

GRADUATE STUDENT SERIES IN PHYSICS

Series Editor:

Professor Douglas F Brewer, MA, DPhil

*Emeritus Professor of Experimental Physics, University of Sussex*

---

# GAUGE THEORIES IN PARTICLE PHYSICS

A PRACTICAL INTRODUCTION  
THIRD EDITION

Volume 2

Non-Abelian Gauge Theories:  
QCD and the Electroweak Theory

IAN J R AITCHISON

*Department of Physics*

*University of Oxford*

ANTHONY J G HEY

*Department of Electronics and Computer Science*

*University of Southampton*

**IOP**

INSTITUTE OF PHYSICS PUBLISHING

Bristol and Philadelphia

# CONTENTS

<b>Preface to Volume 2 of the Third Edition</b>	<b>xiii</b>
Acknowledgments	xvii
<b>PART 5</b>	
<b>NON-ABELIAN SYMMETRIES</b>	<b>1</b>
<b>12 Global non-Abelian Symmetries</b>	<b>3</b>
12.1 The flavour symmetry $SU(2)_f$	5
12.1.1 The nucleon isospin doublet and the group $SU(2)$	5
12.1.2 Larger (higher-dimensional) multiplets of $SU(2)$ in nuclear physics	12
12.1.3 Isospin in particle physics	14
12.2 Flavour $SU(3)_f$	18
12.3 Non-Abelian global symmetries in Lagrangian quantum field theory	24
12.3.1 $SU(2)_f$ and $SU(3)_f$	24
12.3.2 Chiral symmetry	31
Problems	37
<b>13 Local non-Abelian (Gauge) Symmetries</b>	<b>39</b>
13.1 Local $SU(2)$ symmetry: the covariant derivative and interactions with matter	40
13.2 Covariant derivatives and coordinate transformations	48
13.3 Geometrical curvature and the gauge field strength tensor	55
13.4 Local $SU(3)$ symmetry	59
13.5 Local non-Abelian symmetries in Lagrangian quantum field theory	61
13.5.1 Local $SU(2)$ and $SU(3)$ Lagrangians	61
13.5.2 Gauge field self-interactions	63
13.5.3 Quantizing non-Abelian gauge fields	69
Problems	78

<b>PART 6</b>	
<b>QCD AND THE RENORMALIZATION GROUP</b>	<b>81</b>
<b>14 QCD I: Introduction and Tree-Graph Predictions</b>	<b>83</b>
14.1 The colour degree of freedom	84
14.2 The dynamics of colour	87
14.2.1 Colour as an SU(3) group	87
14.2.2 Global SU(3) <sub>c</sub> invariance and ‘scalar gluons’	90
14.2.3 Local SU(3) <sub>c</sub> invariance: the QCD Lagrangian	92
14.3 Hard scattering processes and QCD tree graphs	95
14.3.1 Two-jet events in $\bar{p}p$ collisions	95
14.3.2 Three-jet events	104
14.4 Three-jet events in $e^+e^-$ annihilation	105
Problems	113
<b>15 QCD II: Asymptotic Freedom, the Renormalization Group and Scaling Violations in Deep Inelastic Scattering</b>	<b>115</b>
15.1 QCD corrections to the parton model prediction for $\sigma(e^+e^- \rightarrow$ hadrons)	115
15.2 The renormalization group and related ideas	119
15.2.1 Where do the large logs come from?	119
15.2.2 Changing the renormalization scale	121
15.2.3 The renormalization group equation and large $-q^2$ behaviour in QED	124
15.3 Back to QCD: asymptotic freedom	126
15.4 A more general form of the RGE: anomalous dimensions and running masses	130
15.5 Some technicalities	135
15.6 $\sigma(e^+e^- \rightarrow$ hadrons) revisited	138
15.7 QCD corrections to the parton model predictions for deep inelastic scattering: scaling violations	139
15.7.1 Uncancelled mass singularities	139
15.7.2 Factorization and the DGLAP equation	144
15.7.3 Comparison with experiment	146
Problems	151
<b>16 Lattice Field Theory and the Renormalization Group Revisited</b>	<b>153</b>
16.1 Introduction	153
16.2 Discretization	154
16.3 Gauge invariance on the lattice	158
16.4 Representation of quantum amplitudes	161
16.5 Connection with statistical mechanics	169
16.6 Renormalization and the renormalization group on the lattice	171
16.6.1 Introduction	171
16.6.2 The one-dimensional Ising model	172

16.6.3	Further developments and some connections with particle physics	176
16.7	Numerical calculations	182
	Problems	191
<b>PART 7</b>		
<b>SPONTANEOUSLY BROKEN SYMMETRY</b>		<b>193</b>
<b>17</b>	<b>Spontaneously Broken Global Symmetry</b>	<b>195</b>
17.1	Introduction	195
17.2	The Fabri–Picasso theorem	197
17.3	Spontaneously broken symmetry in condensed matter physics	199
17.3.1	The ferromagnet	199
17.3.2	The Bogoliubov superfluid	202
17.4	Goldstone’s theorem	208
17.5	Spontaneously broken global U(1) symmetry: the Goldstone model	211
17.6	Spontaneously broken global non-Abelian symmetry	215
17.7	The BCS superconducting ground state	218
	Problems	224
<b>18</b>	<b>Chiral Symmetry Breaking</b>	<b>226</b>
18.1	The Nambu analogy	227
18.1.1	Two flavour QCD and $SU(2)_{fL} \times SU(2)_{fR}$	230
18.2	Pion decay and the Goldberger–Treiman relation	234
18.3	The linear and nonlinear $\sigma$ -models	239
18.4	Chiral anomalies	244
	Problems	249
<b>19</b>	<b>Spontaneously Broken Local Symmetry</b>	<b>250</b>
19.1	Massive and massless vector particles	250
19.2	The generation of ‘photon mass’ in a superconductor: the Meissner effect	255
19.3	Spontaneously broken local U(1) symmetry: the Abelian Higgs model	259
19.4	Flux quantization in a superconductor	263
19.5	’t Hooft’s gauges	266
19.6	Spontaneously broken local $SU(2) \times U(1)$ symmetry	269
	Problems	274

<b>PART 8</b>	
<b>WEAK INTERACTIONS AND THE ELECTROWEAK THEORY 275</b>	
<b>20</b>	<b>Introduction to the Phenomenology of Weak Interactions 277</b>
20.1	Fermi's 'current-current' theory of nuclear $\beta$ -decay and its generalizations 278
20.2	Parity violation in weak interactions 281
20.3	Parity transformation of Dirac wavefunctions and field operators 282
20.4	V – A theory: chirality and helicity 285
20.5	Charge conjugation for fermion wavefunctions and field operators 289
20.6	Lepton number 292
20.7	The universal current-current theory for weak interactions of leptons 295
20.8	Calculation of the cross-section for $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ 297
20.9	Leptonic weak neutral currents 301
20.10	Quark weak currents 302
20.11	Deep inelastic neutrino scattering 306
20.12	Non-leptonic weak interactions 314
	Problems 316
<b>21</b>	<b>Difficulties with the Current-Current and 'Naive' Intermediate Vector Boson Models 321</b>
21.1	Violation of unitarity in the current-current model 322
21.2	The IVB model 325
21.3	Violation of unitarity bounds in the IVB model 326
21.4	The problem of non-renormalizability in weak interactions 331
	Problems 334
<b>22</b>	<b>The Glashow-Salam-Weinberg Gauge Theory of Electroweak Interactions 336</b>
22.1	Weak isospin and hypercharge: the $SU(2) \times U(1)$ group of the electroweak interactions: quantum number assignments and W and Z masses 336
22.2	The leptonic currents (massless neutrinos): relation to current-current model 341
22.3	The quark currents 345
22.4	Simple (tree-level) predictions 346
22.5	The discovery of the $W^\pm$ and $Z^0$ at the CERN $p\bar{p}$ collider 351
	22.5.1 Production cross-sections for W and Z in $p\bar{p}$ colliders 351
	22.5.2 Charge asymmetry in $W^\pm$ decay 353
	22.5.3 Discovery of the $W^\pm$ and $Z^0$ at the $p\bar{p}$ collider and their properties 353
22.6	The fermion mass problem 358
22.7	Three-family mixing 364
	22.7.1 Quark flavour mixing 364

22.7.2	Neutrino flavour mixing	369
22.8	Higher-order corrections	370
22.9	The top quark	379
22.10	The Higgs sector	380
22.10.1	Introduction	380
22.10.2	Theoretical considerations concerning $m_H$	383
22.10.3	Higgs phenomenology	385
Problems		388
<b>Appendix M Group Theory</b>		<b>390</b>
M.1	Definition and simple examples	390
M.2	Lie groups	391
M.3	Generators of Lie groups	391
M.4	Examples	393
M.4.1	SO(3) and three-dimensional rotations	393
M.4.2	SU(2)	394
M.4.3	SO(4): The special orthogonal group in four dimensions	395
M.4.4	The Lorentz group	396
M.4.5	SU(3)	397
M.5	Matrix representations of generators and of Lie groups	398
M.6	The Lorentz group	401
M.7	The relation between SU(2) and SO(3)	405
<b>Appendix N Dimensional Regularization</b>		<b>409</b>
<b>Appendix O Grassmann Variables</b>		<b>413</b>
<b>Appendix P Majorana Fermions</b>		<b>418</b>
P.1	Spin- $\frac{1}{2}$ wave equations	418
P.2	Majorana quantum fields	422
<b>Appendix Q Feynman Rules for Tree Graphs in QCD and the Electroweak Theory</b>		<b>425</b>
Q.1	QCD	425
Q.1.1	External particles	425
Q.1.2	Propagators	425
Q.1.3	Vertices	426
Q.2	The electroweak theory	426
Q.2.1	External particles	427
Q.2.2	Propagators	427
Q.2.3	Vertices	428
<b>References</b>		<b>433</b>
<b>Index</b>		<b>443</b>