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GAUGE THEORIES IN PARTICLE PHYSICS

A PRACTICAL INTRODUCTION
THIRD EDITION

Volume 2

Non-Abelian Gauge Theories:
QCD and the Electroweak Theory

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