As before, gauge symmetry must be preserved, but it is not sufficient to fix α_2. The interesting result is that although the shift $k \to k \pm p$ must be allowed in $p_\mu \Pi^{\mu\nu}$, this does not mean that all the surface terms must be set to zero. Actually, they are constrained to respect a definite relation. Even in theories in which there are no parity violating mathematical objects, like the γ_5 matrix, this freedom of fixing one of the surface terms occurs. However, in these cases there is no loss of generality if they are put to zero.

4 Summary and discussion

We have carried out an investigation of the modified Lorentz violating QED with an axial term in the fermionic sector. In particular, attention was paid in the preservation or violation of the gauge symmetry of the model and its relation with the radiative generation of a CS-like term. If we consider the action, we have shown that the quantum corrections must preserve gauge symmetry. If it is broken, then gauge invariance of simple QED is also spoiled. In order to come to this conclusion, the study of the massless model is very instructive, since in this case the functional non-perturbative analysis (in b and in the coupling constant), performed in the beginning of Sect. 2, clearly exhibits this feature. When loop calculations are performed, the surface terms play an important role as parameters to be fixed. Gauge invariance, in this case, requires the possibility of carrying out some shifts. This fact does not restrict all the surface terms to be fixed null, although they are constrained to obey a certain relation. As a conclusion, there is an unavoidable ambiguity in the calculation of the coefficient of the radiatively generated CS-like term.

However, it has not been the main purpose of the present work to show that gauge invariance does not fix the coefficient of the CS-like term. This is only a residual conclusion.

It is obvious that the gauge invariance of the action is not violated by the CS term, whatever its coefficient might be. Our main focus is the quadratic term in b, the background that causes Lorentz violation. Despite the existence of a vast bibliography concerning the radiative generation of the CS-type term, only few publications treat the quadratic piece. Parity arguments have been considered by Altschul [17] to admit the possibility of violation of gauge invariance by the quadratic term and the respective calculation in a specific scheme led indeed to its violation. In further work [18, 19], this result is used to perform another analysis. In Ref. [21], the authors have performed the calculation of the quadratic term in b using a Pauli–Villars regularization and have obtained a gauge invariant result. However, they have used a less general version of this regularization with the constraint that the gauge invariance of the Lagrangian density is preserved. As a consequence, no CS-like term is generated. Therefore, there was left no room for violation of gauge symmetry, and its preservation at b^2 order brings no insight concerning the full amplitude. Moreover, the results of Ref. [17] have not been discussed.

In our present work we adopt the implicit regularization (IR) method to perform a general analysis. For the massless case, for example, we analyze the complete one-loop vacuum polarization tensor (all orders in b). We show that the conditions to adjust the regularization dependent terms in such a way that gauge symmetry is preserved at b^2 order (recall that according to IR these conditions are kept open until the end of the calculation) must be exactly the same as for its preservation at zeroth order in b. In other words, if gauge symmetry is spoiled at b^2 order, it is also spoiled in QED. Furthermore, the functional non-perturbative analysis carried out in the first part of Sect. 2, based on Ref. [22], shows that the model is gauge invariant to all orders, as discussed in the paragraph before (8).

Finally, we have also performed a critical discussion of some procedures that are commonly adopted in the literature when shifts are done in divergent integrals and the corresponding surface terms calculated in a "naive" way. We show by comparing to the results obtained within IR, which is constructed to yield symmetry preserving answers, that a naive implementation of surface terms may lead to inconsistencies and violation of symmetries even in the simplest case (QED).

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