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GAUGE THEORY OF STRONG, WEAK, AND ELECTROMAGNETIC INTERACTIONS*

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ABSTRACT

Starting from the recent $U(3) \otimes U(3)$ gauge model of strong interactions, we discuss the very natural synthesis with Weinberg's model of leptons. Due to the interplay of the two models (a) neutral $\Delta S = 1$ currents are eliminated without enlarging the number of quarks, and (b) a natural $(3,\bar{3}) \oplus (\bar{3},3)$ symmetry breaking emerges for the hadrons. Incorporation of alternative lepton theories is mentioned.

Although there are now a number of renormalizable gauge models of leptons,[1] only one class of such theories has been constructed for hadrons.[2] We have asked the question: Can we effect a marriage between the hadron and lepton theories, such that, for simplicity, we have just $[(3,\bar{3}) \oplus (\bar{3},3)$ broken$]$ $U(3) \otimes U(3)$ (3 quarks) for the hadrons, and only observed leptons? The answer, modulo anomalies, is a very natural yes. Our removal of anomalies, for reasons mentioned below, involves fermionic doubling; this will be discussed after presentation of the model, along with inclusion of other lepton models.

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Our approach to the synthesis is orderly: Starting with the $U(3) \otimes U(3)$ gauge model of hadrons, and Weinberg's $SU(2) \otimes U(1)$ model, we study embedding the latter in progressively larger "primed"[2] groups. Details of this study (and intermediate models), will appear in a larger paper. Here we sketch the emerging picture: The structure of the hadron theory requires the "primed" group at least as large as $SU(3) \otimes SU(3)$,[2] and Weinberg's model can be embedded therein. Unfortunately, models of this type have trouble with strangeness-changing processes. When we embed the leptons in $U(4) \otimes U(4)$ however, everything falls together beautifully, and it is this model we now present.

<u>Groups and Representations</u>. The hadronic group, entirely local, is $U(3)_L \otimes U(3)_R$. We represent its generators $F_{\alpha L}$, $F_{\alpha R}$ by $\frac{1}{2}\lambda^{\alpha}$ ($\alpha = 0,\cdots,8$, left or right), being the usual 3×3 SU(3) matrices. The local leptonic group is $SU(2)_L \otimes U(1)$, embedded in a "primed" $U(4)_L \otimes U(4)_R$. We call the latter's generators $F'_{\beta L}$, $F'_{\beta R}$ ($\beta = 0,\cdots,15$), but only four are realized locally. These are $\tilde{F}'_{kL}(k = 1,2,3)$ and $\tilde{Y} = \tilde{F}'_{3R} + \frac{1}{3}(\tilde{F}'_{0R} + \tilde{F}'_{0L})$, with representations

$$\tilde{t}_{\beta} = R\begin{bmatrix} \frac{1}{2}\tau_{\beta} & 0 \\ 0 & \frac{1}{2}\hat{\tau}_{\beta} \end{bmatrix}R^{-1}, \quad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where τ_{β} ($\beta = 0,1,2,3$, left or right) are the usual Pauli matrices, $\hat{\tau}_{\beta} \equiv \tau_2\tau_{\beta}\tau_2$, and the twiddle operation R is the Cabibbo rotation. The charge operator is

$$Q \equiv (F_{3L} + F_{3R}) + \frac{1}{\sqrt{3}}(F_{8L} + F_{8R}) + (F'_{3L} + F'_{3R}) + \frac{1}{3}(F'_{0L} + F'_{0R}). \tag{2}$$

Neutral operators do not rotate under R: $\tilde{t}_3 = t_3$, $\tilde{Y} = Y$, etc.

Local transformations. We represent the general local operator transformation in a unified super matrix notation

$$\mathcal{U} = \exp i\{\alpha_L\cdot F_L + \alpha_R\cdot F_R + \beta\cdot F'_L + \gamma Y\} \rightarrow S = \begin{bmatrix} S_L & 0 & 0 & 0 \\ 0 & S_R & 0 & 0 \\ 0 & 0 & \tilde{S}'_R & 0 \\ 0 & 0 & 0 & \tilde{S}'_L \end{bmatrix} \quad (3)$$

where, e.g., $\tilde{S}'_L = \exp i(\tilde{t}\cdot\beta + \frac{1}{3}t_0\gamma)$, $\tilde{S}'_R = \exp i(t_3 + \frac{1}{3}t_0)\gamma$, etc.

Fields. Let $V_\alpha{}^\mu$, $A_\alpha{}^\mu$, W_k^μ, B^μ be (respectively) the strong vector and axial vector fields and the weak gauge bosons. Defining $V^\mu_{L,R} \equiv \sum_0^8 (V_\alpha{}^\mu \mp A_\alpha{}^\mu)\frac{\lambda^\alpha}{2}$, $\tilde{W} \equiv \sum_1^3 \tilde{t}_k W_k$, we specify the transformation properties of all gauge bosons at once in terms of a diagonal supermatrix V^μ (like S), with entries[3] $V^\mu:[fV_L{}^\mu, fV_R{}^\mu, g'(t_3 + \frac{1}{3}t_0)B^\mu, g\tilde{W}^\mu + \frac{1}{3}g't_0B^\mu]$. Then $\mathcal{U}V_\mu\mathcal{U}^{-1} = S(V_\mu - i\partial_\mu)S^{-1}$. Similarly, for Weinberg's leptons, we introduce ($\nu_L=\nu_e$, $\nu_R=\nu_\mu{}^c$, D=doublet, S=singlet)

$$\Psi_D = \left[\begin{array}{cc|c} \nu_L & \mu^+_R & \\ e^-_L & -\nu_R & 0 \\ \hline \multicolumn{2}{c|}{0} & ? \end{array}\right], \quad \Psi_S = \left[\begin{array}{cc|c} 0 & \mu^+_L & \\ e^-_R & 0 & 0 \\ \hline \multicolumn{2}{c|}{0} & ? \end{array}\right], \quad \ell = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\Psi}_S & \frac{1}{\sqrt{2}}\tilde{\Psi}_D^c \\ 0 & 0 & \frac{1}{\sqrt{2}}\tilde{\Psi}_D & 0 \end{bmatrix}$$

(?) will be replaced later by heavy leptons to eliminate anomalies and $\psi_D{}^c$ is the charge conjugate of ψ_D; then $\mathcal{U}\,\ell\,\mathcal{U}^{-1} = S\,\ell\,S^{-1}$. There is a great deal of freedom in the quark assignment. We will take here the simplest case, three fractionally charged quarks: (T=transpose) $q^T = (q_L, q_R, 0, 0)$, $\mathcal{U}\,q\,\mathcal{U}^{-1} = S\,q$. The supermatrix notation is most symmetric for the scalar mesons,

$$M = \begin{bmatrix} 0 & \Sigma & 0 & M_L \\ \Sigma^+ & 0 & M_R & 0 \\ 0 & M_R^+ & 0 & \tilde{\emptyset}^+ \\ M_L^+ & 0 & \tilde{\emptyset} & 0 \end{bmatrix}, \qquad \mathcal{U}M\mathcal{U}^{-1} = SMS^{-1}. \quad (5)$$

Here, $\Sigma = \sigma + i\pi$ is the usual $(3,\bar{3}) \oplus (\bar{3},3)$ multiplet of scalars and pseudoscalars; $M_{L,R}$ are three-by-four complex matrices, being the scalars of Ref. (2), now with an extra fourth column. These scalars are the only connections of the weak and electromagnetic interactions with the strong, and will give mass to strong vector mesons;[2] $\emptyset$ is the Weinberg scalar field $\emptyset = \emptyset_0 t_0 + i\emptyset\cdot t$, which we see (in this notation) is to the leptonic system what Σ is to the hadronic.

Covariant derivatives and Lagrangian. The covariant derivatives can be read from the covariant momentum operator

$$\mathcal{P}^\mu \equiv P^\mu + f(V_L{}^\mu\cdot F_L + V_R{}^\mu\cdot F_R) + gW^\mu\cdot\tilde{F}'_L + g'B^\mu Y \quad (6)$$

by commuting this operator with each field. We find, $\Delta_\mu q \equiv \partial_\mu q - iV_\mu q$, $\Delta_\mu\ell \equiv \partial_\mu\ell - i[V_\mu,\ell]$, $\Delta_\mu M \equiv \partial_\mu M - i[V_\mu, M]$ and the usual $F_{\mu\nu}$'s for each gauge meson. Our locally invariant Lagrangian is

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4}\mathrm{Tr}(F_L{}^{\mu\nu}F_{\mu\nu}{}^L + F_R{}^{\mu\nu}F_{\mu\nu}{}^R) - \frac{1}{4}F_{\mu\nu}{}^B F_B{}^{\mu\nu} - \frac{1}{4}\mathrm{Tr}(F_{\mu\nu}{}^W F_W{}^{\mu\nu}) \\ &- i\bar{q}\Delta^\mu\gamma_\mu q - i\,\mathrm{Tr}(\bar{\ell}\Delta^\mu\gamma_\mu\ell) - \frac{1}{4}\mathrm{Tr}\{\Delta^\mu M)^+\Delta_\mu M\} + \alpha(\bar{q}_L\Sigma q_R + \mathrm{h.c.}) \\ &+ \mathrm{Tr}(\tilde{\bar{\psi}}_D\,\tilde{\emptyset}\,\tilde{\psi}_S\,\tilde{G} + \mathrm{h.c.}) + V(M_L) + V(M_R) + V(\Sigma) + V(\emptyset) \\ &+ \mathrm{Tr}(G_2\,\tilde{\emptyset}^+ M_L{}^+ M_L\tilde{\emptyset} + G_2 M_R^+ M_R\tilde{\emptyset}^+\tilde{\emptyset}) + \mathrm{Tr}(G_1\tilde{\emptyset}^+ M_L^+\Sigma M_R + \mathrm{h.c}) \end{aligned} \quad (7)$$

where the $V(\cdots)$'s are the usual quartic, and quadratic terms,[3] and the G "insertions",[2] which are 4×4 diagonal matrices with entries $G: \frac{2}{\lambda}(m_e, m_\mu, ?, ?)$, $G_1: \frac{\sqrt{2}}{2\lambda}\left(\frac{f_\pi m_\pi{}^2}{\kappa_1{}^2}, \frac{f_\pi m_\pi{}^2}{\kappa_1{}^2}, \frac{2f_K m_K{}^2 - f_\pi m_\pi{}^2}{\kappa_2{}^2}, d\right)$, $G_2: (a,a,b,c)$, do not spoil the unified gauge invariance. The interpretation of the parameters in these insertions will be clarified in the following paragraph.

Spontaneous breakdown and symmetries. A detailed study of the complicated scalar system will be presented in a larger paper. Here we sketch the general ideas. First, we use 21 degrees of gauge freedom (all but Q) to eliminate the 3×3 submatrices of $M_L - M_L^+$ and $M_R - M_R^+$, and all the components of ϕ except ϕ_0. Next, in order to give masses to all the gauge fields except the photon, we assign vacuum expectation values $\langle\phi\rangle \equiv \lambda t_0$, $\langle M_L\rangle = \langle M_R\rangle \equiv \kappa$. These then generate a linear term in Σ (last term in $\mathcal{L}$). Thus Σ itself acquires a vacuum expectation value $\langle\Sigma\rangle \equiv v$, which is the usual $(3,\bar{3}) \oplus (\bar{3},3)$ hadronic symmetry breaking in the spirit of Gell-Mann, Oakes, and Renner.[4] It further turns out that the system allows the following arbitrary vacuum expectation values:

$$\kappa = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 \\ 0 & \kappa_1 & 0 & 0 \\ 0 & 0 & \kappa_2 & 0 \end{bmatrix} \qquad v = \frac{1}{\sqrt{2}}\begin{bmatrix} f_\pi & 0 & 0 \\ 0 & f_\pi & 0 \\ 0 & 0 & 2f_K - f_\pi \end{bmatrix}. \tag{8}$$

and no Goldstone bosons. Except for d, the interpretation of the parameters in G_1 and v is standard,[5] while G_2, d, and $V(\cdots)$ can be adjusted to give arbitrarily large masses to ϕ_0 and the remaining scalars in M_L and M_R; hence with Ref. (2), we continue to regard these as unobservable entities. Actually, the model is perhaps more satisfactory with $\kappa_1 = \kappa_2$, leaving $\omega - \phi$ splitting until higher order in strong interactions. For this case we preserve the Weinberg sum rules[6] and so we specialize to $\kappa_1 = \kappa_2$ below.

Photon system and diagonalization. Our spontaneous breakdown is such that the only unbroken gauge symmetry is generated by Q. Rewriting the covariant momentum (6), we isolate the (massless, universal) photon as the coefficient of Q:

$$A^\mu = \cos\eta\,(\sin\phi\, W_3^{\ \mu} + \cos\phi\, B^\mu) + \sin\eta\left(\frac{\sqrt{3}}{2} V_3^{\ \mu} + \frac{1}{2} V_8^{\ \mu}\right)$$
$$e = g\sin\phi\cos\eta; \quad \tan\phi = \frac{g'}{g}, \quad \tan\eta = \frac{2\,g\sin\phi}{\sqrt{3}\,f}. \tag{9}$$

With $f^2/4\pi \sim 2$ and g, g' small, we obtain approximately Weinberg's $e \sim gg'/(g^2 + g'^2)^{\frac{1}{2}}$. This diagonalization induces electromagnetic mixing of bare ρ_0, ϕ, and ω, such that the physical particles have order e^2/f couplings directly to the leptonic electromagnetic currents. This simulates vector-dominated electromagnetic form factors in lowest order, and gives a hadronic correction to the muonic $\frac{g-2}{2}$ which agrees with previous estimates.[7] To keep the usual universality of weak interactions, we do not diagonalize the $W^\pm$-strong vector meson mixings involved in the term

$$\mathcal{L}' = gf\,\mathrm{Tr}\{V_L(M_L + \kappa)\,\tilde{W}(M_L^+ + \kappa)\}\,. \tag{10}$$

Thus, charged currents at low energy proceed via vector dominance in lowest order.

$\Delta S = 1$ neutral currents. Because our Cabibbo rotation rotates only $W^\pm$, we find no neutral $\Delta S = 1$ currents. In this model then, although we need four "things" to eliminate such currents, they are the columns of the unobservable $M_{L,R}$, and not extra quarks.

Fermions and anomalies. As thus far presented, the model has anomalies.[8] Further, there appears to be no way, in the presence of both strong and weak vector mesons, to cancel hadronic against leptonic anomalies. Thus, we mention a flexible doubling scheme that for hadrons is in the spirit of dual models. We introduce q', $\psi'_{S,D}$[9] that couple to gauge bosons just as q, $\psi_{S,D}$, but with the opposite sign of γ_5. In the leptonic system anomalies are cancelled without complication. To avoid suppressing $\pi_0 \to 2\gamma$, however, we also need a new Σ', which transforms like Σ, but by choice, couples only to q'. It is then easy to arrange, with other terms in $\mathcal{L}$ like $\mathrm{Tr}[\tilde{\phi}^+ M_L^+ \Sigma' M_R G_1']$ and $(\alpha' \bar{q}_L' \Sigma' q_R' + \text{h.c.})$, that the masses of q', Σ' are high with negligible effect on V,A masses. Then, $\pi^0 \to 2\gamma$ proceeds only through q. To get an extra factor of three[10] in amplitude, there are a number of choices--the simplest being the introduction of two more "pairs" of cancelling quarks (like q,q') with large mass.[11]

Other lepton models. Among the other lepton models in the literature, one stands out as fitting our hadrons as well as Weinberg's. This is the "second" model of Prentki and Zumino[1] which may confront neutral current measurements more successfully than Weinberg. The Prentki-Zumino leptons fit into our $\psi_{D,S}$ using the lower right-hand corners as well. All other details are essentially the same as above. Not all leptonic theories fit our hadron theory however. For example, the model of Georgi and Glashow,[1] if it fits at all, appears unnatural. In the first place we need a $U(5) \otimes U(5)$ hadronic group (five quarks before anomaly cancellation), and worse, their scalar field transforms such that, without further scalars, we cannot find a $(3,\bar{3}) \oplus (\bar{3},3)$ symmetry breaking term like $\mathrm{Tr}[\tilde{\phi}^+ M_L^+ \Sigma M_R]$.

Conclusions. To the best of our knowledge, our unified model is consistent with known low-energy data, including vector meson dominance at low energies for electromagnetic and weak form factors, and accepted theoretical ideas about broken hadron symmetries etc.--in the presence of explicit hadron dynamics.

The question of deep inelastic scaling for our model (in perturbation theory) remains to be investigated. Although it turns out that

the current algebra generally resembles algebra of fields, we do not expect worse scaling properties than other renormalizable (longitudinally-damped) theories.[12] Possibly relevant to this question is the interesting fact that this unified model can be taken formally scale invariant before spontaneous breakdown: By omitting quadratic mass terms and adding terms of the form $\mathrm{Tr}(M_L M_L^+ \Sigma\Sigma^+)$ etc., all masses for the scalars are generated spontaneously. In this case, scale invariance and internal symmetries are broken together.

We find it very encouraging that a unified renormalizable gauge theory of strong, weak, and electromagnetic interactions exists in which all three forces derive from a single, stringent principle: gauge invariance.

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FOOTNOTES AND REFERENCES

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