Gaussian Multiterminal Source Coding

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Multiterminal Rate-Distortion Theory

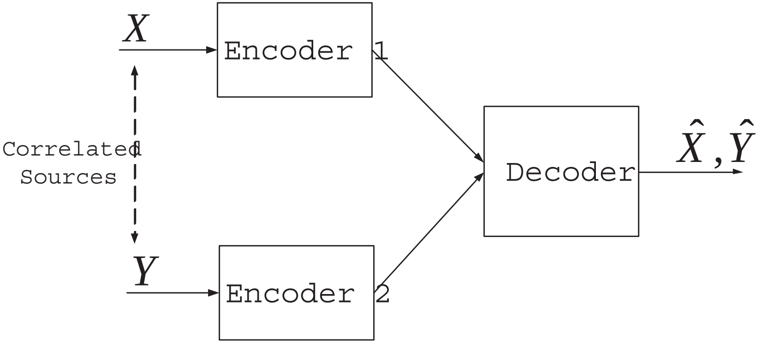

Figure: Distributed source coding with separated encoding and joint decoding

Previous Results

- **Slepian-Wolf:** distortionless coding;
- **Wyner-Ziv:** source coding system with fully observe side information;
- **Berger-Tung:** derive an inner region and an outer region of the rate-distortion region;

Previous Results - cont'd

- **Wyner, Ahlswede and Korner:** distortionless case with partial side information;
- **Berger:** rate-distortion problem with partial side information with finite alphabets.

Contribution of this paper

- determine the rate-distortion region for the case when one source output works as partial side information at the decoder (Gaussian case);
- derive an outer region, demonstrating that the inner region obtained by Berger and Tung is partially tight;
- give a complete proof of the direct coding theorem for Gaussian sources and squared distortions.

Formal Statement of Problem

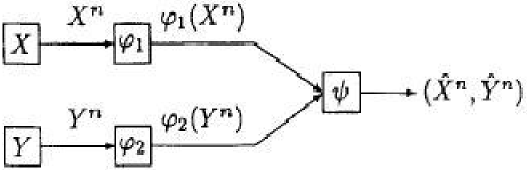

Fig. 1. The separate coding system for two correlated sources.

Formal Statement of Problem - Cont'd

- $\mathcal{F}_{n,\delta}(R_1, R_2)$ - $(\varphi_1, \varphi_2, \psi)$;
-

$$\Delta_1 = E\frac{1}{n}\sum_{t=1} nd_1(X_t, \hat{X}_t)$$

$$\Delta_2 = E\frac{1}{n}\sum_{t=1} nd_2(X_t, \hat{X}_t)$$

- $R(D_1, D_2) = \{(R_1, R_2) : (R_1, R_2) \text{ is admissible}\}$.

Statement of Main Results

- $R_1(D_1) = \{(R_1, R_2) : (R_1, R_2) \in R(D_1, D_2) \text{for some} D_2 > 0\};$
  $R_2(D_2) = \{(R_1, R_2) : (R_1, R_2) \in R(D_1, D_2) \text{for some} D_1 > 0\}.$
- $R_1^*(D_1) = \{(R_1, R_2) : R_1 \geq \frac{1}{2} log^+[\frac{\sigma_X^2}{D_1}(1 - \rho^2 + \rho^2 2^{-2R_2})]\};$
  $R_2^*(D_2) = \{(R_1, R_2) : R_2 \geq \frac{1}{2} log^+[\frac{\sigma_Y^2}{D_2}(1 - \rho^2 + \rho^2 2^{-2R_1})\};$
  $\hat{R}_{12}(D_1, D_2) = \{(R_1, R_2) : R_1 + R_2 \geq \frac{1}{2} log^+[(1 - \rho^2)\frac{\sigma_X^2 \sigma_Y^2}{D_1 D_2}]\};$
  $R_{out}(D_1, D_2) = R_1^*(D_1) \cap R_2^*(D_2) \cap \hat{R}_{12}(D_1, D_2).$
- Theorem: 1) $R_1(D_1) = R_1^*(D_1);$
  2) $R(D_1, D_2) \subseteq R_{out}(D_1, D_2)$

Statement of Main Results - cont'd

$$\beta = \beta(s_1, s_2)$$
$$= 1 + \sqrt{1 + \frac{4\rho^2}{(1-\rho^2)^2} s_1 s_2}$$

$$\begin{aligned} R_{BT}(D_1, D_2) = \{(R_1, R_2) : R_1 &\geq \frac{1}{2} \log^+[(1-\rho^2)(s_1 - \frac{2\rho^2 s_1 s_2}{(1-\rho^2)\beta})^{-1}] \\ R_2 &\geq \frac{1}{2} \log^+[(1-\rho^2)(s_2 - \frac{2\rho^2 s_1 s_2}{(1-\rho^2)\beta})^{-1}] \\ R_1 + R_2 &\geq \frac{1}{2} \log^+[\frac{(1-\rho^2)\beta}{2 s_1 s_2}] \\ \text{for some} 0 < s_1 < \frac{D_1}{\sigma_X^2}&, 0 < s_2 < \frac{D_2}{\sigma_Y^2}\} \quad (1) \end{aligned}$$

Statement of Main Results - cont'd

- $\beta_{max} = \max_{0<s_1<\frac{D_1}{\sigma_X^2}, 0<s_2<\frac{D_2}{\sigma_Y^2}} \beta(s_1, s_2) = \beta(\frac{D_1}{\sigma_X^2}, \frac{D_2}{\sigma_Y^2});$

$$\tilde{R}_{12}(D_1, D_2) = \{(R_1, R_2) : R_1 + R_2 \geq \frac{1}{2} log^+[(1-\rho^2)\frac{\beta_{max}}{2}\frac{\sigma_X^2 \sigma_Y^2}{D_1 D_2}]\};$$

$$R_{in}(D_1, D_2) = R_1^*(D_1) \cap R_2^*(D_2) \cap \tilde{R}_{12}(D_1, D_2).$$

- Theorem:

$$R_{BT}(D_1, D_2) \subseteq R(D_1, D_2) \quad (2)$$

- Proposition: $R_{BT}(D_1, D_2) = R_{in}(D_1, D_2)$