Gaussian imaging transformation for the paraxial Debye formulation of the focal region in a low-Fresnel-number optical system

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The Debye formulation of focused fields has been systematically used to evaluate, for example, the point-spread function of an optical imaging system. According to this approximation, the focal wave field exhibits some symmetries about the geometrical focus. However, certain discrepancies arise when the Fresnel number, as viewed from focus, is close to unity. In that case, we should use the Kirchhoff formulation to evaluate accurately the three-dimensional amplitude distribution of the field in the focal region. We make some important remarks regarding both diffraction theories. In the end we demonstrate that, in the paraxial regime, given a defocused transverse pattern in the Debye approximation, it is possible to find a similar pattern but magnified and situated at another plane within the Kirchhoff theory. Moreover, we may evaluate this correspondence as the action of a virtual thin lens located at the focal plane and whose focus is situated at the axial point of the aperture plane. As a result, we give a geometrical interpretation of the focal-shift effect and present a brief comment on the problem of the best-focus location. © 2000 Optical Society of America
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1. INTRODUCTION

The knowledge of the three-dimensional light distribution in the vicinities of the focus is of particular importance, for example, in estimating the transverse resolution power[1] and the tolerance in the setting of the receiving plane in an optical imaging system.[2] The properties of the out-of-focus monochromatic images of a point source given by a diffraction-limited optical system with a circular exit pupil were treated by Debye,[3] who established that the field is a superposition of plane waves whose propagation vectors fall inside the geometrical cone formed by drawing straight lines from the focal point through the edge of the aperture.[4] Also, he derived certain general features of the diffracted field both near and far away from the focus. For example, he found that the amplitude, and hence also the intensity, possesses inversion symmetry about the focus, where the point of maximum intensity in the focal region is located.[5] This result was later extended to the more general class of monochromatic scalar wave fields that have a focus in the sense of geometrical optics.[6]

In a number of publications that appeared in recent years it was demonstrated that the classic Debye theory regarding the amplitude distribution in the focal region does not predict correct results under all circumstances. Using the Kirchhoff approximation, which assumes that the field inside the aperture is set equal to the field that would exist there in the absence of the aperture and vanishes outside the aperture, Arimoto,[7] and later Stamnes and Spjelkavik[8] and Li and Wolf,[9] found that the intensity distribution about the geometrical focal plane no longer exhibits the well-known symmetry properties. Moreover, the point of maximum intensity of the diffracted wave may not be at the geometrical focus of the incident wave but may be located closer to the aperture. Experimental evidence of this phenomenon has been published elsewhere.[10,11]

This situation may be better understood if we bear in mind that the Debye approximation results when, in addition to the Kirchhoff approximation, we make the assumption that the aperture is infinitely distant from the focal region.[4] Wolf and Li[12] derived a simple condition under which the Debye integral representation may be expected to give a good approximation of the structure of a focused field. For low-angular-aperture systems this condition may be replaced by the requirement that the number of Fresnel zones in the aperture, when viewed from the geometrical focus, be large compared with unity. Simultaneously, they showed[9] that the relative focal shift, that is, the ratio of the shift of the point of maximum intensity to the distance between the geometrical focus and the plane of the aperture, depends only on the Fresnel number. Recently, the concept of the effective Fresnel number that may be applied to any rotationally nonsymmetric scalar field that has a paraxial focus was formulated.[13]

In the case of a circular clear aperture, the expression for the field in the focal region based on the Fresnel–Kirchhoff approximation may be expressed, as with the paraxial Debye approximation, in terms of the Lommel functions, but with arguments that are scaled by a certain factor.[8,14] In the present paper we demonstrate that, from the basis that both formulations are mathematically identical (even with aberrated or apodized focused beams), given a defocused diffraction pattern in the Debye approximation, we find a similar transverse pattern

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in the Kirchhoff approximation but magnified and located at another position. In Section 4 we interpret this three-dimensional mapping as the action of a negative thin lens with a focal length given by the radius of curvature of the incident wave field and situated at the focal plane. Finally, in Section 5 we treat the focal-shift effect from a geometrical point of view, and we discuss the concept of the plane of best focus in image-forming optical systems.

2. DESCRIPTION OF THE FIELD IN THE FOCAL VOLUME

Let us start by considering a scalar monochromatic spherical wave, emerging from an opaque screen of radius a. Let F be the geometrical focus of the wave (see Fig. 1), assumed to be located on the normal of the aperture, through its center O, at a distance f from it.

According to the Huygens–Fresnel principle,[3] the wave field at any point P that is not too close to the plane of the aperture is, as predicted by the Fresnel–Kirchhoff diffraction theory, given by

$$U(P) = \frac{\exp(-ikf)}{i\lambda f} \int\int_W A(S) \frac{\exp(iks)}{s} \mathrm{d}S, \qquad (1)$$

where $k = 2\pi/\lambda$ is the wave number, s is the distance between the point of observation P and a typical point Q on the spherical wave front passing through the center of the aperture, and the integration extends over the wave front. In Eq. (1) we have neglected the well-known obliquity factor that takes values close to unity when the wave field is evaluated in the vicinities of the geometrical focus. The function $A(S)$ stands for the amplitude distribution of the exiting wave field that makes possible the study of general types of focused fields, e.g., diffracted spherical waves in the presence of aberrations[15] and those arising from the focusing of Gaussian laser beams.[16]

First we will determine the diffracted field at the focal plane of the spherical beam. For that purpose, it is usual to use the paraxial approximation, which gives suitable results, assuming that

$$a \gg \lambda, \qquad (a/f)^2 \ll 1. \qquad (2)$$

The approximation is based on the binomial expansion of the distance s in the exponent of Eq. (1), giving

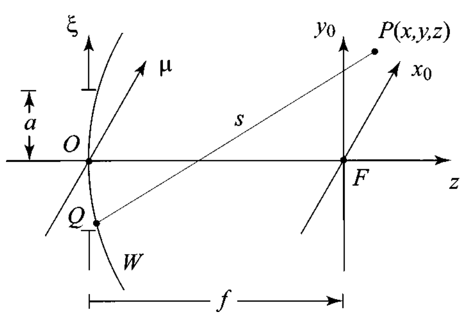

Fig. 1. Schematic diagram of the focusing setup.

$$s \simeq f - \frac{\eta x_0 + \xi y_0}{f} + \frac{{x_0}^2 + {y_0}^2}{2f}. \qquad (3)$$

However, for the s appearing in the denominator, the error introduced by dropping all terms but f is generally acceptably small. The resultant expression for the field at the focal plane therefore becomes

$$U_0(x_0, y_0) = \frac{1}{i\lambda f^2} \exp\left[i\frac{k}{2f}({x_0}^2 + {y_0}^2)\right] \times \int\int_{-\infty}^{\infty} A(\eta, \xi) \exp\left[-i\frac{k}{f}(\eta x_0 + \xi y_0)\right] \mathrm{d}\eta \mathrm{d}\xi. \qquad (4)$$

As is well known, on the focal plane we observe the Fraunhofer diffraction pattern of the field emerging from the plane of the aperture. Also, a quadratic phase factor appears, which is not accessible by direct observation of the intensity distribution of the focal field.

It would seem more interesting to obtain the amplitude distribution of the wave field in adjacent planes of the focal volume. In the Fresnel regime, the amplitude distribution may be obtained by means of the two-dimensional convolution of the wave field at the focal plane and the unit impulse response associated with the free-space propagation;[17] that is,

$$U(P) = \int\int_{-\infty}^{\infty} U_0(x_0, y_0) \frac{1}{i\lambda z} \times \exp\left\{i\frac{k}{2z}[(x - x_0)^2 + (y - y_0)^2]\right\} \mathrm{d}x_0 \mathrm{d}y_0, \qquad (5)$$

where z is the distance between the point of observation P and the focal plane. Equation (5) is itself the Fresnel–Kirchhoff diffraction integral that can be applied over arbitrarily short distances z if the optical beam is truly paraxial. We should mention that an inessential linear phase factor $\exp(ikz)$, which cannot be observed in the intensity distribution of the wave field, has been removed. When we substitute Eq. (4) into Eq. (5), we can express the wave field at point P of the focal region in terms of the amplitude distribution at the illuminated opaque screen. For that purpose, we use the expression[18]

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(iax^2 + i\zeta x) \mathrm{d}x = \frac{1}{\sqrt{2a}} \exp\left(i\frac{\zeta^2}{4a}\right). \qquad (6)$$

After a somewhat long but straightforward calculation, we finally obtain the Fresnel–Kirchhoff diffraction equation for the amplitude distribution in the focal region:

$$U(P) = \frac{1}{i\lambda f(f+z)} \exp\left[i \frac{k}{2(f+z)}(x^2+y^2)\right] \times \int\int_{-\infty}^{\infty} A(\eta, \xi) \exp\left[-i \frac{k}{2f} \frac{z}{f+z} (\eta^2 + \xi^2)\right] \times \exp\left[-i \frac{k}{f+z}(x\eta + y\zeta)\right] \mathrm{d}\eta \mathrm{d}\xi. \quad (7)$$

Finally, the above expression reduces to that given in Eq. (4) when the axial coordinate z is replaced by its value at the focal plane, that is, $z = 0$.

3. DEBYE APPROXIMATION

It is usual to evaluate the diffracted field of a truncated spherical wave within the Debye approximation, according to which the field in the focal region is a superposition of plane waves whose propagation vectors fall inside the geometrical cone formed by drawing straight lines from the focal point through the edge of the aperture. Under the paraxial regime, the field at the focal plane given in Eq. (4) generates the diffraction formula predicted by the Debye theory when the quadratic phase factor outer to the integral is removed, resulting in

$$U_0{}^D(x_0, y_0) = \frac{1}{i\lambda f^2} \int\int_{-\infty}^{\infty} A(\eta, \xi) \times \exp\left[-i \frac{k}{f}(\eta x_0 + \xi y_0)\right] \mathrm{d}\eta \mathrm{d}\xi; \quad (8)$$

that is, the Debye approximation agrees with the Kirchhoff formulation when the quadratic phase factor in Eq. (4) may be neglected.

When the amplitude distributions at planes adjacent to the focal plane in the focal region are evaluated, we may use Eq. (1) and later consider the Debye assumptions concerning the focusing properties of three-dimensional waves given above. However, the resultant amplitude distribution may be obtained instead by free-space propagating the wave field at the focal plane to the observation plane situated at a distance z, with the aid of the two-dimensional convolution given in Eq. (5). Thus we substitute the approximated expression in Eq. (8) into Eq. (5), giving

$$U^D(P) = \frac{1}{i\lambda f^2} \int\int_{-\infty}^{\infty} A(\eta, \xi) \exp\left[-i \frac{k}{2f} \frac{z}{f} (\eta^2 + \xi^2)\right] \times \exp\left[-i \frac{k}{f}(x\eta + y\xi)\right] \mathrm{d}\eta \mathrm{d}\xi. \quad (9)$$

It has been shown[5] that the Debye integral given in Eq. (9) presents some important symmetry properties. For instance, when $A(\eta, \xi)$ is a real function, the amplitude, and hence the intensity, possesses inversion symmetry about the focus. Also, the phase has inversion antisymmetry, apart from an additive term of half a period.[6] Additionally, when the function $A(\eta, \xi)$ is positive, the point of maximum intensity in the focal region is located at the focal point.

Now we will find the restrictions that we must impose to ensure that the Debye approximation will give a reasonably good prediction of the diffracted field. This requirement is satisfied when the quadratic phase factor in Eq. (4) may be neglected. For that purpose, we note that a diffraction-limited optical system, such as that represented schematically in Fig. 1, concentrates most of the light energy at the focal plane in an area about the focus given by $x_0{}^2 + y_0{}^2 \leqslant \lambda^2(f/a)^2$. For example, in the case of a circular clear pupil, the energy encircled within this area represents 90.64% of the emerging radiation energy passing through the aperture. This means that the most representative points contributing to the diffracted field at the focal plane are those satisfying the above inequality. When the phase corresponding to the quadratic term in Eq. (4) is evaluated, the maximum value is given by $\exp(i\pi/N)$, where

$$N = a^2/(\lambda f) \quad (10)$$

stands for the Fresnel number of the focusing geometry, that is, the number of half-waves covered by the diffracting aperture as viewed from the geometrical focus. When the Fresnel number is much higher than unity, it is clear that the quadratic phase factor does not introduce a noticeable variation in the phase of the focal plane, which implies that this term may be ignored. Hence it is concluded that the Debye integral representation of spherical waves should be applied only when the Fresnel number of the focusing geometry is large compared with unity.

Another interesting point is the fact that the Debye approximation results when, in addition to the Kirchhoff approximation, we make the assumption that the aperture is infinitely distant from the focal region. A telecentric optical system fulfills this severe restriction, which implies that Eqs. (8) and (9) hold for this case.[19] However, we will observe the previously mentioned symmetries about the geometrical focus when the wave field is focused onto a region whose axial magnitude is much lower than the focal length. In relation to this point, we can estimate the focal depth of an imaging optical setup in terms of the wavelength and the numerical aperture of the system, N.A. = $\sin\alpha$, as the quantity[3,20]

$$\Delta z = \frac{\lambda}{4\sin^2(\alpha/2)} \simeq \lambda\left(\frac{f}{a}\right)^2, \quad (11)$$

where we have introduced the paraxial approximation. Then we should impose the inequality $\Delta z \ll f$ to guarantee that the Debye approximation is valid. Taking into account that we can express the Fresnel number in terms of the focal depth and the focal length, i.e.,

$$N = f/(\Delta z), \quad (12)$$

we finally obtain that, as deduced previously with alternative reasoning, the Fresnel number should be constrained to values much higher than unity.

4. THREE-DIMENSIONAL MAPPING FOR THE DEBYE REPRESENTATION OF THE FOCAL FIELD

We have shown that to find the focal wave field of a truncated spherical beam it is possible to employ either the Kirchhoff formulation or the Debye approximation. The latter gives suitable results when the wave field is focused mostly on a region about the geometrical focus whose lateral and axial dimensions are much smaller than the aperture diameter and the focal length[14]: meanwhile, noticeable inaccuracies exist when the focal region increases in size. Most investigators have stressed the fact that both theories show such departures.[8,12]

However, we will demonstrate that the two theories provide the same collection of irradiance transverse patterns that constitute the focal region but with a different scale and position. By comparing Eqs. (7) and (9), we observe that both diffraction formulas give an amplitude transverse pattern by performing a two-dimensional Fourier transform of the product of the function $A(\eta, \xi)$ and a quadratic phase factor. This product is usually called the defocused pupil function for the optical system.[21] Thus, when evaluating a given intensity transverse pattern located at a distance z from the focal plane, provided by the Kirchhoff formula in Eq. (9), we may obtain the same transverse distribution within the Debye theory, in a unique plane placed at a distance

$$z_D = \frac{f}{f + z} z \tag{13}$$

but applying a lateral magnification given by

$$M(z) = \frac{f + z}{f}. \tag{14}$$

Such coordinate transformations are successfully utilized in space-variant imaging systems that are to be modeled as an afocal telecentric space-invariant system.[22] The replicated transverse irradiance distributions are obtained when the two defocused pupil functions in Eqs. (7) and (9) coincide, while the lateral magnification arises from the ratio of the two scales in both Fourier transform kernels. Consequently, the focal volume given by the Debye theory is then deformed in such a way that it conserves the same transverse structure, but both the lateral magnification and the axial distribution of the wave field are altered (see Fig. 2).

To interpret adequately the transformation that the field undergoes in the focal region within the Debye approximation, we focus our attention on the amplitude transverse distribution at the focal plane of the focusing setup given by both theories. According to Eqs. (4) and (8), the two expressions differ in the use of a quadratic phase term that multiplies the spectrum of the pupil function $A(\eta, \xi)$. In agreement with the Kirchhoff boundary conditions, a convergent thin lens with a focal length given by f located at the back focal plane of the optical system would compensate the phase modulation of the quadratic factor and produce a collection of amplitude transverse patterns as given by the Debye approximation. Conversely, to obtain the amplitude distribution in the focal plane given by the Fresnel–Kirchhoff theory, we may employ the paraxial Debye approximation and the action of a divergent thin lens of focal length $-f$, that is, with the focal point located on the axial point of the diffracting opaque screen. Also, when the transverse pattern of adjacent planes in the focal region is evaluated, it may be obtained analytically in terms of the Debye formula and later virtually introduce the action of a divergent thin lens. The Gaussian imaging transformation undergone by the diffraction amplitude distribution given by the Debye formula may be mathematically represented as follows:

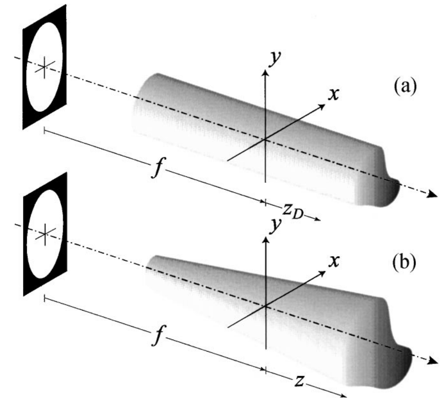

Fig. 2. Three-dimensional mapping of the focal volume provided by (a) the paraxial Debye formulation, thus giving (b) the Fresnel–Kirchhoff representation of the focal wave field.

$$U(x, y, z) = \exp\left[i \frac{k}{2(f + z)} (x^2 + y^2)\right] \times \frac{1}{M} U^D\left(\frac{x}{M}, \frac{y}{M}, \frac{z}{M}\right). \tag{15}$$

We should mention that in the previous three-dimensional mapping there exist a factor $1/M$ and a quadratic phase factor accompanying the amplitude distribution given by the Debye approximation. The first term is associated with the energy conservation law, and the second one is somewhat irrelevant, since it is not observable when intensity is being detected.

The point-spread function of a low-angular-aperture diffraction-limited optical imaging system is usually described in terms of the paraxial Debye integral[23] given in Eq. (9). It may be demonstrated that, in general, the light is concentrated in planes neighboring the focal plane; hence the deformation experienced in the focal volume is unnoticeable. It may be demonstrated that in this region the lateral magnification is closer to unity, $M(z) \simeq 1$, and the axial translation of the irradiance transverse patterns is negligible, $z_D \simeq z$. However, we should remark that this kind of transformation in the diffracted wave field around the focal plane always occurs.

In Fig. 3 we show the symmetries that the focal waves exhibit when the Fresnel number is much higher than unity and hence when the Debye approximation holds. However, when N decreases, the intensity in the focal region begins a process of deformation as deduced from Eq. (15), which becomes more noticeable as the aperture plane comes closer to the focal region. As a result, we observe that the point of maximum intensity of the diffracted wave may not be at the geometrical focus of the incident wave but closer to the aperture. This is the so-called focal-shift effect.

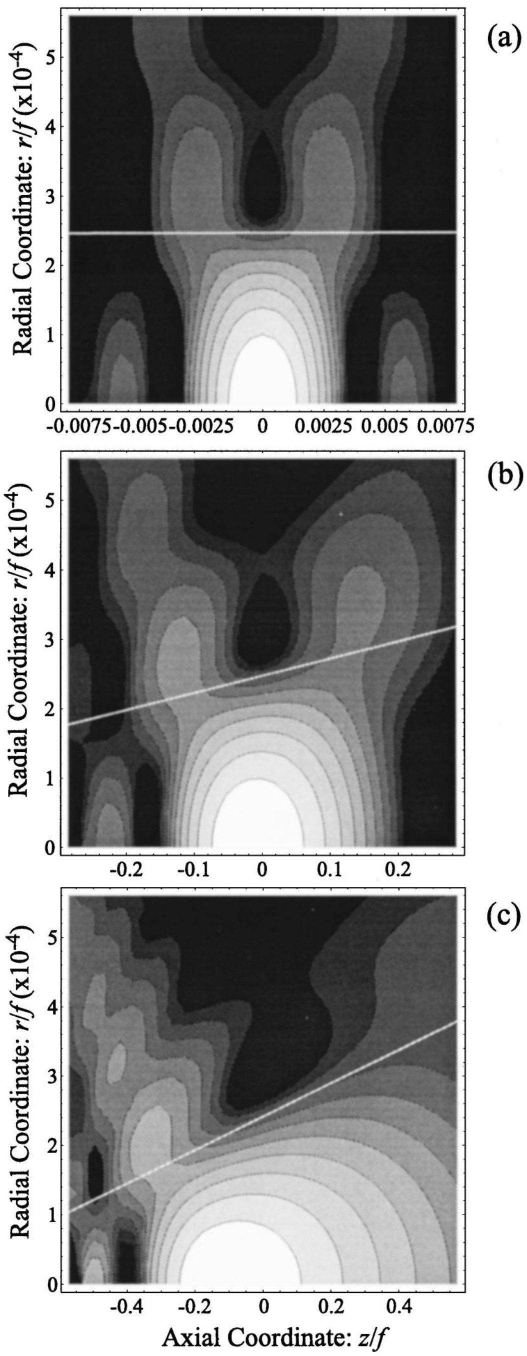

Fig. 3. Diagram of isophotes corresponding to the impulse response of an optical imaging system with a circular clear pupil of radius $a = 1$ mm when the wavelength is given by $\lambda = 500$ nm and the Fresnel number is (a) high ($N = 500$), (b) moderate ($N = 10$), and (c) low ($N = 3$). The continuous white line passes through the axial point of the pupil plane, which gives a rough idea of the relative distance between the focal plane and the aperture plane.

5. GEOMETRICAL INTERPRETATION OF THE FOCAL-SHIFT EFFECT

Much attention has been addressed to the problem concerning the evaluation of the relative focal shift, that is, the ratio of such a shift Δf of the point of maximum axial intensity to the distance f between the geometrical focus and the plane of the aperture.[13,24] This interest is well justified because the determination of the plane at which an imaging system comes to the best focus is of great importance. On the other hand, similar considerations should be taken into account in discussing how to optimally focus a laser beam to illuminate a distant target. In this context, Li and Wolf[9] recognized that the magnitude of the relative focal shift depends solely on the Fresnel number of the focusing geometry. Moreover, they presented a formula for the rapid evaluation of the relative focal shift:

$$\frac{\Delta f}{f} = -\frac{1}{1 + (\pi^2/12)N^2}. \tag{16}$$

This formula gives the focal shift accurately to within 1% when $N \geqslant 12$. However, for lower values of the Fresnel number, Eq. (16) gives a rough estimation of the relative focal shift.

Now we consider the mapping of Eq. (15) that should be applied on the paraxial Debye formulation when the Fresnel number of the focusing setup is comparable to unity. Thus it is possible to obtain the location of the plane provided by the Debye formulation whose conjugate plane corresponds to the transverse pattern in the Kirchhoff approximation that includes the point of maximum intensity along the optic axis. By substituting Eq. (16) into Eq. (13), where the axial coordinate z is given by the focal displacement Δf, we may determine that this plane is then situated at a distance from the focal plane of the spherical beam given by

$$z_D = -\frac{12}{\pi^2}\frac{\Delta z}{N}, \tag{17}$$

where in addition we have made use of Eq. (12).

From Eq. (17) we infer that the plane belonging to the Debye representation that images to the plane in the Kirchhoff formulation of the field that contains the point of maximum intensity on the optical axis is located inside the focal depth. This is true in the range of validity of Eq. (16), that is, for moderate and high values of the Fresnel number. This fact ensures that the plane of best focus, that is, the plane in which the focused beam radius reaches its minimum transverse extension, suffers a defocus aberration that may be neglected. At this point we note that there exists a certain controversy about the appropriate definition of beam radius and hence of plane of best focus. We point out that Mahajan[16,25] used a criterion for the spot size based on the encircled energy. In particular, the maximum of encircled energy occurs where maximum central irradiance is reached for small spot sizes, which agrees with the reasoning presented here.

When the Fresnel number decreases to values lower than unity, the distance z_D of the plane containing the beam waist is located outside the focal region given in the Debye regime. Following Parker Givens,[26] we may conclude that when we deal with an image-forming optical system, we are not able to ensure that the focus of the optical system is effectively shifted to the point of maximum intensity along the axis. However, it should be noted that imaging systems generally have very large Fresnel numbers, and hence the maximum axial irradiance is observed at the geometrical focus. In the case in which the optical system is used to focus light on a target we should consider a different criterion, such as the encircled energy. In particular, we point out that maximum encircled energy is obtained when the beam is focused on the target, even though a higher axial irradiance is obtained at a point closer to the diffracting aperture. As highlighted by Li and Wolf,[14] a somewhat paradoxical aspect of this situation is that as the Fresnel number N decreases, with a and λ kept fixed, the distance f increases; i.e., the geometrical focus F moves farther away from the aperture. However, the point of maximum intensity moves in the opposite direction, i.e., closer to the aperture.

6. DISCUSSION AND CONCLUSIONS

We have proved that both the Debye and the Kirchhoff formulations used for the evaluation of focal waves give the same collection of transverse intensity patterns, except for location and magnification. Deviations from both theories in the focal region arise when the Fresnel number of the focusing geometry is close to unity. However, we claim that the great majority of research performed in the Debye regime are still valuable, even when N is comparable to unity. A simple example represents the great variety of strategies for increasing the power resolution of an imaging system,[27] or the three-dimensional intensity distribution of the point-spread function that characterizes the diffraction behavior of an apodized setup[1] may be applied when the Fresnel number, when viewed from focus, is close to unity.

However, the similarity of the Debye and the Kirchhoff representations of the wave field allows us to point out that there may exist a great discrepancy between the concept of plane of best focus and the plane containing the point of maximum intensity along the axis for Fresnel numbers lower than unity. From our point of view, if the optical system is being used for imaging purposes, we should stress that the two planes do not coincide. To determine the plane of best focus, it is desirable to select a plane belonging to the focal region in the Debye formulation that, when the three-dimensional mapping of Eq. (15) is performed, has the narrower transverse spot light. However, it is possible to demonstrate that, as we will detail in a future paper, the adjacent planes in the focal volume offer practically the same imaging abilities but are characterized by a higher magnification, as given by Eq. (14).

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